5-Minute Check

Over Lesson 8–1

CheckPoint

1 Find the geometric mean between 9 and 13.

A. 2

B. 4

C. $\sqrt{117} \approx 10.8$

D. $\sqrt{250} \approx 15.8$

5-Minute Check Over Lesson 8–1

CheckPoint

1 Find the geometric mean between 9 and 13.

A. 2

B. 4

C. $\sqrt{117} \approx 10.8$

D. $\sqrt{250} \approx 15.8$

5-Minute Check

Over Lesson 8–1

CheckPoint

2 Find the geometric mean between $2\sqrt{5}$ and $5\sqrt{5}$.

A. $3\sqrt{5} \approx 6.7$

B. $10\sqrt{5} \approx 22.4$

C. $\sqrt{50} \approx 7.1$

D. $\sqrt{145} \approx 12.0$

5-Minute Check

Over Lesson 8–1

CheckPoint

2 Find the geometric mean between $2\sqrt{5}$ and $5\sqrt{5}$.

A. $3\sqrt{5} \approx 6.7$

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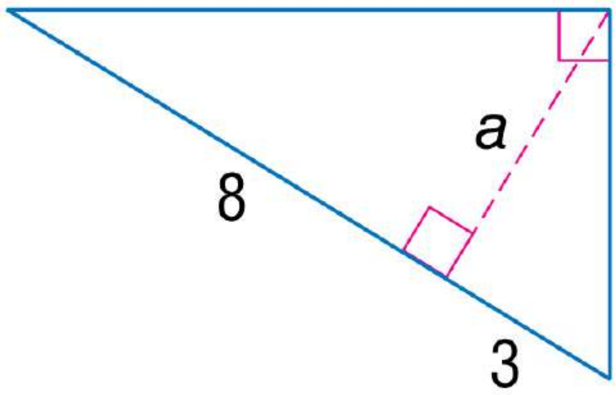
5-Minute Check

