AP Calculus AB - Summer Reading

2021-2022

Hey guys, just looking to get you started in the right direction for next year. I've attached some material from precalculus I need you to know when you walk in the door in August. Regardless of your math background, if your teacher or counselor recommended you for this course then I have confidence in your ability to handle whatever is in this packet, do well in the class, and pass the AP Test. Over the past 10 years, my AP classes have averaged scoring over a "4" on the AP test (well, if you exclude the year when Covid hit and all the seniors were told they could be finished with school on March 13th). Also, the pre-AP math team classes are NOT a prerequisite for AP Calculus and you are not at a disadvantage if you didn't them. In fact, some of the brightest and most successful students every year did not come through our Pre-AP classes, so no worries.

Hopefully, this class will be the best math class you have taken at Homewood. It will take so much of what you have been learning over the past years and apply them in some cool and interesting ways (ok, at least "math" cool). Calculus really is one of the greatest branches of mathematics ever created and its applications in the math, physics, business, & economic worlds are extensive. Seriously… really good stuff.

If you are worried about doing well in here, don't be. Your grade will almost always be a reflection of your work ethic. Students who come to class and are willing to complete homework assignments almost always make A's or B's and will pass the AP test (last semester's class average was an A) and remember we get an extra quality point towards GPA, as well.

In case you are struggling with any of the topics in this packet and can't find help in your old precal notes, try to get help from someone else you know taking this class before you come back this fall – do your best to be prepared! I've never had a student make a D or an F in this class unless they quit working (1 D last year), and very few have made C's (none last year)– and again, remember the class is graded on a 5 point scale. Our AP scores are always *much*, *much* better than the national average so you can have confidence if you do the work your AP score will be solid. Start the year off on a good note by learning the material in this packet this summer.

If you really don't want to do anything your senior year, and you know it, please change your schedule ASAP (well, unless you are reading this in June, July or August because then it would be too late)! That being said, get ready for a great year. The more you try to do your best, the more you will enjoy this class. I've had students succeed in here who were not recommended, but usually your precal teacher's recommendation is a good one.

Table of Contents

Preparation for AP Calculus AB

Finding intercepts of a graph.

To find an *x*-intercept simply plug in 0 for the *y*-values and solve for the remaining *x*-values.

Ex) Find the intercepts of the graph of $y = x^3 - 4x$.

x-intercepts $\Rightarrow y = 0$

$0 = x^3 - 4x$

$0 = x\left(x^2 - 4\right)$

$0 = x\left(x+2\right)\left(x-2\right)$

$x = 0, -2, 2$

$\therefore (0,0), (-2,0), (2,0)$

y-intercepts $\Rightarrow x = 0$

$y = 0^3 - 4 \cdot 0$

$y = 0 - 0$

$y = 0$

$\therefore (0,0)$

Graphically, you could use your calculator to enter $\mathrm{Y}_1 = x^3 - 4x$ or $f1(x) = x^3 - 4x$, and then adjust to get a decent "window."

- To find *x*-intercepts (where *y* is 0), use the CALC, ZERO function (TI-84) or use the MENU,ANALYZE GRAPH,ZERO function to find the zeros (TI-NSPIRE).

Find points of intersection / Find solutions to a system of equations

Manually

Solve the system of equations below:

$x^2 - y = 3$ and $x - y = 1$

By substitution...

$x^2 - y = 3$ and $y = x - 1$

$\therefore \quad x^2 - (x-1) = 3$

$x^2 - x + 1 - 3 = 0$

$x^2 - x - 2 = 0$

$\Rightarrow$

$(x-2)(x+1) = 0$

$x = 2$ and $x = -1$

$\therefore\ y = 1$ and $\therefore\ y = -2$

$(2,1), (-1,-2)$

Graphically: if the equations can be written in $y =$ form, then enter the equations as $\mathrm{f}1(x)$ and $\mathrm{f}2(x)$ or if you have a TI-84 as Y_1 and Y_2, then calculate where they INTERSECT. The intersections are the solutions! Or, Graph $Y_1 - Y_2 = 0$ and then use CALC, ZEROS to find the roots. The roots, *x*-values, can be used to find the *y*-values.

Symmetry and EVEN/ODD

Symmetry typically goes hand in hand with a function being even or odd.

A function with ***y*-axis symmetry** is always an **even** function.

A function with **origin symmetry** is always an **odd** function.

y-axis symmetry test: replace *x* with –*x* & simplify $\Rightarrow$ same equation

origin symmetry test: replace *x* with –*x* & *y* with –*y* & simplify $\Rightarrow$ same equation

Tests for symmetry, continued…

Ex) Show that the graph of $y = x^4 - 3x^2 + 5$ is symmetric with respect to the ***y*-axis**.

$$y = (-x)^4 - 3(-x)^2 + 5$$

$$y = x^4 - 3x^2 + 5 \qquad \therefore \text{ ended with the same equation!}$$

Ex) Show that the graph of $y = 2x^3 - 5x$ is symmetric with respect to the **origin**.

$$(-y) = 2(-x)^3 - 5(-x)$$

$$-y = -2x^3 + 5x$$

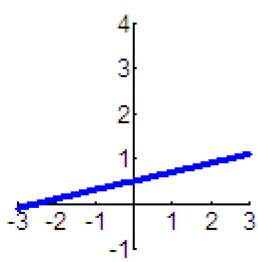
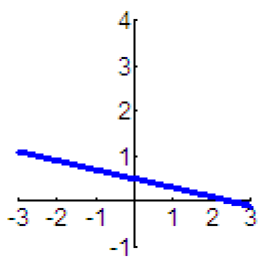
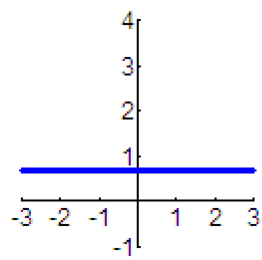
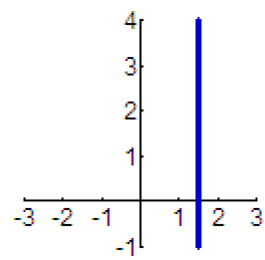
$$y = 2x^3 - 5x \qquad \therefore \text{ ended with the same equation!}$$

Slope

$$m = \frac{rise}{run} = \frac{\text{change along vertical axis}}{\text{change along horizontal axis}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

If…

$\boxed{m > 0}$	$\boxed{m < 0}$	$\boxed{m = 0}$	$\boxed{m \text{ is undefined}}$
Positive slope	*Negative slope*	*0 slope*	*No slope*
Then…	Then…	Then…	Then…
Line rises from left to right	Line falls from left to right	Line is horizontal	Line is vertical

Point-Slope form - *for the love of all that is good and true, please know how to find an equation of a line using point-slope form. This is usually asked multiple times on every AP test!*

$$y - y_1 = m(x - x_1)$$

Where m is slope and (x_1, y_1) is a point

Ex) You are given two points $(-1,3)$ *and* $(2,4)$.
Find the equation of the line that contains both of them.

1st, find the slope… $m = \dfrac{(4)-(3)}{(2)-(-1)} = \dfrac{1}{3}$

2nd, enter slope & point into point-slope equation $y-(3) = \frac{1}{3}(x-(-1))$ ⇨ $\boxed{y-3 = \frac{1}{3}(x+1)}$

*Don't bother changing this to another form unless you are required to.

Parallel and Perpendicular Lines:

Parallel lines have slopes that are equal to each other.

Perpendicular lines have slopes that are negative reciprocals of each other.

