Agenda

Homework (reg)
Pg.142 #2.29
Pg.147 #2.32, 2.33

- Warm Up 10 min
- Checkup 5 min
  - *Check, copies*
- Introduce z 10 min
  - Activity w/ 2 types of nonsensical tests
  - Calculate mean & s.d., come back on E.P.
- Lecture, z-score 10 min
- Whiteboards, what's my z-score 20 min
- Using z-scores to calculate percentiles
- Exit Pass 5 min
  - *Update!*

Warm Up

Sketch a density curve that is:

1. Skewed left
2. Skewed right
3. Bimodal
4. Uniform
5. Symmetric but not uniform
6. Skewed right with an extreme outlier

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7. Johnny scores in the 65^{th} percentile on the Unit 1 quiz. What does that mean?

Check-up time

Notes

Z-scores (super-important)

- The "z-score" of an individual is the number of standard deviations away from the mean.
- *The average American male weighs 170 pounds, with a standard deviation of 30 pounds. If I weigh 155 pounds, what is my z-score?*

$$z = \frac{(observation - mean)}{s.d.} \qquad z = \frac{(x - \bar{x})}{\sigma}$$

- <u>Only</u> with symmetric distributions. Why?

Example with quiz scores (P.2)

Awesomeness	
Josh	8
Taylor	6
Hayley	5
Syed	5
Shayanne	4
Aleftina	3
Ari	2
Allyson	2
Melissa	2
Karina	2
Anthony	0
Kayla	0

Mean	**3.25**
St.dev	**2.31**

Mean	**3.82**
St.dev	**1.41**

IQ	
Joey	6
Lisette	5
Darya	5
Gurpreet	5
Jamie	4
Spencer	4
Beloved	4
Claudia	3
Riya	3
Camille	2
Daniel	1

Door/Window....What's my z-score?

- Tell partner:
  - "The distribution has mean ___ and standard deviation ___."
  - "I'm ____."
  - "What is my z-score?"
- Each person practice <u>five times</u>.
- *Example.*

Do both.

Mr. Colligan weighed 13.9 lbs at 3 months. The national average for the weight of a 3-month baby is 12.5 lbs, with standard deviation 1.5 lbs.

1. Determine the *z*-score of Mr. Colligan's weight.

For a 6-month-old, the national average weight is 17.25 lbs, with a standard deviation of 2.0 lbs.

2. Determine what Mr. Colligan's weight would have been at age 6 months for him to have the same *z*-score that he did at 3 months.

Door/Window

Suppose that your score on an exam has a positive z-score compared to your classmates. Explain your answers to the following:

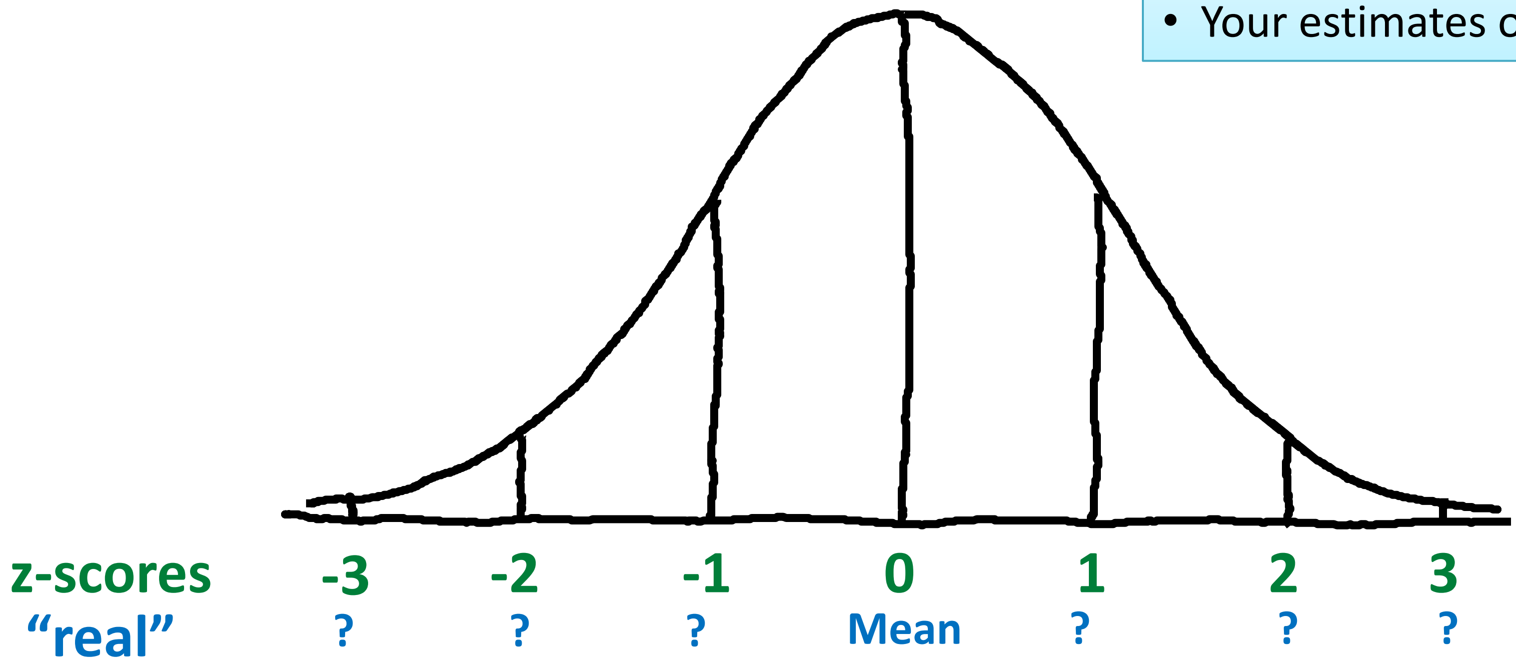
1. Is it possible that your score is below the mean score? Window
2. Is it possible that your score is below the median score? Door
3. Does this imply that your score is more than one standard deviation above the mean? Window
4. Is it possible that your score is the largest in the class? Door

Notes

"Normal" Distribution (use with *z*)

- Symmetric, single-peaked, bell-shaped
  - Sometimes called "bell curve"
- Important because it describes lots of real-world situations
- BUT…..non-Normal data are common. Don't assume Normality.
- Label with z-scores and real values

- Your guesses of my weight
- Your guesses of my age
- Your estimates of 1 minute



Is it Normal?

- Option 1
  - It says so in the problem. Done.
- Option 2
  - Construct a *normal quantile/prob. plot* (graph type #6)

    x axis → x   y axis → z

    Linear = Normal
- Option 3
  - Compare to Empirical rule

41	120
76	55
-10	7
-61	20
-13	95
22	-44
-16	1
-17	-37
8	-27
-50	-94
33	-57
-125	8
93	45
-84	-31
-10	69

Try it!

