AP Calculus AB

The Calculus courses are the Advanced Placement topical outlines and prepare students for a successful performance on both the Advanced Placement Calculus exam and their college calculus experience, whether they pursue additional mathematics courses beyond calculus or receive full credit in their major field for their Advanced Placement Calculus exam grade. The following philosophy is from the course description published by the College Board for these courses.

Calculus AB and Calculus BC are primarily concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multirepresentational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important.

Calculus BC is an extension of Calculus AB rather than an enhancement; common topics require a similar depth of understanding. Both courses are intended to be challenging and demanding.

Broad concepts and widely applicable methods are emphasized. The focus of the courses is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems, or problem types. Thus, although facility with manipulation and computational competence are important outcomes, they are not the core of these courses.

Technology should be used regularly by students and teachers to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation and to assist in interpreting results.

Through the use of the unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all the functions listed in the prerequisites.

The Regular Calculus course covers all topics in the AP Calculus AB course except the pace is slower since there is no review for the Advanced Placement exam nor is there a need to complete the course by the Advanced Placement exam date which is usually in the first week of May. In both Advanced Placement Calculus courses, there is an approximate 4.5-week period of review before the Advanced Placement Calculus exam. Finally, the Advanced Placement Calculus BC course contains approximately 40% additional material that is not included in the Advanced Placement Calculus AB course; therefore, more college credit is given for the same grade on the Advanced Placement Calculus BC exam than for the Advanced Placement Calculus AB exam.

❑ Denotes a topic for Regular Calculus, AP Calculus AB, and AP Calculus BC
*** Denotes a topic for AP Calculus AB and BC**
+ Denotes a topic for AP Calculus BC only

Students will:

Functions, Graphs, and Limits

1. Analyze graphs and equations of families of functions. (I-1)
   a. Determine the effects of shifts, reflections, and dilations on families of functions
   b. Identify domain and range
   c. Apply laws of exponents and logarithms

d. Determine the inverse of a function
e. ***With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.**

2. Evaluate limits of functions (including one-sided limits.) (I-2)
   a. Calculate limits using algebra
   b. Estimate limits from graphs or tables of data
   c. ***Understand intuitively the limiting process**

3. Determine asymptotic and unbounded behavior. (I-3)
   a. Determine vertical, horizontal, and oblique asymptotes from a function or a graph
   b. Describe asymptotic behavior in terms of limits involving infinity
   c. ***Understand asymptotes in terms of graphical behavior**
   d. ***Describe asymptotic behavior in terms of limits involving infinity**
   e. ***Compare relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)**

4. Demonstrate continuity as a property of functions. (I-4)
   a. Understand continuity in terms of limits
   b. Understand graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)
   c. ***Understand intuitively, continuity (the function values can be made as close as desired by taking sufficiently close values of the domain)**

5. **+ Demonstrate continuity with parametric, polar and vector functions. (I-5)**
   a. **+ Analyze planar curves including those given in parametric form, polar form, and vector form**

Derivatives

6. Determine the derivative as an instantaneous rate of change graphically, numerically, and analytically. (II-1)
   a. Present the derivative graphically, numerically, and analytically
   b. Interpret derivatives as an instantaneous rate of change
   c. Define the derivative as the limit of the difference quotient
   d. ***Determine the relationship between differentiability and continuity**

7. Determine the derivative at a point. (II-2)
   a. Find the slope of a curve at a point.
   b. Find the tangent line to a curve at a point and local linear approximation
   c. Find instantaneous rate of change as the limit of average rate of change
   d. Approximate a rate of change from graphs and tables of values
   e. ***Find the slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents**

8. Analyze the derivative as a function and identify the corresponding characteristics of the graphs of the function and derivative. (II-3)
   a. Identify corresponding characteristics of graphs of f and
   b. Identify the relationship between the increasing and decreasing behavior of f and the sign of f'

c. Use the Mean Value Theorem and its geometric interpretation
d. Translate verbal descriptions into equations involving derivatives and vice versa

9. Identify the corresponding characteristics of the graphs of the function and the second derivative. (II-4)
   a. Identify the corresponding characteristics of the graphs of f, f', and f''
   b. Identify the relationship between the concavity of f and the sign of f''
   c. Find points of inflection as places where concavity changes

10. Apply derivatives as a rate of change in a variety of application problems. (II-5)
    a. Determine optimization, both absolute (global) and relative (local) extrema
    b. Model rates of change, including related rates problems
    c. Use implicit differentiation to find the derivative of an inverse function
    d. Use the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration

11. ***Solve applications of derivatives. (II-5)**
    a. ***Analyze curves, including the notions of monotonicity and concavity**
    b. **+ Analyze planar cures given in parametric, polar, and vector functions**
    c. ***Determine optimization, both absolute (global) and relative (local) extrema**
    d. ***Model rates of change, including related rates problems**
    e. ***Use implicit differentiation to find the derivative of an inverse function**
    f. ***Interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration**
    g. ***Interpret differential equations geometrically via slope fields and distinguishing between slope fields and solution curves for differential equations**
    h. **+ Find the numerical solution of differential equations using Euler's method**
    i. **+ L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series**

12. Use the rules of differentiation (sum, product, quotient, chain rule, and implicit differentiation) to compute derivatives including the derivative of basic function power, exponential, logarithmic, and trig functions. (II-6)
    a. Know derivatives of basic functions, including power, exponential, logarithmic, trigonometric functions
    b. Use the derivative rules for sums, products, and quotients of functions
    c. Use the chain rule and implicit differentiation to compute derivatives
    d. +**Find derivatives of parametric, polar, and vector functions**

Integrals

13. Compute Riemann sums and definite integrals. (III-1)
    a. Computing the definite integral as a limit of Riemann sums
    b. Solving a definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: $\int_a^b f'(x)dx = f(b) - f(a)$
    c. Computing Riemann sums using left, right, and midpoint evaluations
    d. ***Using basic properties of definite integrals including additivity and linearity**

14. Apply integrals to model physical, biological, or economic situations, including finding the area of a region, the volume of a solid with known cross sections, the average value of a function, and the distance traveled by a particle along a line. (III-2)
    a. Applying integrals to model physical, biological, or economic situations
    b. Finding the area of a region using integrals
    c. Finding the volume of a solid of revolution or a solid with known cross sections using integration
    d. Finding the average value of a function using integration
    e. Finding the total distance traveled by a particle along a line and the displacement using integration
    f. ***Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.**

    g. **+Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, the length of a curve (including a curve given in parametric form), and accumulated change from a rate of change.**

15. Use the Fundamental Theorem of Calculus to evaluate definite integrals. (III-3)
    a. Using the Fundamental Theorem to evaluate definite integrals
    b. ***Using the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined**

16. Apply the techniques of antidifferentiation. (III-4)
    a. Recognizing and applying the basic functions of antidifferentiation
    b. Solving antiderivatives by substitution of variables
    c. ***Solving antiderivatives by substitution of variables (including change of limits for definite integrals)**

d. **+Solving antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only)**
e. **+Solving improper integrals (as limits of definite integrals)**

17. Apply antiderivatives using initial conditions, including applications to motion along a line. (III-5)
    a. Finding specific antiderivatives using initial conditions, including applications to motion along a line
    b. Solving separable differential equations and using them in modeling
    c. **Solving separable differential equations and using them in modeling including the studyof the equation $y' = ky$ and exponential growth**
    d. **+Solving logistic differential equations and using them in modeling**

18. Compute numerical approximations to definite integrals. (III-6)
    a. Using Riemann (left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

Polynomial Approximations and Series

19. **+Understand and integrate the concept of series.** (IV-1)
    a. **A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence or divergence**

20. **+Formulate series of constants.** (IV-2)
    a. **Use examples, including decimal expansion**
    b. **Solve geometric series with applications**
    c. **Develop and apply the harmonic series**
    d. **Formulate an alternating series and calculate error bound**
    e. **Use terms of series as areas of rectangles and relate their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series**
    f. **Use the ratio test for convergence and divergence**
    g. **Compare series to test for convergence or divergence**

21. **+Taylor series** (IV-3)
    a. **Computing a Taylor polynomial approximation with graphical demonstration of convergence. (For example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve.)**
    b. **Constructing a Maclaurin series and the general Taylor series centered at** $x = a$
    c. **Developing and recognizing the Maclaurin series for the functions** e^x**,** $\sin x$**,** $\cos x$**, and** $\frac{1}{1-x}$
    d. **Manipulating a Taylor series and shortcuts to compute a Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series**
    e. **Defining functions by power series**
    f. **Finding the radius and interval of convergence of a power series**
    g. **Calculating the Lagrange error bound for Taylor polynomials**