Ex) Find the equation of the line that contains $(3,-6)$ and is…

…parallel to $y=-3x+4$

…perpendicular to $y=-3x+4$

PARALLEL - *same slope*

$m=-3$

$y-(-6)=-3(x-(3))$

$y+6=-3(x-3)$

PERPENDICULAR – *negative reciprocal slope*

$m=\frac{1}{3}$

$y-(-6)=\frac{1}{3}(x-(3))$

$y+6=\frac{1}{3}(x-3)$

Rates of Change

Slope as a **Ratio**
…*x* & *y* axes have the same units

Ex) On a roof, if the height of the roof rises 5 ft for every 12 ft of length we say …

$$m=ratio=\frac{5\ \cancel{ft}.}{12\ \cancel{ft}.}=\frac{5}{12}$$

Slope as a **Rate of change**
…*x* & *y* axes have different units

Ex) The population of the United States was 281,000,000 in 2000 and was 308,000,000 in 2010.

$$m=rate\ of\ \ change=\frac{308,000,000-281,000,000\text{ people}}{2010-2000\text{ year}}$$

$$=\frac{27,000,000\text{ people}}{10\text{ years}}=270,000\text{ people/year during this span}$$

Functions & Their Graphs

Function – for each *x*-value there exists exactly one *y*-value (passes the Vertical Line Test).

Independent variable: "*x*"

dependent variable: "*y*"

Implicit form of an equation: $x^2+2y=1$

Basically, *x*'s and *y*'s can be found on the same side of an equation and or both sides of an equation.

Explicit form of an equation: $y=\frac{1}{2}(1-x^2)$ or $y=-\frac{1}{2}x^2+\frac{1}{2}$ (basically in $y=$ or $f(x)=$ form)

Basically, this is "function notation," where *y* or $f(x)$ is written in terms of *x*.

<u>*Function notation*</u>: $f(x)=\frac{1}{2}(1-x^2)$ or $f(x)=-\frac{1}{2}x^2+\frac{1}{2}$ (Explicit, but with $f(x)$ instead of *y*).

Know how to evaluate functions…

Ex) If $f(x) = x^2 - 3x - 2$ then evaluate…

1. $f(4) = ?$

$$= (4)^2 - 3(4) - 2$$
$$= 16 - 12 - 2$$
$$= \boxed{2}$$

2. $f(2a) = ?$

$$f(2a) = (2a)^2 - 3(2a) - 2$$
$$f(2a) = \boxed{4a^2 - 6a - 2}$$

don't go on to factor & solve this!

Ex) If $f(x) = x^2 - 3x - 2$ then evaluate…

3. $f(b-1) = ?$

$$f(b-1) = (b-1)^2 - 3(b-1) - 2$$
$$f(b-1) = b^2 - 2b + 1 - 3b + 3 - 2$$
$$f(b-1) = \boxed{b^2 - 5b + 2}$$

don't go on to factor & solve this!

Ex) If $f(x) = x^2 - 3x - 2$ then evaluate…

4. $$\frac{f(x+h) - f(x)}{h} = \frac{\left[(x+h)^2 - 3(x+h) - 2\right] - \left[(x)^2 - 3(x) - 2\right]}{h}$$

$$= \frac{\left[x^2 + 2xh + (h)^2 - 3x - 3h - 2\right] - \left[(x)^2 - 3(x) - 2\right]}{h}$$

$$= \frac{\cancel{x^2} + 2xh + (h)^2 \cancel{-3x} - 3h \cancel{-2} \cancel{-x^2} \cancel{+3x} \cancel{+2}}{h}$$

$$= \frac{2xh + (h)^2 - 3h}{h} = \frac{\cancel{h}(2x + h - 3)}{\cancel{h}}$$

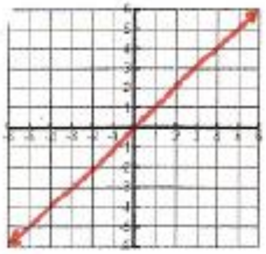
$$= \boxed{2x + h - 3}, \quad h \neq 0$$

don't go on to factor & solve this!

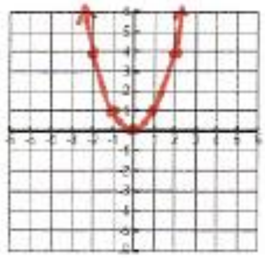
Graphs of basic functions. Be able to graph #1-5, 7-13, & the "look" of #14-16.

Know how to sketch the graph & know how to recognize the equation from the graph.