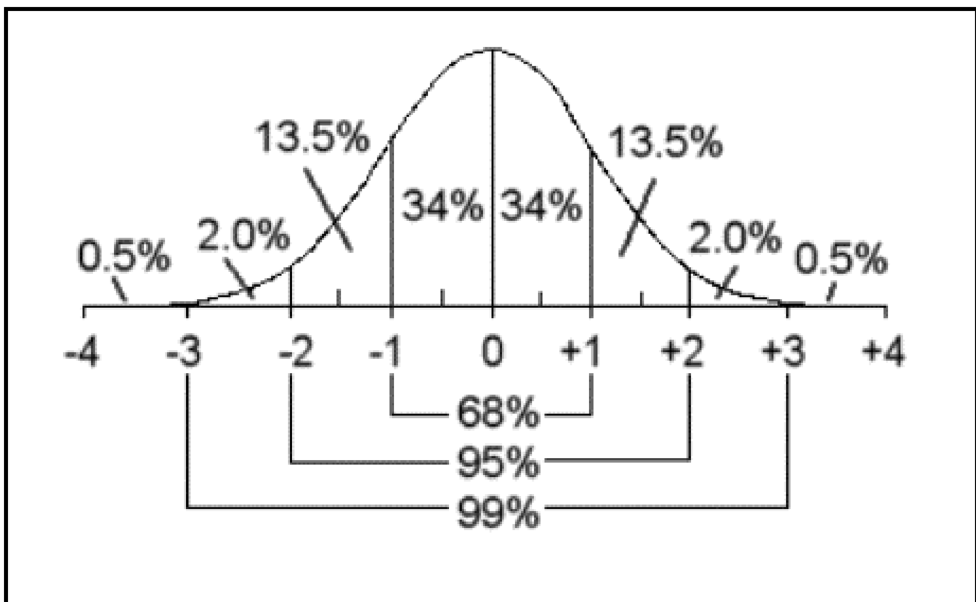
n=17

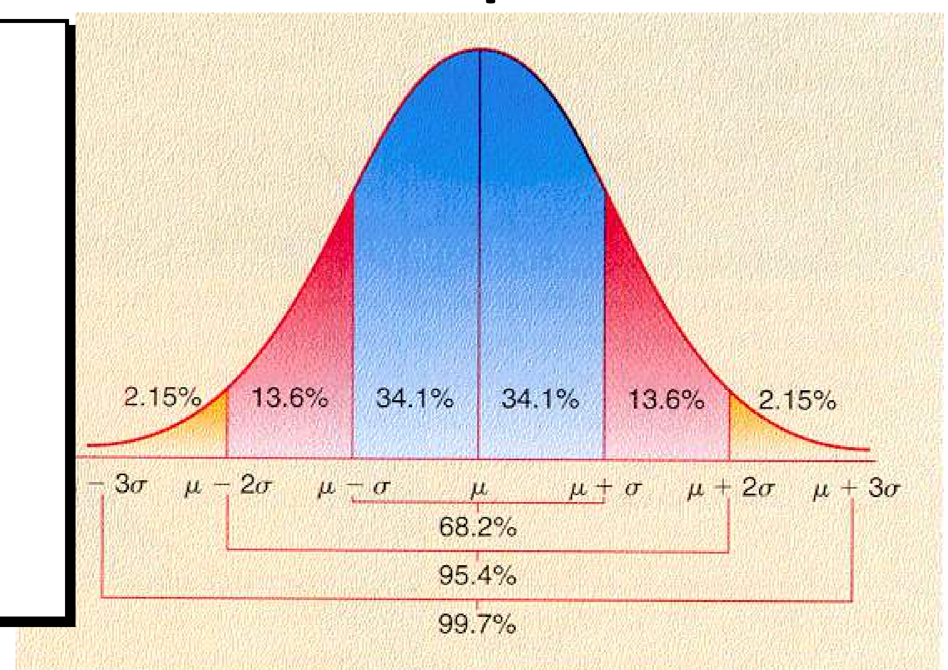
- Construct a normal quantile plot for the heights of my students.

56
58
61
61
62
62
63
63.5
65
66
67
67
67
67
70
72
75

Empirical Rule (68-95-99)

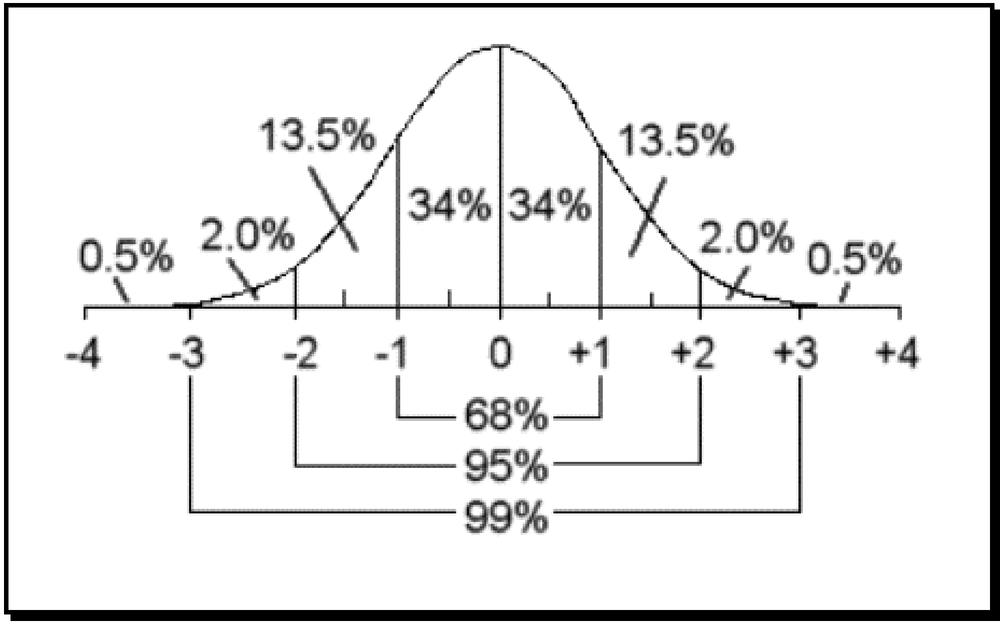
- In a Normal distribution with mean **μ** and standard deviation **σ**….
  - A fair amount of data **(68%)** fall within **one σ** of **μ**
  - Lots of data **(95%)** fall within **two σ** of **μ**
  - Most data **(99%)** fall within **three σ** of **μ**





