Addition Information on AP Calculus AB

Course Overview

This is a college-level calculus course designed to meet the Advanced Placement curriculum requirements for Calculus AB (equivalent to a one-semester college course). The major topics of this course are limits, derivatives, integrals, and the Fundamental Theorem of Calculus. We will investigate and analyze course topics using equations, graphs, tables, and words, with the particular emphasis on a conceptual understanding of calculus. Additional calculus topics will be introduced after the AP Exam.

The objective of this course is to give students the understanding of calculus concepts, related mathematical skills and appropriate technology necessary for success on the Advanced Placement Exam and in subsequent college mathematics courses.

Technology Requirement

You will need a handful graphing calculator every day which is required to demonstrate concepts and calculations. When students obtain solutions analytically, they will support, confirm, and interpret their results using both TI-84 and TI-NSpire CAS in a variety of ways including: graphing functions, solving equations, finding limits, derivatives and integrals, areas and volumes, and supporting solutions found analytically. Any AP-approved calculator will work.

The teacher has both TI-84 and TI-Nspire calculators available for class work and projects.

[CR3a]: Students have access to graphing calculators

Classroom Expectations

Every student is an important member of this class. You are expected to actively participate, stay engaged, and discuss ideas. There will be group as well as individual work. Groups will be asked to present their work.

The teacher will be available during school and after school if requested for extra help. Calculus is a challenging, rigorous course. We have to work together to have a great success.

<u>Textbook</u>

The textbook used for this course is Larson, Hostetler, and Edwards. *Calculus—of* a single variable 8^{th} edition.

[CR4]: Students and teachers have access to a college-level calculus book.

Prerequisites

Successful completion of the following yearlong courses.

- 1. Algebra 1
- 2. Geometry
- 3. Algebra 2 (which includes analytic geometry and logarithms)
- 4. Precalculus (which includes elementary functions and trigonometry)

Course Outline

Section	Торіс	Problems
1.2	Find limits graphically and	p.55 #3,7,8,11,13,15,17
	numerically	
1.3	Evaluate limits	p.67
	analytically.	#11,14,26,35,39,43,52,55,60,83,86,101,102.
1.4	Continuity and one-sided	p.78
	limits	#4,8,14,15,26,30,31,36,37,40,43,47,49,60,83,84
1.5	Infinite limits and vertical asymptotes	p.88 #11,15,18,21,24,33,35,39,43,47,53,56,63
Chapter 1	Review	p.91
Ĩ		#11,13,15,19,29,30,31,33,41,45,47,53,56,61,63
Chapter 1	Test	

Unit One: Limits and Their Properties/Continuity (3 weeks)

Extra Materials for Limits using released items of AP Calculus Exams from College Board.

✓ Intermediate Value Theorem

[CR1a]: The course is structured around the enduring understanding within Big Idea 1: Limits.

Unit Two: Differentiation (4 weeks)

2.1	The derivative and	p.103 #7,9,13,19,26,27,33,37,39-42,47,62,63
	the tangent line problem	
2.2	Basic differentiation rules and rates of change (average and instantaneous) The derivative and the tangent line problem	p.115 #3,7,9,13,15,19,21,23,24,35,39,43,47,49,51,53,56,63,87,89,91.

2.3	Product and Quotient Rules and Higher Order derivatives	p.126 #3,7,14,17,25,29,35,39,43,47,49,53,59,61,69,75,77,85,88,95- 100,101.
2.4	The Chain Rule	p.137 #9,13,15,21,25,29,33,47,51,55,59,61,63,67,73,78,83- 86,92,98.
2.5	Implicit differentiation	p.146 #5,8,11,13,22,24,27,28,30,35,37,43,44,47,48
2.6	Related Rates And Problem Solving	p.154 #3,4,5,7,13,15,19,22,23,24,27,30,31,32,43,44.
Chapter 2	Review	p.158 #3,9,13,25,29,31,33,37,43,51,57,61,63,65,71,77,79,91,101,103,107
Chapter 2	Test	

Unit Three: Applications of Differentiations. (5 weeks)