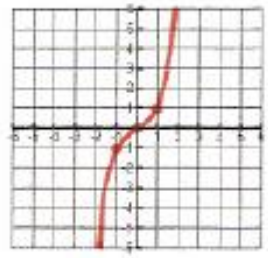
1. $y = x$



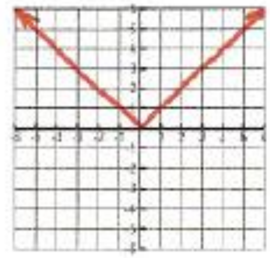
2. $y = x^2$



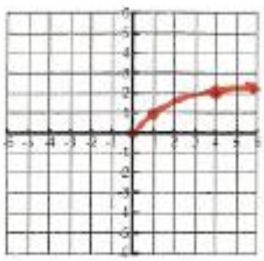
3. $y = x^3$



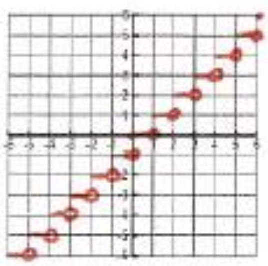
4. $y = |x|$



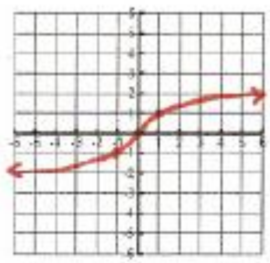
5. $y = \sqrt{x}$



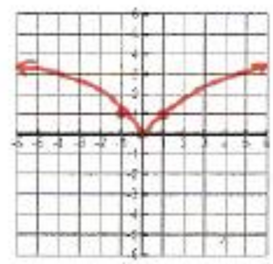
6. $y = [\![x]\!]$



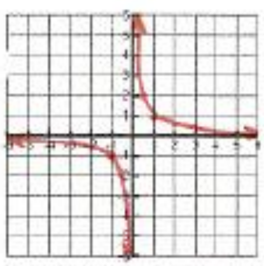
7. $y = x^{\frac{1}{3}}$



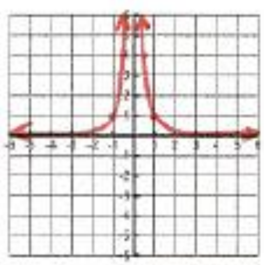
8. $y = x^{\frac{2}{3}}$



9. $y = \frac{1}{x}$

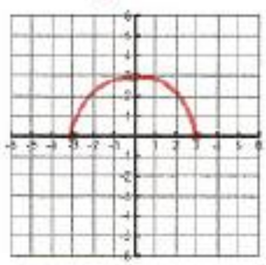


10. $y = \frac{1}{x^2}$

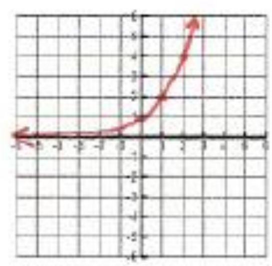


11. $y = \sqrt{r^2 - x^2}$

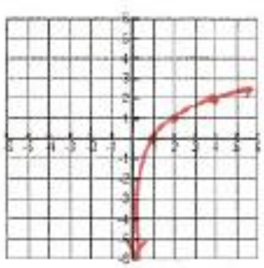
if $r = 3$



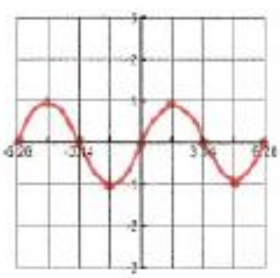
12. $y = 2^x$ or $y = a^x$



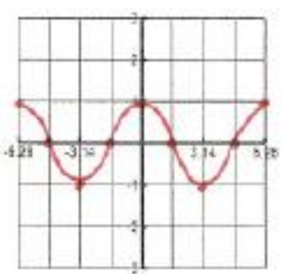
13. $y = \log_2 x$ or $y = \log_a x$



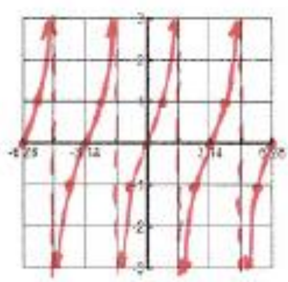
14. $y = \sin x$



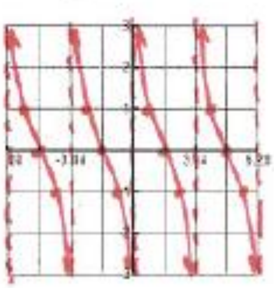
15. $y = \cos x$



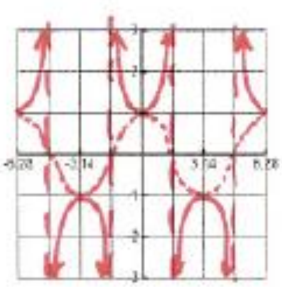
16. $y = \tan x$



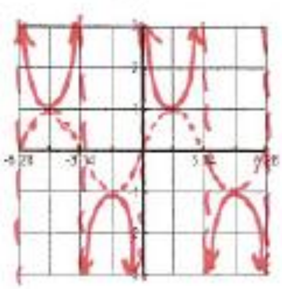
17. $y = \cot x$



18. $y = \sec x$

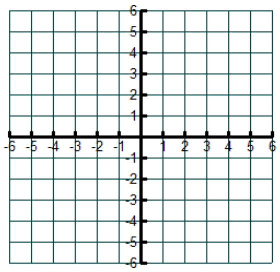


19. $y = \csc x$

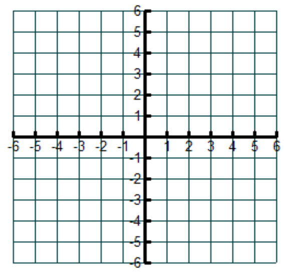


Practice sketching these basic functions.

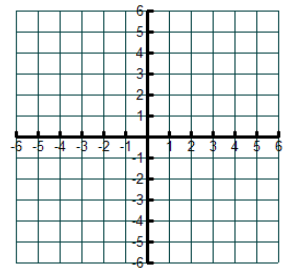
1. $y = x$



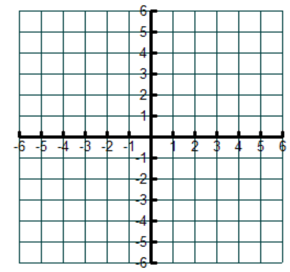
2. $y = x^2$



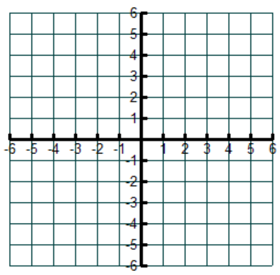
3. $y = x^3$



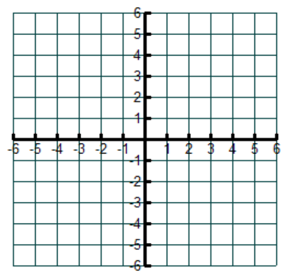
4. $y = |x|$



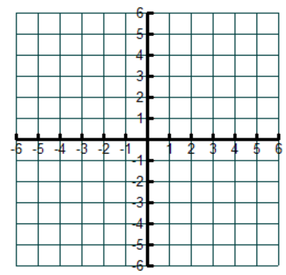
5. $y = \sqrt{x}$



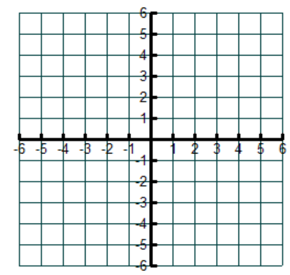
6. OMIT



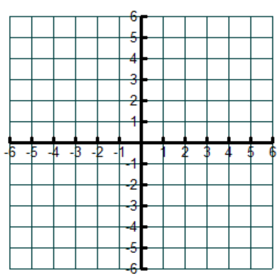
7. $y = x^{\frac{1}{3}}$



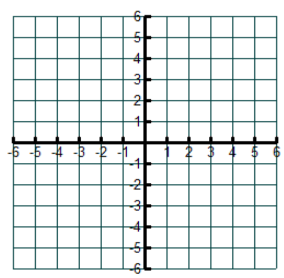
8. $y = x^{\frac{2}{3}}$



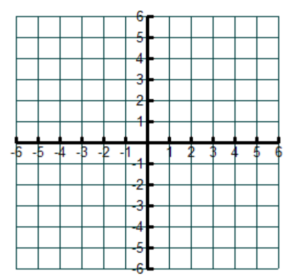
9. $y = \frac{1}{x}$



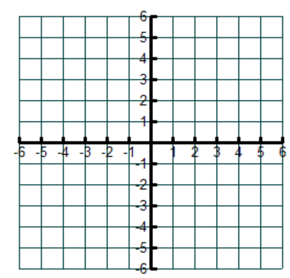
10. $y = \frac{1}{x^2}$



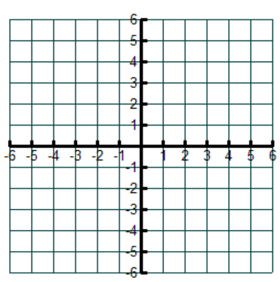
11. $y = \sqrt{16 - x^2}$



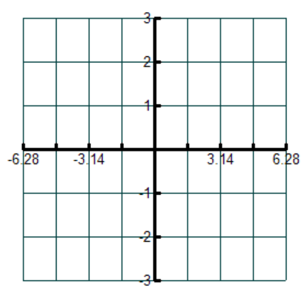
12. $y = e^x$



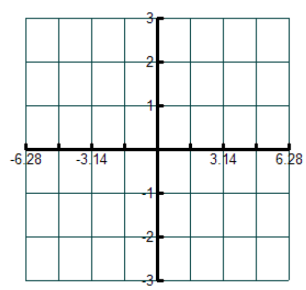
13. $y = \ln x$



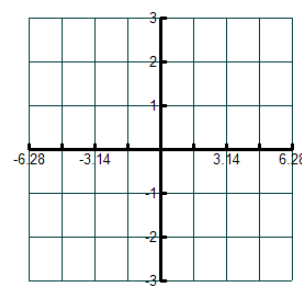
14. $y = \sin x$



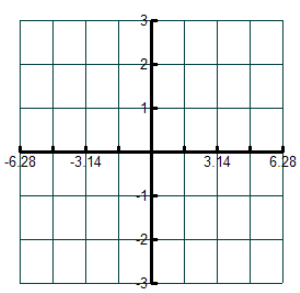
15. $y = \cos x$



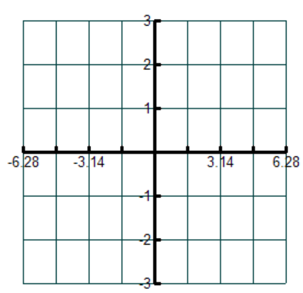
16. $y = \tan x$



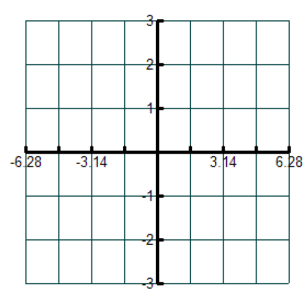
17. OMIT



18. OMIT



19. OMIT



Interval Notation

Writing solutions that have an infinite number of values like…

$x > 4$ $\quad$ $x \le 7$ $\quad$ $-2 < x \le 5$ $\quad$ $-3 \le x < 2$ or $x \ge 5$