Using *z*-scores to get proportions

- Each SAT section is curved so μ=500 and σ=100 (which is awesome for AP Statistics teachers)
- I earn 600 on the Math portion. What is the z-score?
  - What is the *probability* of earning at least 600 on the Math section of the SAT?
- What is the probability of earning at least 625?



Using *z*-scores to get proportions

- A chart of *standard Normal probabilities* will tell you the **area <u>below</u> any z-score**
- Area = proportion = probability = percentile

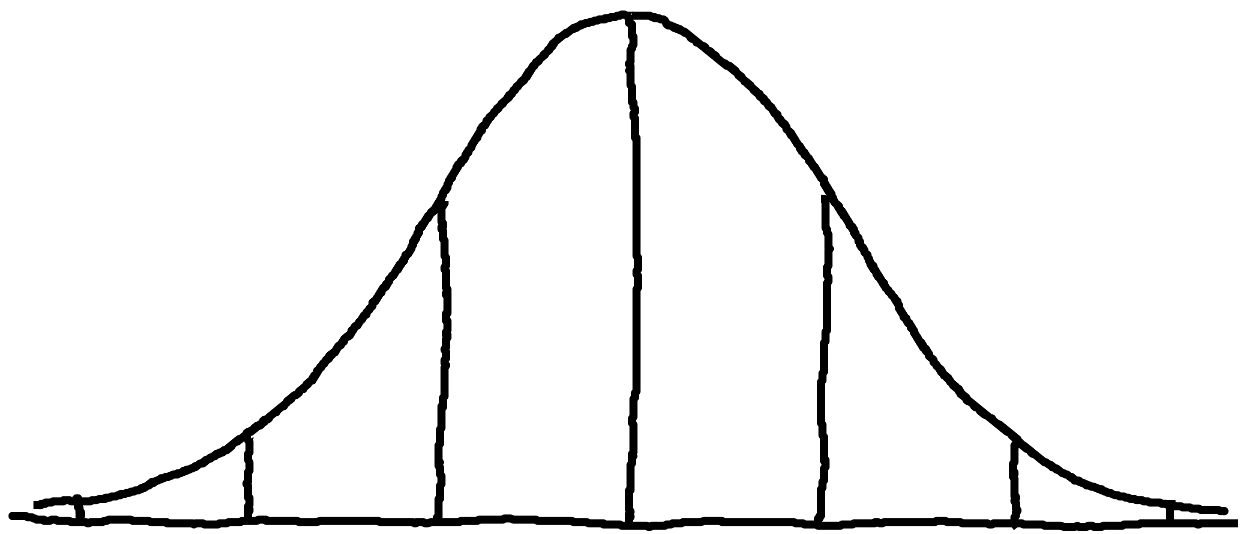


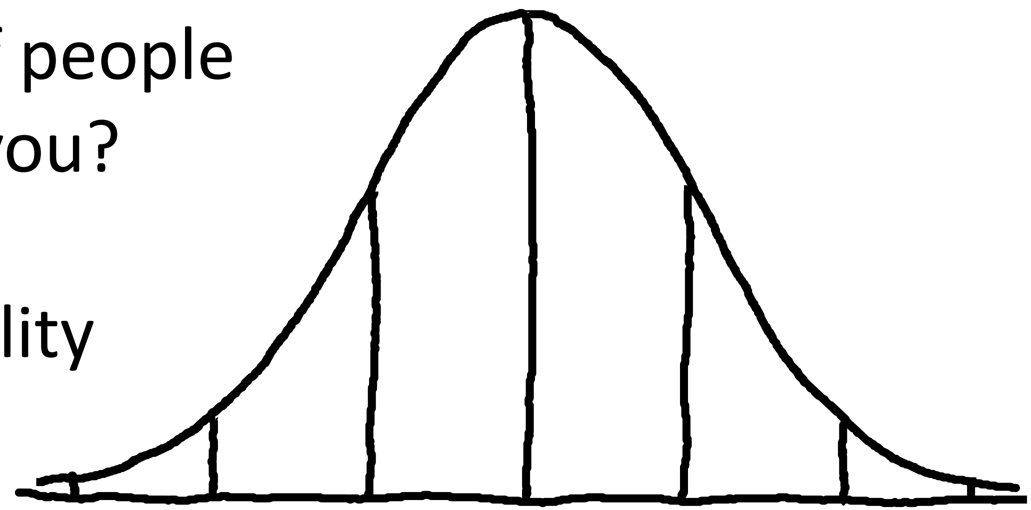
Table C (z-chart)

z	0	0.01	0.02	0.03	0.04	0.05
0	0.5	0.50399	0.50798	0.51197	0.51595	0.51994
0.1	0.53983	0.5438	0.54776	0.55172	0.55567	0.55962
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871
0.3	0.61791	0.62172	0.62552	0.6293	0.63307	0.63683
0.4	0.65542	0.6591	0.66276	0.6664	0.67003	0.67364
