5-Minute Check

Over Chapter 7

CheckPoint

1 Solve the proportion $\frac{13}{6} = \frac{x}{8}$.

A. 15

B. 16.5

C. $\frac{52}{3}$

D. 18

5-Minute Check Over Chapter 7

CheckPoint

1 Solve the proportion $\frac{13}{6} = \frac{x}{8}$.

A. 15

B. 16.5

C. $\frac{52}{3}$

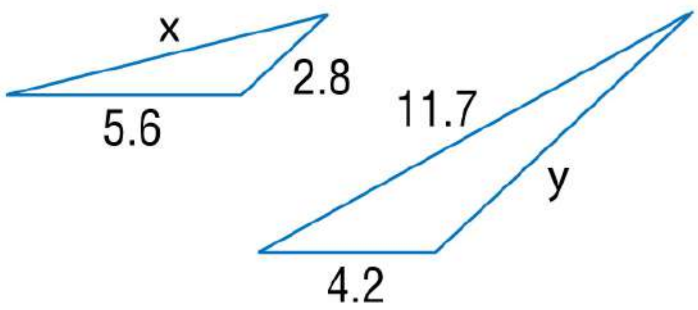
D. 18

5-Minute Check

Over Chapter 7

CheckPoint

2 The triangles at the right are similar. Find *x* and *y*.



A. $x = 16.1$, $y = 1.6$

B. $x = 15.6$, $y = 2.1$

C. $x = 7.8$, $y = 8.4$

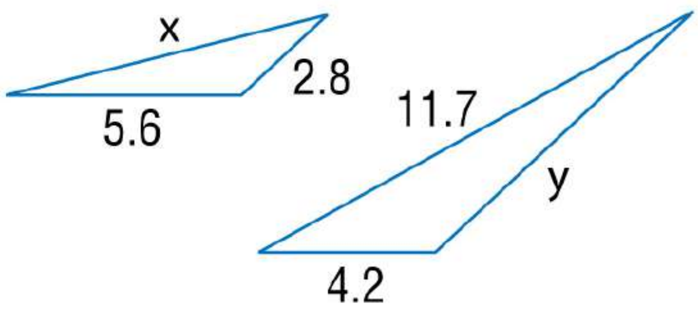
D. $x = 17.6$, $y = 3.7$

5-Minute Check

Over Chapter 7

CheckPoint

2 The triangles at the right are similar. Find *x* and *y*.



A. $x = 16.1$, $y = 1.6$

B. $x = 15.6$, $y = 2.1$

C. $x = 7.8$, $y = 8.4$

D. $x = 17.6$, $y = 3.7$

KeyConcept Geometric Mean

Words The geometric mean of two positive numbers a and b is the number x such that $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

Example The geometric mean of $a = 9$ and $b = 4$ is 6, because $6 = \sqrt{9 \cdot 4}$.

EXAMPLE 1 **Geometric Mean**

Find the geometric mean between 2 and 50.

Let *x* represent the geometric mean.

$\frac{2}{x} = \frac{x}{50}$	Definition of geometric mean
$x^2 = 100$	Cross products
$x = \sqrt{100}$	Take the positive square root of each side.
$x = 10$	Simplify.

Answer: The geometric mean is 10.

