Forces

- Newton's Laws of Motion

- Weight

- Free fall

- Force and motion problems in 1-D

- Normal force

- Tension

- Free body diagrams

- Atwood device

- Static and kinetic friction

- Coefficients of friction

- Air resistance

- Terminal velocity

Examples of Forces

- A force is just a push or pull. Examples:
  - an object's weight
  - tension in a rope
  - a left hook to the schnozola
  - friction
  - attraction between an electron and proton
- Bodies don't have to be in contact to exert forces on each other, e.g., gravity.

Fundamental Forces of Nature

- Gravity
  - Attraction between any two bodies w/ mass
  - Weakest but most dominant
- Electromagnetic
  - Forces between any two bodies w/ charge
  - Attractive or repulsive
- Weak nuclear force – responsible for radioactive decay
- Strong nuclear force – holds quarks together (constituents of protons and neutrons)

Newton's Laws of Motion

1. Inertia: "An object in motion tends to stay in motion. An object at rest tends to stay at rest."

2. $F_{net} = ma$

3. Action – Reaction: "For every action there is an equal but opposite reaction."

1ˢᵗ Law: Inertia

"An object in motion tends to stay in motion; an object at rest tends to stay at rest."

- A moving body will continue moving in the same direction with the same speed until some net force acts on it.

- A body at rest will remain at rest unless a net force acts on it.

- Summing it up: It takes a net force to change a body's *velocity*.

Inertia Example 1
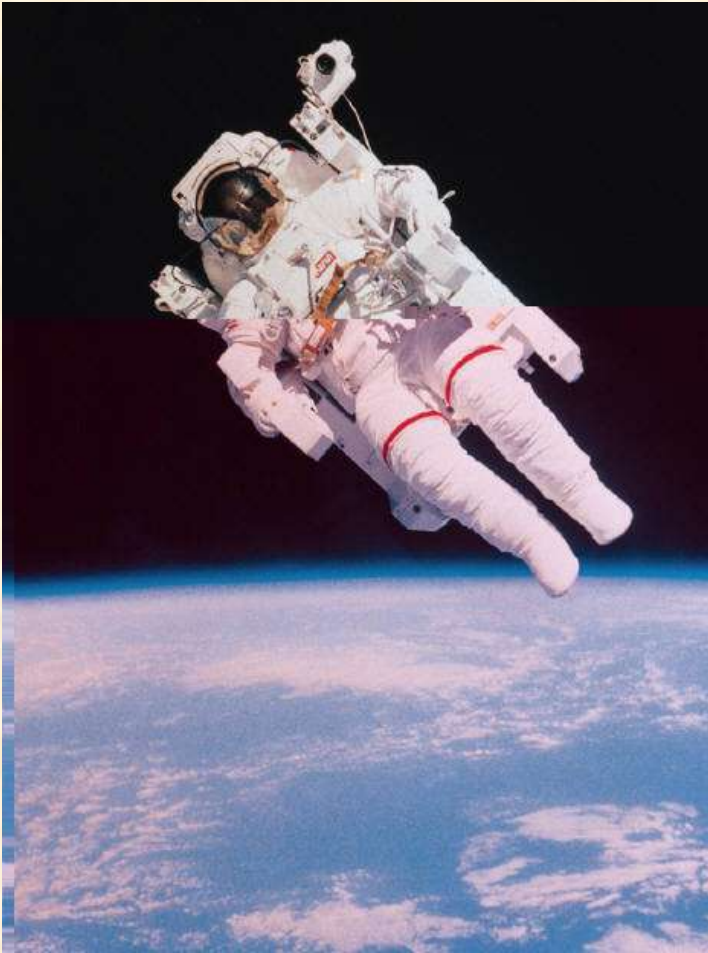
An astronaut in outer space will continue drifting in the same direction at the same speed indefinitely, until acted upon by an outside force.

Inertia Example 2

If you're driving at 65 mph and have an accident, your car may come to a stop in an instant, while your body is still moving at 65 mph. Without a seatbelt, your inertia could carry you through the windshield.

2ⁿᵈ Law: $F_{net} = ma$

- The acceleration an object undergoes is directly proportion to the net force acting on it.
- Mass is the constant of proportionality.
- For a given mass, if F_{net} doubles, triples, etc. in size, so does a.
- For a given F_{net} if m doubles, a is cut in half.
- F_{net} and a are vectors; m is a scalar.
- F_{net} and a always point in the same direction.
- The 1ˢᵗ law is really a special case of the 2ⁿᵈ law (if net force is zero, so is acceleration).

What is Net Force?

$\mathbf{F}_1$

$\mathbf{F}_2$

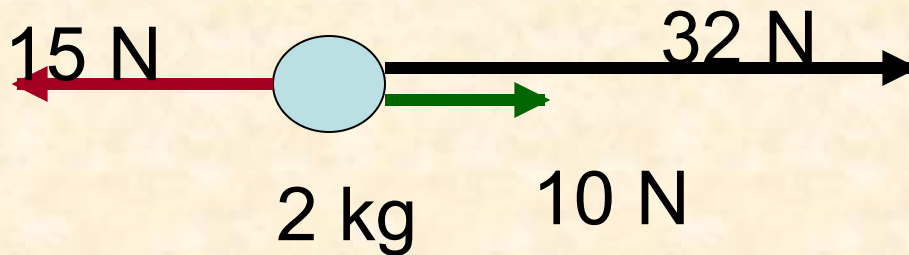$\mathbf{F}_3$

$\mathbf{F}_{net}$
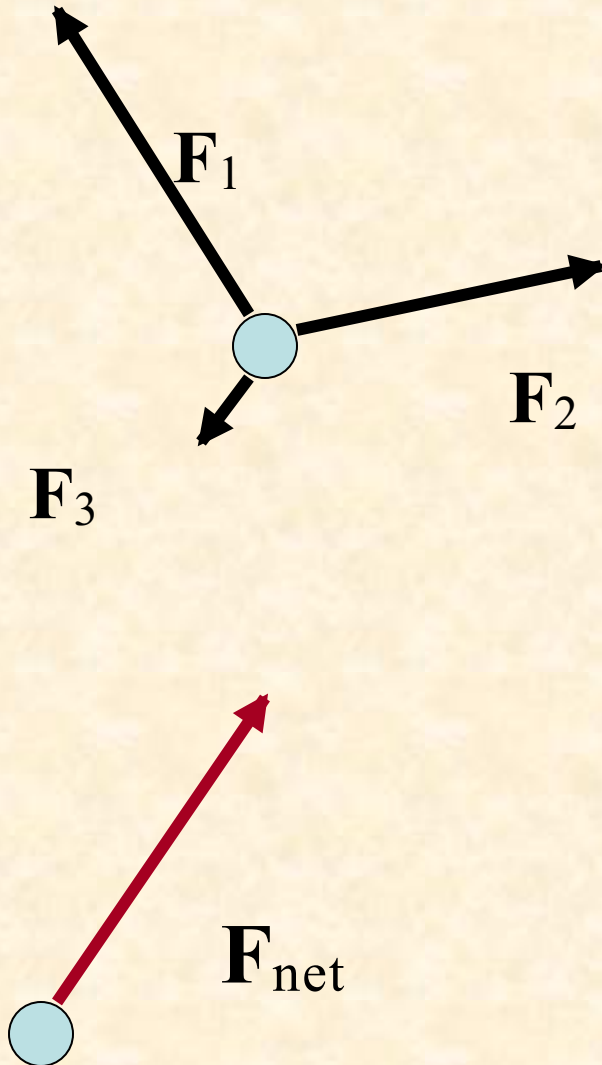
When more than one force acts on a body, the net force (resultant force) is the vector combination of all the forces, i.e., the "net effect."

Net Force & the 2nd Law

For a while, we'll only deal with forces that are horizontal or vertical.

When forces act in the same line, we can just add or subtract their magnitudes to find the net force.

15 N   32 N

2 kg   10 N

F_{net} = 27 N to the right

a = 13.5 m/s^2

Units

$$F_{net} = ma$$
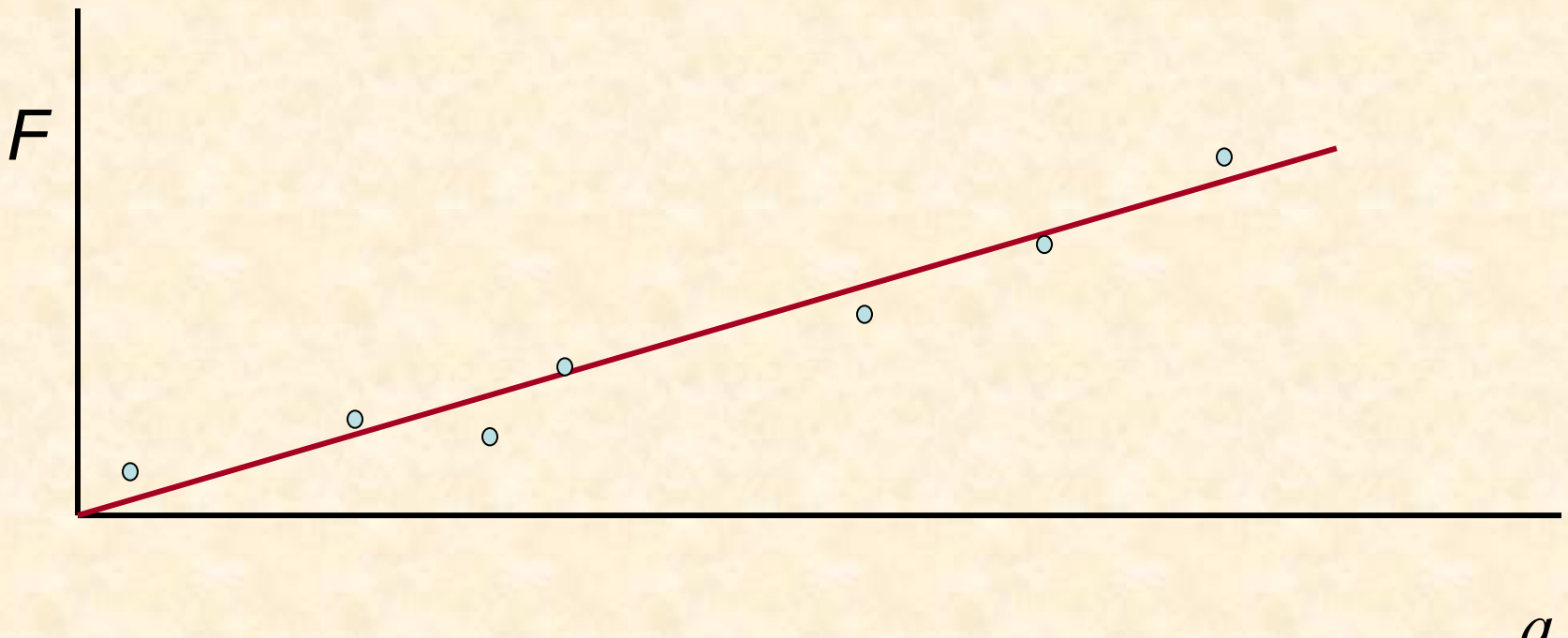
$$1\ N = 1\ kg\ m/s^2$$

The SI unit of force is the Newton.

A Newton is about a quarter pound.

1 lb = 4.45 N

Graph of *F* vs. a

In the lab various known forces are applied—one at a time, to the same mass—and the corresponding accelerations are measured. The data are plotted. Since *F* and a are directly proportional, the relationship is linear.

