FOUNDATIONS OF ALGEBRA
Lesson 1: Variables, expressions, and properties

LESSON 1: OPENER

1. Follow these steps and record your work.

Directions		How your number changes
Step 1.	Write down your age in years.	
Step 2.	Multiply your age by 2.	
Step 3.	Add 10.	
Step 4.	Multiply by 5.	
Step 5.	Add the number of brothers and/or sisters you have.	
Step 6.	Subtract 50. The result is your ending number.	

2. Compare your ending number with your partner's ending number. What do you notice when you look at the final answer?

3. Discuss with your partner why this particular magic number puzzle works. Then record your ideas in the space provided. It's okay if your ideas are only educated guesses.

LESSON 1: CORE ACTIVITY

1. Here is a new magic number puzzle.

    a. In each column, write a different starting number in Step 1, then record your work for the remaining steps and note your ending number.

Directions	Pick any starting number	Pick a different starting number	Pick a negative number	Pick a fraction
Step 1. Write down a number.				
Step 2. Double it.				
Step 3. Add 4.				
Step 4. Divide by 2.				
Step 5. Subtract your original number.				

    b. What is always true about the ending number in this puzzle, no matter what number you start with?

2. The following table contains the steps for a new magic number puzzle.

   a. For each column, follow the steps to find the ending number for the given starting number.

Directions	How your number changes		
			Pick your own number ↓
Step 1. Write down a number.	2	4	-7	
Step 2. Multiply by 3.	6			
Step 3. Add 12.	18			
Step 4. Divide by 3.	6			

   b. What is always true about the ending number in this puzzle, no matter what number you start with?

3. For the following magic number puzzles, complete the column "For any number, *n*."

   a.

Directions	For any number, *n*
Step 1. Write down a number.	*n*
Step 2. Multiply by -2.	
Step 3. Add 5.	

   b.

Directions	For any number, *n*
Step 1. Write down a number.	*n*
Step 2. Add 5.	
Step 3. Multiply by -2.	

4. For each of the following shape equations, determine whether the equation is true or false. Provide an explanation of why you believe it is true or false. You may want to use numerical examples to support your argument.

a.	◆ + ■ $\overset{?}{=}$ ■ + ◆	(circle one): **True** **False** Explanation:
b.	◆ − ■ $\overset{?}{=}$ ■ − ◆	(circle one): **True** **False** Explanation:
c.	◆ × ■ $\overset{?}{=}$ ■ × ◆	(circle one): **True** **False** Explanation:
d.	◆ ÷ ■ $\overset{?}{=}$ ■ ÷ ◆	(circle one): **True** **False** Explanation:
e.	(◆ × ■) × ● $\overset{?}{=}$ ◆ × (■ × ●)	(circle one): **True** **False** Explanation:
f.	(◆ + ■) + ● $\overset{?}{=}$ ◆ + (■ + ●)	(circle one): **True** **False** Explanation:

LESSON 1: CONSOLIDATION ACTIVITY

Example 1:
Follow the steps in the table to find the magic number expression.

Directions	For any number, n
Step 1. Choose a number.	n
Step 2. Multiply by 2.	$2n$
Step 3. Add 3.	$2n + 3$
Magic number expression: $2n + 3$	

Example 2:
Write the magic number expression that results from following these directions:
Subtract 4 from any number, then divide your answer by 3.

Answer: $\dfrac{n - 4}{3}$

1. Follow the steps in each table to find the magic number expression. (Refer to Example 1.)

Directions	For any number, n
Step a. Choose a number.	
Step b. Multiply by -2.	
Step c. Subtract 9.	
Magic number expression:	

Directions	For any number, n
Step a. Choose a number.	
Step b. Subtract 7.	
Step c. Divide by 5.	
Magic number expression:	

2. Write the magic number expression that results from each set of directions. (Refer to Example 2.)

    a.   Multiply any number by 2, then add 4.

    b.   Add 4 to any number, then multiply your answer by 2.

    c.   Divide any number by 5, then add 6.

    d.   Add 6 to any number, then square your answer.

    e.   Subtract 5 from a starting number, then divide your
          answer by that starting number.

3. Alisha and Brianna completed magic number puzzles for homework. Although their puzzles had the same steps, the two girls got different results. The work for each puzzle is shown side-by-side. For each set of puzzles, tell whose result is correct, then explain why that result is correct. Your explanation should include a numerical example that follows the steps in the puzzle to support your case.

a.

Alisha

Directions	For any number, n
Step 1. Write down a number.	n
Step 2. Add -7.	$n - 7$

Brianna

Directions	For any number, n
Step 1. Write down a number.	n
Step 2. Add -7.	$7 - n$

Whose result is correct? Be sure to provide an explanation with example(s) to support your case:

b.

Alisha

Directions	For any number, n
Step 1. Write down a number.	n
Step 2. Add -5.	$n + (-5)$

Brianna

Directions	For any number, n
Step 1. Write down a number.	n
Step 2. Add -5.	$-5 + n$

Whose result is correct? Be sure to provide an explanation with example(s) to support your case:

c.

Alisha

Directions	For any number, n
Step 1. Write down a number.	n
Step 2. Add 8.	$n + 8$
Step 3. Subtract the starting number.	$n + 8 - n$

Brianna

Directions	For any number, n
Step 1. Write down a number.	n
Step 2. Add 8.	$n + 8$
Step 3. Subtract the starting number.	8

Whose result is correct? Be sure to provide an explanation with example(s) to support your case:

LESSON 1: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

1. The following table contains the steps for a new magic number puzzle.

    a. Complete the table columns for the starting numbers 3, -2, 0.5, and a number that you choose.

Directions		How Your Number Changes			
					Pick your own number
Step 1. Write down a number.		3	-2	0.5	
Step 2. Add 5.					
Step 3. Multiply by 2.					
Step 4. Subtract 10.					
Step 5. Divide by 2.					

    b. How is the starting number related to the ending number for this puzzle?

2. For these magic number puzzles, complete the column "For any number, *n*."

a.

Directions	For any number, *n*
Step 1. Write down a number.	*n*
Step 2. Multiply by 4.	
Step 3. Add -5.	

b.