0.5	0.69146	0.69497	0.69847	0.70194	0.7054	0.70884
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215
0.7	0.75804	0.76115	0.76424	0.7673	0.77035	0.77337
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894
1	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314
1.1	0.86433	0.8665	0.86864	0.87076	0.87286	0.87493
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435
1.3	0.9032	0.9049	0.90658	0.90824	0.90988	0.91149
1.4	0.91924	0.92073	0.9222	0.92364	0.92507	0.92647
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943

Using *z*-scores (example)

Each SAT test is curved so the mean 500 with σ=100. Let's say you score 625 on the SAT Math.

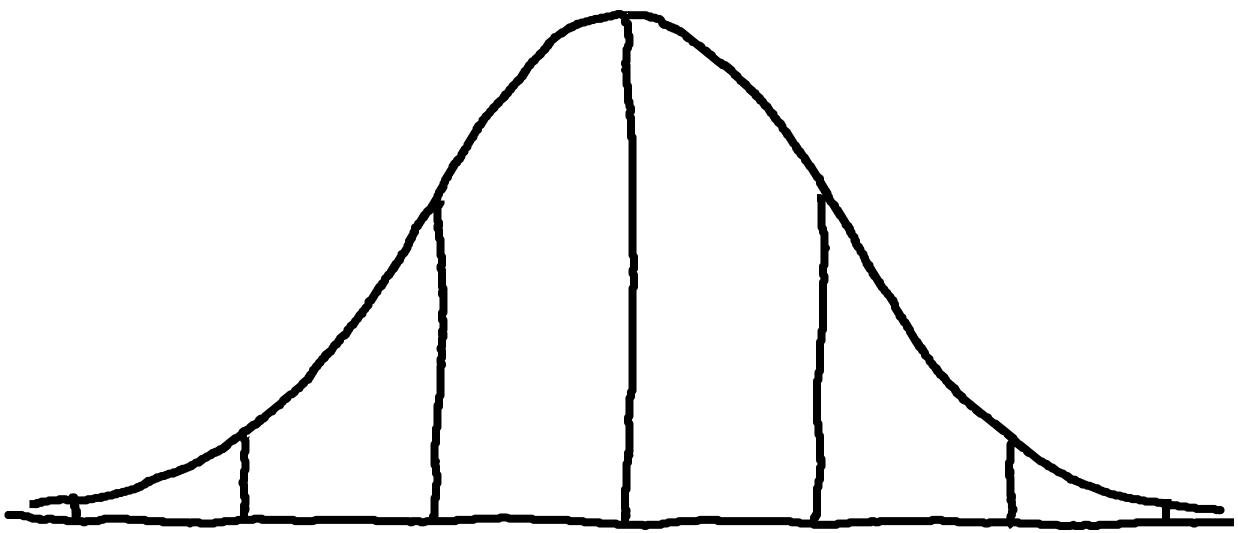
1. What is my z-score? $\dfrac{625-500}{100}=1.25$
2. What percentage of people scored worse than you?
3. What is the probability of doing better than 625?