Slope

Since slope = rise / run = $\Delta F / \Delta a$, the slope is equal to the mass. Or, think of $y = mx + b$, like in algebra class. y corresponds to force, m to mass, x to acceleration, and b (the y-intercept) is zero.
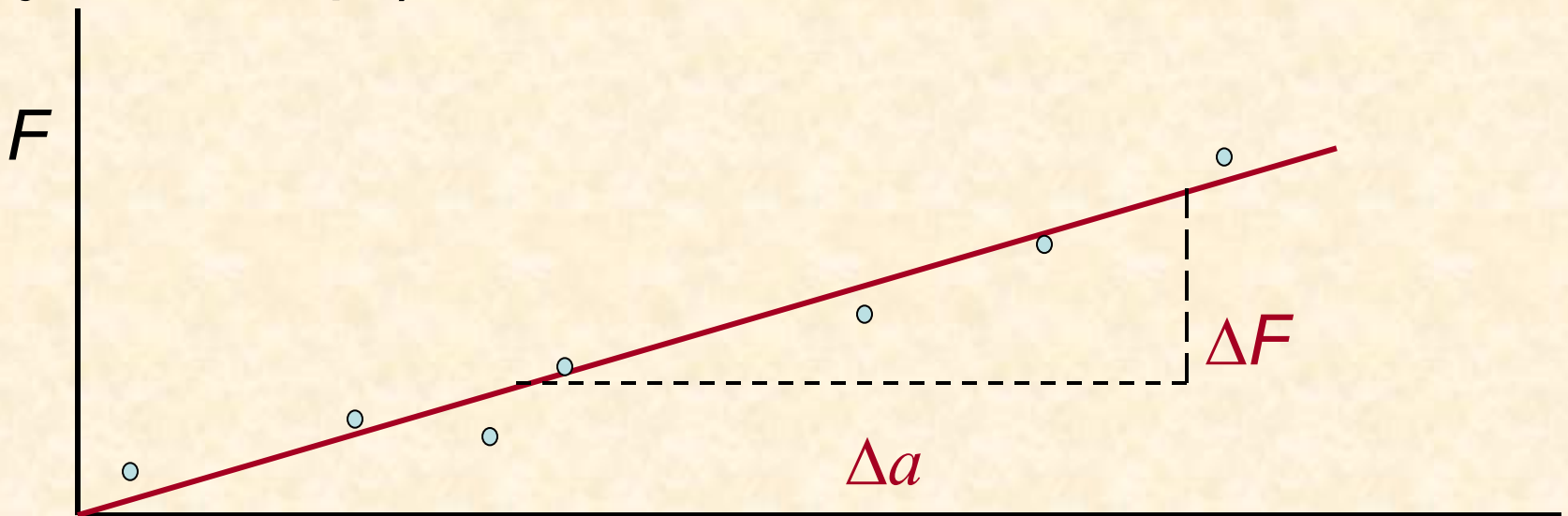
W = mg

- Weight = mass × acceleration due to gravity.

- This follows directly from $F = ma$.

- Weight is the force of gravity on a body.

- Near the surface of the Earth, $g = 9.8$ m/s^2.

Two Kinds of Mass

- Inertial mass: the net force on an object divided by its acceleration. $m = F_{net}/a$

- Gravitational mass: Compare the gravitational attraction of an unknown mass to that of a known mass, usually with a balance. If it balances, the masses are equal.

m

?

Balance
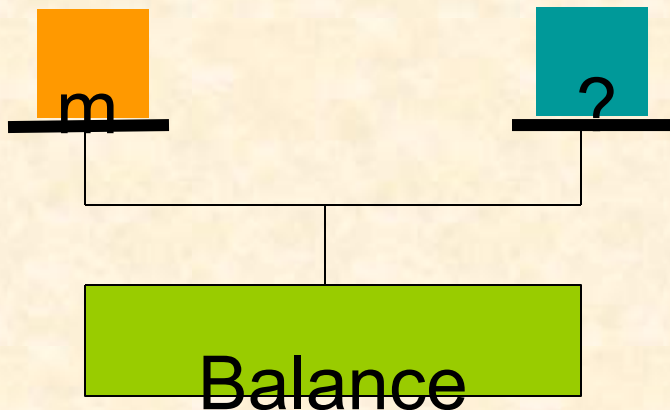
Einstein asserted that these two kinds of masses are equivalent.

Action - Reaction

"For every action there's an equal but opposite reaction."

- If you hit a tennis ball with a racquet, the force on the ball due to the racquet is the same as the force on the racquet due to the ball, except in the opposite direction.

- If you drop an apple, the Earth pulls on the apple just as hard as the apple pulls on the Earth.

- If you fire a rifle, the bullet pushes the rifle backwards just as hard as the rifle pushes the bullet forwards.

Earth / Apple

How could the forces on the tennis ball, apple, and bullet, be the same as on the racquet, Earth, and rifle? The 3rd Law says they must be, the effects are different because of the 2nd Law!

apple 0.40 kg
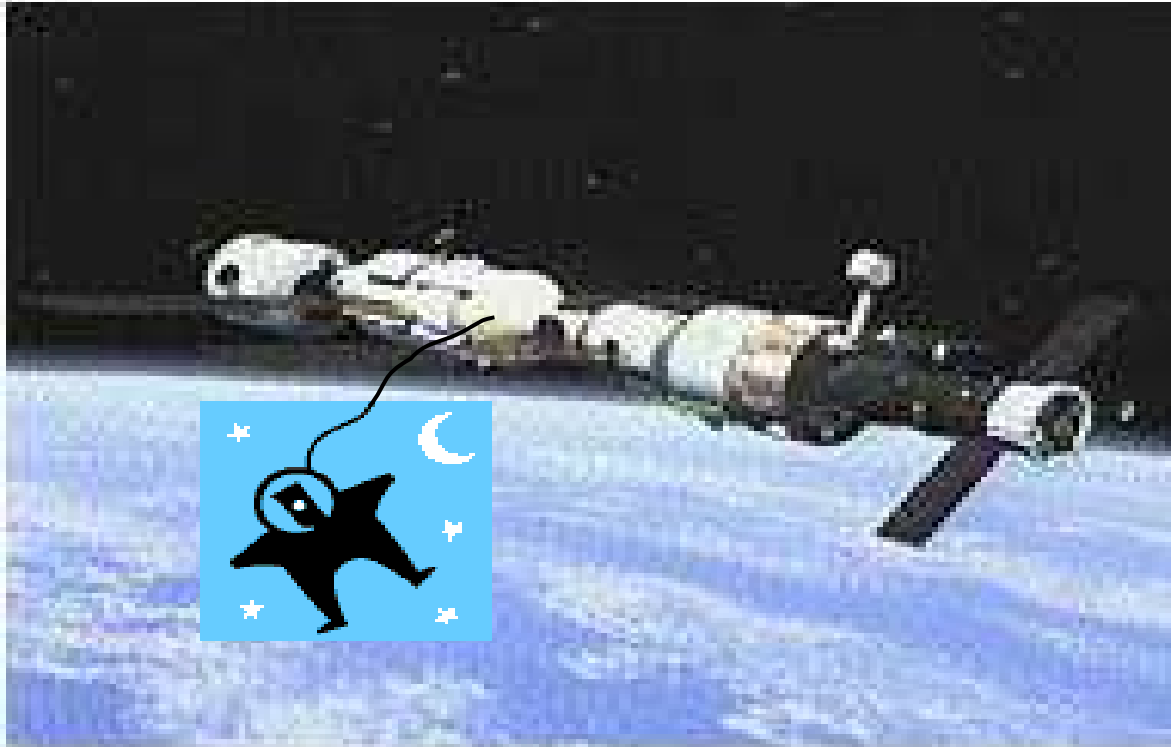
3.92 N

Earth

3.92 N

5.98 × 10^{24} kg
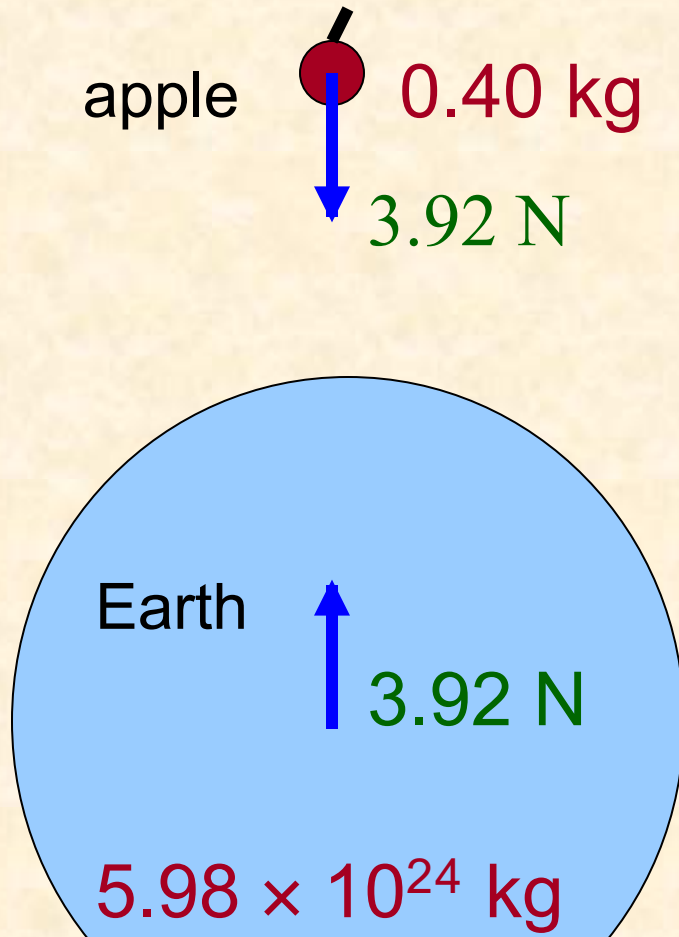
A 0.40 kg apple weighs 3.92 N ($W = mg$). The apple's weight is Earth's force on it. The apple pulls back just as hard. So, the same force acts on both bodies. Since their masses are different, so are their accelerations (2nd Law). The Earth's mass is so big, it's acceleration is negligible.

Earth / Apple (cont.)

The products are the same, since the forces are the same.

m **a** = **m** *a*

Apple's little mass

Apple's big acceleration

Earth's big mass

Earth's little acceleration

Lost in Space

Suppose an International Space Station astronaut is on a spacewalk when her tether snaps. Drifting away from the safety of the station, what might she do to make it back?

Swimming

Due to the 3rd Law, when you swim you push the water (blue), and it pushes you back just as hard (red) in the forward direction. The water around your body also produces a drag force (green) on you, pushing you in the backward direction. If the green and red cancel out, you don't accelerate (2nd Law) and maintain a constant velocity.
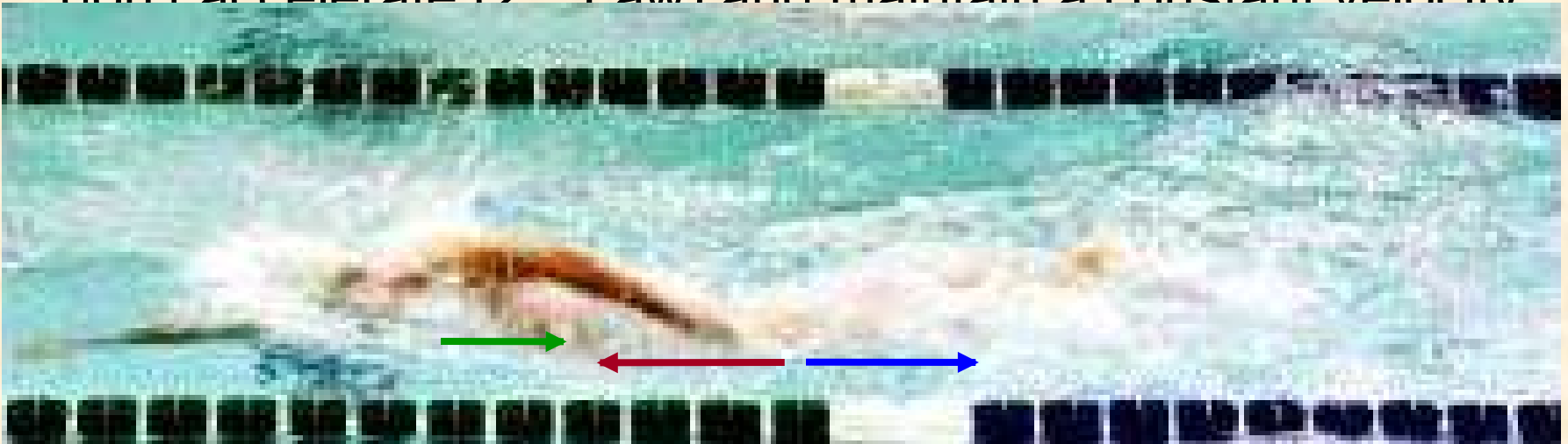
Note: The blue vector is a force on the water, not the on swimmer! Only the green and red vectors act on the swimmer.