Directions	For any number, *n*
Step 1. Write down a number.	*n*
Step 2. Multiply by -2.	
Step 3. Subtract 6.	

c.

Directions	For any number, *n*
Step 1. Write down a number.	*n*
Step 2. Subtract 6.	
Step 3. Multiply by -2.	

d.

Directions	For any number, *n*
Step 1. Write down a number.	*n*
Step 2. Multiply by 2.	
Step 3. Subtract the starting number.	

e.

Directions	For any number, *n*
Step 1. Write down a number.	*n*
Step 2. Add the starting number.	
Step 3. Add 3.	

f.

Directions	For any number, *n*
Step 1. Write down a number.	*n*
Step 2. Add the starting number.	
Step 3. Multiply by 2.	

3. This magic number puzzle shows how the steps are applied using a starting value of *n*, but the steps themselves are missing.
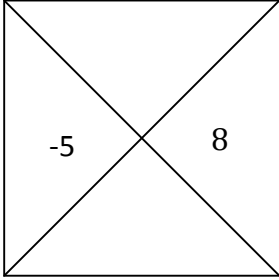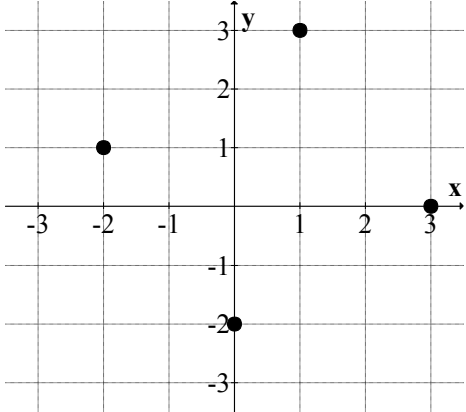
Directions	For any number, *n*
Step 1.	*n*
Step 2.	*n* + 6
Step 3.	4*n* + 24
Step 4.	4*n*

a. Write the missing directions for each step in the puzzle.

b. Describe in words how the starting number is related to the ending number for this puzzle.

4. Complete the following math journal. You can refer to your notes and work in the Student Activity Book, but be sure to state ideas in your own words and provide your own examples.

Vocabulary term	My understanding of what the term means	An example that shows the meaning of the term
a. Variable		
b. Algebraic expression		
c. Equation		
d. Commutative property		
e. Associative property		

LESSON 1: STAYING SHARP

Practicing skills & concepts

1. Complete the Square Box Problem.

-5   8

2. Follow the steps of this magic number puzzle for two starting numbers of your choice. One of your numbers should be negative. Also write the steps for any number, **n**, in column 3.

Directions	Results
Write down a number.	
Add -5.	
Add the original number.	

How is the ending number related to the starting number?

Preparing for upcoming lessons

3. Write the next three terms of the following sequence:

160, 136, 112, 88, …

Explain your reasoning or the pattern you found:

4. Enter the coordinates of the points plotted on this graph into the table.

x	y

Reviewing ideas from earlier grades

5. Find the value of each expression:

$-12 \times -2 =$

$9 + -3 =$

$12 - (-2) =$

$-7 - 3 =$

$-9 \div 3 =$

$2 \div (-2) =$

$4 \times -11 =$

$5 + -6 =$

$27 \div -9 =$

$-5 \times -20 =$

$-6 + -17 =$

6. Complete each sentence about adding signed numbers.

a. The sum of two positive numbers is…

b. The sum of two negative numbers is…

c. The sum of a positive number and a negative number is positive when…

d. The sum of a positive number and a negative number is negative when…

Lesson 2: The distributive property

LESSON 2: OPENER

1. Alana and José completed magic number puzzles for homework. Although their puzzles had the same steps, Alana and José came out with different results. The work for each puzzle is shown side-by-side. Tell whose result is correct and explain why that result is correct. Your explanation should include a numerical example that follows the steps in the puzzle to support your case.

Alana

Directions	For any number, n
Step 1. Write down a number.	n
Step 2. Add 3.	$n + 3$
Step 3. Multiply by 4.	$4n + 3$

José

Directions	For any number, n
Step 1. Write down a number.	n
Step 2. Add 3.	$n + 3$
Step 3. Multiply by 4.	$4n + 12$

Whose result is correct? Be sure to provide an explanation with example(s) to support your case:

LESSON 2: CORE ACTIVITY

Answer the following questions using the distributive property.

> You can state the **distributive property of multiplication over addition** algebraically by using variables:
> $$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$$
> The distributive property tells you that $a \cdot (b + c)$ and $(a \cdot b) + (a \cdot c)$ are **equivalent algebraic expressions**.

1. Use the distributive property to write the following expression in an equivalent form: 3 • (5 + 25).

2. Create an example based on making a purchase at a candy store to model the expression in question 1.

3. Use the distributive property to write the following expression in an equivalent form: (6 • 8) + (6 • 10).

4. Create an example based on making a purchase at a candy store to model the expression in question 3.

You can state the **distributive property of multiplication over subtraction** algebraically:

$$a \cdot (b - c) = (a \cdot b) - (a \cdot c)$$

The distributive property tells you that $a \cdot (b - c)$ and $(a \cdot b) - (a \cdot c)$ are equivalent algebraic expressions.

5. Use the distributive property to write the following expression in an equivalent form: $9 \cdot (10 - 3)$.

6. Use the distributive property to write the following expression in an equivalent form: $(3 \cdot 10) - (3 \cdot 4)$.

7. Create your own situation that illustrates the distributive property of multiplication over subtraction. You may want to use the candy store situation as a model.

LESSON 2: CONSOLIDATION ACTIVITY

To illustrate the distributive property, Carla created a single rectangle modeling $5 \cdot 14$.

