What you will learn about:
Higher Roots

The Nth Root of A Number

If $b^n = a$, is the ***nth root of a number*** a.
The principal nth root of a is written $\sqrt[n]{a}$.
n is called the **index** of the radical.

$$4^3 = 64 \qquad \sqrt[3]{64} = 4$$
$$3^4 = 81 \qquad \sqrt[4]{81} = 3$$
$$(-2)^5 = -32 \qquad \sqrt[5]{-32} = -2$$

PROPERTIES OF $\sqrt[n]{a}$

When n is an even number and

- $a \geq 0$, then $\sqrt[n]{a}$ is a real number
- $a < 0$, then $\sqrt[n]{a}$ is not a real number

When n is an odd number, $\sqrt[n]{a}$ is a real number for all values of a.

$\sqrt[n]{a}$

$\sqrt{\ }$ → radical

a → randicand

n → index

$2^3 = 8$

$\sqrt[3]{8} = 2$

Simplify: $(3)^4 = 81$

$\sqrt[3]{8} = 2$ $\qquad \sqrt[4]{81} = 3$ $\qquad \sqrt[5]{32} = 2$

$\sqrt[3]{27} = 3$ $\qquad \sqrt[4]{256} = 4$ $\qquad \sqrt[5]{243} = 3$

$\sqrt[3]{1000}$ 10

$\sqrt[4]{16}$ 2

$\sqrt[5]{32}$ 2

$\sqrt[3]{-125}$ -5

$\sqrt[4]{-16}$ No Solution

$\sqrt[5]{-243}$ -3

$\sqrt[3]{-216}$ -6

$\sqrt[4]{-81}$ No Real #

$\sqrt[5]{-1024}$ -4

SIMPLIFYING ODD AND EVEN ROOTS

For any integer $n \geq 2$,

when n is odd $\sqrt[n]{a^n} = a$

when n is even $\sqrt[n]{a^n} = |a|$

We must use the absolute value signs when we take an even root of an expression with a variable in the radical.

Simplify:

$\sqrt{x^2} = |x|$ $\quad \sqrt[3]{n^3} = n$ $\quad \sqrt[4]{p^4} = |p|$ $\quad \sqrt[5]{y^5} = y$

$\sqrt[3]{y^{18}} = y^6$ $\quad \sqrt[4]{z^4} = |z|$ $\quad \sqrt[5]{c^{20}} = c^4$ $\quad \sqrt[6]{d^{24}} = |d^4|$

$\sqrt[3]{27x^{27}}$ $3x^9$ $\quad \sqrt[5]{243q^{25}}$ $3q^5$ $\quad \sqrt[4]{81x^{28}}$ $|3x^7|$ $\quad \sqrt[3]{125p^9}$ $5p^3$

$\sqrt[3]{x^4} = \sqrt[3]{x^3} \cdot \sqrt[3]{x}$

$\sqrt[3]{x^4}$ $\quad$ $\sqrt[4]{x^7}$ $\quad$ $\sqrt[5]{p^8}$ $\quad$ $\sqrt[6]{t^{13}}$

$x\sqrt[3]{x}$ $\quad$ $\sqrt[4]{x^4}\cdot\sqrt[4]{x^3}$ $\quad$ $\sqrt[5]{p^5}\cdot\sqrt[5]{p^3}$ $\quad$ $\sqrt[6]{t^{12}}\cdot\sqrt[6]{t}$

$|x|\sqrt[4]{x^3}$ $\quad$ $p\sqrt[5]{p^3}$ $\quad$ $|t^2|\sqrt[6]{t}$

$\sqrt[3]{16}$ $\quad$ $\sqrt[4]{243}$ $\quad$ $\sqrt[3]{625}$ $\quad$ $\sqrt[4]{729}$

$\sqrt[3]{8}\cdot\sqrt[3]{2}$ $\quad$ $\sqrt[4]{81}\cdot\sqrt[4]{3}$ $\quad$ $\sqrt[3]{125}\cdot\sqrt[3]{5}$ $\quad$ $\sqrt[4]{81}\cdot\sqrt[4]{9}$

$2\sqrt[3]{2}$ $\quad$ $|3|\sqrt[4]{3}$ $\quad$ $5\sqrt[3]{5}$ $\quad$ $|3|\sqrt[4]{9}$

$\sqrt[3]{24}\cdot\sqrt[3]{x^7}$

$\sqrt[3]{8}\cdot\sqrt[3]{3}\cdot\sqrt[3]{x^6}\cdot\sqrt[3]{x}$

$2\sqrt[3]{3}\cdot x^2\sqrt[3]{x}$

$\sqrt[3]{24x^7}$ $\quad$ $\sqrt[4]{80y^{14}}$ $\quad$ $\sqrt[3]{54p^{10}}$ $\quad$ $\sqrt[4]{162n^7}$

$(2x^2)\sqrt[3]{3x}$ $\quad$ $\sqrt[4]{16}\cdot\sqrt[4]{5}\cdot\sqrt[4]{y^{12}}\cdot\sqrt[4]{y^2}$

$|2y^3|\sqrt[4]{5y^2}$ $\quad$ $|3n|\sqrt[4]{2n^3}$

$\sqrt[3]{-27}$ $\quad$ $\sqrt[3]{-108}$ $\quad$ $\sqrt[3]{-625}$

$\sqrt[4]{\frac{x^7}{x^3}} = \sqrt[4]{x^4}$ $\quad$ $\sqrt[3]{\frac{m^{13}}{m^7}} = \sqrt[3]{m^6}$ $\quad$ $\sqrt[4]{\frac{y^{17}}{y^5}} = \sqrt[4]{y^{12}}$