EXAMPLE 1 Check Your Progress

CheckPoint

A. Find the geometric mean between 3 and 12.

A. 3.9

B. 6

C. 7.5

D. 4.5

EXAMPLE 1 Check Your Progress

CheckPoint

A. Find the geometric mean between 3 and 12.

A. 3.9

B. 6

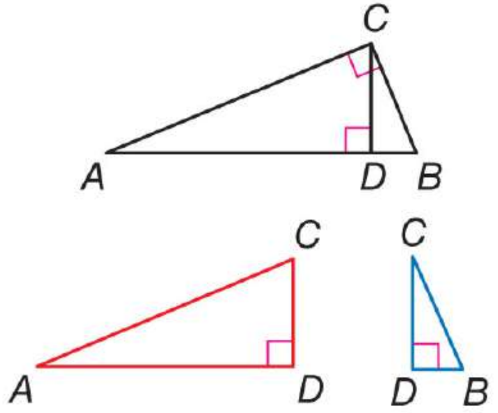
C. 7.5

D. 4.5

Theorem 8.1

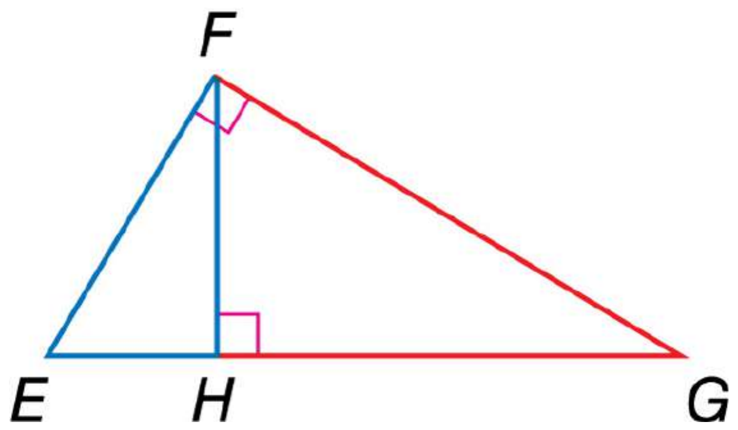
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\triangle ACD \sim \triangle ABC$, $\triangle CBD \sim \triangle ABC$, and $\triangle ACD \sim \triangle CBD$.



EXAMPLE 2 Identify Similar Right Triangles

Write a similarity statement identifying the three similar triangles in the figure.

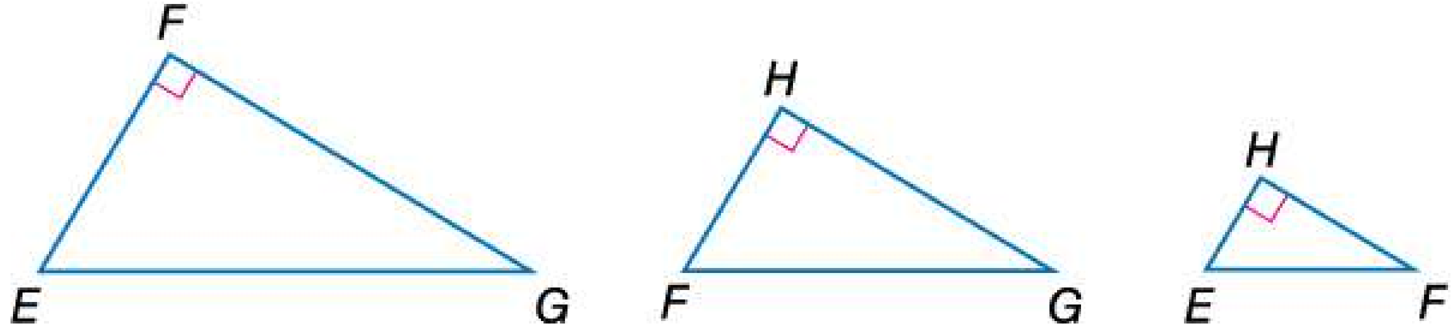


Separate the triangles into two triangles along the altitude.

EXAMPLE 2

Identify Similar Right Triangles

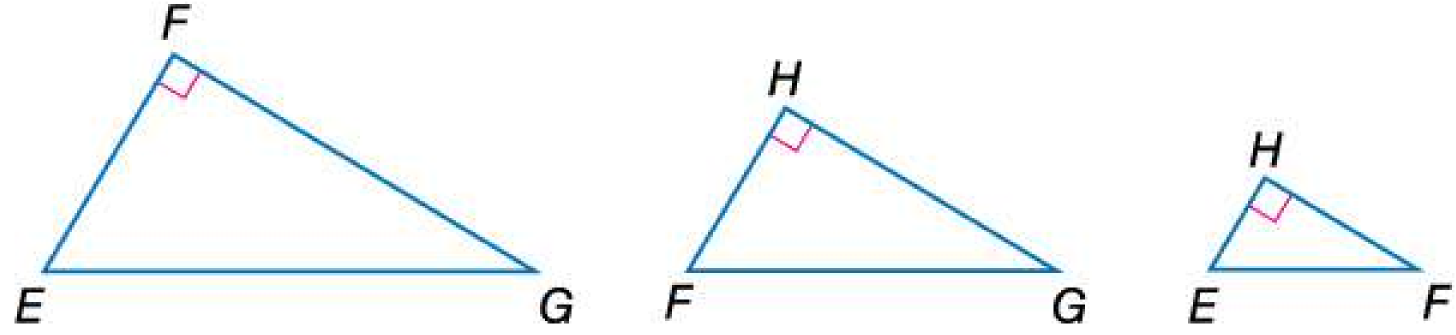
Then sketch the three triangles, reorienting the smaller ones so that their corresponding angles and sides are in the same position as the original triangle.