1. How does Carla's diagram illustrate the distributive property?
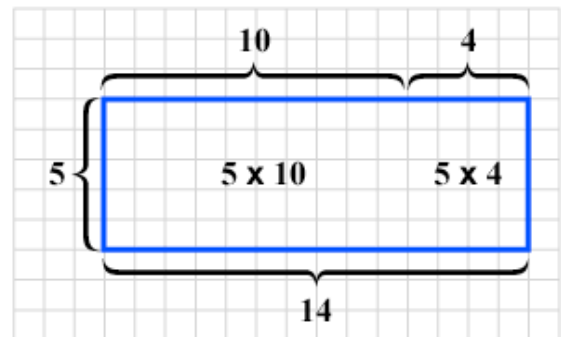
2. Using Carla's work as an example, create a single rectangle to model $7 \cdot 13$, illustrating the distributive property. Then write the equivalent expressions shown by your model.

Equivalent expressions:
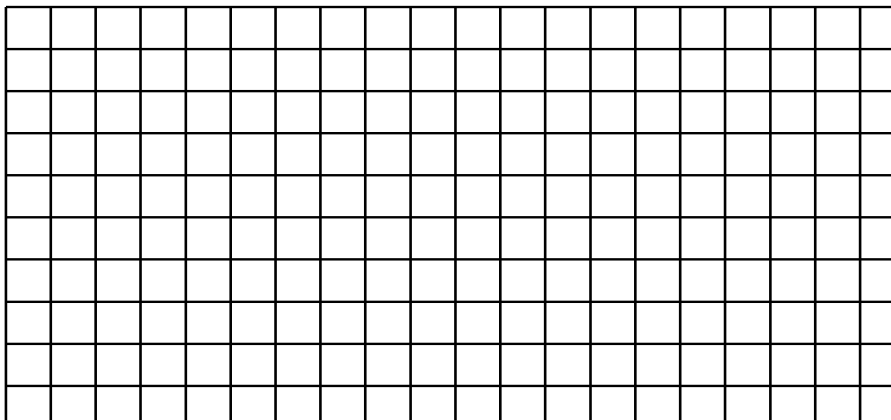
3. Rectangle models of the distributive property are often called **area models**. Why does the name **area model** make sense?

Next, you will use area models to apply the distributive property to find the product of a constant and a binomial. A **binomial** is an algebraic expression that has two terms. For example, $x + 5$ and $4x - 7$ are both binomials.

Marty and Jim made area models to show $4(x + 8)$.

Marty's Model   **Jim's Model**
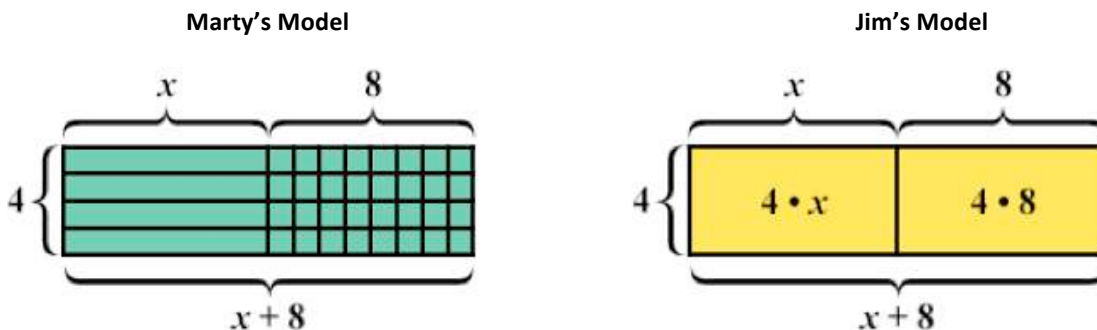
4. How does each model show that $4(x + 8) = (4 \cdot x) + (4 \cdot 8) = 4x + 32$?

5. Create an area model to represent the product of $3(x + 4)$. Sketch your model using the rectangle provided. Clearly label the factors and record the product.

6. Lara wants to multiply $5(3x - 10)$. She says, "I know how to use an area model for addition, but not for subtraction." Without drawing an area model, use what you have learned about the distributive property to explain to Lara how to find the product of $5(3x - 10)$.

7. Amanda claims to know two amazing multiplication tricks. They are shown below. Are these really tricks, or can they be explained by mathematics, as you did with magic number puzzles? Use mathematics to explain why Amanda's multiplication tricks work.

   a. To find 101 times any number, multiply the number by 100, then add the original number.

   b. To find 99 times any number, multiply the number by 100, then subtract the original number.

LESSON 2: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Alisha and Brianna are working on more magic number puzzles. Again, the puzzles have the same steps, but Alisha and Brianna come up with different results. The girls' work for each puzzle is shown side-by-side. For each set of puzzles, tell whose result is correct and explain why that result is correct.

a.

Alisha

Directions	For any number, n
Step 1. Write down a number.	n
Step 2. Add 2.	$n + 2$
Step 3. Multiply by 3.	$3n + 2$

Brianna

Directions	For any number, n
Step 1. Write down a number.	n
Step 2. Add 2.	$n + 2$
Step 3. Multiply by 3.	$3n + 6$

Whose result is correct? Be sure to provide an explanation with example(s) to support your case:

b.

Alisha

Directions	For any number, n
Step 1. Write down a number.	n
Step 2. Subtract 4.	$n - 4$
Step 3. Divide by 2.	$\dfrac{n - 4}{2}$

Brianna

Directions	For any number, n
Step 1. Write down a number.	n
Step 2. Subtract 4.	$n - 4$
Step 3. Divide by 2.	$n - \dfrac{4}{2}$

Whose result is correct? Be sure to provide an explanation with example(s) to support your case:

c.

Alisha

Directions	For any number, n
Step 1. Write down a number.	n
Step 2. Subtract 4.	n − 4
Step 3. Multiply by 7.	$7(n - 4)$

Brianna

Directions	For any number, n
Step 1. Write down a number.	n
Step 2. Subtract 4.	n − 4
Step 3. Multiply by 7.	$7n - 28$

Whose result is correct? Be sure to provide an explanation with example(s) to support your case:
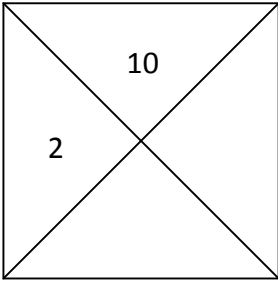
2. a. Use the distributive property to write the following expression in an equivalent form: 6•(10 + 25).

    b. Create an example based on making a purchase at a candy store to model the expression in part a.

    c. Use the distributive property to write the following expression in an equivalent form: (12 • 5) + (12 • 10).

    d. Create an example based on making a purchase at a candy store to model the expression in part c.