Over Lesson 8–1

CheckPoint

3 Find the altitude *a*.



A. 4

B. $\sqrt{24}$

C. 6

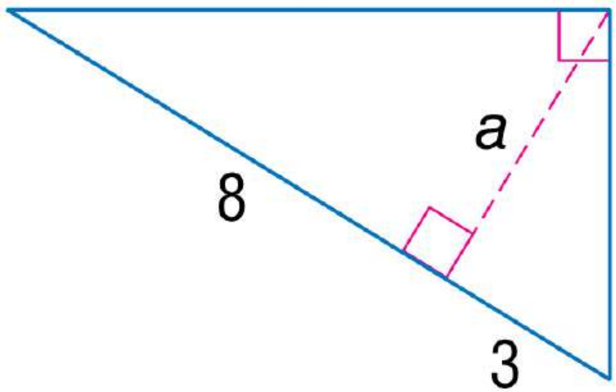
D. $\sqrt{11}$

5-Minute Check

Over Lesson 8–1

CheckPoint

3 Find the altitude *a*.



A. 4

B. $\sqrt{24}$

C. 6

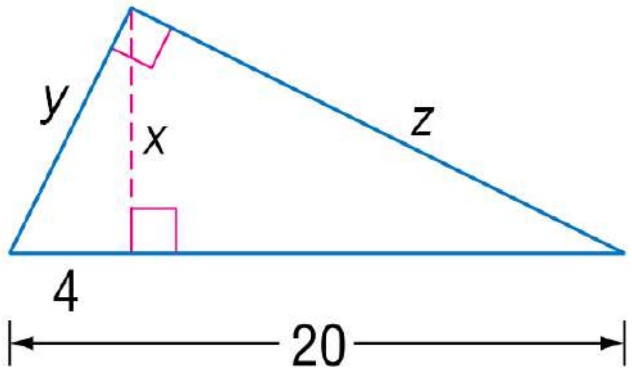
D. $\sqrt{11}$

5-Minute Check

Over Lesson 8–1

CheckPoint

4 Find x, y, and z to the nearest tenth.



A. $x = 6$, $y = 8$, $z = 12$

B. $x = 7$, $y = 8.5$, $z = 15$

C. $x = 8$, $y \approx 8.9$, $z \approx 17.9$

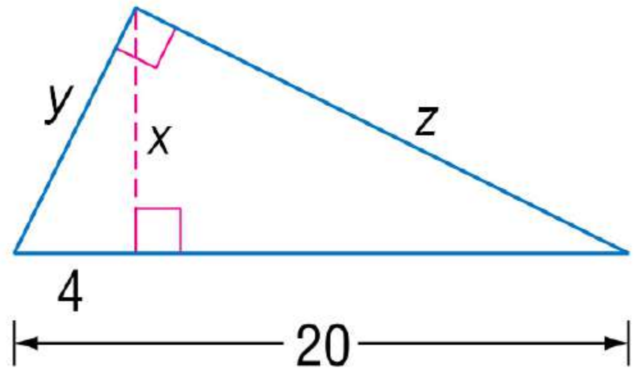
D. $x = 9$, $y \approx 10.1$, $z = 23$

5-Minute Check

Over Lesson 8–1

CheckPoint

4 Find x, y, and z to the nearest tenth.



A. $x = 6$, $y = 8$, $z = 12$

B. $x = 7$, $y = 8.5$, $z = 15$

→ C. $x = 8$, $y \approx 8.9$, $z \approx 17.9$

D. $x = 9$, $y \approx 10.1$, $z = 23$

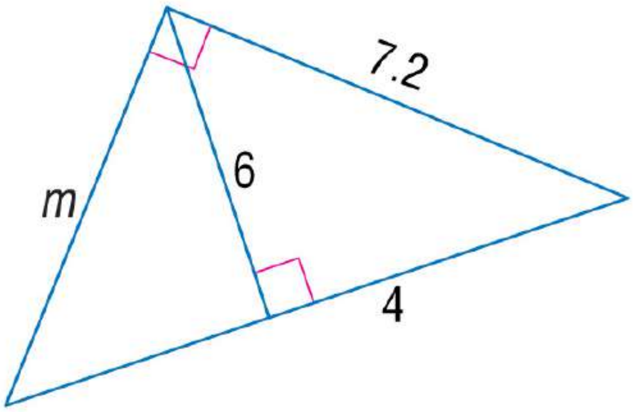
5-Minute Check

Over Lesson 8–1

CheckPoint

Standardized Test Practice

5 Which is the best estimate for *m*?



A. 9

B. 10.8

C. 12.3

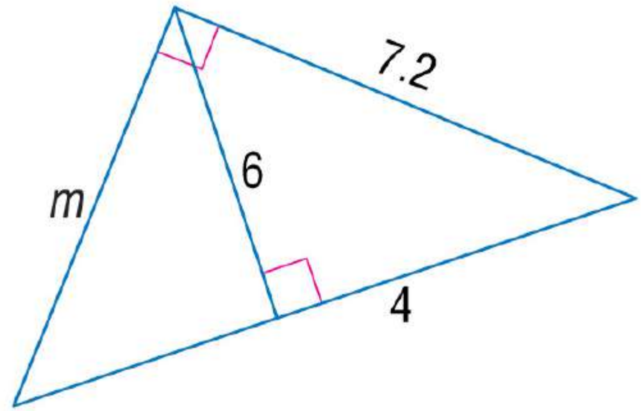
D. 13

5-Minute Check

Over Lesson 8–1

Standardized Test Practice

5 Which is the best estimate for *m*?



A. 9

B. 10.8

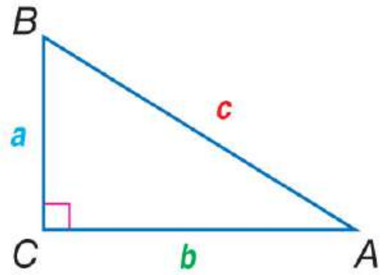
C. 12.3

D. 13

Theorem 8.4 Pythagorean Theorem

Words In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Symbols If $\triangle ABC$ is a right triangle with right angle C, then $a^2 + b^2 = c^2$.