$= |x|$ $\quad$ $= m^2$ $\quad$ $= |y^3|$

$\frac{\sqrt[3]{-108}}{\sqrt[3]{2}}$ $\quad$ $\frac{\sqrt[4]{96x^7}}{\sqrt[4]{3x^2}}$ $\quad$ $\frac{\sqrt[3]{-192}}{\sqrt[3]{3}} = \sqrt[3]{-64}$

$\sqrt[3]{-54}$ $\quad$ $\sqrt[4]{32x^5}$ $\quad$ $= -4$

$\sqrt[3]{-27}\cdot\sqrt[3]{2}$ $\quad$ $\sqrt[4]{16}\cdot\sqrt[4]{2}$

$-3\sqrt[3]{2}$ $\quad$ $|2x|\sqrt[4]{2x}$

	$\sqrt[3]{\frac{24x^7}{y^3}}$ $\sqrt[4]{\frac{48x^{10}}{y^8}}$ $\sqrt[3]{\frac{108c^{10}}{d^6}}$ Simplify: $\sqrt[3]{4x} + 4\sqrt[3]{4x}$ $4\sqrt[4]{8} - 2\sqrt[4]{8}$ $5\sqrt[3]{4x}$ $\sqrt[3]{192} - \sqrt[3]{81}$ $\sqrt[4]{48} + \sqrt[4]{243}$ $\sqrt[3]{24x^4} - \sqrt[3]{-81x^7}$ $\sqrt[4]{243r^{11}} + \sqrt[4]{768r^{10}}$

What you will learn about:
Rational Exponents

Rational Exponent $a^{\frac{1}{n}}$

If $\sqrt[n]{a}$ is a real number $n \geq 2$, $a^{\frac{1}{n}} = \sqrt[n]{a}$.

Write as a radical expression:

$x^{\frac{1}{2}} = \sqrt{x}$ $y^{\frac{1}{3}} = \sqrt[3]{y}$ $z^{\frac{1}{4}} = \sqrt[4]{z}$

$\sqrt{x^4}$
$= (x^6)^{\frac{1}{2}}$

Write as rational exponents:

$\sqrt{t} = t^{1/2}$ $\sqrt[3]{h} = h^{1/3}$ $\sqrt[4]{m}$ $m^{1/4}$

Simplify:

$25^{\frac{1}{2}} = \sqrt{25}$
$= 5$

$8^{\frac{1}{3}} = \sqrt[3]{8}$
$= 2$

$16^{\frac{1}{4}}$

$(-64)^{\frac{1}{3}}$ $-64^{\frac{1}{3}}$ $(64)^{-\frac{1}{3}}$

$a^{\frac{m}{n}}$ m → Power, n → root	$(-16)^{\frac{1}{4}}$ $\quad -16^{\frac{1}{4}}$ $\quad (16)^{-\frac{1}{4}}$ Simplify Expressions with $a^{\frac{m}{n}}$. Write with rational exponents: $\sqrt{x^5} = x^{\frac{5}{2}}$ $\quad \sqrt[4]{z^3} = z^{3/4}$ $\quad \sqrt[6]{p^7} = p^{\frac{7}{6}}$ Simplify: $9^{\frac{3}{2}} = (\sqrt{9})^3 = 27$ $\quad 27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 9$ $\quad 625^{\frac{3}{4}}$ $16^{-\frac{3}{2}}$ $\quad 32^{-\frac{2}{5}}$ $\quad 8^{-\frac{5}{3}}$

	Simplify $2^{\frac{1}{2}} \cdot 2^{\frac{5}{2}}$ $\quad$ $x^{\frac{2}{3}} \cdot x^{\frac{4}{3}}$ $\quad$ $z^{\frac{3}{4}} \cdot z^{\frac{5}{4}}$ $(x^4)^{\frac{1}{2}}$ $\quad$ $(y^6)^{\frac{1}{3}}$ $\quad$ $\left(z^{\frac{2}{3}}\right)^9$ $\frac{u^{\frac{5}{4}}}{u^{\frac{1}{4}}}$ $\quad$ $\frac{v^{\frac{3}{5}}}{v^{\frac{2}{5}}}$ $\quad$ $\frac{x^{\frac{2}{3}}}{x^{\frac{5}{3}}}$ $\left(27u^{\frac{1}{2}}\right)^{\frac{2}{3}}$ $\quad$ $\left(8v^{\frac{1}{4}}\right)^{\frac{2}{3}}$ $\quad$ $\left(32x^{\frac{1}{3}}\right)^{\frac{3}{5}}$ $\left(81n^{\frac{2}{5}}\right)^{\frac{3}{2}}$ $\quad$ $(m^3n^9)^{\frac{1}{3}}$ $\quad$ $(p^4q^8)^{\frac{1}{4}}$

	$\frac{x^{\frac{3}{4}} \cdot x^{-\frac{1}{4}}}{x^{-\frac{6}{4}}}$ $\qquad$ $\frac{y^{\frac{4}{3}} \cdot y}{y^{-\frac{2}{3}}}$ $\frac{m^{\frac{2}{3}} \cdot m^{-\frac{1}{3}}}{m^{-\frac{5}{3}}}$ $\qquad$ $\frac{u^{\frac{4}{5}} \cdot u^{-\frac{2}{5}}}{u^{-\frac{13}{5}}}$