3. Create an area model to represent each of the following products. Sketch your model using the rectangles provided. Clearly label the factors and record the product.

    a. 3 • 13

    b. 3(x + 2)
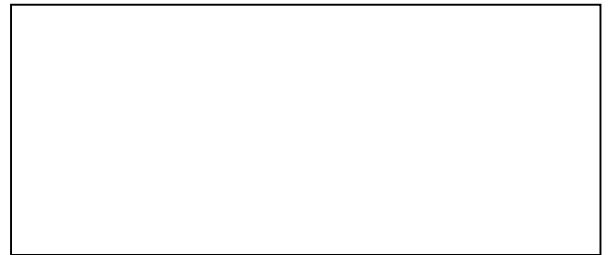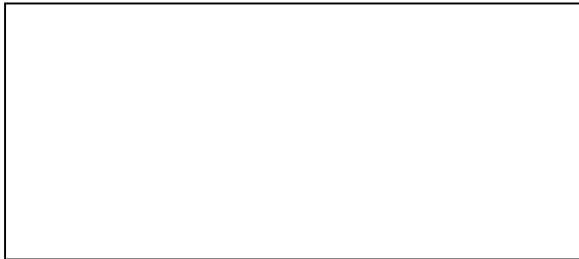
4. Complete the following math journal. You can refer to your notes and work in the Student Activity Book, but be sure to state ideas in your own words and provide your own examples.

Vocabulary term	My understanding of what the term means	An example that shows the meaning of the term
Distributive property		

LESSON 2: STAYING SHARP

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<td rowspan="2">Practicing skills & concepts</td>
<td>

1. Complete the Square Box Problem.

</td>
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2. Follow the steps of this magic number puzzle for two starting numbers of your choice. One of your numbers should be negative. Then write the steps for any number, **n**, in column 3.

Directions	Results		
Write down a number.			
Add 10.			
Multiply by 2.			
Subtract 2.			
Divide by 2.			

How is the ending number related to the starting number?

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<td>Preparing for upcoming lessons</td>
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3. Write the next three terms of the following sequence:

   4, 5, 7, 10, 14, …

   Explain your reasoning or the pattern you found.

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4. Use the following input-output rule to fill in the table.

   Rule: **Output** = -2(**Input**) + 4

Input	Process	Output
3		
-2		
1.5		
-1		
5		

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<td>Reviewing ideas from earlier grades</td>
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5. Find the value that makes each statement true.

   $2 \cdot \underline{\quad} = 2$

   $-2 \cdot \underline{\quad} = -2$

   $2 \cdot \underline{\quad} = 1$

   $-2 \cdot \underline{\quad} = 1$

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6. Complete the following statements.

   a. When you multiply a number by 1…

   b. When you multiply a number by its reciprocal…

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Lesson 3: Working with algebraic expressions

LESSON 3: OPENER

1. For each shape equation, determine the missing value in the equation by substituting the values given for the shapes.

   a. If $\triangle = -3$ and $\pentagon = -7$, then $\triangle - \pentagon =$ _____.

   b. If $\pentagon = 2$, then $\pentagon^3 =$ _____.

   c. If $\triangle = 4$ and $\hexagon = 3$, then $\triangle^2 + \hexagon^2 =$ _____.

   d. If $\triangle = 4$ and $\hexagon = 3$, then $(\triangle + \hexagon)^2 =$ _____.

2. The following problems are similar, except one has parentheses and the other does not. Are the answers the same? Why or why not?

   a. If $\star = 2$ and $\diamond = 5$, then $(\star + \diamond) \cdot \star =$ _____.

   b. If $\star = 2$ and $\diamond = 5$, then $\star + \diamond \cdot \star =$ _____.

LESSON 3: CORE ACTIVITY

For her daughter's birthday, Ms. Francis is planning a party in a recreation room. She is considering different food, beverage, and entertainment options.

The following list of variables uses symbols to represent some quantities associated with the birthday party.

K	Number of **K**ids attending the party	A	Number of **A**dults attending the party
R	Number of chai**R**s for the party	T	Number of **T**ables for the party
P	Number of **P**izzas ordered for the party	C_P	**C**ost (in dollars) for one **P**izza
S	Number of cases (24 cans) of **S**oft drinks ordered for the party	C_S	**C**ost (in dollars) for a case of **S**oft drinks
B	Number of **B**unches of balloons ordered for the party	C_B	**C**ost (in dollars) for a **B**unch of balloons
D	Number of hours the **DJ** will play at the party	C_D	Total cost (in dollars) for the **DJ** hired for the party
H	Number of bags of c**H**ips ordered for the party	C_H	**C**ost (in dollars) for a bag of c**H**ips

1. Tell whether the algebraic expressions given below have practical meaning. If the answer is "Yes," then summarize the meaning of the algebraic expression.

	Algebraic expression	Practical meaning?		If "Yes," description of expression's practical meaning
a.	$H \cdot C_H$	Yes	No	
b.	$\dfrac{K}{C_S}$	Yes	No	
c.	$\dfrac{(K + A)}{T}$	Yes	No	

2. Invent three meaningful algebraic expressions of your own from the list of variables and enter them into the table below. Write a description of the practical meaning of each expression in the last column.

	Algebraic expression	**Description of expression's practical meaning**
a.		
b.		
c.		

3. Consider the expression $P \cdot C_P$.

   a. Write a description of the practical meaning of this expression in the context of the party situation.

   b. Evaluate the expression if $P = 7$ and $C_P = 7$.

   c. Evaluate the expression if $P = 9$ and $C_P = 5.50$.

4. Consider the expression $\dfrac{C_D}{D}$.

   a. Write a description of the practical meaning of the expression.

   b. Evaluate the expression if $C_D = 72$ and $D = 3$.

   c. Evaluate the expression if $C_D = 64$ and $D = 4$.