Free fall

- An object is in free fall if the only force acting on it is gravity.

- It doesn't matter which way it's moving.

- A shell in a cannon is not in freefall until it leaves the barrel of the cannon. (There are other forces acting on it while inside the barrel.)

- For an object in free fall, $a = -g$, if:
  - we ignore air resistance.
  - don't stray too far from Earth.

Freefall (cont.)

- Any launched object is in freefall the entire time it's in the air, if:
  - we ignore air resistance.
  - it has no propulsion system.
- With the previous condition met, $a = -g = -9.8$ m/s^2 everywhere:
  - on the way up
  - at its peak
  - on the way down

Hippo & Ping Pong Ball

In a vacuum, all bodies fall at the same rate.
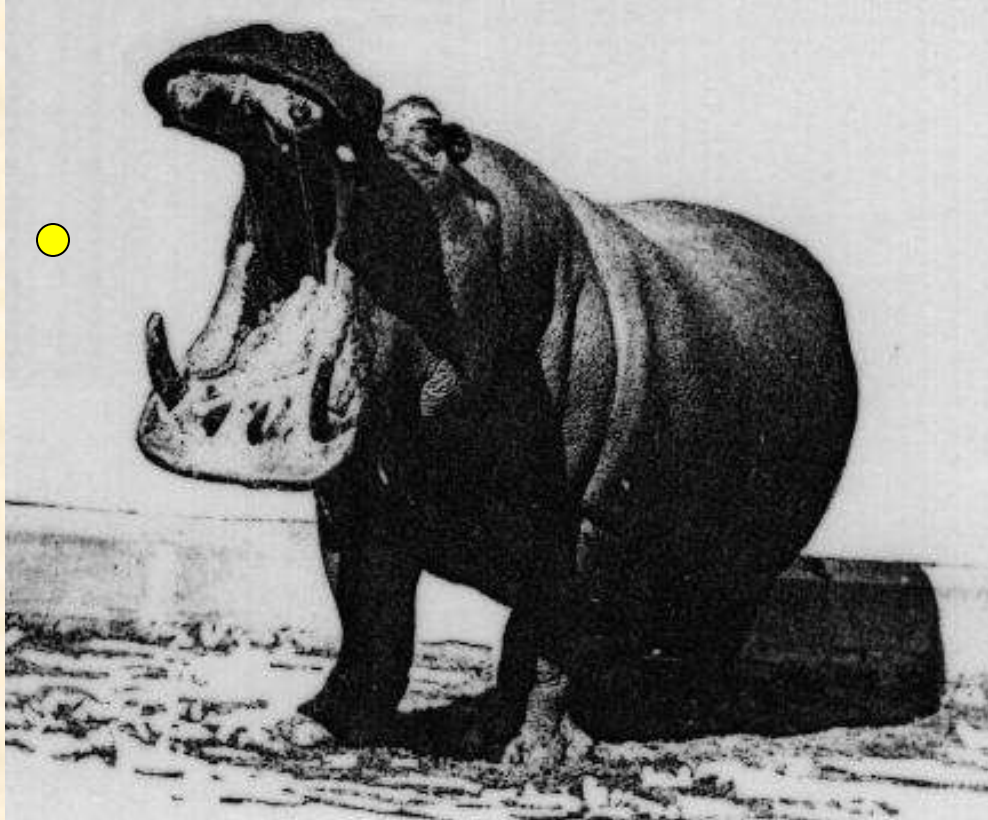
If a hippo and a ping pong ball were dropped from a helicopter in a vacuum (assuming the copter could fly without air), they'd land at the same time.

When there's no air resistance, size and shape don't matter!

Misconceptions

- If an object is moving, there must be some force making it move. **Wrong! It could be moving without accelerating.**

- If $v = 0$, then a and F_{net} must be zero. **Wrong! Think of a projectile shot straight up at its peak.**

- An object must move in the direction of the net force. **Wrong! It must <u>accelerate</u> that way but not necessarily move that way.**

Misconceptions (cont.)

- Heavy objects must fall faster than light ones. **Wrong! The rate is the same in a vacuum.**

- When a big object collides with a little one, the big one hits the little one harder than the little one hits the big one. **Wrong! The 3rd Law says they hit it each other with the same force.**

- If an object accelerates, its speed must change. **Wrong! It could be turning at constant speed.**

Projectile confusion

$a \neq 0$ at the vertex (peak) of a projectile's trajectory. Velocity can be zero there, but not acceleration!

If a were zero at the vertex, F_{net} would have to be zero as well (by the 2nd law), which means gravity would have to be turned off!

$a = -g$ throughout the whole trip, including the high point !

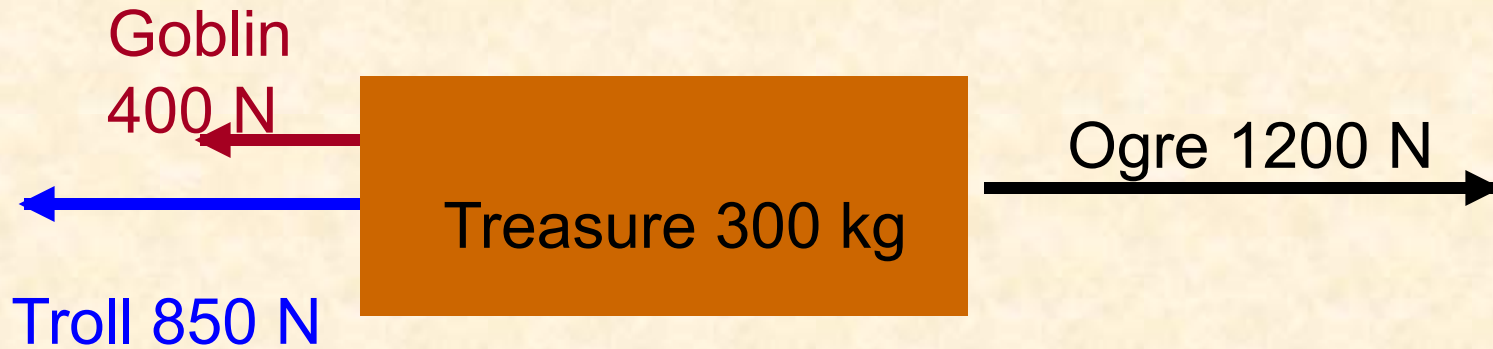# Forces & Kinematics

To solve motion problems involving forces:

1. *Find net force (by combining vectors).*
2. *Calculate acceleration (using 2nd law).*
3. *Use kinematics equations:*

$$v_f = v_0 + a\,t$$

$$\Delta x = v_0\,t + \tfrac{1}{2}\,a\,t^2$$

$$v_f^2 - v_0^2 = 2\,a\,\Delta x$$

Sample Problem 1

Goblin
400 N

Treasure 300 kg

Ogre 1200 N

Troll 850 N

A troll and a goblin are fighting with a big, mean ogre over a treasure chest, initially at rest. Find:

1. F_{net} = 50 N left

2. a = 0.167 m/s² left

3. v after 5 s = 0.835 m/s left

4. Δx after 5 s = 2.08 m left

A 3 kg watermelon is launched straight up by applying a 70 N force over 2 m. Find its max height. Hints:

Phase I: the launch

1. Draw pic and find net force.   40.6 N up

2. Calculate a during launch.   +13.5333 m/s^2

3. Calculate v_f at the end of the launch (after 2 m).
    +7.3575 m/s

Phase II: freefall

4. Draw pic and think about what a is now.   -9.8 m/s^2

5. v_f from phase I is v_0 for phase II.   -9.8 m/s^2

6. What is v_f for phase II?   zero

7. Calculate max height & add 2 m.   4.76 m

Normal force

- When an object lies on a table or on the ground, the table or ground must exert an upward force on it, otherwise gravity would accelerate it down.

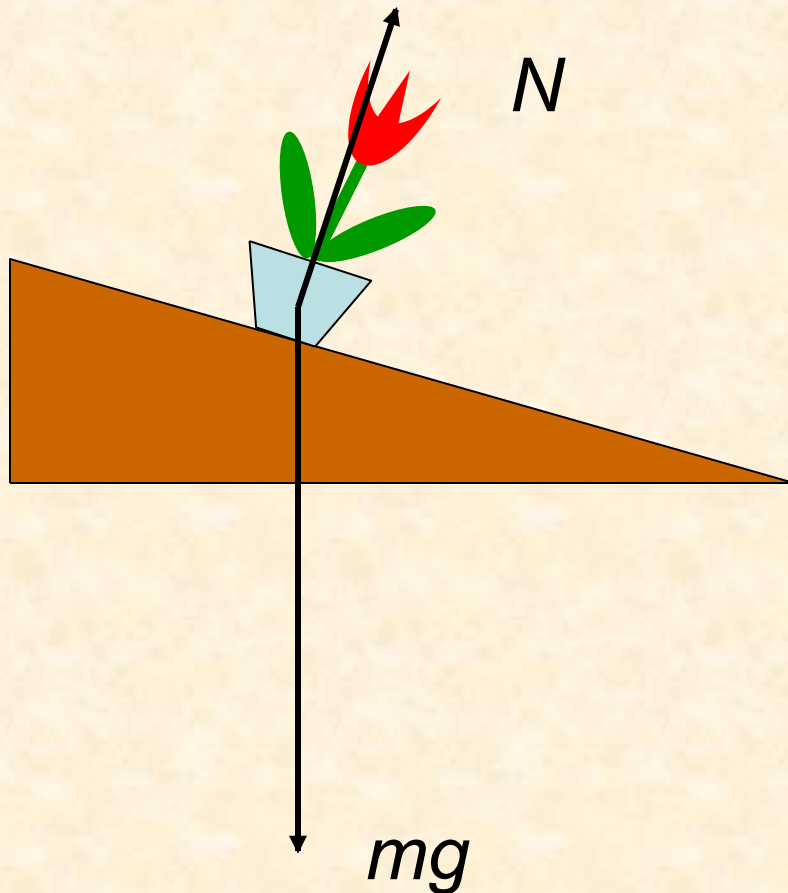- This force is called the normal force.

N

m

mg
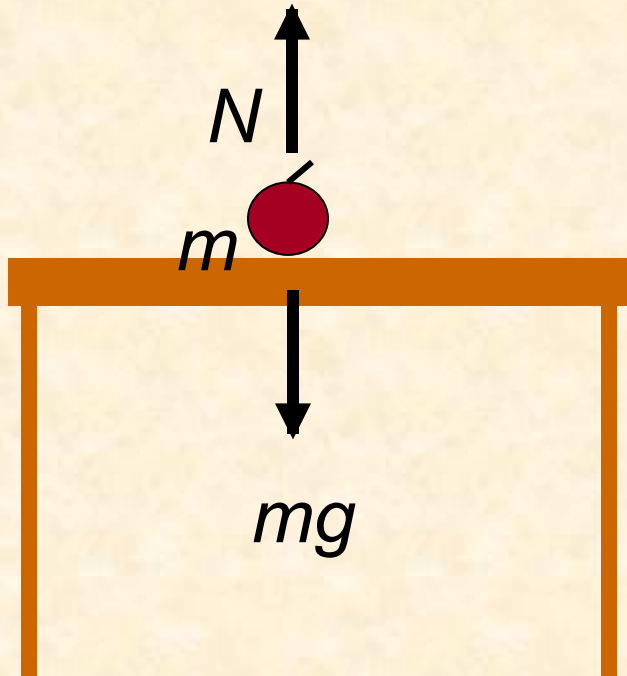
In this particular case, $N = mg$.

