SOLUTIONS - Practice Problems

Solutions can be found at the end of the packet.

FOR YOUR SAKE…

Don't just copy down the solutions. Make a note beside each problem you don't understand and talk with a friend about it or come see me before or after school during the first few days of class. I will continually refer to these topics throughout the year. If you don't understand this stuff, you won't be able to effectively do the calculus (even when the calculus concepts are easy).

Need to be able to do these WITHOUT A CALCULATOR

Unit Circle Trig. - Evaluate. Express your answers as exact values (no decimals, no rounding).

1. $\sin\frac{\pi}{6}$ $= \frac{1}{2}$

2. $\cos\frac{2\pi}{3}$ $= -\frac{1}{2}$

3. $\tan\frac{5\pi}{6}$ $= -\frac{\sqrt{3}}{3}$

4. $\sin\frac{7\pi}{4}$ $= -\frac{\sqrt{2}}{2}$

5. $\cos\frac{3\pi}{2}$ $= 0$

6. $\tan\frac{5\pi}{3}$ $= -\sqrt{3}$

7. $\cot\frac{3\pi}{4}$ $= -1$

8. $\sec\frac{4\pi}{3}$ $= -2$

9. $\csc\pi$ $=$ DNE

10. $\sin\frac{5\pi}{3}$ $= -\frac{\sqrt{3}}{2}$

11. $\cos\pi$ $= -1$

12. $\tan\frac{3\pi}{4}$ $= -1$

13. $\cos\frac{\pi}{2}$ $= 0$

14. $\sin\frac{\pi}{3}$ $= \frac{\sqrt{3}}{2}$

15. $\tan\frac{\pi}{2}$ $=$ DNE

16. $\cot\frac{\pi}{6}$ $= \sqrt{3}$

17. $\sec\frac{7\pi}{6}$ $= \frac{-2\sqrt{3}}{3}$

18. $\csc\frac{2\pi}{3}$ $= \frac{2\sqrt{3}}{3}$

19. $\sin\frac{8\pi}{3}$ $= \frac{\sqrt{3}}{2}$

20. $\cos 11\pi$ $= -1$

21. $\tan\frac{19\pi}{6}$ $= \frac{\sqrt{3}}{3}$

22. $\sin\left(-\frac{3\pi}{4}\right)$ $= \frac{-\sqrt{2}}{2}$

23. $\cos\left(-\frac{5\pi}{6}\right)$ $= -\frac{\sqrt{3}}{2}$

24. $\sec\left(-\frac{11\pi}{4}\right)$ $= \sqrt{2}$

25. $\cos 2\pi$ $= 1$

Inverse Trig Expressions. * Remember, with **inverse trig functions** the domain & ranges are limited (see the notes about these limits), so there is only one possible value for each inverse.

26. $\sin^{-1}\left(\frac{1}{2}\right)=\boxed{\frac{\pi}{6}}$

27. $\arccos\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}$

28. $\tan^{-1}(1)=\frac{\pi}{4}$

29. $\arctan\left(\sqrt{3}\right)=\frac{\pi}{3}$

30. $\arccos\left(-\frac{1}{2}\right)=\boxed{\frac{2\pi}{3}}$

31. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\frac{-\pi}{3}$

32. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\frac{5\pi}{6}$

33. $\tan^{-1}(0)=0$ rad

34. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)=\frac{-\pi}{4}$

35. $\sin^{-1}\left(-\frac{1}{2}\right)=\frac{-\pi}{6}$

36. $\arccos(-1)=\pi$

37. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)=\frac{-\pi}{6}$

38. $\tan^{-1}\left(-\sqrt{3}\right)=\frac{-\pi}{3}$

39. $\cos^{-1}(2)$ undefined

b/c $-1\le \#\le 1$

40. $\sin^{-1}\left(\sin\left(\frac{7\pi}{3}\right)\right)$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$=\frac{\pi}{3}$

Solving each logarithmic or exponential equation for x

41. $\ln(5x-1)=4$

$e^{\ln(5x-1)}=e^4$

$5x-1=e^4$

$x=\boxed{\frac{e^4+1}{5}}$

42. $\frac{3\ln(3x+14)}{3}=\frac{18}{3}$

$e^{\ln(3x+14)}=e^6$

$3x+14=e^6$

$\frac{3x}{3}=\frac{e^6-14}{3}$

$x=\boxed{\frac{e^6-14}{3}}$

43. $\frac{4\left(e^{2x-1}\right)}{4}=\frac{20}{4}$

$\ln e^{2x-1}=\ln 5$

$2x-1=\ln 5$

$\frac{2x}{2}=\frac{\ln 5+1}{2}$

$x=\boxed{\frac{\ln 5+1}{2}}$

44. $\ln\sqrt{x+2}=1$

$e^{\ln\sqrt{x+2}}=e^1$

$\left(\sqrt{x+2}\right)^2=(e)^2$

$x+2=e^2$

$x=\boxed{e^2-2}$

Solve this equation for y in terms of x.