Answer:

EXAMPLE 2 Identify Similar Right Triangles

Then sketch the three triangles, reorienting the smaller ones so that their corresponding angles and sides are in the same position as the original triangle.

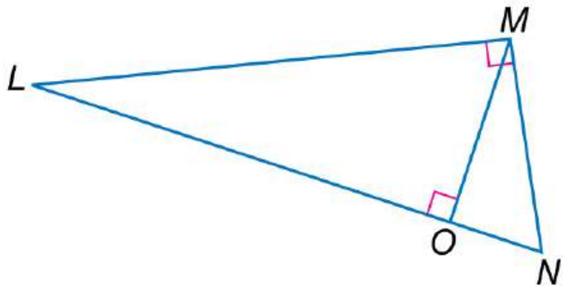


Answer: So, by Theorem 8.1, $\Delta EGF \sim \Delta FGH \sim \Delta EFH$.

EXAMPLE 2 Check Your Progress

CheckPoint

Write a similarity statement identifying the three similar triangles in the figure.



A. ΔLNM ~ ΔMLO ~ ΔNMO

B. ΔNML ~ ΔLOM ~ ΔMNO

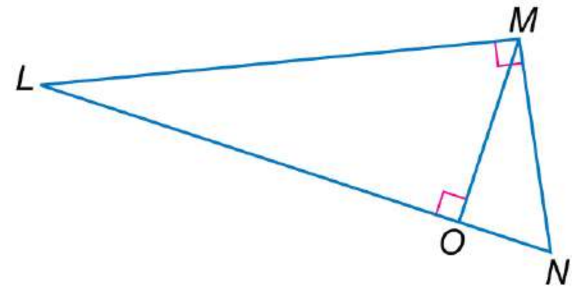
C. ΔLMN ~ ΔLOM ~ ΔMON

D. ΔLMN ~ ΔLMO ~ ΔMNO

EXAMPLE 2 Check Your Progress

CheckPoint

Write a similarity statement identifying the three similar triangles in the figure.



A. $\Delta LNM \sim \Delta MLO \sim \Delta NMO$

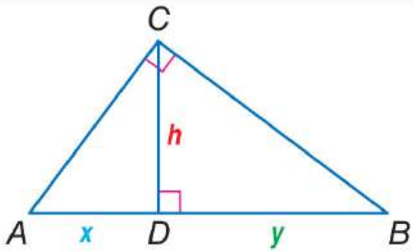
B. $\Delta NML \sim \Delta LOM \sim \Delta MNO$

C. $\Delta LMN \sim \Delta LOM \sim \Delta MON$

D. $\Delta LMN \sim \Delta LMO \sim \Delta MNO$

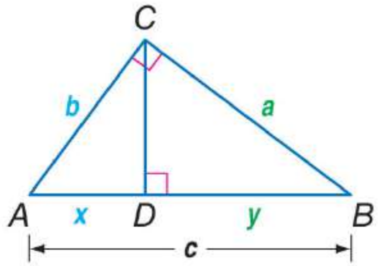
Theorems Right Triangle Geometric Mean Theorems

8.2 Geometric Mean (Altitude) Theorem The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.



Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\frac{x}{h} = \frac{h}{y}$ or $h = \sqrt{xy}$.