…can also be written using "***interval notation***" like…

$(4, \infty)$ $\quad$ $(-\infty, 7]$ $\quad$ $(-2, 5]$ $\quad$ $[-3, 2) \cup [5, \infty)$

<u>AP uses both of these ways of showing inequalities</u>

Notice that…

parentheses are used with numbers associated with $<, >, \infty$ and $-\infty$

brackets are used with numbers associated with $\le$ and $\ge$

Always list values from smallest to greatest number.

$*$ never write $[5, -\infty)$ instead write $(-\infty, 5]$

DOMAIN – The possible values for x when you are dealing with a function, an equation, or a point (ordered pair). The domain represents the x-values that are "allowed" to be substituted in for x in an equation.

When dealing with the equation itself, and not the graph, sometimes it is easier to "see" the domain by observing what *x* – values cannot be plugged into an equation and then choosing everything but these values.

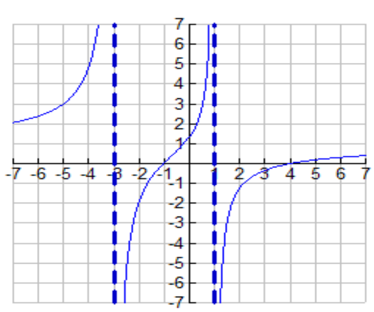
Ex1) In the equation $y = \sqrt{x}$, you can see that any negative value for x will give us imaginary values for y and therefore can't be included in the domain. We would say that the domain is:

	D:	All Real numbers except the negative numbers	←	*meh*
Or	D:	$x \ge 0$	←	great
Or	D:	$[0, \infty)$	←	great

Ex2) In the equation $y = \frac{1}{x}$, you can see that if I substitute 0 in for x in the denominator I will have an undefined value for y and therefore cannot be included in the domain. We would say that the domain is:

	D:	All Real numbers except 0	←	*meh*
	D:	$\mathbb{R};\ x \ne 0$	←	*meh*
	D:	$x < 0$ or $x > 0$	←	great
Or	D:	$(-\infty, 0), (0, \infty)$	←	great

Ex3) Looking at the graph below, I can see a point above or below every x-value represented except at $x = -3$ and at $x = 1$. Therefore, I would say that the domain is:



D: $\quad x < -3$ or $-3 < x < 1$ or $x > 1$

Or

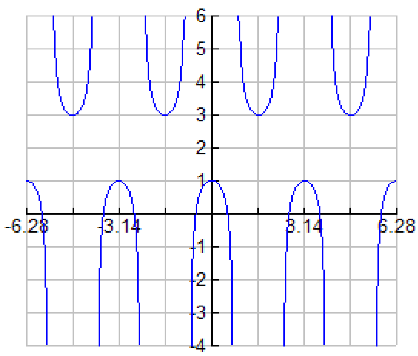
D: $\quad (-\infty, -3) \cup (-3, 1) \cup (1, \infty)$

Both of these options for writing the domain are great.

RANGE – The possible values for y that are a *result* of numbers in the Domain being plugged in for x.

I feel it is easier to look at the graph of the equation to "see" the Range. To find the range by looking at the graph, see which *y* values are represented on the graph (Does the graph move continuously up and down? If not, where do *y*-values exist?)

Ex) Looking at the graph below I can see that although the graph does move up continuously and down continuously, it has a break in the middle, between a y-value of 1 and a y-value of 3 (remember, with range we don't care about the x-values). This means that every y-value (except in this broken interval) has an x-value associated with it. Therefore, I would say that the range is:



R: $y \le 1$ or $y \ge 3$

Or

R: $(-\infty, 1] \cup [3, \infty)$

Either of these options for writing the range are great.

Definition of Even and Odd functions

Even Functions

$f(x) = f(-x)$ *All even functions have y-axis symmetry

Basically means: "When you plug in opposite x-values you still get the same y-values."

Odd Functions

$-f(x) = f(-x)$ *All odd functions have origin symmetry

Basically means: "When you plug in opposite x-values you get opposite y-values, as well."

Ex) Determine whether the functions are even, odd, or neither.

1. $f(x) = 5x^2 - \cos x$

Does $f(x)$ equal $f(-x)$?

$f(-x) = 5(-x)^2 - \cos(-x)$

$f(-x) = 5x^2 - \cos x$

$\therefore \; f(x) = f(-x)$

$\therefore$ Even function

2. $f(x) = 2x^3 - 5x$

Does $f(x)$ equal $-f(-x)$?

$-f(-x) = -\left[2(-x)^3 - 5(-x)\right]$

$-f(-x) = -\left[-2x^3 + 5x\right]$

$-f(-x) = 2x^3 - 5x$

$\therefore \; f(x) = -f(-x)$

$\therefore$ Odd function

Also, remember: ALL **even** functions have y-axis symmetry, therefore you can use the ***y*-axis** symmetry test.
And, remember: ALL **odd** functions have origin symmetry, therefore you can use the **origin** symmetry test.
And, finally: NO **functions** have x-axis symmetry, therefore you don't have to worry about this happening.
Again...

***y*-axis symmetry test:** replace x with $-x$ $\Rightarrow$ same equation
origin symmetry test: replace x with $-x$ & y with $-y$ $\Rightarrow$ same equation

Inverse Functions

Can be written as ordered pairs… **or** **…as functions**

$f:\{(1,6),(2,-3),(5,1),(-4,7)\}$

$f^{-1}:\{(6,1),(-3,2),(1,5),(7,-4)\}$

$f(x)=2x^3-5$

$f^{-1}(x)=\sqrt[3]{\dfrac{x+5}{2}}$

Important to note about functions, f, and their inverses, f^{-1}.

If $f(a)=b$,

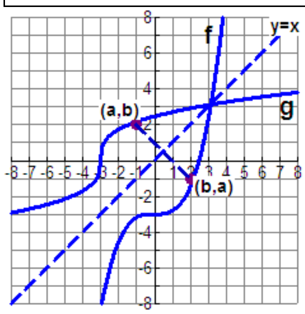
then $f^{-1}(b)=a$.

So, if I ask you to determine $f^{-1}(6)=?$ and you know that $f(1)=6$, then you know that $f^{-1}(6)=1$

Key things to know…

- Since you are essentially just switching the x & y values, the domain of a function is the same as the range of its inverse (and vice versa). Big concept for later.
- If $f(g(x))=x$ and $g(f(x))=x$, then f and g are inverses of each other.
- You could think of an inverse function as "undoing" what was done by the function.
- If you graph f and f^{-1} you will see they are reflections about the $y=x$ line. This will be valuable next year.
- A function will only have an inverse if it is "one-to-one." Essentially, one-to-one means that a function passes both the vertical line test (VLT) & the horizontal line test (HLT).

Ex) Let $f(x)=\frac{1}{5}x^3-3$ and $g(x)=\sqrt[3]{5x+15}$ be inverse functions.