3.1	Extrema on an interval	p.169 #2,3,11,14,17,19,21,23,25,27,29,37,51-54,55,58
3.2	Rolle's Theorem and the Mean Value Theorem	p.176 #9,13,17,20,32,38,43,44,47,48.
3.3	Increasing and decreasing functions The First Derivative Test	p.186 #3,5,7,9,13,17,23,27,31,33,35,43,45,47,57,65.
3.4	Concavity and points of inflection The Second Derivative Test	p.195 #3,7,9,11,21,26,27,31,37,39,51,53,55,61,65,67,69
3.5	Limits at Infinity (horizontal asymptotes)	p.205 #2,3,6,13,17,20,21,25,29,30,31,79
3.6	Summary of Curve Sketching (including monotonicity)	p.215 #1-6,23,25,29,30,33,40,41,43,51-55
3.7	Optimization problems	p.223 #3,6,11,13,16,18,20,23,29,33,39,41,60
3.9	Differentials	p.240 #3,4,9,10,13,14,17,18,23,24,27,28,31,32,34,36,46,47
Chapter 3	Review	p.342 #5,9,15,18,19,23,25,33,35,45,49,51,65,81,83
Chapter 3	Test	

[CR1b]: The course is structured around the enduring understanding within Big Idea 2: Derivatives

Ollit Iour. Introd	idenoii to integral Caleu	ius (5 6 weeks)
4.1	Antiderivatives and indefinite integration	p. 255 #15,19,23,27,29,31,35,37,41,49,53,57,59,61,63,70,73,78,81,83
4.3	Reimann sums Definite integrals solved using geometric formulas	p.278 #14,17,18,22,23,25,29,31,35,39,42,43,45
4.4	The Fundamental Theorem of Calculus Average value of a function Second Fundamental Theorem of Calculus	p.291 #7,13,21,37,43,46,47,50,51,54- 60,70,73,77,80,83,84,89,90,95,103
4.5	Integration using u- substitution	p.305 #9,13,17,21,22,25,29,33,37,39,45,49,51,57,59,67,71,77,79,81
4.6	Trapezoidal Rule	p.314 #7,9,13,14,42,44
Chapter 4	Review	p.316 #5,7,9,10,13,17,18,29,32,36,40,43,46,49,53,57,61,63,68,71,73,81
Chapter 4	Test	

Unit four: Introduction to Integral Calculus (5-6 weeks)

[CR1c]: The course is structured around the enduring understanding within Big Idea 3: Integrals and Fundamental Theorem of Calculus.

Unit i ive and six. I	ranscendental r unetions	(7 weeks)
5.1	The Natural	p.329 #39,41,47,51,55,59,63,67,71,73,77,81,87,91
	Logarithmic Function	
	and Differentiation	
5.2	The Natural	p.338
	Logarithmic Function	#5,9,10,12,13,18,19,23,29,33,41,42,43,47,67,68,72,74,79,82
	and Integration	
5.4	Exponential	p.356
	Functions:	#37,41,47,51,57,59,65,69,87,91,95,99,103,107,111,115
	Differentiation and	

Unit Five and six: Transcendental Functions (7 weeks)

	Integration	
5.5	Bases other than <i>e</i> and applications	p.366 #43,45,47,51,53,55,57,59,63,65,67,77,79,87,89
5.6	Inverse trigonometric functions and Differentiation	p.377 #17,19,21,23,25,27,29,41,45,49,53,57,59.
5.7	Inverse trigonometric functions and Integration	p.385 #5,11,17,19,23,27,33,37,43,45
6.1	Slope fields and Euler's Methods	p.409 #3,5,7,9,11,13,15,19,21,23,25,27,37,39,41,43,45,47,49- 5657,59,71,73,75,77
6.2	Differential equations: Growth and decay	p.418 #3,5,7,9,11,13,15,17,19,21,23,25,27,63,64,71
6.3	Differential equations: Separation of variables	p.429 #3,5,7,9,11,13,15,17,19,21,23,27,31,33,37,49,51.
Chapter5	Review	
Chapter 5	Test	

Unit seven: Applications of Integration (2-3 weeks)