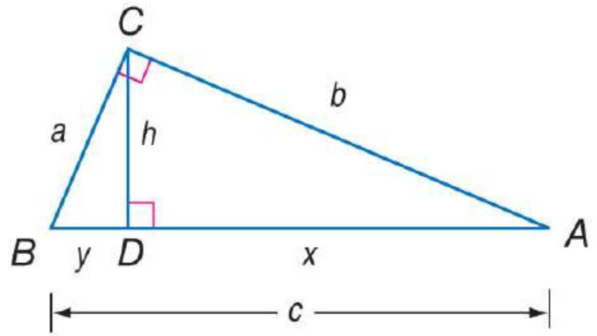


Proof Pythagorean Theorem

Given: $\triangle ABC$ with right angle at C

Prove: $a^2 + b^2 = c^2$

Proof:



Draw right triangle ABC so C is the right angle. Then draw the altitude from C to $\overline{AB}$. Let $AB = c$, $AC = b$, $BC = a$, $AD = x$, $DB = y$, and $CD = h$. Two geometric means now exist.

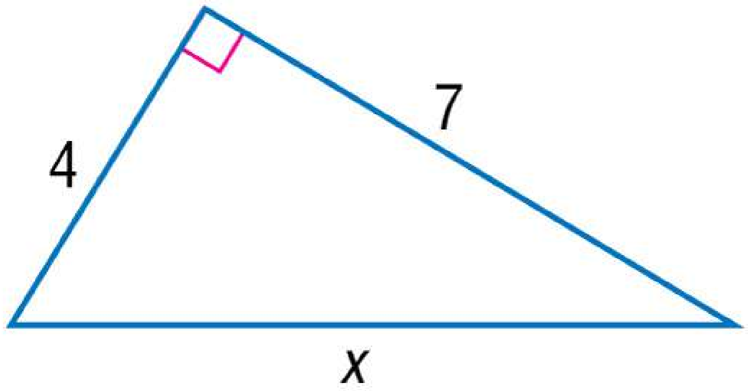
$\frac{c}{a} = \frac{a}{y}$ and	$\frac{c}{b} = \frac{b}{x}$	Geometric Mean (Leg) Theorem
$a^2 = cy$	$b^2 = cx$	Cross products

$a^2 + b^2 = cy + cx$	Add the equations.
$a^2 + b^2 = c(y + x)$	Factor.
$a^2 + b^2 = c \cdot c$	Since $c = y + x$, substitute c for $(y + x)$.
$a^2 + b^2 = c^2$	Simplify.

EXAMPLE 1

Find Missing Measures Using the Pythagorean Theorem

A. Find *x*.



The side opposite the right angle is the hypotenuse, so $c = x$.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$4^2 + 7^2 = c^2$ $a = 4$ and $b = 7$

EXAMPLE 1

Find Missing Measures Using the Pythagorean Theorem

$65 = c^2$ Simplify.

Take t $\sqrt{65} = c$ square root of each side.

Answer:

EXAMPLE 1

Find Missing Measures Using the Pythagorean Theorem

$65 = c^2$ Simplify.

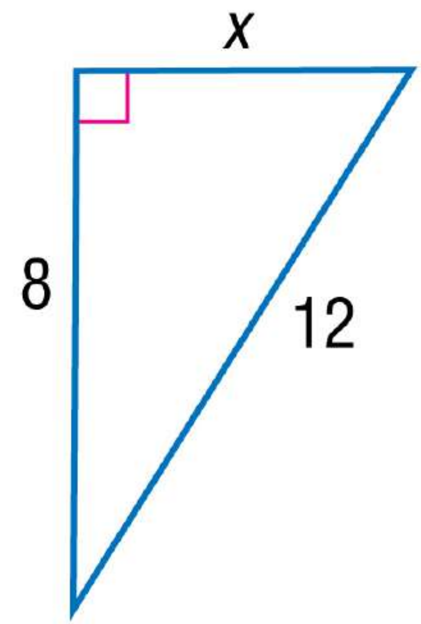
Take the square root of each side. $\sqrt{65} = c$

Answer: $c = \sqrt{65}$

EXAMPLE 1

Find Missing Measures Using the Pythagorean Theorem

B. Find *x*.



The hypotenuse is 12, so $c = 12$.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$x^2 + 8^2 = 12^2$ $b = 8$ and $c = 12$

EXAMPLE 1

Find Missing Measures Using the Pythagorean Theorem

$x^2 + 64 = 144$ Simplify.