Notice that the graphs of inverses are reflections about the $y=x$ line.
Also, notice that if (a,b) lies on one graph, then (b,a) will lie on its inverse.

To find the inverse of a function, go through the following steps…

1. Interchange x and y 2. Solve for the "new" y. 3. Replace y with $f^{-1}(x)$.

Ex) Find the inverse of $f(x)=\dfrac{5x+4}{2}$

1. $x=\dfrac{5y+4}{2}$

2. $2\cdot x=\cancel{2}\cdot\dfrac{5y+4}{\cancel{2}}$

$2x-4=5y+\cancel{4-4}$

$\dfrac{2x-4}{5}=\dfrac{\cancel{5}y}{\cancel{5}}$

3. $f^{-1}(x)=\dfrac{2x-4}{5}$

Check to see if $f(x)$ and $f^{-1}(x)$ are inverses…

is $f\left(f^{-1}(x)\right)=x$?

$$f\left(f^{-1}(x)\right)=\frac{5\left(\frac{2x-4}{5}\right)+4}{2}$$

$$=\frac{\cancel{5}\left(\frac{2x-4}{\cancel{5}}\right)+4}{2}$$

$$=\frac{2x-4+4}{2}=\frac{2x}{2}=x$$ Yes!

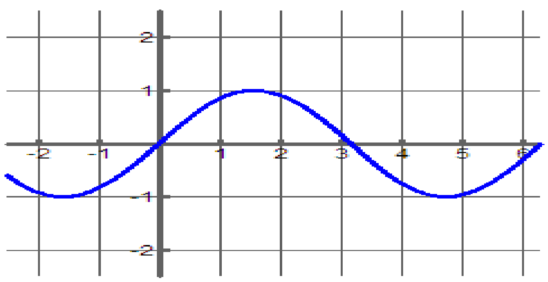
Inverse Trig Functions

So that the inverse trig functions will be actual functions (pass the V.L.T.), we have to limit their domain and range values (essentially making them one-to-one).

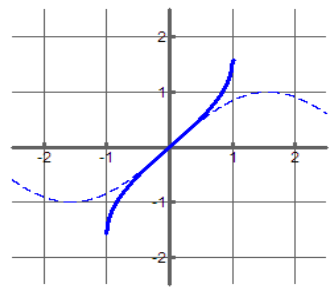
$y = \arcsin x = \sin^{-1} x$	$y = \arccos x = \cos^{-1} x$	$y = \arctan x = \tan^{-1} x$
$-1 \le x \le 1$	$-1 \le x \le 1$	$-\infty \le x \le \infty$
$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$	$0 \le y \le \pi$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

When you limit the domain & range values you get the following inverse trig function.
Notice how the inverse trig function still has $y = x$ symmetry with the original function (on the new domain & range)