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7.1	Area of a region between two	p. 452 #1,3,5,6,13,14, 17,18,22,24,26,30,33-
	curves	36,57,59,88,93.
7.2	Volume: Disc method/The	p.463. #1-10,
	washer Method	11,13,15,17,18,20,21,23,27,29,31,33,35,37,39,53,56,
	Volume: Known cross-sections	61
7.3	Volume: Shell method	p. 472 #1,4,5,6,9,11,12,14,15,16,20,21,23,25,29,46
7.4	Arc length and surfaces of	p.483 #5,6,9,11, 12,39,41,43.
	Revolution.	
8.7	L'Hopital's Rule	574. #5,7,9,11-36 (odd), 87-90, 97-99,
Review		
Test		

Mid Term Covering Unit One to Unit four.

AP Review (3 to 4 weeks)

- ✓ College Board Multiple Choice practice released Tests.
 ✓ Free response questions; 2000 to present

Unit Seven: Projects (3 to 4 weeks)

- ✓ Volume of a Known Cross Section Model
- ✓ Vegetable Lab
- ✓ Oral presentation: An Architectural Wonder and Its Calculus

Final Exam covering the entire year

Exams: The **Midterm and Final** exam includes problems from past AP exams that test the students' abilities to connect concepts graphically, analytically, numerically, and verbally. These exams determine 20% of the student's final grade.

Curricular Requirement

CR1a: The course is structured around enduring understanding within Big Idea 1: Limits.

CR1b: The course is structured around enduring understanding within Big Idea 2: Derivatives.

CR1c: The course is structured around enduring understanding within Big Idea 3: Integrals and the Fundamental theorem of Calculus.

CR2a: The course provides opportunities for students to reason with definitions and theorems.

CR2b: The course provides opportunities for students to connect concepts and processes.

CR2c: The course provides opportunities for students to implement algebraic/computational processes.

CR2d: The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.

CR2e: The course provides opportunities for students to build notational fluency.

CR2f: The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.

CR3a: Students have access to graphing calculators.

CR3b: Students have opportunities to use calculators to solve problems.

CR3c: Students have opportunities to use a graphing calculator to explore and interpret calculus concepts.

CR4: Students and teachers have access to a college-level calculus textbook.

Brief description of some of Activities included in the course.

1. **CR2a:** The course provides opportunities for students to reason with definitions and theorems.

Problems that include Intermediate Value Theorem, Mean Value Theorem, L'Hopital's Rule Theorem, Rolle's Theorem etc. students are required to demonstrate verbally and/ or in writing that the hypotheses of the theorem are met in order to justify the use of the appropriate theorem. Example:

- 1. In a group class-activity students are given a worksheet with several functions on specified domain. They will decide in the hypotheses of Mean Value Theorem and Intermediate Value Theorem apply. Students then, discus their answers and try to fix any of their mistakes.
- 2. Consider function

 $f(x) = 3\cos^2(\frac{\pi x}{2})$

- a) Use a graphing utility to graph f and f'
- b) Explain in writing: Is f a continuous function? Is f' a continuous function?
- c) Does Rolle's Theorem apply on the interval [-1, 1]? Does it apply on interval [1, 2]? Explain in writing.
- 2. **CR2b:** The course provides opportunities for students to connect concepts and processes.

Example:

 f, f', f" activity: Students will be given a set of cards that include graphs of f, f', f" and written descriptions of f, f', f". They will work in groups to match each f, f', f" set both graphically and verbally. Then they discuss their answers to come to agreement.

3. **CR2c:** The course provides opportunities for students to implement algebraic/computational processes. Example:

- 1. A worksheet paper is given to students with a free response problem released from the College Board. The problem contains a table of values of a function f and students are asked to compute the right and the left-hand Riemann sums. This could be used as a review for the exam.
- A worksheet is given to students containing a variety of problems; many of them involve the indeterminate form 0/0 or ∞/∞ and require using L'Hopital's Rule. Students are asked to find the limits.

4.CR2d: The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.

Example:

 A function f(x) = 4ln(3 - x) is given. Students are asked to: a) graph the function.
 b) Estimate the area of the region R bounded by the function, the x-axis and y-axis. (first quadrant) using the midpoint Riemann sums with four subintervals of equal length.