$x^2 = 80$ Subtract 64 from each side.

Take the positive square root of each side and simplify. $x = \sqrt{80}$ or $4\sqrt{5}$

Answer:

EXAMPLE 1

Find Missing Measures Using the Pythagorean Theorem

$x^2 + 64 = 144$ Simplify.

$x^2 = 80$ Subtract 64 from each side.

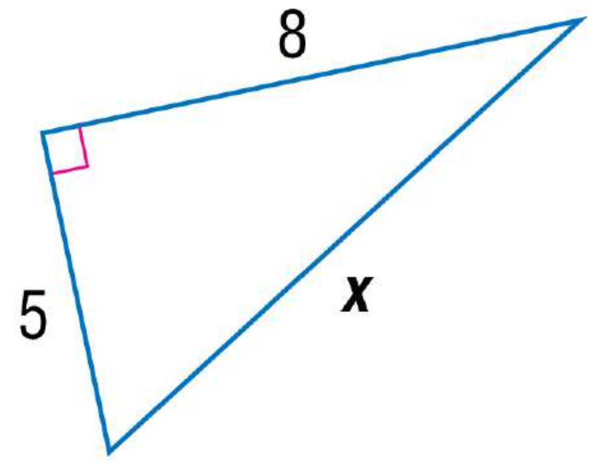
Take the positive square root of each side and simplify. $x = \sqrt{80}$ or $4\sqrt{5}$