So, $F_{net} = 0$; hence $a = 0$.

Normal forces aren't always up

"Normal" means perpendicular. A normal force is always perpendicular to the contact surface.

N

mg

For example, if a flower pot is setting on an incline, *N* is not vertical; it's at a right angle to the incline. Also, in this case, $mg > N$.

Normal force directions

- Up
  - You're standing on level ground.
  - You're at the bottom of a circle while flying a loop-the-loop in a plane.

- Sideways
  - A ladder leans up against a wall.
  - You're against the wall on the "Round Up" ride when the floor drops out.

- At an angle
  - A race car takes a turn on a banked track.

- Down
  - You're in a roller coaster at the top of a loop.

Cases in which $N \neq mg$

1. Mass on incline

2. Applied force acting on the mass

3. Nonzero acceleration, as in an elevator or launching space shuttle
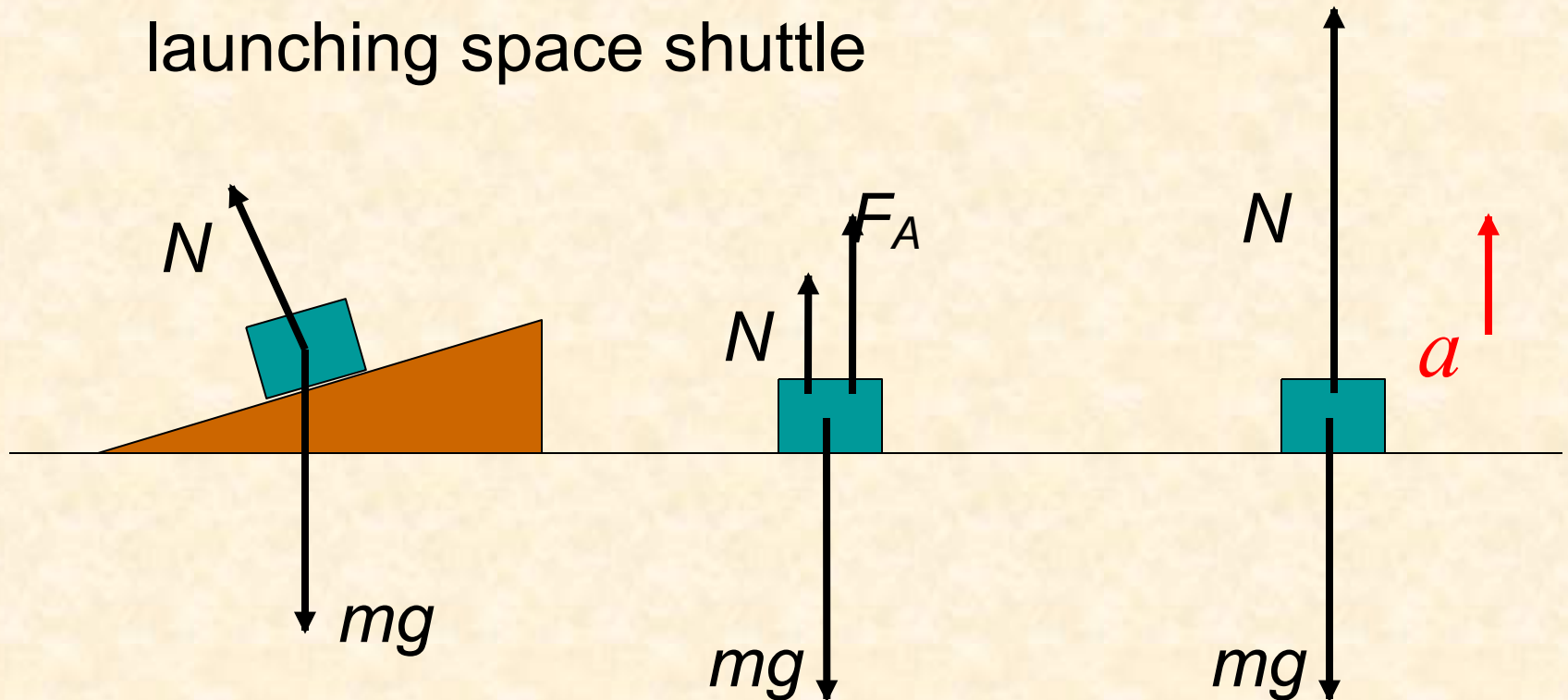
When does $N = mg$?

If the following conditions are satisfied, then $N = mg$:

- The object is on a level surface.

- There's nothing pushing it down or pulling it up.

- The object is not accelerating vertically.

Box / Tension Problem

frictionless floor
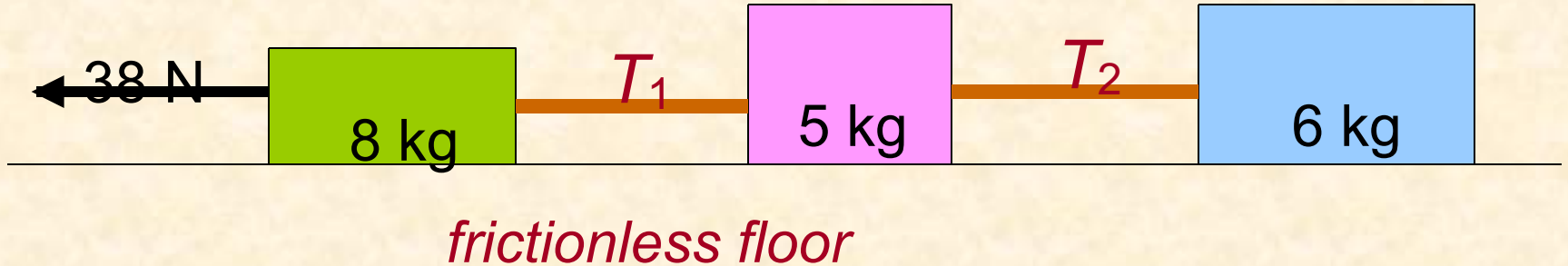
- A force is applied to a box that is connected to other boxes by ropes. The whole system is accelerating to the left.

- The problem is to find the tensions in the ropes.

- We can apply the 2nd Law to each box individually as well as to the whole system.

Box / Tension Analysis

38 N   8 kg T_1 5 kg T_2 6 kg

frictionless floor

- T_1 pulls on the 8-kg box to the right just as hard as it pulls on the middle box to the left.

- T_1 must be < 38 N, or the 8-kg box couldn't accelerate.

- T_2 pulls on the middle box to the right just as hard as it pulls on the 6-kg box to the left.

- T_1 must be > T_2 or the middle box couldn't accelerate.

Free Body Diagram – system
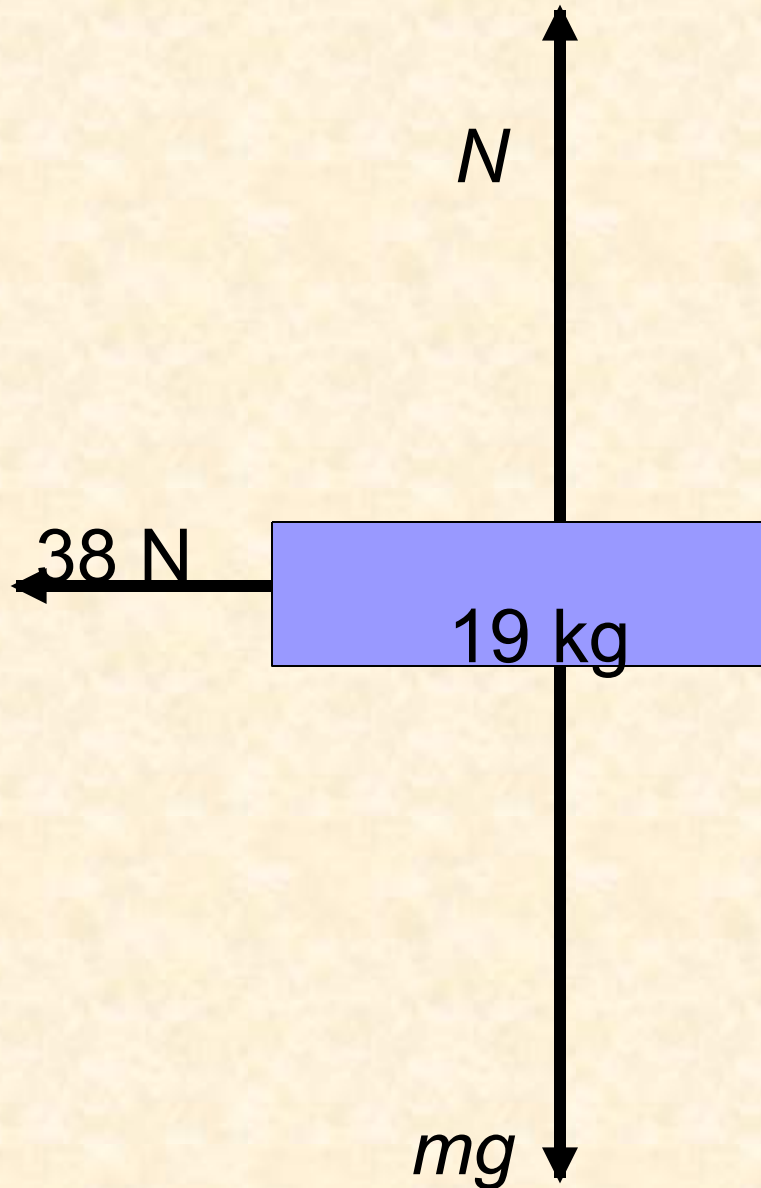
For convenience, we'll choose left to be the positive direction.

N

38 N

19 kg

mg

The total mass of all three boxes is 19 kg.

N and **mg** cancel out.

$F_{net} = ma$ implies
a = 2.0 m/s^2

Since the ropes don't stretch, a will be 2.0 m/s^2 for all three boxes.
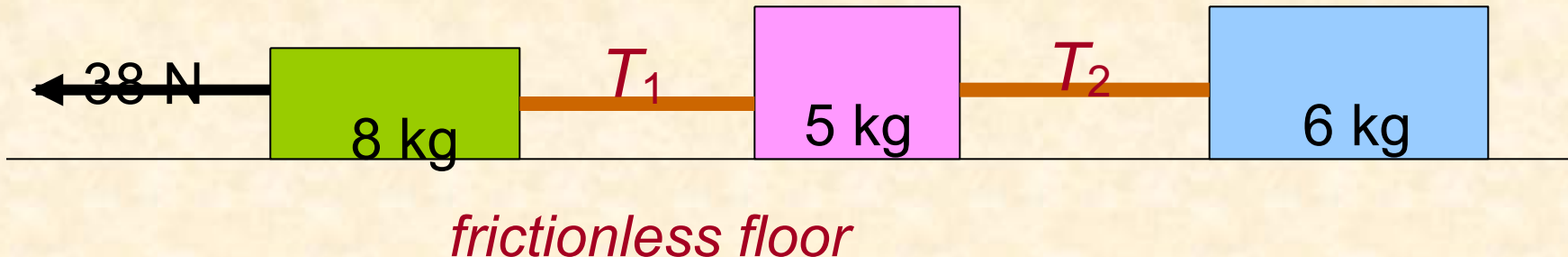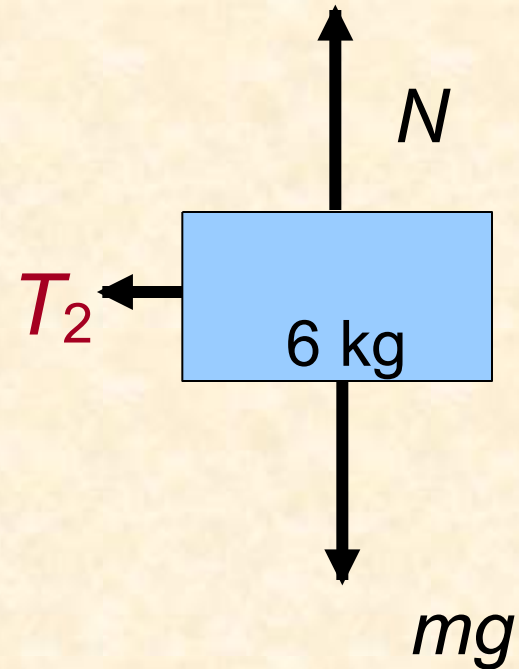
Free Body Diagram – right box

N and *mg* cancel out.