8.3 Geometric Mean (Leg) Theorem The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

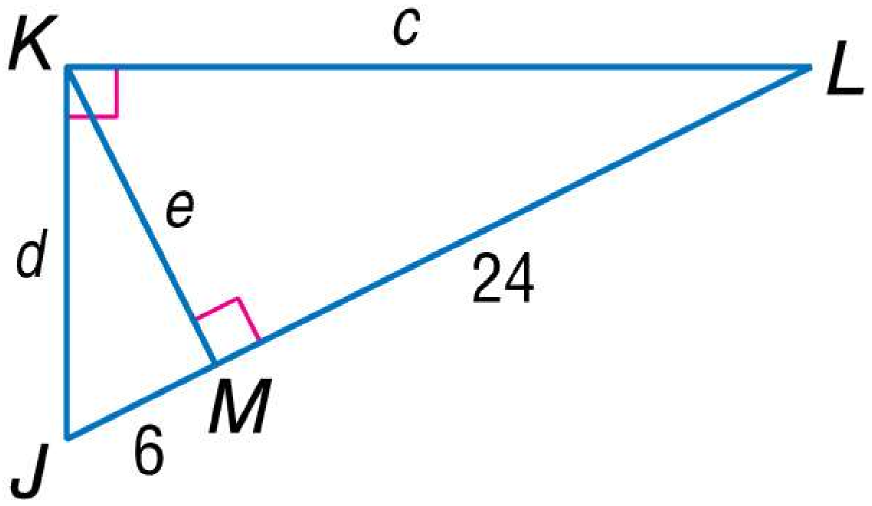


Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\frac{c}{b} = \frac{b}{x}$ or $b = \sqrt{xc}$ and $\frac{c}{a} = \frac{a}{y}$ or $a = \sqrt{yc}$.

EXAMPLE 3

Use Geometric Mean with Right Triangles

Find *c*, *d*, and *e*.



EXAMPLE 3 Use Geometric Mean with Right Triangles

Since *e* is the measure of the altitude drawn to the hypotenuse of right Δ*JKL*, *e* is the geometric mean of the lengths of the two segments that make up the hypotenuse, *JM* and *ML*.

$e = \sqrt{JM \bullet ML}$ n
(Altitude) Theorem

$= \sqrt{6 \bullet 24}$

$= \sqrt{144}$ or 12

EXAMPLE 3 **Use Geometric Mean with Right Triangles**

Since *d* is the measure of leg $\overline{JK}$, *d* is the geometric mean of *JM*, the measure of the segment adjacent to this leg, and the measure of the hypotenuse *JL*.

$$d = \sqrt{JM \bullet JL}$$

(Leg) Theorem

$$= \sqrt{6 \bullet (6 + 24)}$$

$$= \sqrt{180} \text{ or about } 13.4$$

simplify.

EXAMPLE 3 Use Geometric Mean with Right Triangles

Since *c* is the measure of leg $\overline{KL}$, *c* is the geometric mean of *ML*, the measure of the segment adjacent to *KL*, and the measure of the hypotenuse *JL*.

$$c = \sqrt{ML \bullet JL}$$

(Leg) Theorem

$$= \sqrt{24 \bullet (24 + 6)}$$

$$= \sqrt{720} \text{ or about } 26.8$$

simplify.

Answer:

EXAMPLE 3 Use Geometric Mean with Right Triangles

Since *c* is the measure of leg $\overline{KL}$, *c* is the geometric mean of *ML*, the measure of the segment adjacent to *KL*, and the measure of the hypotenuse *JL*.

$$c = \sqrt{ML \bullet JL}$$

(Leg) Theorem

$$= \sqrt{24 \bullet (24 + 6)}$$

$$= \sqrt{720} \text{ or about } 26.8$$

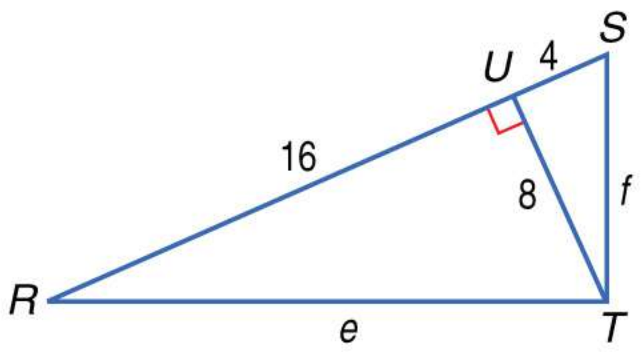
simplify.

Answer: *e* = 12, *d* ≈ 13.4, *c* ≈ 26.8

EXAMPLE 3 Check Your Progress

CheckPoint

Find *e* to the nearest tenth.