Answer: $x = 4\sqrt{5}$

EXAMPLE 1 Check Your Progress

CheckPoint

A. Find *x*.



A. $\sqrt{13}$

B. $\sqrt{39}$

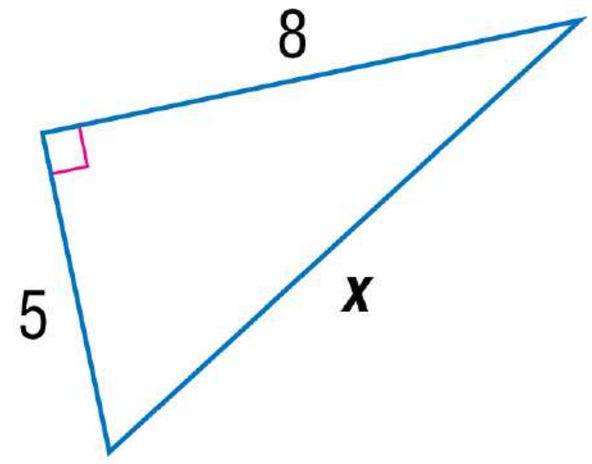
C. $\sqrt{89}$

D. $\sqrt{169}$

EXAMPLE 1 Check Your Progress

CheckPoint

A. Find *x*.



A. $\sqrt{13}$

B. $\sqrt{39}$

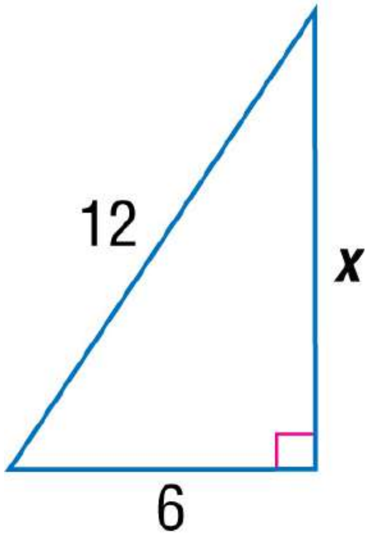
C. $\sqrt{89}$

D. $\sqrt{169}$

EXAMPLE 1 Check Your Progress

CheckPoint

B. Find *x*.



A. $3\sqrt{6}$

B. $6\sqrt{3}$

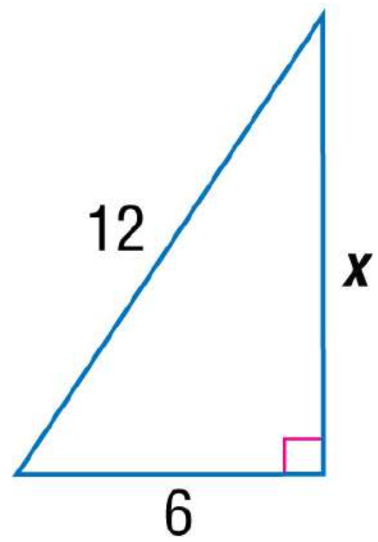
C. $6\sqrt{2}$

D. $6\sqrt{5}$

EXAMPLE 1 Check Your Progress

CheckPoint

B. Find *x*.



A. $3\sqrt{6}$

B. $6\sqrt{3}$

C. $6\sqrt{2}$

D. $6\sqrt{5}$

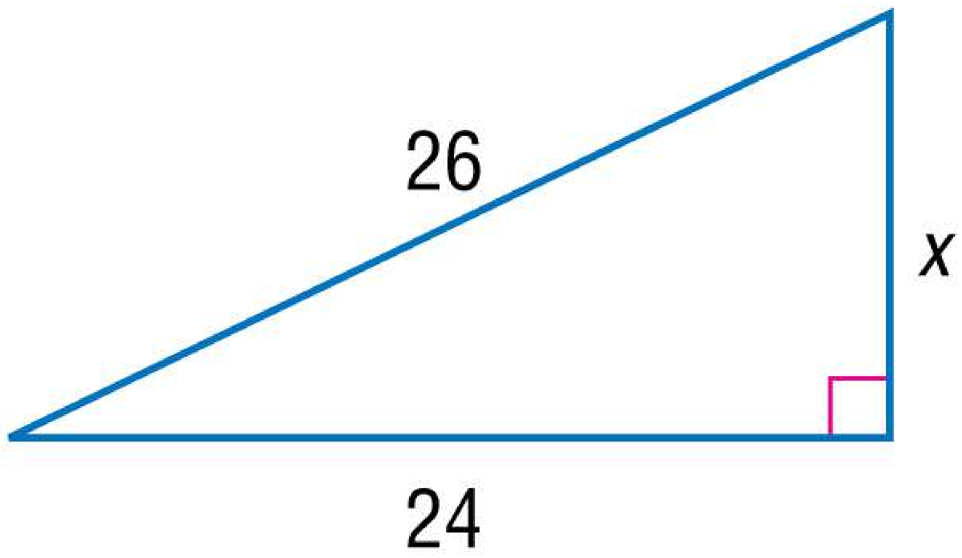
KeyConcept Common Pythagorean Triples

3, 4, 5	5, 12, 13	8, 15, 17	7, 24, 25
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
$3x$, $4x$, $5x$	$5x$, $12x$, $13x$	$8x$, $15x$, $17x$	$7x$, $24x$, $25x$

EXAMPLE 2

Use a Pythagorean Triple

Use a Pythagorean triple to find *x*. Explain your reasoning.



EXAMPLE 2

Use a Pythagorean Triple

Notice that 24 and 26 are multiples of 2: 24 = 2 ● 12 and 26 = 2 ● 13. Since 5, 12, 13 is a Pythagorean triple, the missing leg length *x* is 2 ● 5 or 10.

Answer:

EXAMPLE 2

Use a Pythagorean Triple

Notice that 24 and 26 are multiples of 2: 24 = 2 ● 12 and 26 = 2 ● 13. Since 5, 12, 13 is a Pythagorean triple, the missing leg length *x* is 2 ● 5 or 10.