5. Write a description of the practical meaning of the expression $(P \cdot C_P) + (S \cdot C_S) + (H \cdot C_H)$.

LESSON 3: CONSOLIDATION ACTIVITY

The student council has decided to host a school dance. They are considering decoration, refreshment, and entertainment options. The following list of variables uses symbols to represent some quantities associated with the dance.

C_R	**C**ost for **R**efreshments for the dance	T	Number of **T**ables to set up at the dance
C_D	**C**ost for **D**ecorations for the dance	H	Number of c**H**airs to set up for the dance
C_L	**C**ost for **L**ighting for the dance	S	Number of **S**tudents who attend the dance
C_J	**C**ost for the D**J** to play at the dance	P	**P**rice of a student ticket to the dance
M	Number of teachers who attend the dance and **M**onitor the students		

1. Tell whether the algebraic expressions given below have practical meaning. If the answer is "Yes," summarize the meaning of the algebraic expression. If the answer is "No," explain why the expression does not make sense.

	Algebraic expression	Practical meaning?		Summary or explanation
a.	$S \bullet P$	Yes	No	
b.	$\dfrac{S}{C_J}$	Yes	No	
c.	$\dfrac{(S + M)}{T}$	Yes	No	

2. Consider the expression $S + M$.

   a. Write a description of the practical meaning of the expression in the context of the dance situation.

   b. Evaluate the expression if $S = 135$ and $M = 10$.

   c. Evaluate the expression if $S = 75$ and $M = 8$.

3. Consider the expression $\dfrac{H}{T}$.

   a. Write a description of the practical meaning of the expression in the context of the dance situation.

   b. Evaluate the expression if $H = 120$ and $T = 12$.

   c. Evaluate the expression if $H = 152$ and $T = 19$.

4. Consider the expression $(C_R + C_D + C_L + C_J)$.

   a. Write a description of the practical meaning of the expression in the context of the dance situation.

   b. Evaluate the expression if $C_R = 253$, $C_D = 105$, $C_L = 98$, and $C_J = 84$.

   c. Evaluate the expression if $C_R = 329$, $C_D = 78$, $C_L = 212$, and $C_J = 160$.

5. Consider the expression $(S \bullet P) - (C_R + C_D + C_L + C_J)$. Write a description of the practical meaning of the expression in the context of the dance situation.

6. Invent your own meaningful algebraic expression from the list of variables. Then, write a description of the practical meaning of the expression.

Algebraic expression	Description of expression's practical meaning

LESOSN 3: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

The freshman students of Frederick Douglass High School are going on a field trip to the movie theater to watch a documentary for their American History class. Below is a list of variables for things associated with the field trip. The meaning of each variable is defined in the table.

H_A	Number of students in **H**omeroom **A** going on field trip	D_L	Cost (in dollars) of a **L**arge soft **D**rink at the theater
H_B	Number of students in **H**omeroom **B** going on field trip	D_S	Cost (in dollars) of a **S**mall soft **D**rink at the theater
H_C	Number of students in **H**omeroom **C** going on field trip	N_L	**N**umber of ounces in a **L**arge soft drink
R	Number of teache**R**s going on field trip	N_S	**N**umber of ounces in a **S**mall soft drink
C	Number of parent **C**haperones going on field trip	P_L	Cost (in dollars) of a **L**arge **P**opcorn at the theater
T_A	Cost (in dollars) of an **A**dult movie **T**icket	P_S	Cost (in dollars) of a **S**mall **P**opcorn at the theater
T_S	Cost (in dollars) of a **S**tudent movie **T**icket	S	Number of **S**eats in the movie theater

Use this list of variables to answer questions 1 through 3.

1. Tell whether the algebraic expressions given below have practical meaning. If the answer is "Yes," then summarize the meaning of the algebraic expression.

	Algebraic expression	Practical meaning?		If "Yes,", description of expression's practical meaning
a.	$(R + C) \bullet T_A$	Yes	No	
b.	$\dfrac{D_L}{N_L}$	Yes	No	
c.	$R \bullet S$	Yes	No	
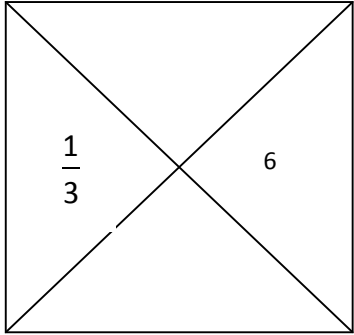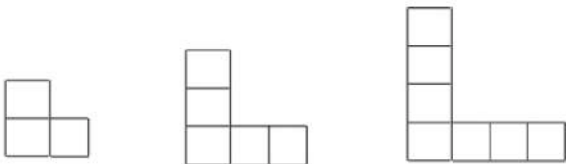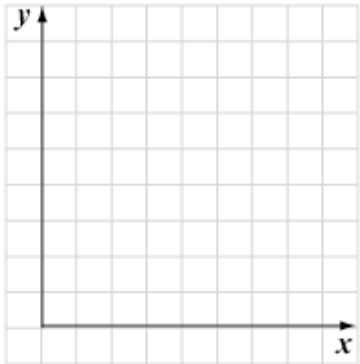
2. For each of the algebraic expressions below, write a description of its practical meaning, then evaluate it for the given values of the variables.

	Algebraic expression	Description of expression's practical meaning	Evaluate the expression for the given values of the variables
a.	$H_A \bullet P_S$		If $H_A = 25$ and $P_S = 2.50$
b.	$\dfrac{H_A + H_B + H_C}{R + C}$		If $H_A = 25$, $H_B = 21$, $H_C = 27$, $R = 3$, and $C = 6$
c.	$\dfrac{H_A + H_B + H_C}{S}$		If $H_A = 30$, $H_B = 28$, $H_C = 32$, and $S = 300$

3. Write an algebraic expression for each of the descriptions below.

	Description of expression's practical meaning	Algebraic expression
a.	Cost of buying a large drink for each student in Homeroom B	
b.	Money saved if each student in Homeroom C buys a small popcorn instead of a large popcorn	
c.	Total cost of tickets for the field trip	

LESSON 3: STAYING SHARP

Practicing skills & concepts

1. Complete the Square Box Problem.

$\frac{1}{3}$ 6

2. The steps of a magic number puzzle are:
- Pick a starting number.
- Multiply by 5.
- Subtract 2.

Follow the steps to complete this table.