For this particular box,
$F_{net} = ma$ implies:

$T_2 = 6a = 6(2) = 12$ N.
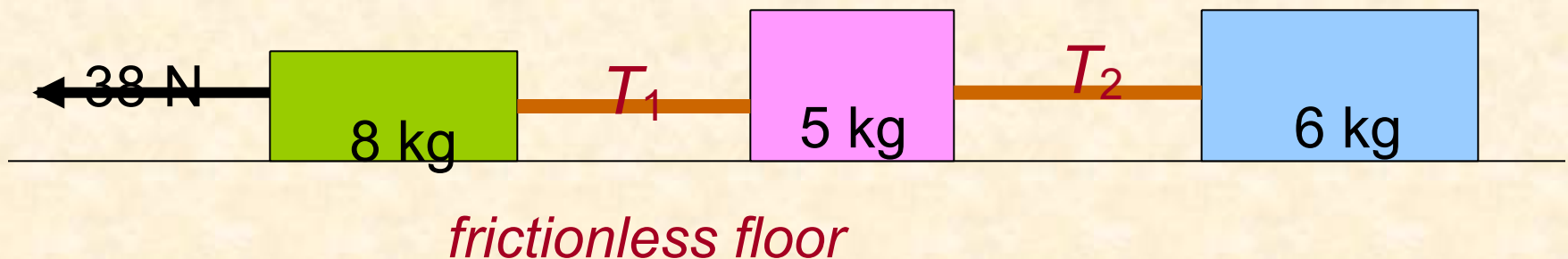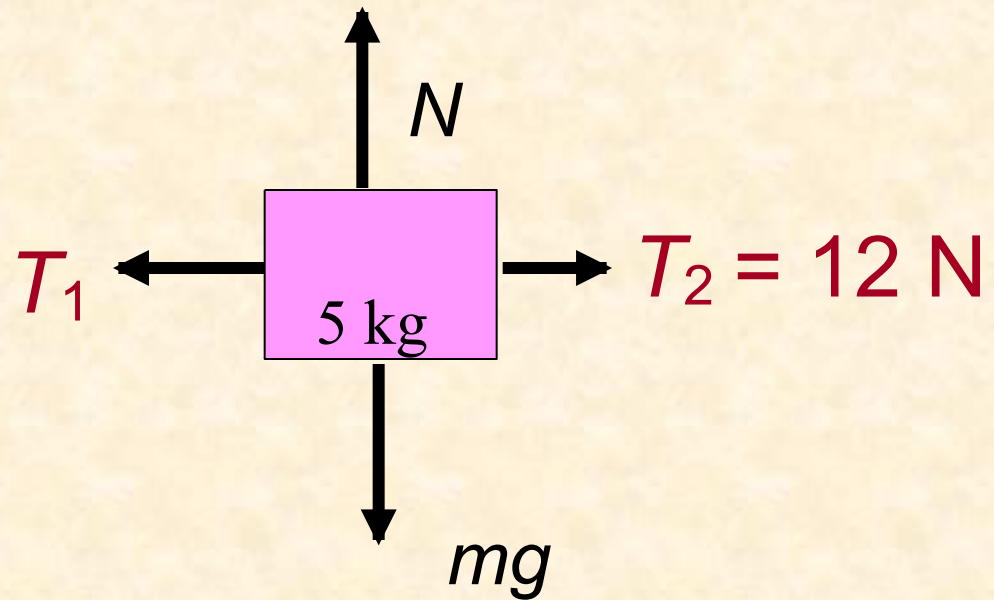
(Remember, $a = 2$ m/s^2 for all three boxes.)

frictionless floor

Free Body Diagram – middle box

N and **mg** cancel out again.

$F_{net} = ma$ implies:

$T_1 - T_2 = 5a$. So,
$T_1 - 12 = 5(2)$, and
$T_1 = 22$ N

$T_1 \longleftarrow$ [5 kg] $\longrightarrow T_2 = 12$ N

N

mg

38 N $\longleftarrow$ [8 kg] — T_1 — [5 kg] — T_2 — [6 kg]

frictionless floor

Free Body Diagram – left box

N

38 N

$T_1 =$ 22 N

8 kg

mg
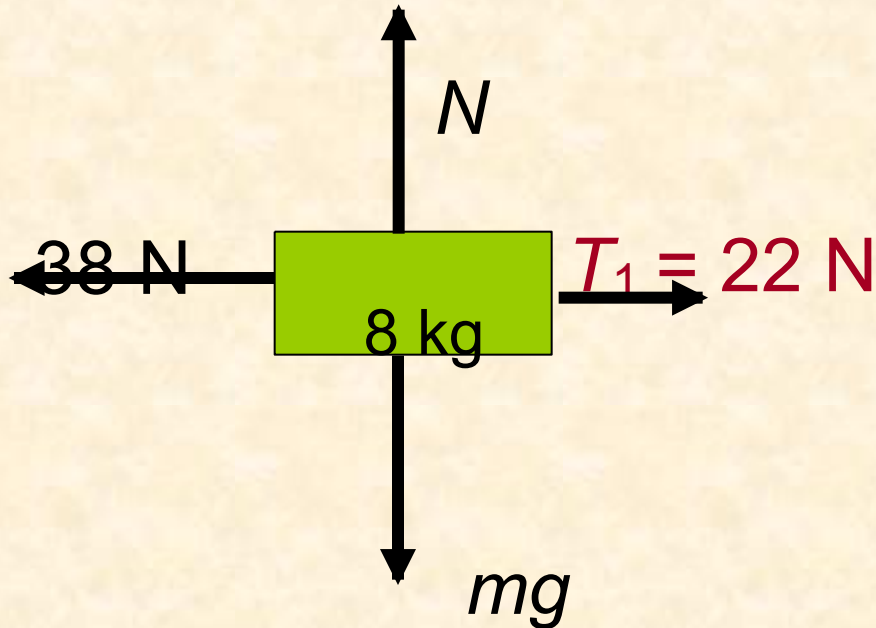
Let's check our work using the left box.

N and mg cancel out here too.

$F_{net} = ma$ implies:

38 - 22 = ma = 8(2).
16 = 16.

38 N

T_1

T_2

8 kg
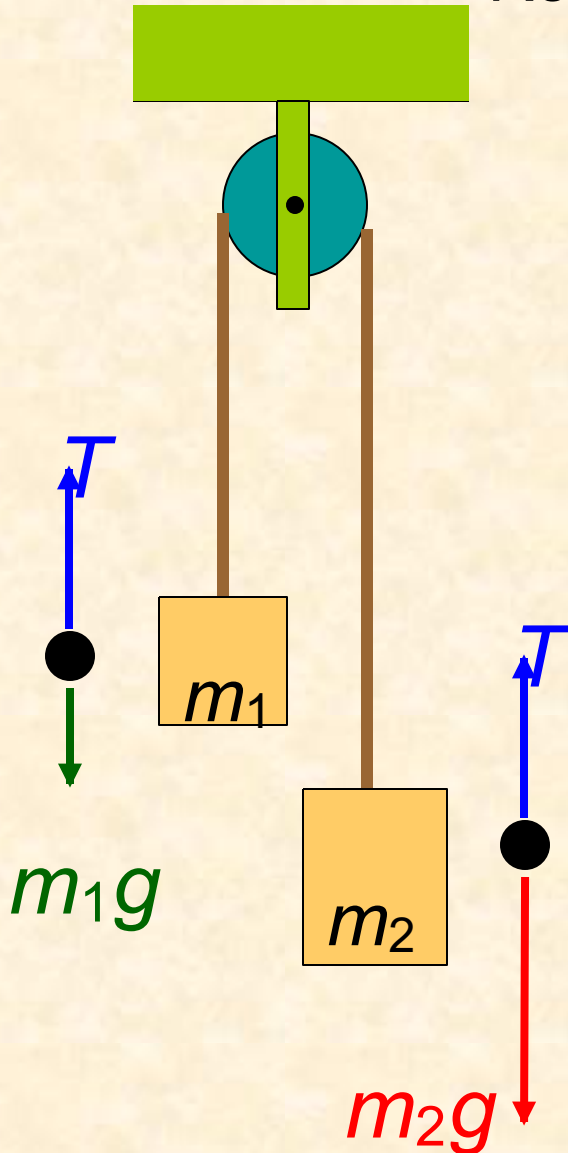
5 kg

6 kg

Atwood Device
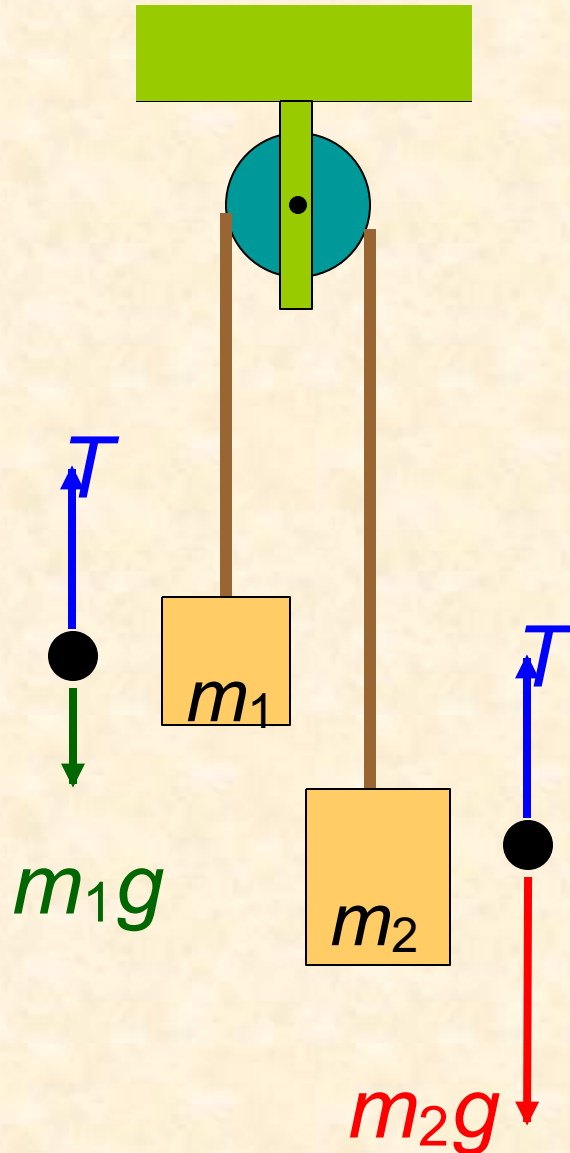
Assume $m_1 < m_2$ and that the clockwise direction is **+**.

If the rope & pulley have negligible mass, and if the pulley is frictionless, then T is the same throughout the rope.

If the rope doesn't stretch, a is the same for both masses.

T

m_1

m_1g

T

m_2

m_2g

Atwood Analysis

Remember, clockwise has been defined as **+**.