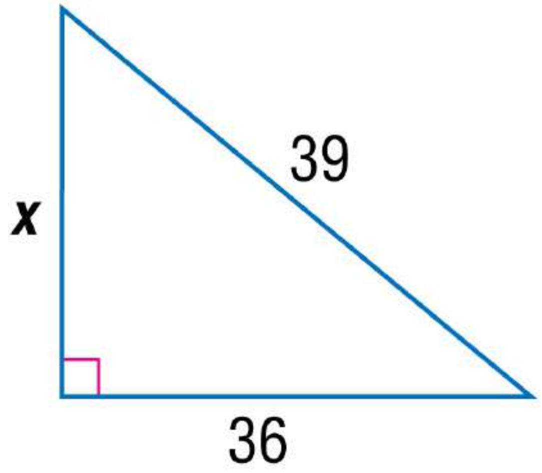
Answer: $x = 10$

Check: $24^2 + 10^2 = 26^2$ Pythagorean Theorem

676 = 676 Simplify. ✓

EXAMPLE 2 Check Your Progress

Use a Pythagorean triple to find *x*.



A. 10

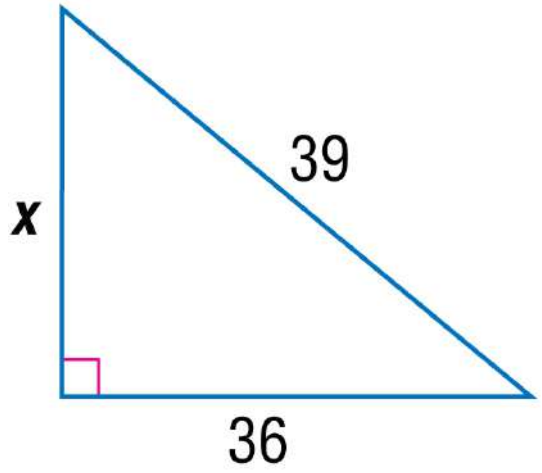
B. 15

C. 18

D. 24

EXAMPLE 2 Check Your Progress

Use a Pythagorean triple to find *x*.



A. 10

B. 15

C. 18

D. 24

STANDARDIZED TEST EXAMPLE 3

Check Your Progress

CheckPoint

A 10-foot ladder is placed against a building. The base of the ladder is 6 feet from the building. How high does the ladder reach on the building?

A. 6 ft

B. 8 ft

C. 9 ft

D. 10 ft

STANDARDIZED TEST EXAMPLE 3

Check Your Progress

CheckPoint

A 10-foot ladder is placed against a building. The base of the ladder is 6 feet from the building. How high does the ladder reach on the building?

A. 6 ft

B. 8 ft

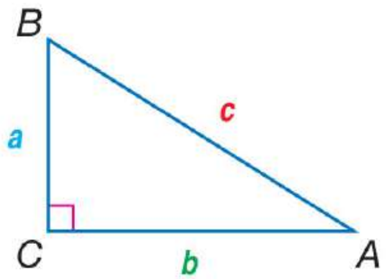
C. 9 ft

D. 10 ft

Theorem 8.5 Converse of the Pythagorean Theorem

Words If the sum of the squares of the lengths of the shortest sides of a triangle is equal to the square of the length of the longest side, then the triangle is a right triangle.

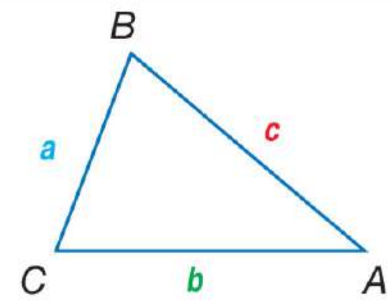
Symbols If $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right triangle.



Theorems Pythagorean Inequality Theorems

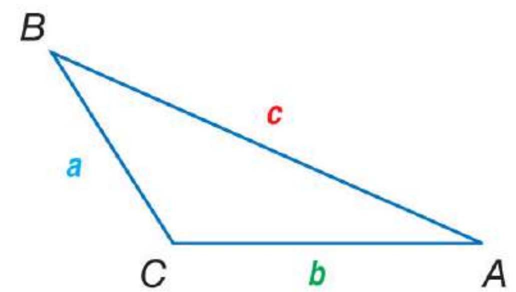
8.6 If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is an acute triangle.