Starting number	Ending number
0	
1	
4	
7	
10	
20	

Preparing for upcoming lessons

3. How many squares would be in term 12 of this pattern?

Term 1   Term 2   Term 3

Answer with supporting work:

4. Use the coordinates in the table to make a graph.

x	y
8	2
5	9
0	3
4	1

Reviewing ideas from earlier grades

5. Find the value of each expression. Then write an expression that has the same value, using addition of related numbers.

Expression	Value	Addition expression
7 – (-3)		
7 – 3		
59 – (-1)		
-17 – (-6)		

Complete the following statement: Subtracting a number is the same as adding its…

6. Find the value of each expression:

-1 - (-10) =

$3 \times (-3) =$

-3 - (-4) =

$6 \times (-11) =$

-12 ÷ -3 =

20 + -8 =

-4 • -5 =

28 ÷ -7 =

-18 + 10 =

7 - 15 =

Lesson 4: More about expressions

LESSON 4: OPENER

The Smithville City Council has decided to build a new recreation center. The city has already purchased a 450-foot by 600-foot lot on which to construct the recreation center. A plan of the lot is shown here.

1. What is the perimeter of the lot?   2. What is the area of the lot?

LESSON 4: CORE ACTIVITY

The city wants to ensure that the rooms in the recreation center are large enough that they can be used for lots of different purposes. The architect draws a floor plan to help the project manager make decisions about the dimensions of two of the rooms.
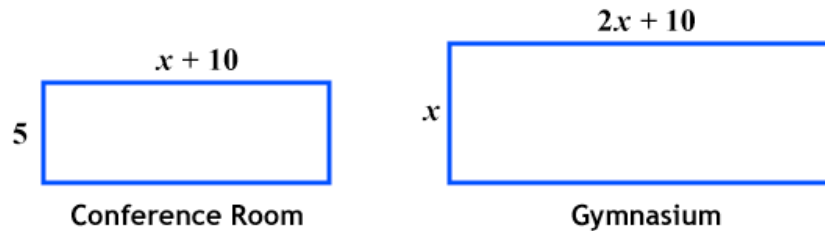
1. Determine the dimensions of the conference room and the gymnasium for different values of x. Then, find the perimeter and area.

Value of x	Room	Length of shortest side in feet	Length of longest side in feet	Perimeter	Area
10	Conference room				
	Gymnasium				
50	Conference room				
	Gymnasium				

2. Consider the conference room.

    a.   Write an algebraic expression for the perimeter of the conference room.

    b.   Check your expression by evaluating it for x = 50 and comparing the answer you get to the one in the table above.

3. Consider the gymnasium.

    a.   Write an algebraic expression for the perimeter of the gymnasium.

    b.   Check your expression by evaluating it for x = 50 and comparing the answer you get to the one in the table above.

4. Write an algebraic expression for the perimeter of each of the rooms shown below.
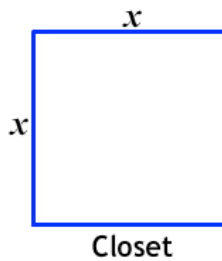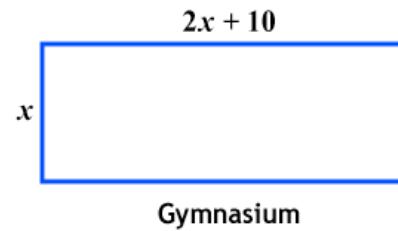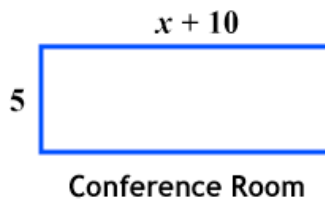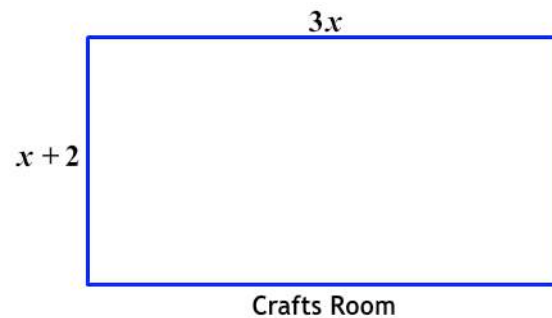
    a.   b.

    x

    x

    Closet

    $3x$

    $x + 2$

    Crafts Room
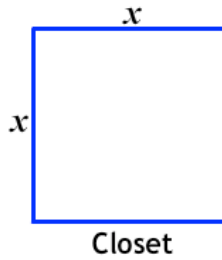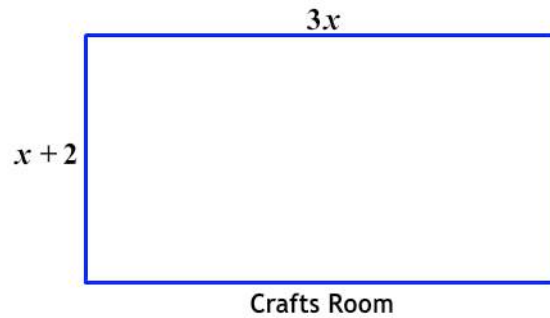
5. Write an algebraic expression for the area of each of the rooms shown below.

    a.   b.

    x

    x

    Closet

    $2x + 10$

    x

    Gymnasium

    c.   d.

    $x + 10$

    5

    Conference Room

    $3x$

    $x + 2$

    Crafts Room

6. Consider the gymnasium.

    a. Write the algebraic expression for the area of the gymnasium.

    b. Check your expression by evaluating it for x = 50 and comparing the answer you get to the one in the table from question 1.

LESSON 4: CONSOLIDATION ACTIVITY

PART I.