> * This problem requires analytical, numerical, graphical, connection between analytical and graphical as well as verbal skills. There will be several problems like this; some of them will require the use of the calculator and some of them will be non-calculator use. These practices will take place 2 or 3 times before the AP Review and then occur twice a week until the AP exam. Special emphasis will be placed on helping students learn to justify answers in complete sentence form.

2. Volume of a known cross-section model project: Students will build a 3-D model of an assigned solid and will calculate the volume numerically using a spreadsheet and cross-sections and also analytically using the cross-section formula.

5. CR2e: The course provides opportunities for students to build notational fluency. <u>Example.</u>

A piecewise function is given:

 $f(x) = \begin{cases} 2x - 2 & if \ x < 3\\ 2x - 4 & if \ x \ge 3 \end{cases}$

Which of the following statements are true:

$$\lim_{h \to 0^{-}} \left(\frac{f(3+h) - f(3)}{h} \right) = 2$$

$$\lim_{h \to 0^+} \left(\frac{f(3+h) - f(3)}{h} \right) = 2$$

III. $f^{I}(3) = 2$

(A) None

- (B) II Only
- (C) I and II only
- (D) I, II and III

In such problems students not only will connect concepts (MPAC2) but they will build notational fluency as well. (MPAC5). Students will have to realize that the instantaneous rate of change of a function at a point can be expressed by

$$\lim_{h \to 0+} \left(\frac{f(a+h) - f(a)}{h} \right)_{\text{or}}$$
$$\lim_{x \to a} \left(\frac{f(x) - f(a)}{x - a} \right)_{\text{pro}}$$

 $x \rightarrow a$ provided that the limit exist. These are common forms of the definition of the derivative and are denoted as f'(a). The concept of the limit can be extended to include one-sided limits, limits at infinity, and infinite limits.

6. CR2f: The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.

Example:

- 1. In the end of each unit students are asked to describe in well-written sentences the main ideas of a particular Theorem included in that unit. For example: Rolle's Theorem and MVT are part of Unit 3. Students may choose one of them and write the main ideas in the unit on the Rolle's or MV Theorem.
- 2. At the end of the year students are asked to write a paper choosing one the topic such as: foundation and history of calculus, how limits are used in calculus, and the relation slope-derivative. Then, they make an oral presentation of their paper to the students who will be taking AP Calculus in the following year (usually Pre-calculus students)

7. CR3a: Students have access to graphing calculators.

Example: Under the <u>Technology Requirement.</u>

- 8. CR3b: Students have opportunities to use calculators to solve problems. <u>Examples:</u>
 - 1. Students are given a worksheet with given functions in given intervals and are asked to use their calculator to determine the absolute extrema, or increase decrease intervals.
 - 2. On a classroom assignment, students are asked to find the area between two curves. Students will be required to put the two functions in the calculator and find the points of intersection of the curves and then evaluate the definite integrals numerically using the calculator's integration feature.
 - 3. For Review for Exam use the released Free-Response Question that requires the use of the calculator.

9. CR3c: Students have opportunities to use a graphing calculator to explore and interpret calculus concepts.

Example:

1. Classwork activity: Students explore the limit of a function at values close to some x value, using the zoom, trace, and table features of the calculator. They explore the algebra that leads to calculating the limit. They write about and

discuss how the question "How close is close?" applies to a function and its tangent lines. The students use the calculator to then explore the tangent lines and how the graph is "locally linear."

2. Activity in class: Students will investigate rates of change in such topics as population growth, interest income, productions costs, physics, volume and distance.

Textbook and Resources

Larson, Ron, Robert P. Hostetler, and Bruce H. Edwards. <u>Calculus of a Single Variable</u>. Boston: Houghton Mifflin, 1998, Eighth edition.

Barron's AP Calculus....The Leader in Test Preparation

Fast Track to a 5. Preparing for the AP Calculus AB and BC Examinations. AP Calculus. Multi-Day Workshop Handbook and Resources.

Videos: The Standard Deviants: Calculus Part 1 and 2 1998 Cerebellum Corporation

The Most Astounding Structures 2001 A&E Television Networks

Web-Sites: Calculus in motion,

CR4: Students and teachers have access to a college-level calculus textbook.

Possibly