2$^{\text{nd}}$ Law on m_1: $T - m_1g = m_1a$

2$^{\text{nd}}$ Law on m_2: $m_2g - T = m_2a$
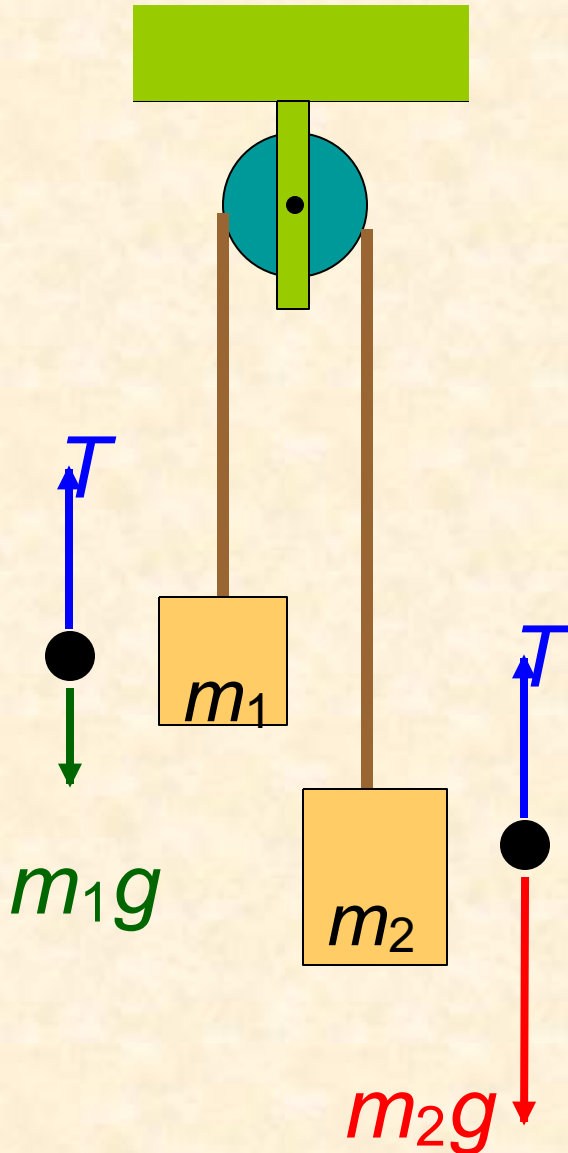
Add equations:

$$m_2g - m_1g = m_1a + m_2a$$

(The T's cancel out.)

Solve for a:

$$a = \frac{m_2 - m_1}{m_1 + m_2}g$$

Atwood as a system

Treated as a system (rope & both masses), tension is internal and the *T*'s cancel out (one clock-wise, one counterclockwise).

F_{net} = (total mass) $\times a$ implies

(force in **+** direction) **-**
      (force in **-** direction)

$= m_2 g - m_1 g = (m_1 + m_2)a.$

Solving for a gives the same result. Then, knowing a, *T* can be found by substitution.

T

T

m_1

m_2

$m_1 g$

$m_2 g$

Atwood: Unit Check

$$a = \frac{\cancel{m_2 - m_1}}{m_1 + m_2} \, g$$

units: $\dfrac{\cancel{kg} - \cancel{kg}}{kg + kg} \cdot \dfrac{\cancel{m}}{s^2} = \dfrac{\cancel{m}}{s^2}$

Whenever you derive a formula you should check to see if it gives the appropriate units. If not, you screwed up. If so, it doesn't prove you're right, but it's a good way to check for errors. Remember, you can multiply or divide scalar quantities with different units, but you can only add or subtract quantities with the same units!

Atwood: Checking Extremes

Besides units, you should also check a formula to see if what happens in extreme & special cases makes sense.

$$a = \frac{m_2 - m_1}{m_1 + m_2} g$$

$m_2 \gg m_1$: In this case, m_1 is negligible compared to m_2. If we let $m_1 = 0$ in the formula, we get $a = (m_2 / m_2)g = g$, which makes sense, since with only one mass, we have freefall.

$m_2 \ll m_1$: This time m_2 is negligible compared to m_1, and if we let $m_2 = 0$ in the formula, we get $a = (-m_1 / m_1)g = -g$, which is freefall in the negative (counterclockwise) direction.

$m_2 = m_1$: In this case we find $a = 0 / (2m_1)g = 0$, which is what we would expect considering the device is balanced. Note: The masses in the last case can still move but only with constant velocity!
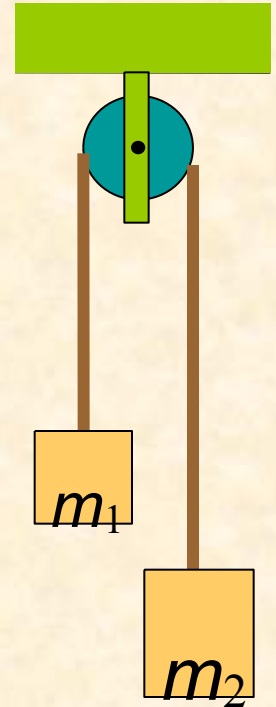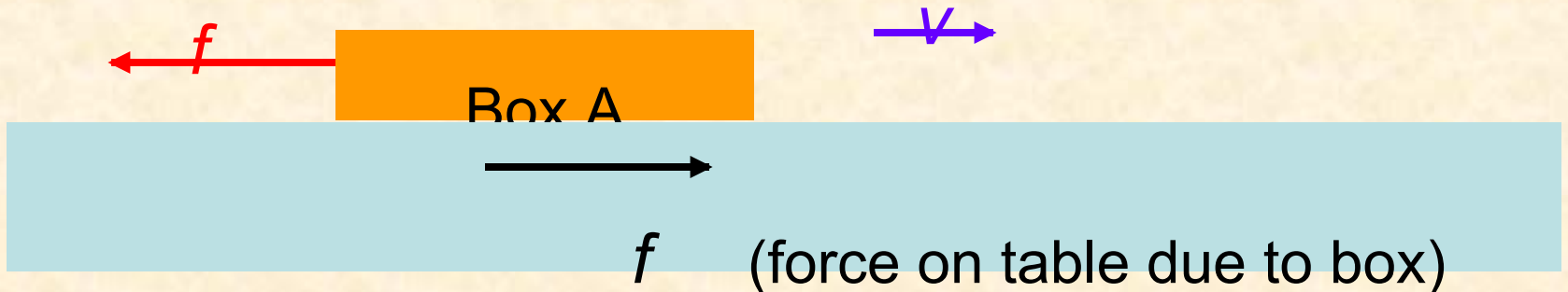
Friction

Friction is the force bodies can impart on each other when they're in contact.

The friction forces are parallel to the contact surface and occur when…

- One body slides over the other, or…
- They cling together despite and external force.

The forces shown are an action-reaction pair.

(force on box due to table)
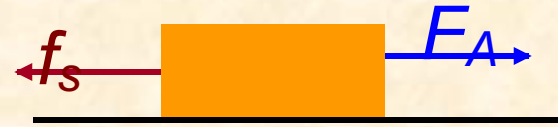
f

Box A

f (force on table due to box)

Friction Facts

- Friction is due to electrostatic attraction between the atoms of the objects in contact.

- It can speed you up, slow you down, or make you turn.

- It allows you to walk, turn a corner on your bike, warm your hands in the winter, and see a meteor shower.

- Friction often creates waste heat.

- It makes you push harder / longer to attain a given acceleration.

- Like any force, it always has an action-reaction pair.

Two Kinds of Friction (or 3)

- Static friction
  - Must be overcome in order to budge an object
  - Present only when there is **no** relative motion between the bodies, e.g., the box & table top

- Kinetic friction
  - Weaker than static friction
  - Present only when objects are moving with respect to each other (skidding)

f_s F_A

Objects are still or moving together. $F_{net} = 0$.

f_k F_A

F_{net} is to the right.

a is to the right.

v is left or right.

Friction Strength

The magnitude of the friction force is proportional to:

- how hard the two bodies are pressed together (the normal force, N).
- the materials from which the bodies are made (the coefficient of friction, μ).

Attributes that have little or no effect:

- sliding speed
- contact area

Coefficients of Friction

- Static coefficient … μ_s.
- Kinetic coefficient … μ_k.
- Both depend on the materials in contact.
  - Small for steel on ice or scrambled egg on Teflon frying pan
  - Large for rubber on concrete or cardboard box on carpeting
- The bigger the coefficient of friction, the bigger the frictional force.

Static Friction Force

$$f_s \leq \mu_s N$$

static
frictional
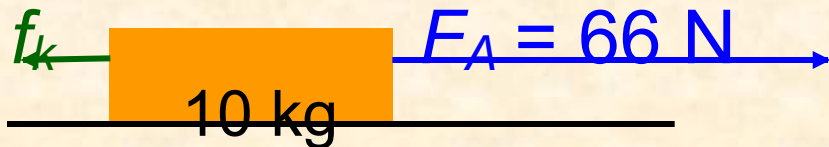force
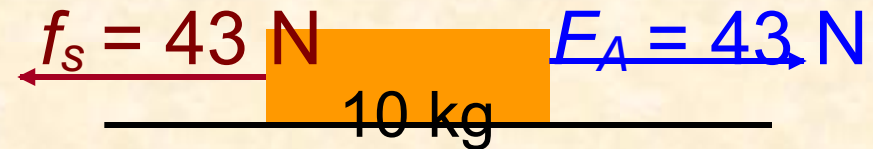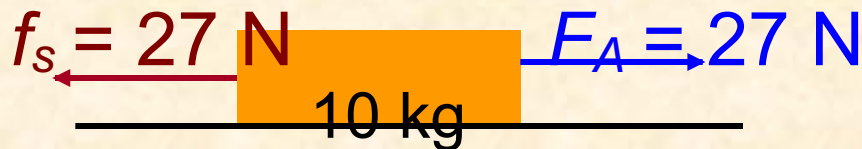
coefficient of
static friction

normal
force

$$f_{s,max} = \mu_s N$$

maximum
force of static
friction

$f_{s,max}$ is the force you must exceed in order to budge a resting object.

Static friction force varies

- $f_{s,max}$ is a constant in a given problem, but f_s varies.

- f_s matches F_A until F_A exceeds $f_{s,max}$.

- Example: In the picture below, if μ_s for a wooden crate on a tile floor is 0.6, $f_{s,max}$ = 0.6 (10) (9.8) = 58.8 N.

f_s = 27 N F_A = 27 N

10 kg

f_s = 43 N F_A = 43 N

10 kg

f_k F_A = 66 N

10 kg

The box finally budges when F_A surpasses $f_{s,max}$. Then kinetic acts on the box

Kinetic Friction

$$f_k = \mu_k N$$

kinetic frictional force

coefficient of kinetic friction

normal force

- Once object budges, forget about μ_s.

- Use μ_k instead.

- f_k is a constant so long as the materials involved don't change.

- There is no "maximum f_k."

μ values

- Typically, $0 < \mu_k < \mu_s < 1$.

- This is why it's harder to budge an object than to keep it moving.

- If $\mu_k > 1$, it would be easier to lift an object and carry it than to slide across the floor.

- Dimensionless (μ's have no units, as is apparent from $f = \mu N$).