1. For each card numbered 1 through 12, find a match using the lettered cards. Note that there are extra lettered cards. Tape the matching cards together.

2. As a class, discuss the results of the matching activity. Once you agree to the answers for the matching activity, proceed to Part II.

PART II.

1. For each of the expression cards that you matched in Part I:

   a. Evaluate the expression for x = 10. Write the answer on the space next to Answer 1 on the card.

   b. Now, order your results from the highest numerical value to the lowest. Write the order of your cards in the space provided here.

Letter												
Value												

2. For each of the expression cards that you matched in Part I:

   a. Evaluate the expression for x = −1. Write the answer on the space next to Answer 2 on the card.

   b. Now, order your results from the highest numerical value to the lowest. Write the order of your cards in the space provided here.

Letter												
Value												

3. Was the order of the cards the same when you substituted x = 10 as when you substituted x = −1? Explain why or why not.

LESSON 4: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Consider the rectangle shown.

    a. Find the length of the rectangle if x = 10.

    b. Find the perimeter of the rectangle if x = 10.

    5

    c. Write an expression using the variable x to represent
        the perimeter of this rectangle.

    x + 2

    d. Evaluate your expression from part c for x = 10, then compare the result to your answer for part b.

2. Write expressions for the perimeter and the area
    of the rectangle.

    x

    x + 10
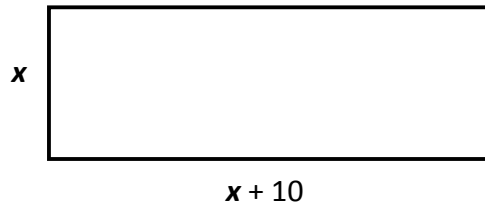
3. Write expressions for the perimeter and the area
    of the rectangle.

    12

    x − 2
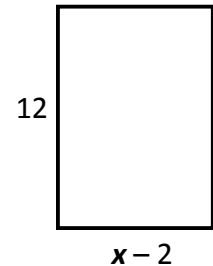
4. Sarah and John each found expressions for the perimeter of this rectangle. Their work is shown below.
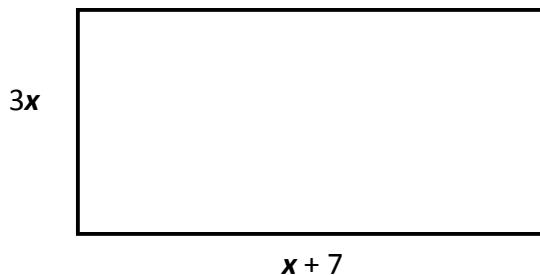
    3x

    x + 7

    Sarah:

    $P = 3x + x + 7 + 3x + x + 7$
    $\quad = 6x + 14$
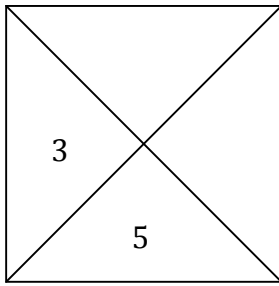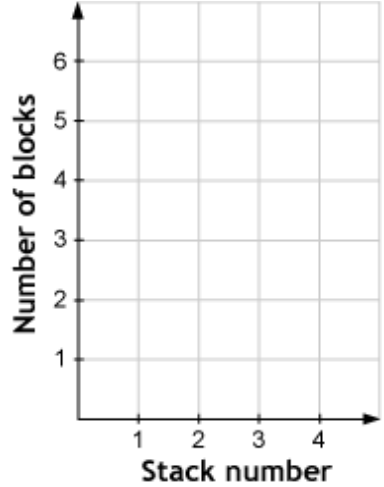
    John:

    $P = 2(x + 7) + 2(3x)$
    $\quad = 2x + 7 + 6x$
    $\quad = 8x + 7$

    Who is correct (Sarah, John, or neither)? Explain your answer, and correct all incorrect work.

LESSON 4: STAYING SHARP

Practicing skills & concepts

1. Complete the Square Box Problem.

2. The steps of a magic number puzzle are:
- Pick a starting number.
- Square the number.
- Subtract 2.

Follow the steps to complete this table.

Starting number	Ending number
1	
2	
3	
4	
7	
9	

Preparing for upcoming lessons

3. Plot points on the graph to represent the block pattern.

Stack Number 1   Stack Number 2   Stack Number 3

Reviewing ideas from earlier grades

4. Find the value that makes each statement true.

3 + ___ = 3	-3 + ___ = -3
3 − ___ = 3	-3 − ___ = -3
3 + ___ = 0	-3 + ___ = 0
5 + 11 = 11 + ___	7 − 1 = -1 − ___

5. Complete the following statements and answer the following questions.

a. When you add 0 to a number…

b. When you add a number to its opposite…

c. What is the relationship between $a + b$ and $b + a$?

d. What is the relationship between $a − b$ and $b − a$?

Lesson 5: Using expressions to represent relationships

LESSON 5: OPENER

Magic number puzzles use algebraic expressions to show how a starting number is related to an ending number. For example, if the expression for a puzzle is 3*n*, this tells you that the ending number is three times the starting number.

You can represent the relationship between the starting number and the ending number with a mathematical rule. For the puzzle with expression 3*n*, you can write the following rule:

Ending number = 3(**Starting number**)

1. Use the rule to complete the following statements:

   - If the starting number is 5, the ending number is _____.

   - If the starting number is -2, the ending number is _____.

   - If the starting number is 4.5, the ending number is _____.

   - If the starting number is *n*, the ending number is _____.

2. Organize the information from question 1 into a table.

Starting number	Ending number
5	
-2	
4.5	
n	

LESSON 5: CORE ACTIVITY

1. Use the rule to determine the output value for each input value in the table.

   Rule: **Output** = 3(**Input**) + 1

Input	Process	Output
-2		
4		
0		
1.5		
10		

2. Use the information in the table to determine whether the rule works for the given input-output pair. Put "Yes" if the rule works for the input-output pair and "No" if it does not. Once you have found a rule that works for all input-output pairs, write the rule on the line provided below the table.