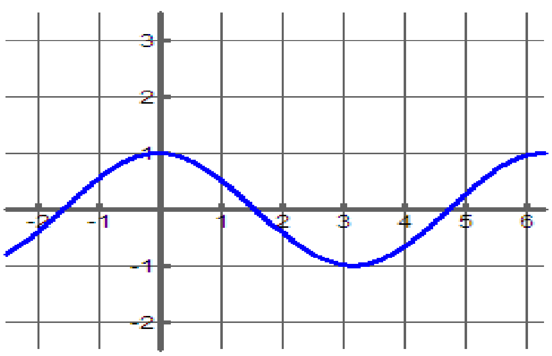
$y = \sin x$



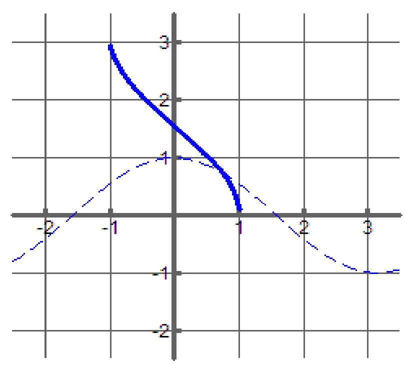
$y = \sin^{-1} x$



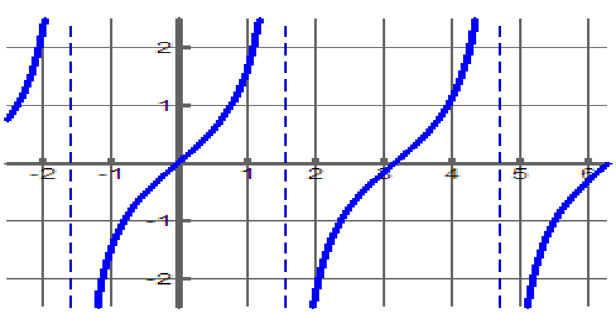
$y = \cos x$



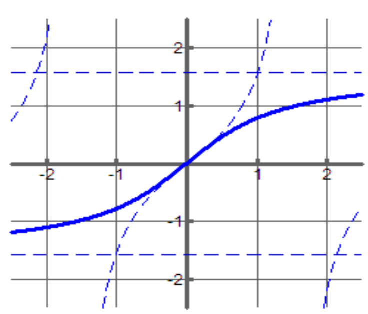
$y = \cos^{-1} x$



$y = \tan x$



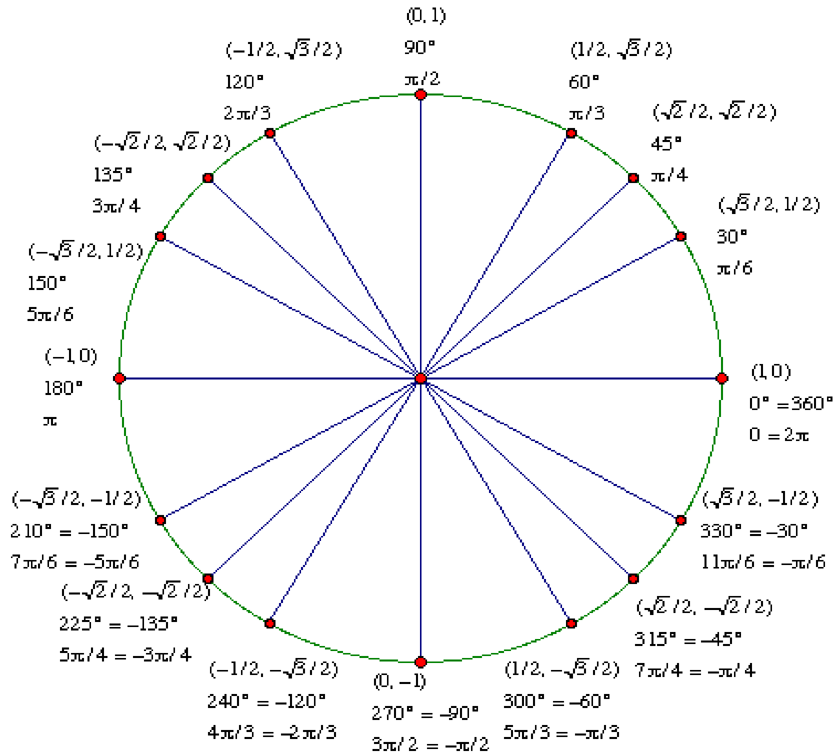
$y = \tan^{-1} x$



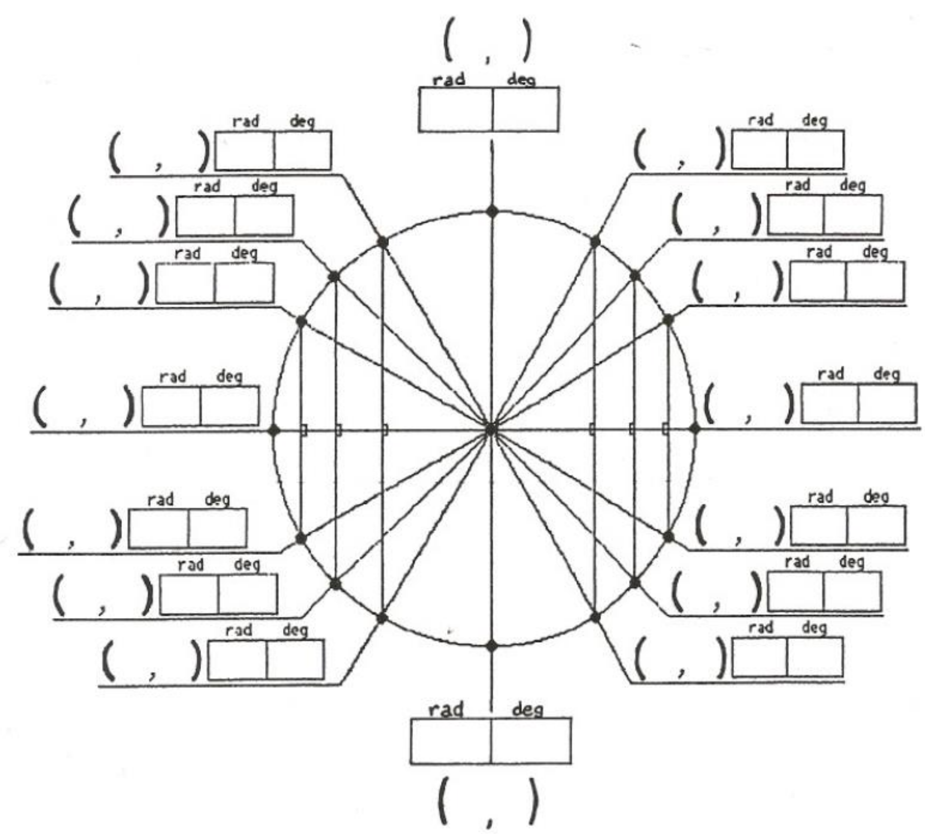
Basically just KNOW the domain & range intervals/limits of each inverse trig function which are in the boxed region above.

The Unit Circle

In case you are a little rusty, you really need to know the values of the unit circle. The A.P. test (and my tests) will leave angles in **RADIAN MEASURES ONLY!**, so it is imperative you know the radian values especially well. You **REALLY** need to know the values of the trig functions of any radian measure from this circle.



For Practice…



Trigonometric Identities

You will need to know the following trig identities (at least these for now)…

Fundamental Identities

$$\sin\theta = \frac{y}{r} = \frac{opp}{hyp}$$
$$\cos\theta = \frac{x}{r} = \frac{adj}{hyp}$$
$$\tan\theta = \frac{y}{x} = \frac{opp}{adj}$$
$$\cot\theta = \frac{x}{y} = \frac{adj}{opp}$$
$$\sec\theta = \frac{r}{x} = \frac{hyp}{adj}$$
$$\csc\theta = \frac{r}{y} = \frac{hyp}{opp}$$

$$\sin\theta = \frac{1}{\csc\theta}$$
$$\cos\theta = \frac{1}{\sec\theta}$$
$$\tan\theta = \frac{1}{\cot\theta}$$
$$\cot\theta = \frac{1}{\tan\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\csc\theta = \frac{1}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

Double Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$

** *Don't confuse* $\cos^2\theta + \sin^2\theta = 1$ *with* $\cos^2\theta - \sin^2\theta = \cos(2\theta)$. *Be careful out there !!!*

Properties of Exponents

For all positive a and b, and all real x and y…

$$a^0 = 1 \qquad a^x a^y = a^{x+y} \qquad (ab)^x = a^x b^x \qquad (a^x)^y = a^{xy}$$

$$a^{-x} = \frac{1}{a^x} \qquad \frac{1}{a^{-x}} = a^x \qquad \frac{a^x}{a^y} = a^{x-y} = \frac{1}{a^{y-x}} \qquad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$x^{\frac{a}{b}} = \sqrt[b]{x}^{\,a} \qquad x^{\frac{1}{2}} = \sqrt{x} \qquad x^{\frac{4}{3}} = \sqrt[3]{x}^{\,4} \qquad 8^{\frac{4}{3}} = \sqrt[3]{8}^{\,4} = 2^4 = 16$$

Logarithms

A logarithm is the inverse of an exponential function. You need to know the following properties of logarithms as well as be able to sketch a logarithm's graph.

If b and y are positive numbers ($b \neq 1$),

then $\ln y = x$

iff $y = e^x$

Ex)

$$\ln y = 3$$
$$\cancel{e}^{\cancel{\ln} y} = e^3$$
$$y = e^3$$

Laws of Logarithms

Examples

$\ln MN = \ln M + \ln N$ — $\ln 36 = \ln 4 + \ln 9 = \ln 2 + \ln 18 = etc$

$\ln \frac{M}{N} = \ln M - \ln N$ — $\ln 4 = \ln 8 - \ln 2 = \ln 12 - \ln 3 = etc$

$\ln M^k = k \ln M$ — $\ln 81 = \ln 3^4 = 4 \ln 3$

$\ln\left(\frac{1}{M}\right) = -\ln M$ — $\ln\left(\frac{1}{2}\right) = -\ln 2$

Properties of Logarithms

Examples

$\ln e^y = y$ — $\ln e^3 = 3$

$e^{\ln x} = x$ — $e^{\ln 12} = 12$

$\ln e = 1$ — $\ln e = 1$

$\ln 1 = 0$ — $\ln 1 = 0$

$\ln b$ is the "natural log"

If $\ln a$, then $a > 0$ — $\ln(-2) \Rightarrow DNE$

Be careful: $e^{4\ln x} \neq 4x$. You must "swing" the 4 up first… $e^{4\ln x} = e^{\ln x^4} = \cancel{e}^{\cancel{\ln} x^4} = x^4$

★ ***If you don't have a TI-NSPIRE CX II, you ought to try to get one. TI-nspire's are better than the TI-84's and they make certain calculus applications possible because of the speed at which its operating system calculates. However, if your calculator is a TI-84, you'll be ok, it'll just be slower with occasional applications, especially when graphing.***

The number *e*

Know that the number $e \approx 2.718...$

It is also equal to … (the first definition below is the most common)

$$\boxed{e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x} \qquad e = \lim_{x\to 0}(1+x)^{\frac{1}{x}} \qquad e = \sum_{n=0}^{\infty}\frac{1}{n!}$$

If you hated logs in precal, don't worry. The logarithms you will need in AP Calculus are pretty basic.

Rational Functions

Unlike a polynomial function, $f(x) = a_1x^n + a_2x^{n-1} + a_3x^{n-2} + \ . \ . \ . \ + a_{n+1}$, a rational function is written as the quotient of two polynomial functions, $f(x) = \dfrac{g(x)}{h(x)} = \dfrac{a_1x^n + a_2x^{n-1} + a_3x^{n-2} + \ . \ . \ . \ + a_{n+1}}{b_1x^n + b_2x^{n-1} + b_3x^{n-2} + \ . \ . \ . \ + b_{n+1}}$.

Holes-

After factoring the numerator (top) and the denominator (bottom) completely, if there is a common factor in the numerator and the denominator, this will indicate that there is a hole in the graph. A hole does not act like a vertical asymptote. It is simply a missing point in an otherwise continuous section of the function. In AP Calculus we also refer to holes as "***removable discontinuities***."