Friction Example 1

You push a giant barrel o' monkeys setting on a table with a constant force of 63 N. If μ_k = 0.35 and μ_s =0.58, when will the barrel have moved 15 m?

answer:

Never, since this force won't even budge it!
$$63 < 0.58\,(14.7)\,(9.8) \approx 83.6 \text{ N}$$
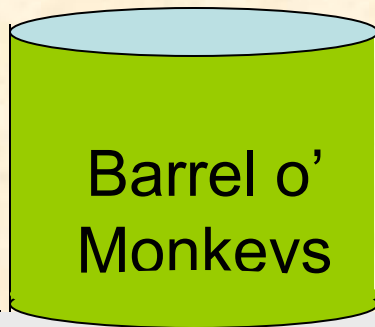
Barrel o' Monkevs 14.7 kg

Friction Example 2

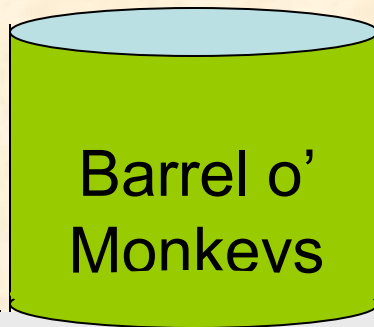Same as the last problem except with a bigger F_A: You push the barrel o' monkeys with a constant force of 281 N. μ_k = 0.35 and μ_s =0.58, same as before. When will the barrel have moved 15 m?

step 1: $f_{s,max}$ = 0.58 (14.7)(9.8) $\approx$ 83.6 N

step 2: F_A= 281N > $f_{s,max}$. Thus, it budges this time.
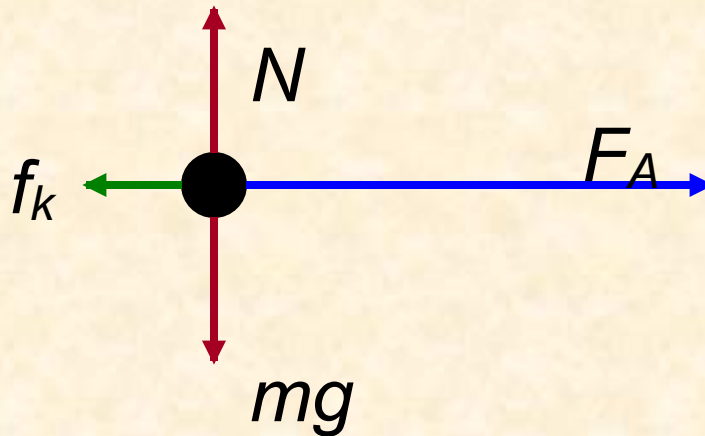
step 3: Forget f_s and calculate f_k:

$$f_k = 0.35(14.7)(9.8) = 50.421 \text{ N}$$

Barrel o' Monkeys

14.7 kg

(continued on next slide)

step 4: Free body diagram while sliding:

N

f_k

F_A

mg

step 5: $F_{net} = F_A - f_k$ = 281 - 50.421 = 230.579 N

Note: To avoid compounding of error, do not round until the end of the problem.

step 6: $a = F_{net} / m$ = 230.579 / 14.7 = 15.68564 m/s^2

step 7: Kinematics: Δx = +15 m, v_0 = 0,

a = +15.68564 m/s^2, t = ?

$\Delta x = v_0 t + \frac{1}{2} a t^2$ $\Rightarrow$ $t = \sqrt{2 \Delta x / a}$ ≈ 1.38 s

Friction as the net force

A runner is trying to steal second base. He's running at a speed v; his mass is m. The coefficient of kinetic friction between his uniform and the base pass is μ. How far from second base should he begin his slide in order to come to a stop right at the base?

Note: In problems like these where no numbers are given, you are expected to answer the questions in terms of the given parameters and any constants. Here, the given parameters are m, μ, and v. Constants may include g, π, and "regular" numbers like 27 and –1.86.

N

f_k

mg
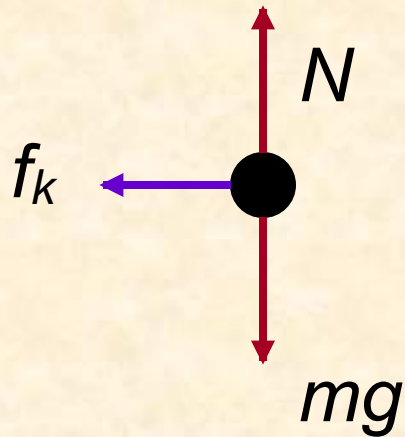
Once the slide begins, there is no applied force. Since N and mg cancel out, f_k is the net force. So Newton's 2nd Law tells us:

$f_k = ma$. But the friction force is also given by $f_k = \mu N = \mu mg$.

Therefore, $\mu mg = ma$. Mass cancels out, meaning the distance of his slide is completely independent of how big he is, and we have $a = \mu g$. (Note that the units work out since μ is dimensionless.) This is just the magnitude of a. If the forward direction is positive, his acceleration (which is always in the direction of the net force) must be negative. So, $a = -\mu g$.

Since he comes to rest at 2nd base, $v_f = 0$.

$$v_f^2 - v_0^2 = 2\,a\,\Delta x$$

$$\Rightarrow \quad 0 \; - \; v^2 \; = -2\,\mu g\,\Delta x$$

$$\Rightarrow \quad \Delta x = v^2 / (2\,\mu g)$$

Unit check: $(m/s)^2 / (m/s^2) \; = \; m^2 / m = m$

Note the slide distance is inversely proportional to the coefficient of friction, which makes sense, since the bigger μ is, the bigger f is.

Note also that here v and F_{net} are in opposite directions, which is perfectly fine.

Scales

- A scale is *NOT* necessarily a weight meter.
- A scale is a normal force meter.
- A scale might lie about your weight if
  - you're on an incline.
  - someone pushes down or pulls up on you.
  - you're in an elevator.
- You're actual weight doesn't change in the above cases.

Weight in a Rocket
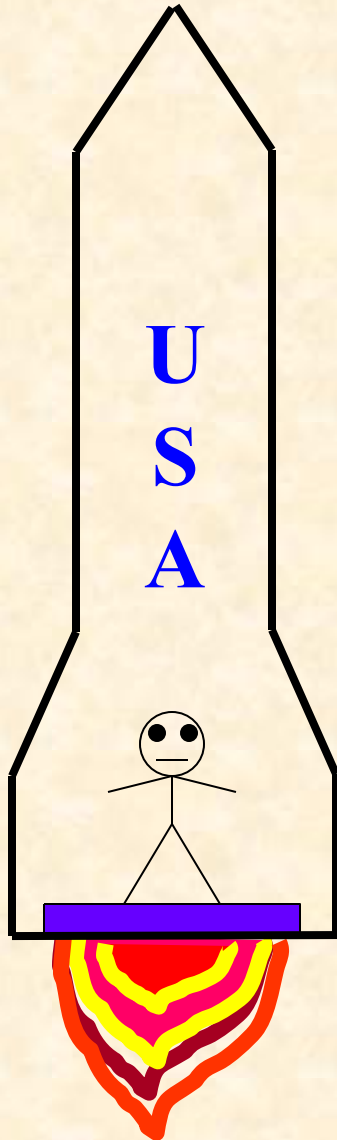
You're on a rocket excursion standing on a purple bathroom scale. You're still near enough to the Earth so that your actual weight is unchanged.
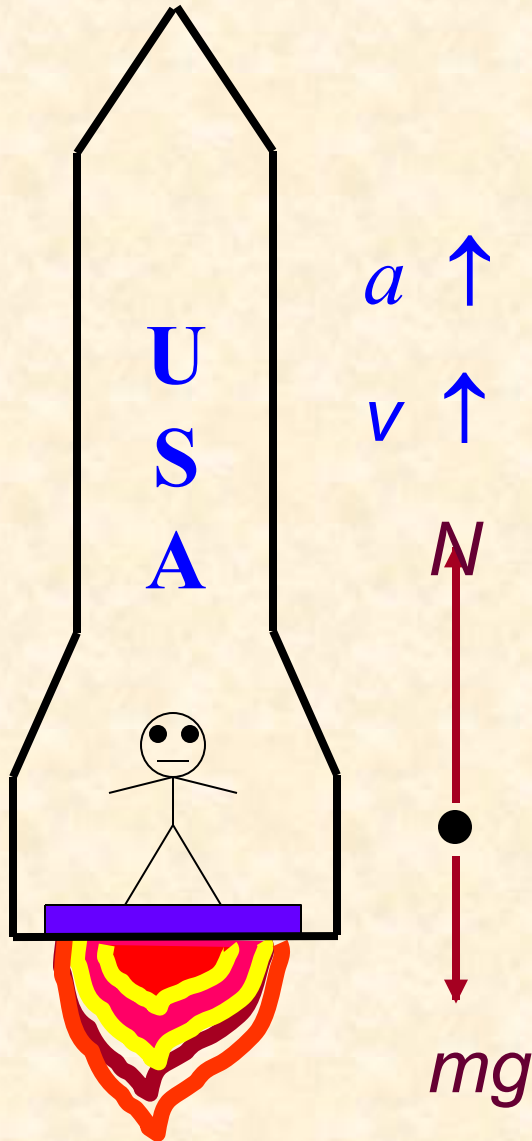
The scale, recall, measures normal force, not weight. Your apparent weight depends on the acceleration of the rocket.

At rest on the launch pad

$a = 0$

$v = 0$

N

m

mg

U
S
A
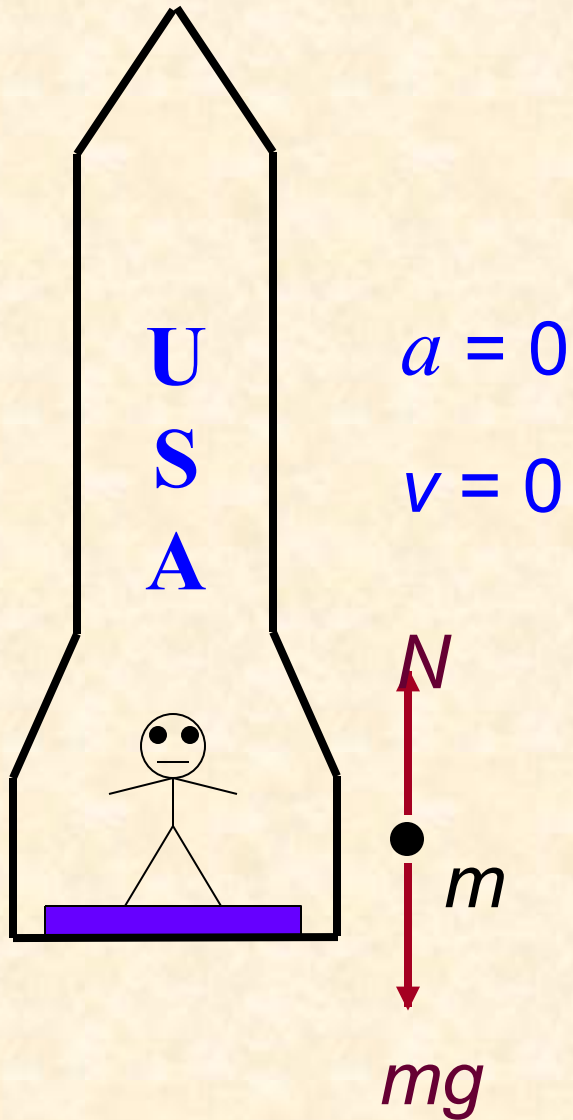
During the countdown to blast off, you're not accelerating. The scale pushes up on you just as hard as the Earth pulls down on you. So, the scale reads your actual weight.