45. $\ln(y-2)=x-3$

$e^{\ln(y-2)}=e^{x-3}$

$y-2=e^{x-3}$

$\boxed{y=e^{x-3}+2}$

Solve this equation for x in terms of y.

46. $y=3\sqrt{x}-4$

$\frac{y+4}{3}=\frac{3\sqrt{x}}{3}$

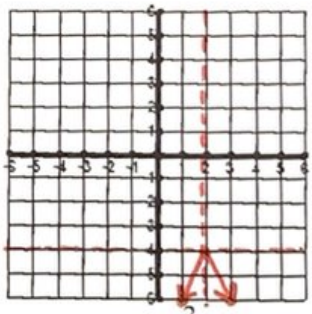
$\left(\frac{y+4}{3}\right)^2=\left(\sqrt{x}\right)^2$ $\therefore$ $\boxed{x=\left(\frac{y+4}{3}\right)^2}$ or $\boxed{x=\frac{y^2+8y+16}{9}}$

Characteristics of some of the "Basic" Functions.

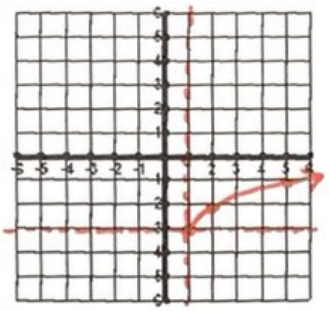
Function	Domain	Range	Symmetry	Odd/Even	Roots
47. $f(x)=x$	$(-\infty,\infty)$	$(-\infty,\infty)$	origin	odd	$x=0$
48. $f(x)=x^2$	$(-\infty,\infty)$	$[0,\infty)$	y-axis	even	$x=0$
49. $f(x)=x^3$	$(-\infty,\infty)$	$(-\infty,\infty)$	origin	odd	$x=0$
50. $f(x)=\lvert x\rvert$	$(-\infty,\infty)$	$[0,\infty)$	y-axis	even	$x=0$
51. $f(x)=\sqrt{x}$	$[0,\infty)$	$[0,\infty)$	neither	neither	$x=0$
52. $f(x)=x^{1/3}$	$(-\infty,\infty)$	$(-\infty,\infty)$	origin	odd	$x=0$
53. $f(x)=x^{2/3}$	$(-\infty,\infty)$	$[0,\infty)$	y-axis	even	$x=0$
54. $f(x)=\frac{1}{x}$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,0)\cup(0,\infty)$	origin	odd	none
55. $f(x)=\frac{1}{x^2}$	$(-\infty,0)\cup(0,\infty)$	$(0,\infty)$	y-axis	even	none
56. $f(x)=\sqrt{9-x^2}$	$[-3,3]$	$[0,3]$	y-axis	even	$x=-3,3$
57. $f(x)=e^x$	$(-\infty,\infty)$	$(0,\infty)$	neither	neither	none
58. $f(x)=\ln x$	$(0,\infty)$	$(-\infty,\infty)$	neither	neither	$x=1$
59. $f(x)=\sin x$	$(-\infty,\infty)$	$[-1,1]$	origin	odd	$\ldots -\pi, 0, \pi, \ldots$
60. $f(x)=\cos x$	$(-\infty,\infty)$	$[-1,1]$	y-axis	even	$\ldots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$
61. $f(x)=\tan x$	All $\mathbb{R}$ except $\ldots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2} \ldots$ (asymptotes are here!)	$(-\infty,\infty)$	origin	odd	$\ldots -\pi, 0, \pi, \ldots$

Transformations of functions - Sketch without the aid of a graphing calculator.