To find the hole(s) set the cancelled factor = 0 to get the *x*-coordinate of the hole, then plug this *x*-value back into the uncancelled part of the function in order to find the *y*-value.

**Once you have cancelled the common factors and found the hole(s), don't look at these factors again for the rest of the problem – you are done with them and they will not effect the graph in any other way.*

Horizontal Asymptotes-

what the graph looks as if it is doing as it moves to the right toward ∞ and to the left toward $-\infty$, if the conditions below exist. <u>*The graph is able to cross and or bounce off of a horizontal asymptote.*</u>

To find the H.A.'s If $f(x) = \dfrac{ax^m + \ . \ . \ .}{bx^n + \ . \ . \ .}$, and if . . .

1. If $m < n$, (i.e. if the degree of the denominator is bigger than the degree of the numerator) then the horizontal asymptote is ***y = 0***.
2. If $m = n$, (i.e. if the degree of the numerator and the denominator is the same) then the horizontal asymptote is $\boldsymbol{y = \dfrac{a}{b}}$.
3. If $m > n$, (if the degree of the numerator is bigger than the degree of the denominator) then a horizontal asymptote **DOES NOT EXIST,** ***but most likely a slant asymptote will exist.***

Slant /oblique asymptotes -

Slant asymptotes are similar to H.A.'s, but rather they are not horizontal, they are sloped. Either a horizontal or a slant asymptote can occur, but not both. Oblique asymptotes are asymptotes shaped like parabolas or cubics, but we will not do any graphing with these. <u>*A graph CAN cross and or bounce off a slant or oblique asymptote.*</u>

To find a <u>slant/</u>oblique asymptote"- If $f(x) = \dfrac{ax^m + \ . \ . \ .}{bx^n + \ . \ . \ .}$, and if $m > n$, then a horizontal asymptote D.N.E. and there WILL BE a slant or an oblique asymptote. Divide the numerator by the denominator. The result (not including the remainder) when you set it = to *y* is the slant/oblique asymptote.

Vertical Asymptotes-

these are the zeros of the denominator of the function (after you have cancelled all of the holes). This is where the equation is undefined, therefore, where the graph can not exist. Vertical asymptotes all have the property that they "draw" or "pull" the graph either upward or downward as the graph nears the asymptote. <u>*The graph can never cross over a vertical asymptote, although it will appear on both sides of the V.A*</u>. In AP Calculus we also refer to vertical asymptotes as "***infinite discontinuities***."

To find the V.A.'s If $f(x) = \dfrac{ax^m + \ . \ . \ .}{bx^n + \ . \ . \ .}$, then set what is left of the denominator (after you have cancelled out the "holes") equal to zero and solve.

These are the vertical asymptotes, UNLESS THERE IS ALSO A FACTOR JUST LIKE IT IN THE NUMERATOR which cancels the factor in the denominator (see "Holes").

Practice Problems

Solutions can be found at the end of the packet.

FOR YOUR SAKE…

Don't just copy down the solutions. Make a note beside each problem you don't understand and talk with a friend about it or come see me before or after school during the first few days of class. I will continually refer to these topics throughout the year. If you don't understand this stuff, you won't be able to effectively do the calculus (even when the calculus concepts are easy).

<u>Need to be able to do these WITHOUT A CALCULATOR</u>

Unit Circle Trig. - Evaluate. Express your answers as exact values (no decimals, no rounding).

1. $\sin\frac{\pi}{6}$
2. $\cos\frac{2\pi}{3}$
3. $\tan\frac{5\pi}{6}$
4. $\sin\frac{7\pi}{4}$
5. $\cos\frac{3\pi}{2}$
6. $\tan\frac{5\pi}{3}$
7. $\cot\frac{3\pi}{4}$
8. $\sec\frac{4\pi}{3}$
9. $\csc\pi$
10. $\sin\frac{5\pi}{3}$
11. $\cos\pi$
12. $\tan\frac{3\pi}{4}$
13. $\cos\frac{\pi}{2}$
14. $\sin\frac{\pi}{3}$
15. $\tan\frac{\pi}{2}$
16. $\cot\frac{\pi}{6}$
17. $\sec\frac{7\pi}{6}$
18. $\csc\frac{2\pi}{3}$
19. $\sin\frac{8\pi}{3}$
20. $\cos 11\pi$
21. $\tan\frac{19\pi}{6}$
22. $\sin\left(-\frac{3\pi}{4}\right)$
23. $\cos\left(-\frac{5\pi}{6}\right)$
24. $\sec\left(-\frac{11\pi}{4}\right)$
25. $\cos 2\pi$

Inverse Trig Expressions. * Remember, with **inverse trig functions** the domain & ranges are limited (see the notes about these limits), so there is only one possible value for each inverse.

26. $\sin^{-1}\left(\frac{1}{2}\right)=\boxed{\frac{\pi}{6}}$

27. $\arccos\left(\frac{\sqrt{3}}{2}\right)$

28. $\tan^{-1}(1)$

29. $\arctan\left(\sqrt{3}\right)$

30. $\arccos\left(-\frac{1}{2}\right)=\boxed{\frac{2\pi}{3}}$

31. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

32. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

33. $\tan^{-1}(0)$

34. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

35. $\sin^{-1}\left(-\frac{1}{2}\right)$

36. $\arccos(-1)$

37. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

38. $\tan^{-1}\left(-\sqrt{3}\right)$

39. $\cos^{-1}(2)$

40. $\sin^{-1}\left(\sin\left(\frac{7\pi}{3}\right)\right)$

Solving each logarithmic or exponential equation for *x*

41. $\ln(5x-1)=4$

42. $3\ln(3x+14)=18$

43. $4\left(e^{2x-1}\right)=20$

44. $\ln\sqrt{x+2}=1$

Solve this equation for *y* in terms of *x*. ...for *x* in terms of y.

45. $\ln(y-2)=x-3$

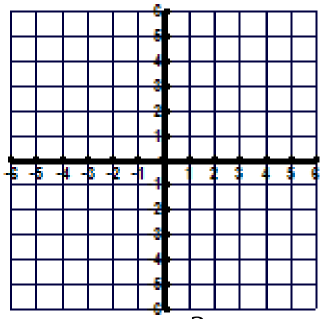
46. $y=3\sqrt{x}-4$

Characteristics of some of the "Basic" Functions.

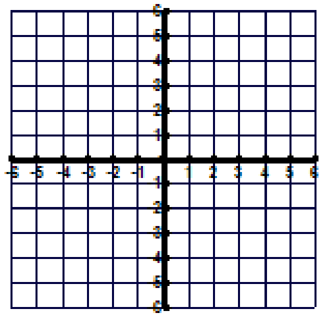
Function	Domain	Range	Symmetry	Odd/Even	Roots
47. $f(x) = x$					
48. $f(x) = x^2$					
49. $f(x) = x^3$					
50. $f(x) = \lvert x \rvert$					
51. $f(x) = \sqrt{x}$					
52. $f(x) = x^{1/3}$					
53. $f(x) = x^{2/3}$					
54. $f(x) = \dfrac{1}{x}$					
55. $f(x) = \dfrac{1}{x^2}$					
56. $f(x) = \sqrt{9 - x^2}$					
57. $f(x) = e^x$					
58. $f(x) = \ln x$					
59. $f(x) = \sin x$					
60. $f(x) = \cos x$					
61. $f(x) = \tan x$					

Transformations of functions - Sketch without the aid of a graphing calculator.