A. 13.9

B. 24

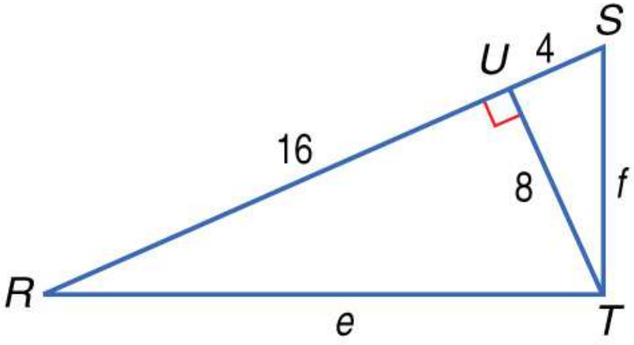
C. 17.9

D. 11.3

EXAMPLE 3 Check Your Progress

CheckPoint

Find *e* to the nearest tenth.



A. 13.9

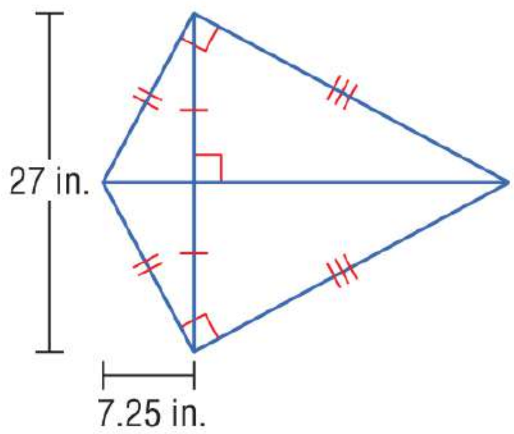
B. 24

C. 17.9

D. 11.3

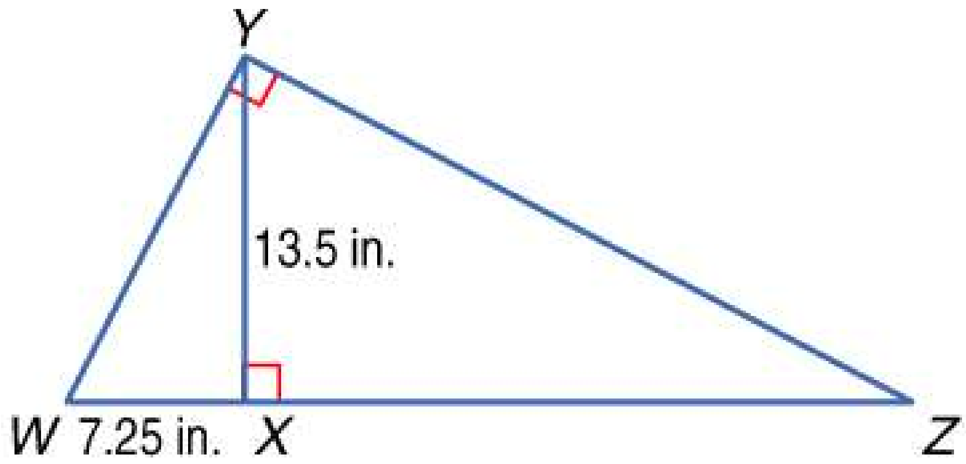
Real-World Example 4 Indirect Measurement

KITES Ms. Alspach is constructing a kite for her son. She has to arrange two support rods so that they are perpendicular. The shorter rod is 27 inches long. If she has to place the short rod 7.25 inches from one end of the long rod in order to form two right triangles with the kite fabric, what is the length of the long rod?



Real-World Example 4 — Indirect Measurement

Draw a diagram of one of the right triangles formed.



Let $\overline{YX}$ be the altitude drawn from the right angle of ΔWYZ.

Real-World Example 4 Indirect Measurement

G $YX = \sqrt{WX \cdot XZ}$

(Altitude) Theorem

S $13.5 = \sqrt{7.25 \cdot XZ}$

S $182.25 = 7.25 \cdot XZ$

D $25.14 \approx XZ$ .25.

Answer:

Real-World Example 4 Indirect Measurement

G (Altitude) Theorem

$$YX = \sqrt{WX \cdot XZ}$$

S

$$13.5 = \sqrt{7.25 \cdot XZ}$$

S

$$182.25 = 7.25 \cdot XZ$$

D .25.

$$25.14 \approx XZ$$

Answer: The length of the long rod is 7.25 + 25.14, or about 32.39 inches long.