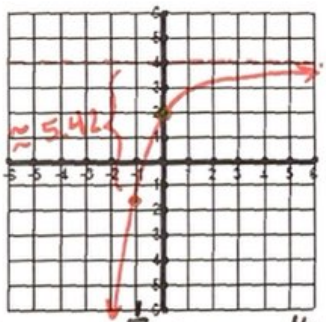
62. $y = -2|x-2|-4$



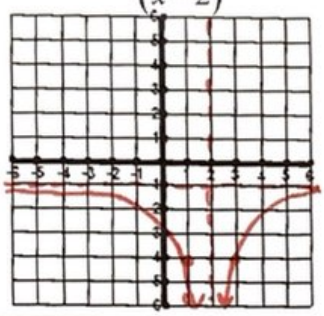
63. $y = \sqrt{x-1}-3$



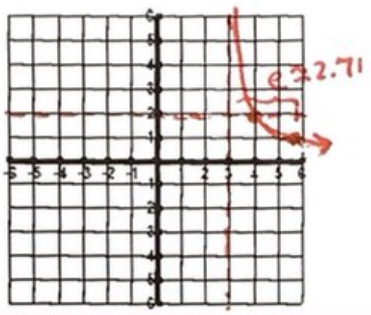
64. $y = -2 \cdot e^{-x} + 4$



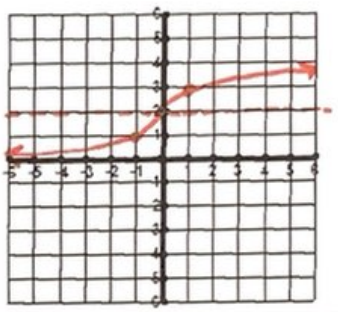
65. $y = -\dfrac{3}{(x-2)^2} - 1$



66. $y = -\ln(x-3)+2$



67. $y = x^{\frac{1}{3}} + 2$ ← $x^{1/3} + 2$



Slope and the equation of a line.

Let $A(-7,4)$ and $B(5,-12)$ be points in the plane.

68. Find the slope of the line that contains A and B.

$m = \dfrac{(4)-(-12)}{(-7)-(5)} = \dfrac{16}{-12} = \boxed{-\dfrac{4}{3}}$

69. Find an equation of the line that passes through A and B.

$\boxed{y-4 = -\frac{4}{3}(x+7)}$ or $\boxed{y+12 = -\frac{4}{3}(x-5)}$

70. What are the intercepts of this line?

x-int ($y=0$)

$0-4 = -\frac{4}{3}(x+7)$

$-4 = -\frac{4}{3}x - \frac{28}{3}$

$-12 = -4x - 28$

$-4 = x$

y-int ($x=0$)

$y-4 = -\frac{4}{3}(0+7)$

$y = -\frac{28}{3} + 4$

$y = -\frac{16}{3}$

71. Determine the slope that is perpendicular to the line between A and B.

$m_{\perp} = \boxed{\dfrac{3}{4}}$

Miscellaneous Problems

72. If $f(2) = -7$, then $f^{-1}(-7) = \boxed{2}$

73 & 74. Determine the location (x-values) of the removable (hole) and infinite discontinuities (V.A.) of: $f(x) = \dfrac{x-5}{(x-5)(x+3)} = \dfrac{1}{(x+3)}$

hole is located at $x=5$. The hole is $(5, \frac{1}{8})$

V.A. is located at $x=-3$. The V.A. is $x=-3$

Miscellaneous Problems

Determine the domain of each function without using a graphing calculator.

76. $f(x)=\sqrt{x-10}$

$x-10 \geq 0$

$\boxed{x \geq 10}$

77. $y=\dfrac{1}{2x^2+1}$

$2x^2+1 \neq 0$

$2x^2 \neq -1$

$x^2 \neq -\frac{1}{2}$

imag $\therefore$ domain is $\boxed{\mathbb{R}}$

78. $h(x)=\dfrac{10}{x^2-2x}$

$x^2-2x \neq 0$

$x(x-2) \neq 0$

$\therefore x \neq 0, 2$

$\boxed{(-\infty,0) \cup (0,2) \cup (2,\infty)}$

Determine the inverse of each function.

79. ~~$f(x)$~~ $y = \dfrac{5-3x}{2}$

$x = \dfrac{5-3y}{2}$

$2x = 5-3y$

$\dfrac{2x-5}{-3} = y$

$\therefore f^{-1}(x) = \boxed{-\dfrac{2x-5}{3}}$ or $\boxed{\dfrac{-2x+5}{3}}$

80. ~~$f(x)$~~ $y = \sqrt{2x-3}$

$(x)^2 = (\sqrt{2y-3})^2$

$x^2 = 2y-3$

$\dfrac{x^2+3}{2} = y$

$\therefore \boxed{f^{-1}(x) = \dfrac{x^2+3}{2},\ x \geq 0}$

Determine the vertical & horizontal/slant asymptotes of each rational function (if any) without using a graphing calculator.

81. $f(x)=\dfrac{2x^2}{x^2-1}$

$(x-1)(x+1)$

V.A. $\Rightarrow$ $x=1$, $x=-1$

H.A./~~S.A.~~ $\Rightarrow$ $y=2$

82. $y=\dfrac{x^2+x-2}{x^2-x-6}$

$(x+2)(x-1)$

$(x-3)(x+2)$

V.A. $\Rightarrow$ $x=3$

H.A./~~S.A.~~ $\Rightarrow$ $y=1$

83. $g(x)=\dfrac{x^2-x-2}{x-1}$

$(x-2)(x+1)$

$x=1$ $\therefore$

$1 \mid 1 \quad -1 \quad -2$

$ \quad 1 \quad 0$

$y = 1x+0 \quad -2$

V.A. $\Rightarrow$ $x=1$

~~H.A.~~/S.A. $\Rightarrow$ $y=x$

Determine whether each function is even, odd, or neither.