Symbols If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.



8.7 If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is an obtuse triangle.

Symbols If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.



EXAMPLE 4 Classify Triangles

A. Determine whether 9, 12, and 15 can be the measures of the sides of a triangle. If so, classify the triangle as *acute, right,* or *obtuse*. Justify your answer.

Step 1 Determine whether the measures can form a triangle using the Triangle Inequality Theorem.

$9 + 12 > 15$ ✓ $9 + 15 > 12$ ✓ $12 + 15 > 9$ ✓

The side lengths 9, 12, and 15 can form a triangle.

EXAMPLE 4

Classify Triangles

Step 2 Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$c^2 \stackrel{?}{=} a^2 + b^2$ Compare c^2 and $a^2 + b^2$.

$15^2 \stackrel{?}{=} 12^2 + 9^2$ Substitution

$225 = 225$ Simplify and compare.

Answer:

EXAMPLE 4

Classify Triangles

Step 2 Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$c^2 \stackrel{?}{=} a^2 + b^2$ Compare c^2 and $a^2 + b^2$.

$15^2 \stackrel{?}{=} 12^2 + 9^2$ Substitution

$225 = 225$ Simplify and compare.

Answer: Since $c^2 = a^2 + b^2$, the triangle is a right triangle.

EXAMPLE 4

Classify Triangles

B. Determine whether 10, 11, and 13 can be the measures of the sides of a triangle. If so, classify the triangle as *acute, right,* or *obtuse*. Justify your answer.

Step 1 Determine whether the measures can form a triangle using the Triangle Inequality Theorem.

10 + 11 > 13 ✓ 10 + 13 > 11 ✓ 11 + 13 > 10 ✓

The side lengths 10, 11, and 13 can form a triangle.

EXAMPLE 4

Classify Triangles

Step 2 Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$c^2 \stackrel{?}{=} a^2 + b^2$ Compare c^2 and $a^2 + b^2$.

$13^2 \stackrel{?}{=} 11^2 + 10^2$ Substitution

$169 < 221$ Simplify and compare.

Answer:

EXAMPLE 4

Classify Triangles

Step 2 Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$c^2 \stackrel{?}{=} a^2 + b^2$ Compare c^2 and $a^2 + b^2$.

$13^2 \stackrel{?}{=} 11^2 + 10^2$ Substitution

$169 < 221$ Simplify and compare.

Answer: Since $c^2 < a^2 + b^2$, the triangle is acute.

EXAMPLE 4 Check Your Progress

CheckPoint

A. Determine whether the set of numbers 7, 8, and 14 can be the measures of the sides of a triangle. If so, classify the triangle as *acute, right,* or *obtuse*. Justify your answer.

A. yes, acute

B. yes, obtuse

C. yes, right

D. not a triangle

EXAMPLE 4 Check Your Progress

CheckPoint

A. **Determine whether the set of numbers 7, 8, and 14 can be the measures of the sides of a triangle. If so, classify the triangle as *acute, right,* or *obtuse*. Justify your answer.**

A. yes, acute

B. yes, obtuse

C. yes, right

D. not a triangle

EXAMPLE 4 Check Your Progress

CheckPoint

B. Determine whether the set of numbers 14, 18, and 33 can be the measures of the sides of a triangle. If so, classify the triangle as *acute, right,* or *obtuse*. Justify your answer.

A. yes, acute

B. yes, obtuse

C. yes, right

D. not a triangle

EXAMPLE 4 Check Your Progress

CheckPoint

B. Determine whether the set of numbers 14, 18, and 33 can be the measures of the sides of a triangle. If so, classify the triangle as *acute, right,* or *obtuse*. Justify your answer.

A. yes, acute

B. yes, obtuse

C. yes, right

D. not a triangle