Notes?

More examples

1. $z \leq 1.5$
2. $z \geq 1.5$
3. $-1.5 \leq z \leq 1.5$
4. $-0.7 \leq z \leq 1.5$



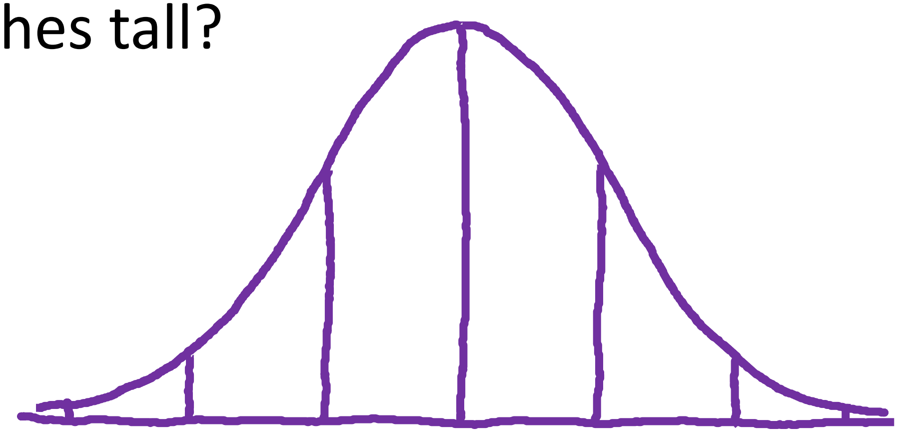
Door/Window

- Door: Say any ***two*** z-scores between -3 and 3
- Door/Window: Find the probability of falling within those two z-scores.
- Door/Window: Compare answers.
- Window: Say two new z-scores between -3 and 3.
- Four times total.

Notes?

Example of test problem

The heights of American men are approximately normally distributed, with a mean of μ=70 inches and a standard deviation of σ=3 inches. What percentage of American men are at least as tall as Mr. Colligan, who is 75 inches tall?



$$z = \frac{75-70}{3} = 1.67$$

Area under 1.67 → .9525

Area above 1.67 → 1 - .9525 = .0475

About 4.75% of American men are at least as tall as Mr. Colligan.

You try

The level of cholesterol in the blood is important because high cholesterol levels may increase the risk of heart disease. The distribution of blood cholesterol levels in a large population of people of the same age and sex is roughly Normal. For 14-year-old boys, the mean is $\mu=170$ milligrams of cholesterol per deciliter of blood (mg/dl) and the standard deviation is $\sigma=30$ mg/dl. Levels above 240 mg/dl may require medical attention. What percent of 14-year-old boys have more than 240 mg/dl of cholesterol?

1. Draw and label a Normal curve.
2. Use the table to calculate the z-score and associated percentile.
3. State your conclusion in context.

Door/Window Competition

- Each of you writes a z-score on your whiteboard
- Show each other
- Calculate the area between those z-scores.
- Winner → First with the correct answer written on their whiteboard.
- 3 rounds

Exit Pass (P.2)

Homework (reg)
Pg.142 #2.29
Pg.147 #2.32, 2.33

1. What is the standardized (z) score you earned on your quiz from the beginning of class? Also state which quiz you took, “IQ” or “Awesome”.
2. What would you have to score to earn a z-score of 0 on your quiz?
3. What would you have to score to earn a z-score of 2 on your quiz?

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Example with quiz scores (P.2)

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Beloved	4
Claudia	3
Riya	3
Camille	2
Daniel	1