84. $f(x)=x^6-2x^2+3x^0$

even

85. $f(t)=t^2+2t^1-3t^0$

neither

86. $g(x)=x\sqrt{1-x^2}$

odd · even = odd

odd

Symmetry – Test each equation for symmetry (x-axis, y-axis, origin)

87. $y=x^3+3x^1$

odd $\therefore$

origin symmetry

88. $x=y^2-1$

$x=(-y)^2-1$

$\therefore x=y^2-1$

same $\therefore$

x-axis symmetry

89. $x^2+2y^2=1$

x-axis, y-axis, & origin symmetry!

Simplify the following expressions

90. $e^{3\ln 4} = e^{\ln 4^3}$

$= 8^{\ln 64}$

$= \boxed{64}$

91. $5e^{\ln 7}$

$= 5 \cdot 8^{\ln 7}$

$= 5 \cdot 7$

$= \boxed{35}$

92. $\ln\left(\left(e^x\right)^2\right) = \ln e^{2x}$

$= \boxed{2x}$

93. Starting with the graph of a function f, which sequence of transformations produces the graph of $g(x) = -5f(x-4)+1$?

(a) Shift 1 unit up, then shift 4 units right, stretch vertically by a factor of 5, and reflect about the x-axis

(b) Shift 1 unit up, then shift 4 units left, stretch vertically by a factor of 5, and reflect about the x-axis

(c) Shift 4 units right, then stretch vertically by a factor of 5, reflect about the y-axis, and shift 1 unit up

(d) Shift 4 units left, then stretch vertically by a factor of 5, reflect about the x-axis, and shift 1 unit up

94. If $f(x) = -x^2 - 2x + 8$, evaluate $\dfrac{f(1+h) - f(1)}{h}$

$= \dfrac{\left[-(1+h)^2 - 2(1+h) + 8\right] - \left[-1^2 - 2\cdot 1 + 8\right]}{h} = \dfrac{\left[-1 - 2h - h^2 - 2 - 2h + 8 - 5\right]}{h}$

$= \dfrac{-4h - h^2}{h} = \boxed{-4-h}$ or $\boxed{-h-4}$

(a) -4 (b) $h-4$ (c) $4-h$ (d) $-4-h$

95. Given $f(x) = \begin{cases} x^2 - 1 & \text{if } x \geq 2 \\ \ln(3-x) + 2 & \text{if } x < 2 \end{cases}$, evaluate $f(2)$.

$f(2) = 2^2 - 1 = \boxed{3}$

(a) 3 (b) -4 (c) 2 (d) -3

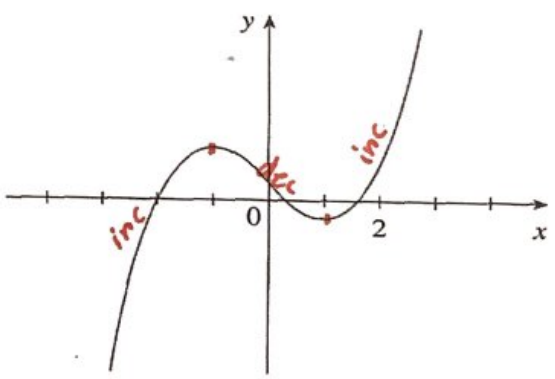
96. Pick the letter which indicates whether the function is even, odd, or neither; and determine the interval(s) over which the function is increasing.

(a) Even, $(-\infty, \infty)$

(b) Neither, $(-1,1)$

(c) Odd, $(-\infty,-1)$ and $(1,\infty)$

(d) Neither, $(-\infty,-1)$ and $(1,\infty)$



Complete the table below. If it helps, use the graphs below to sketch the graphs first.

		Domain	Range	Zeros / roots / x-intercepts (if any)	y-intercepts	Symmetry? (y-axis, origin, or none)
97.	$y = -2\|x-1\|+3$	$\mathbb{R}$	$(-\infty, 3]$	$x = \frac{5}{2}, \frac{-1}{2}$	$y = 1$	none
ex.	$y = (x-3)^{\frac{3}{2}} - 2$	All Reals. $\mathbb{R}$	$[-2, \infty)$ or $y \geq -2$	$(3 \pm \sqrt{2}, 0)$	7	NO
98.	$y = 3\sqrt{x-1} - 3$	$[1, \infty)$	$[-3, \infty)$	$x = 2$	none	none
99.	$y = e^x$	$(-\infty, \infty)$	$(0, \infty)$	none	$y = 1$	none
100.	$y = \ln x$	$(0, \infty)$	$(-\infty, \infty)$	$x = 1$	none	none

Sketch the graph of each equation above on the planes below if it helps to fill in the blanks in the table.

97. ex. 98. 99. 100.