Blasting Off

USA

a ↑

v ↑

N

mg

During blast off your acceleration is up, so the net force must be up (no matter which way **v** is).
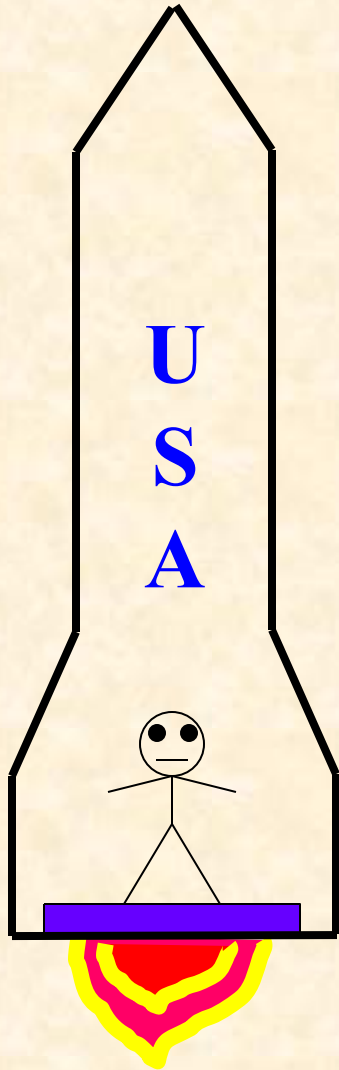
$F_{net} = ma$

$\Rightarrow\ N - mg = ma$

$\Rightarrow\ N = m(a + g) > mg$

$\Rightarrow$ Apparent weight > Actual weight

Cruising with constant velocity

U S A

$a = 0$

$v \uparrow$

N

m

mg

If v = constant, then a = 0.
If a = 0, then F_{net} = 0 too.
If F_{net} = 0, then N must be equal in magnitude to mg.

This means that the scale reads your normal weight (same as if you were at rest) regardless of how fast you're going, so long as you're not accelerating.
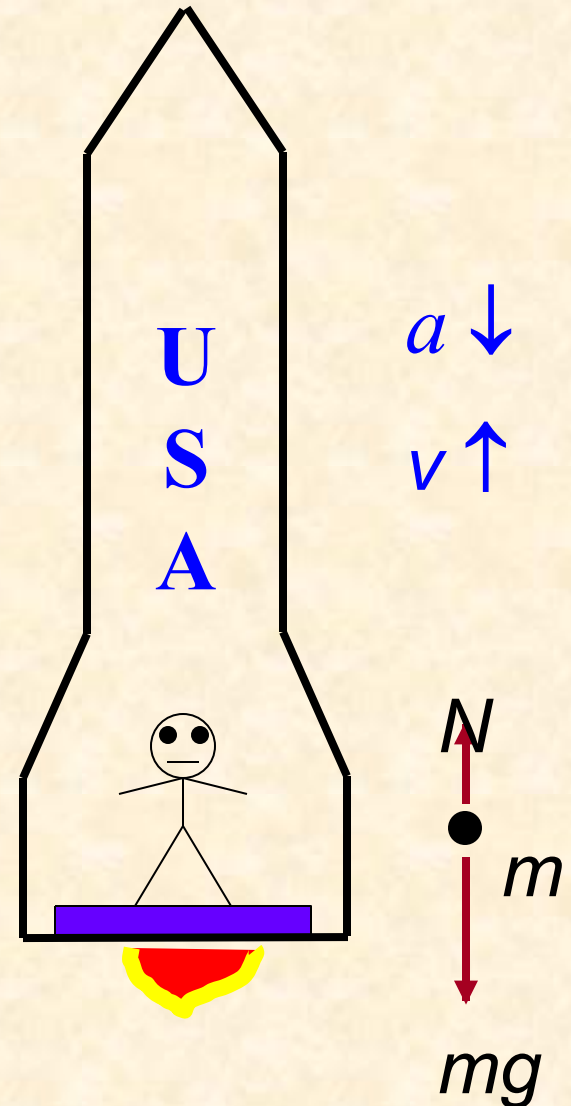
Rocket: Engines on low

As soon as you cut way back on the engines, the Earth pulls harder on you than the scale pushes up. So you're acceleration is down, but you'll still head upward for a while. Choosing down as the positive direction,
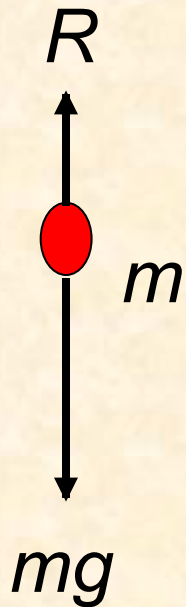
$F_{net} = ma$

$\Rightarrow mg - N = ma$

$\Rightarrow N = m(g - a) < mg$

$\Rightarrow$ *Apparent weight < Actual weight*

U
S
A

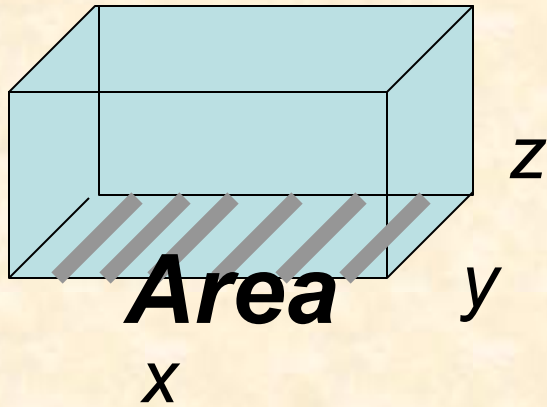$a \downarrow$

$v \uparrow$

N

m

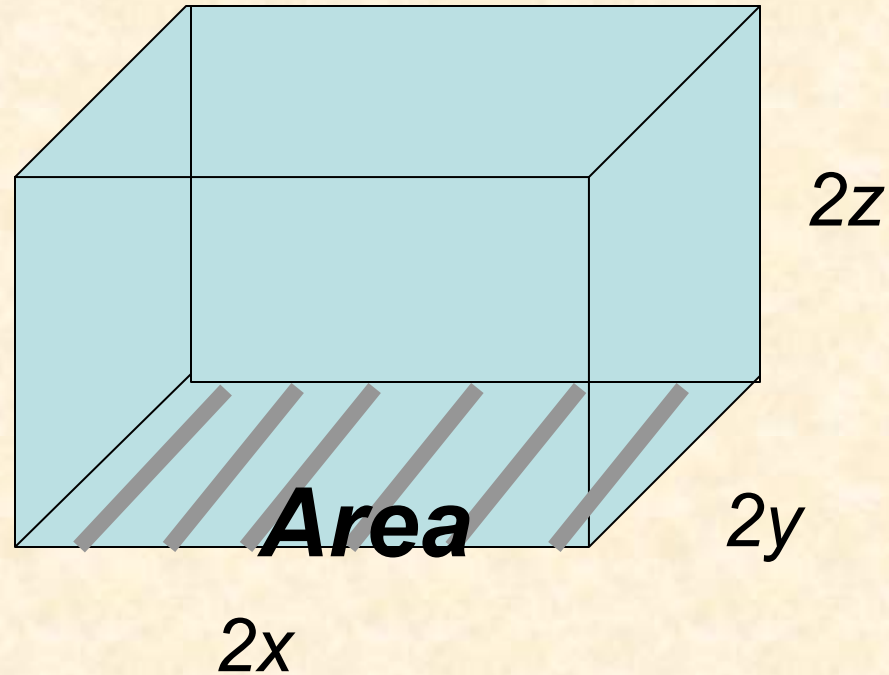mg

Air Resistance

R

m

mg

- Although we often ignore it, air resistance, R, is usually significant in real life.

- R depends on:
  - speed (approximately proportional to v^2)
  - cross-sectional area
  - air density
  - other factors like shape

- R is not a constant; it changes as the speed changes
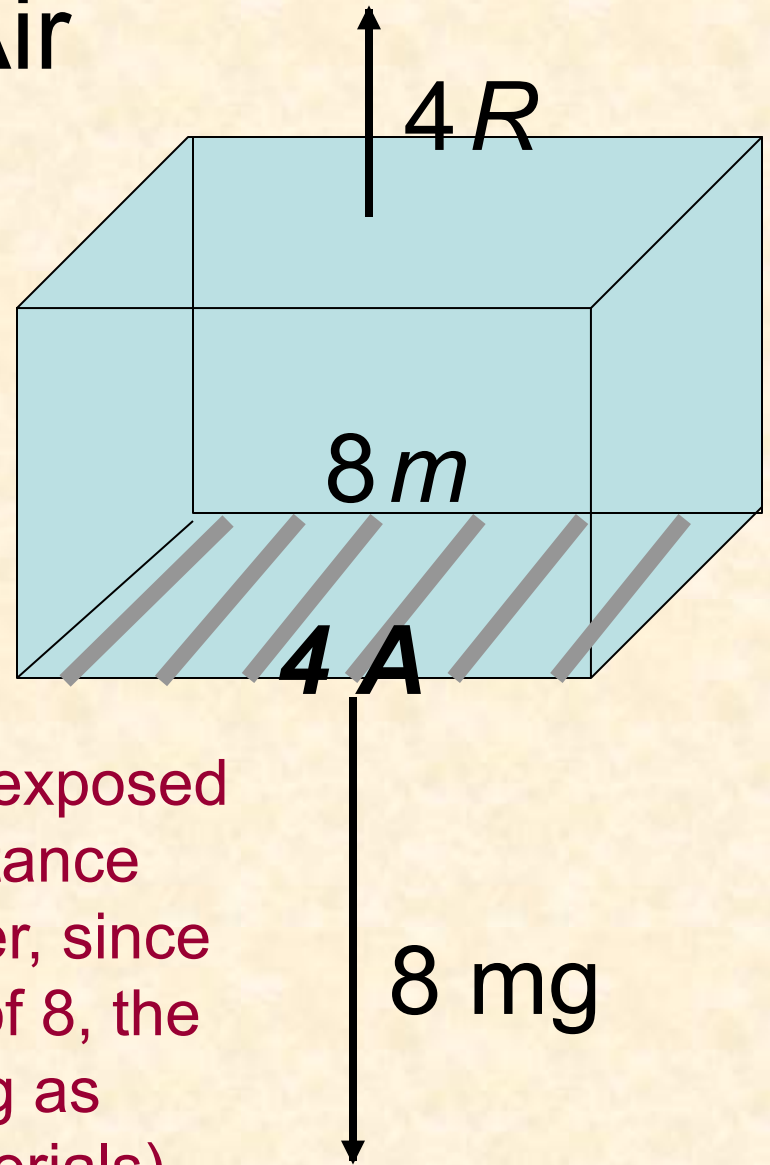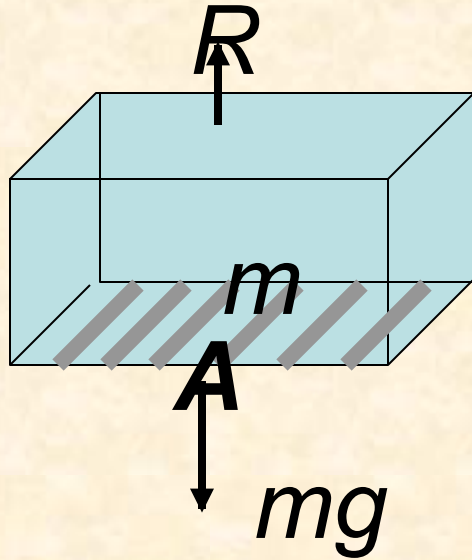
Volume & Cross-sectional Area

Volume = xyz
Area = xy

Volume = 8 xyz
Area = 4 xy

If all dimensions of an object are doubled the cross-sectional area gets 4 times bigger, but the volume goes up by a factor of 8.

Falling in Air

R

m

A

mg

$4\,R$

$8\,m$

$4\,A$

$8\ mg$

With all sides doubled, the area exposed to air is quadrupled, so the resistance force is 4 times greater. However, since the volume goes up by a factor of 8, the weight is 8 times greater (as long as we're dealing with the same materials).

Terminal Velocity

Suppose a daredevil frog jumps out of a skyscraper window. At first $v = 0$, so $R = 0$ too, and $a = -g$. As the frog speeds up, R increases, and his acceleration diminishes. If he falls long enough his speed will be big enough to make R as big as mg. When this happens the net force is zero, so the acceleration must be zero too. This means this frog's velocity can't change any more. He has reached his terminal velocity. Small objects, like raindrops and insects, reach terminal velocity more quickly than large objects.

R

mg