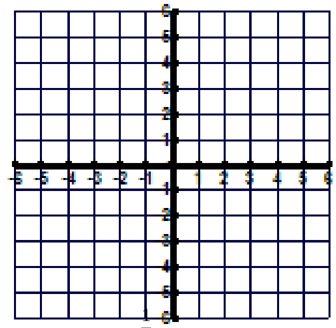
62. $y = -2|x-2| - 4$



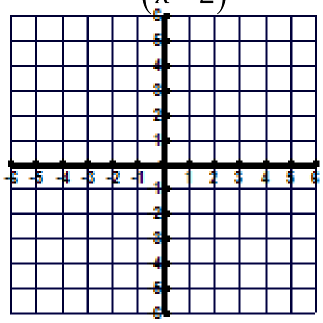
63. $y = \sqrt{x-1} - 3$



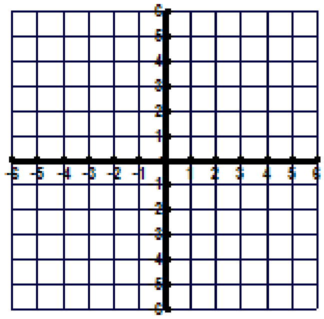
64. $y = -2 \cdot e^{-x} + 4$



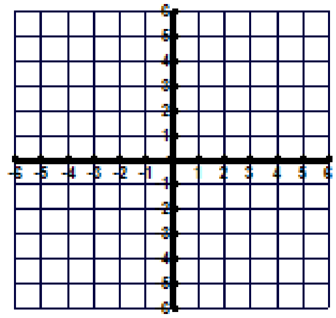
65. $y = -\dfrac{3}{(x-2)^2} - 1$



66. $y = -\ln(x-3) + 2$



67. $y = x^{\frac{1}{3}} + 2$



Slope and the equation of a line.

Let $\mathrm{A}(-7,4)$ and $\mathrm{B}(5,-12)$ be points in the plane.

68. Find the slope of the line that contains A and B.

69. Find an equation of the line that passes through A and B.

70. What are the intercepts of this line?

71. Determine the slope that is perpendicular to the line between A and B.

***f* and its inverse…**

72. If $f(2) = -7$, then $f^{-1}(-7) = ?$

Discontinuities…

73 & 74. Determine the location (x-values) of the removable *and* infinite discontinuities of: $f(x) = \dfrac{x-5}{(x-5)(x+3)}$

Determine the domain of each function without a graphing calculator.

76. $f(x)=\sqrt{x-10}$

77. $y=\dfrac{1}{2x^2+1}$

78. $h(x)=\dfrac{10}{x^2-2x}$

Determine the inverse of each function.

79. $f(x)=\dfrac{5-3x}{2}$

80. $f(x)=\sqrt{2x-3}$

Determine the vertical & horizontal/slant asymptotes (if any) without a graphing calculator.

81. $f(x)=\dfrac{2x^2}{x^2-1}$

V.A. $\Rightarrow$

H.A./S.A. $\Rightarrow$

82. $y=\dfrac{x^2+x-2}{x^2-x-6}$

V.A. $\Rightarrow$

H.A./S.A. $\Rightarrow$

83. $g(x)=\dfrac{x^2-x-2}{x-1}$

V.A. $\Rightarrow$

H.A./S.A. $\Rightarrow$

Determine whether each function is even, odd, or neither.

84. $f(x)=x^6-2x^2+3$

85. $f(t)=t^2+2t-3$

86. $g(x)=x\sqrt{1-x^2}$

Symmetry – Test each equation for symmetry (*y*-axis, origin, neither, or both)

87. $y=x^3+3x$

88. $x=y^2-1$

89. $x^2+2y^2=1$

Simplify the following expressions

90. $e^{3\ln 4}=$

91. $5e^{\ln 7}$

92. $\ln\left(e^x\right)^2$

Transformations

93. Starting with the graph of a function f, which sequence of transformations produces the graph of $g(x) = -5f(x-4)+1$?

(a) Shift 1 unit up, then shift 4 units right, stretch vertically by a factor of 5, and reflect about the x-axis

(b) Shift 1 unit up, then shift 4 units left, stretch vertically by a factor of 5, and reflect about the x-axis

(c) Shift 4 units right, then stretch vertically by a factor of 5, reflect about the y-axis, and shift 1 unit up

(d) Shift 4 units left, then stretch vertically by a factor of 5, reflect about the x-axis, and shift 1 unit up

Determine the difference quotient of a function

94. If $f(x) = -x^2 - 2x + 8$, evaluate $\dfrac{f(1+h) - f(1)}{h}$

(a) -4 (b) $h-4$ (c) $4-h$ (d) $-4-h$

Evaluate a piecewise function

95. Given $f(x) = \begin{cases} x^2 - 1 & \text{if } x \ge 2 \\ \ln(3-x) + 2 & \text{if } x < 2 \end{cases}$, evaluate $f(2)$.

(a) 3 (b) -4 (c) 2 (d) -3

Even, Odd, or Neither, and Increasing intervals of a function.

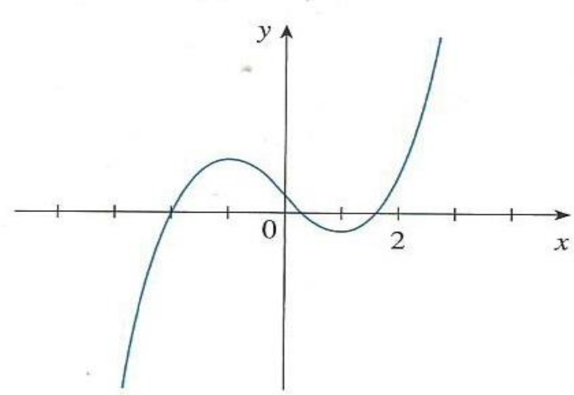
96. Pick the letter which indicates whether the function is even, odd, or neither; and determine the interval(s) over which the function is increasing.

(a) Even, $(-\infty, \infty)$

(b) Neither, $(-1, 1)$

(c) Odd, $(-\infty, -1)$ and $(1, \infty)$

(d) Neither, $(-\infty, -1)$ and $(1, \infty)$

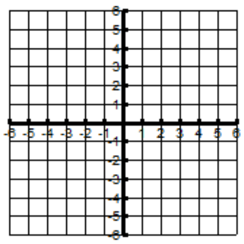


Complete the table below. If it helps, use the graphs below to sketch the graphs first.

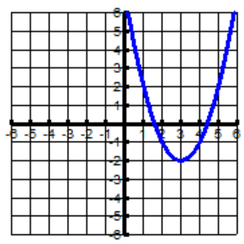
		Domain	**Range**	**Zeros / roots / x-intercepts (if any)**	**y-intercepts**	**Symmetry? (y-axis, origin, or none)**
97.	$y=-2\lvert x-1\rvert+3$					
ex.	$y=(x-3)^2-2$	All Reals, $\mathbb{R}$	$[-2,\infty)$ or $y\ge -2$	$(3\pm\sqrt{2},0)$	7	NO
98.	$y=3\sqrt{x-1}-3$					
99.	$y=e^x$					
100.	$y=\ln x$					

Sketch the graph of each equation above on the planes below if it helps to fill in the blanks in the table.

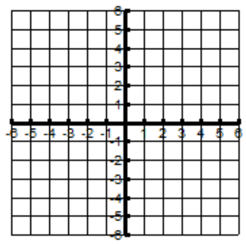
97.



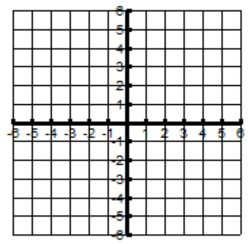
ex.



98.



99.



100.