

**CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Watts and Kennedy**  
**Chapter 3: Derivatives Derivatives from a table of values**

What you'll Learn About  
 How to find the derivative at a point given a table of values

2013 BC3

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time  $t$ ,  $0 \leq t \leq 6$ , is given by a differentiable function  $C$ , where  $t$  is measured in minutes. Selected values of  $C(t)$ , measured in ounces, are given in the table.

|             |   |     |     |      |      |      |      |
|-------------|---|-----|-----|------|------|------|------|
| t (minutes) | 0 | 1   | 2   | 3    | 4    | 5    | 6    |
| C(t) ounces | 0 | 5.3 | 8.8 | 11.2 | 12.8 | 13.8 | 14.5 |

a) Use the data in the table to approximate  $C'(5.5)$ . Show the computations that lead to your answer, and indicate units of measure.

$$C'(5.5) = \frac{14.5 - 13.8}{6 - 5} = \frac{0.7}{1} = 0.7 \text{ oz/min}$$

2011 #2

|                |    |    |    |    |    |
|----------------|----|----|----|----|----|
| t (minutes)    | 0  | 2  | 5  | 9  | 10 |
| H(t) degrees C | 66 | 60 | 52 | 44 | 43 |

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function  $H$  for  $0 \leq t \leq 10$  where time  $t$  is measured in minutes and temperature  $H(t)$  is measured in degrees Celsius. Values of  $H(t)$  at selected values of time  $t$  are shown in the table above

Use the data in the table to approximate the rate at which the temperature of the tea is changing at time  $t = 9.5$ . Show the computations that lead to your answer.

$$H'(9.5) = \frac{43 - 44}{10 - 9} = \frac{-1}{1} = -1$$

## 2012 #1

|                |      |      |      |      |      |
|----------------|------|------|------|------|------|
| t(minutes)     | 0    | 4    | 9    | 15   | 20   |
| W(t) degrees F | 55.0 | 57.1 | 61.8 | 67.9 | 71.0 |

The temperature of water in a tub at time  $t$  is modeled by a strictly increasing, twice differentiable function,  $W$ , where  $W(t)$  is measured in degrees Fahrenheit and  $t$  is measured in minutes. At time  $t = 0$ , the temperature of the water is  $55^\circ\text{F}$ . The water is heated for 30 minutes, beginning at time  $t = 0$ . Values of  $W(t)$  at selected times  $t$  for the first 20 minutes are given in the table above.

- a) Use the data in the table to estimate  $W'(17.5)$ . Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$w'(17.5) = \frac{71 - 67.9}{20 - 15} = \frac{3.1}{5} = .62^\circ\text{F}/\text{min}$$

At  $t = 17.5$  the rate that the temperature of the water is increasing is  $.62^\circ\text{F}/\text{min}$

## 2010 #2

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ( $t=0$ ) and 8 P.M. ( $t=8$ ). The number of entries in the box  $t$  hours after noon is modeled by a differentiable function  $E$  for  $0 \leq t \leq 8$ . Values of  $E(t)$ , in hundreds of entries, at various times  $t$  are shown in the table.

|                               |   |   |    |    |    |
|-------------------------------|---|---|----|----|----|
| t(hours)                      | 0 | 2 | 5  | 7  | 8  |
| E(t)<br>(hundreds of entries) | 0 | 4 | 13 | 21 | 23 |

- b) Use the data in the table to approximate the rate in hundreds of entries per hour, at which entries were being deposited at time  $t = 7.5$ . Show the computations that lead to your answer.

$$E'(7.5) = \frac{23 - 21}{8 - 7} = \frac{2}{1}$$

~~$$E'(7.5) = \frac{200}{100} =$$~~

2016 BC 1

|                     |      |      |     |     |     |
|---------------------|------|------|-----|-----|-----|
| t(hours)            | 0    | 1    | 3   | 6   | 8   |
| R(t)<br>liters/hour | 1340 | 1190 | 950 | 740 | 700 |

Water is pumped into a tank at a rate modeled by  $W(t) = 2000e^{-t^2/20}$  liters per hour for  $0 \leq t \leq 8$ , where  $t$  is measured in hours. Water is removed from the tank at a rate modeled by  $R(t)$  liters per hour, where  $R$  is differentiable and decreasing on  $0 \leq t \leq 8$ . Selected values of  $R(t)$  are shown in the table above. At time  $t = 0$ , there are 50,000 liters of water in the tank.

- a) Estimate  $R'(2)$ . Show the work that leads to your answer. Indicate units of measure.
- d) For  $0 \leq t \leq 8$ , is there a time when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank. Explain why or why not?

$$R'(2) = \frac{950 - 1190}{3 - 1} = \frac{-240}{2} = -120 \frac{\text{liters}}{(\text{hour})^2}$$

$$w(t) = 2000e^{-t^2/20}$$

$$\begin{matrix} R(0) = 1340 & w(0) = 2000 \\ R(8) = 700 & w(8) = 81.524 \end{matrix}$$

yes, since  $R(t)$  and  $w(t)$  are differentiable between  $0 \leq t \leq 8$  and

2012 #4

The function  $f$  is twice differentiable for  $x > 0$  with  $f(1.2) = 5$  and  $f''(1) = 20$ . Values  $f'$ , the derivative of  $f$ , are given for selected values of  $x$  in the table.

|         |   |     |     |     |      |
|---------|---|-----|-----|-----|------|
| x       | 1 | 1.1 | 1.2 | 1.3 | 1.4  |
| $f'(x)$ | 8 | 10  | 12  | 13  | 14.5 |

- a) Write an equation for the line tangent to the graph of  $f$  at  $x = 1.2$ . Use this line to approximate  $f(1.4)$ .

$$f(x) = 5 + 12(x - 1.2)$$

$$(1.2, 5) \quad m = 12$$

$$y = 5 + 12(x - 1.2) \checkmark$$

$$f(1.4) = 5 + 12(1.4 - 1.2)$$

$$y = 5 + 12(1.4 - 1.2) \checkmark$$

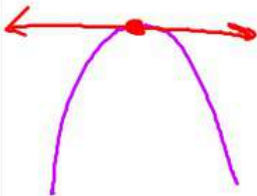
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The rate at which water is being removed ~~is~~ is decreasing at a rate of 120 liter/hour<sup>2</sup>

$$y = f(x)$$

What you'll Learn About  
 The derivative represents velocity  
 The second derivative represents acceleration

position of rock  
 at any time,  $t$



13a) Lunar Projectile Motion: A rock thrown vertically upward from the surface of the moon at a velocity of 20 m/sec reaches a height of  $s = 20t - .8t^2$  in  $t$  seconds.

a) Find the rock's velocity and acceleration as functions of time.

$$s(t) = 20t - .8t^2 \quad \text{meters}$$

$$s'(t) = v(t) = 20 - 1.6t \quad \text{meters/sec}$$

$$s''(t) = a(t) = -1.6 \quad \text{meters/sec}^2$$

b) How long did it take the rock to reach its highest point?

$$v(t) = 0$$

$$20 - 1.6t = 0$$

$$\frac{20}{1.6} = \frac{1.6t}{1.6}$$

$$t = 12.5 \text{ sec}$$

c) When did the rock reach half its maximum height?

$$s(12.5) = 20(12.5) - .8(12.5)^2 = 125$$

$$62.5 = 20t - .8t^2$$

d) How long was the rock aloft?

$$s(t) = 0$$

$$20t - .8t^2 = 0$$

$$t = 25$$