Pythagorean Theorem



In all right triangles: $a^2 + b^2 = c^2$

Example A

The square pyramid shown below has a base with sides of 10 units. The slant height of the pyramid is 8 units.



What is the vertical height, *h*? Round your answer to the nearest tenth.

Example B

What is the length of the diagonal, d, of the rectangular prism shown below? Round your answer to the nearest tenth.



Can you see that this section is half of the side marked 10 units? That is 5 units long.

We will use Pythagorean Theorem: $a^2 + b^2 = c^2$ for this triangle.

a = *h*, b= 5 and c = 8

 $h^2 + 5^2 = 8^2$

h² + 25 = 64 subtract 25 from

both sides of the equation.

 $h^2 = 39$ now take the square root of both sides

h = 6.2449.... rounded = 6.2

It will take two steps to solve this problem.

Step 1 find the length of the diagonal 'c' We will use Pythagorean Theorem: $a^2 + b^2 = c^2$ for this triangle.



$$2^{2} + 8^{2} = C^{2}$$

$$4 + 64 = C^{2}$$

$$68 = C^{2}$$

$$\sqrt{68} = \sqrt{c^{2}}$$

$$8.246211251... = c$$

STEP 2 We now can find 'd' using this triangle



Example C

A builder makes staircases where each step is 28 cm long and rises 18 cm so people don't trip. Every staircase has a stringer – a diagonal support connecting all of the steps.



A certain staircase will have 6 steps.



What is the length ℓ of the stringer for this taircase? Round your answer to the nearest tenth of a centimeter.

We will use Pythagorean Theorem: $a^2 + b^2 = c^2$ for this triangle. $18^2 + 28^2 = 1^2$

324 + 784 = I²

 $1108 = I^2$ Take the square root of both sides of the equation to get 33.2866... = I But we need 6 of this length, so multiply 33.266 by 6 to get 199.7198... rounded it will be 199.7cm

Example D

The Museu Blau in Barcelona, Spain, is a natural science museum built in the shape of a triangular prism. The base of the building is an equilateral triangle 180 meters on each side. The height of the prism is 25 m.



Look at the base of the triangle, it is 180, but it is split in half by the vertical line marked 'h.' So half of 180 would be 90 and we can use that to find 'h' with the Pythagorean Theorem. We will use Pythagorean Theorem: $a^2 + b^2 = c^2$ for this triangle.



We only need half of the entire triangle to figure out the height 'h'

 $a^2 + b^2 = c^2$ which is $h^2 + 90^2 = 180^2$

 h^2 + 8100 = 32400 subtract 8100 from both sides

 h^2 = 24300 take the square root of both sides

EXAMPLE E

Julian jogs 2 kilometers east, 4 kilometers north, and then 7 kilometers west.



How far is Julian from his starting position? Round your answer to the nearest tenth of a kilometer.

Let's make this easier by drawing in some lines...



Now we can use $a^2 + b^2 = c^2$ to find the distance from 'Start' to 'End' $4^2 + 5^2 =$ our distance² $16 + 25 = d^2$ $41 = d^2$ take the square root of both sides 6.40312... = d rounded is 6.4

Special Right Triangles



EXAMPLE 1



How long is 'BC'?

Using the 45-45-90 triangle above as a model we will get

 $x\sqrt{2} = 12$ divide both side by $\sqrt{2}$ $x = \frac{12}{\sqrt{2}}$ Multiply numerator and denominator by $\sqrt{2}$ $x = \frac{12\sqrt{2}}{2}$ simplify $x = 6\sqrt{2}$ = ANSWER

EXAMPLE 2

In the right triangle shown, $m \angle K = 60^{\circ}$ and KL = 2.



Using the model above we can see the x will be $2\sqrt{3}$, because

The "KL" segment is 2 and the "LJ" segment is always the same length, but times $\sqrt{3}$

ANSWER = $2\sqrt{3}$ = JL

How long is JL?