Input	Output	*Output* = *Input* + 2	*Output* = 2(*Input*) - 2	*Output* = (*Input*)^2 + 2
4	6	_____	_____	_____
0	2	_____	_____	_____
-6	-4	_____	_____	_____
7.1	9.1	_____	_____	_____

   Rule: _____

3. Use the information in the table to determine whether the rule works for the given input-output pair. Put "Yes" if the rule works for the input-output pair and "No" if it does not. Once you have found a rule that works for all input-output pairs, write the rule on the line provided below the table.

Input	Output	*Output* = *Input* + 1	*Output* = -3(*Input*) + 9	*Output* = (*Input*)^2 − 1
2	3	_____	_____	_____
-5	24	_____	_____	_____
-1	0	_____	_____	_____
4	15	_____	_____	_____

Rule: _____

4. Look at the first row in the table below. Make a list of possible input-output rules that could represent the relationship between this input-output pair.

Input	Output
1	3
2	6
3	9
4	12
5	15

Possible rules:

5. What is the rule that represents the input-output relationship shown in the table in question 4? How do you know?

6. Write a rule for the input and output values in each table. Use the word *Input* in your rule.

a. Rule: *Output* =

Input	Process	Output
5		25
7		35
10		50
12		60

b. Rule: *Output* =

Input	Process	Output
-2		-6
3		9
0.5		1.5
7		21

c. Rule: *Output* =

Input	Process	Output
1		2
2		5
3		10
4		17
5		26

d. Rule: *Output* =

Input	Process	Output
-2		-15
0		-5
3		10
4		15
8		35

7. Jonah was given the two tables below and asked to find the rule that represents the input-output relationship for each table. The rules he came up with are written above each table. Are the rules correct? Explain.

a. Rule: Output = 3(Input) - 3

Input	Output
1	0
2	3
3	8
4	15

b. Rule: Output = $(Input)^2$

Input	Output
1	1
2	4
3	9
4	16

8. Three students came up with rules for the table below. Each rule is different, and the students are trying to decide who is right. Explain to the students who is correct and justify your response.

Input	Output
-1	-2
0	1
1	4
2	7

Student 1: Output = 3(Input) + 1

Student 2: Output = $-3(Input)^2$ + 1

Student 3: Output = (Input) + (Input) + (Input) + 1

LESSON 5: CONSOLIDATION ACTIVITY

1. Because there is only one row in each of the following tables, it is possible to come up with many possible input-output rules. For each table, state three possible input-output rules. Your rules should use the words **Input** and **Output**.

a.

Input	Output
4	1

Three possible rules:

b.

Input	Output
-2	4

Three possible rules:

c.

Input	Output
-3	-9

Three possible rules:

2. More rows have been added to each of the tables from question 1. Do any of the input-output rules you developed in question 1 still work? If so, explain how you know the rules work. If not, find an input-output rule that describes the relationship. Your rules should use the words **Input** and **Output**.

a.

Input	Output
4	1
6	3
7	4
10	7

Rule:

b.

Input	Output
-2	4
3	-6
8	-16
-5	10

Rule:

c.

Input	Output
-3	-9
-1	-1
0	0
2	-4

Rule:

3. Write a rule for the input and output values in each table below. Use the word **Input** in your rules.

    a. Rule: **Output** =

Input	Process	Output
1		4
2		9
3		14
4		19
5		24

    b. Rule: **Output** =

Input	Process	Output
1		2
2		5
3		10
4		17
5		26

4. There is a relationship between input and output in the tables below, even though the tables do not only involve numbers. Write an input-output rule in words to describe each relationship.

    a.

Input	Output
apple	e
egg	g
hour	r
and	d
eight	t

Rule (in words):

    b.

Input	Output
sing	16
chair	25
is	4
a	1
revolve	49

Rule (in words):

5. For the following tables, it is not possible to determine an input-output rule. For each case, explain why finding a rule is impossible.

    a.

Input	Output
3	6
2	4
5	10
3	8
2	6

Explanation:

    b.

Input	Output
1	5
6	8
6	5
24	8
15	5

Explanation:

LESSON 5: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Find a rule for the following table. Write your rule as a sentence relating the input and the output.

   Rule:

Input	Output
	59
	29
	39
	119
	No output
	39

2. Explain why it is not possible to find a rule for the following table.

Input	Output
-2	4
3	9
6	36
-2	9
8	64

   Explanation:

3. Because there is only one row in each of the following tables, it is possible to come up with many possible input-output rules. For each table, state three possible input-output rules. Your rules should use the words **Input** and **Output**.

   a.
Input	Output
-3	2

   Three possible rules:

   b.
Input	Output
-1	-5

   Three possible rules:

   c.
Input	Output
-1	2

   Three possible rules:

4. More rows have been added to each of the tables from question 2. Do any of the input-output rules you developed in question 2 still work? If so, explain how you know the rules work. If not, find an input-output rule that describes the relationship. Your rules should use the words **Input** and **Output**.

   a.
Input	Output
-3	2
-1	4
0	5
2	7
3	8

   Rule:

   b.
Input	Output
-1	-5
0	-1
1	3
2	7
3	11
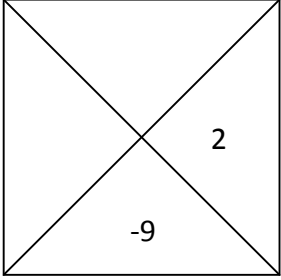
   Rule:

   c.
Input	Output
-2	8
-1	2
0	0
1	2
2	8

   Rule:

LESSON 5: STAYING SHARP

Practicing skills & concepts

1. Complete the Square Box Problem.

2

-9

2. Use the rule $5n - 2$ to complete the missing entries in the input-output table below.

Input	Output
0	-2
1	
4	
	33
	48
20	

Preparing for upcoming lessons

3. Let **n** represent the row number in this pattern. Write a rule that tells you the number of dots, **d**, in row **n**. (Hint: Your rule should begin with "**d =**".)

Row 1
Row 2
Row 3
Row 4

4. Describe the relationship represented by the table. Write your description in a complete sentence.

Hours rented	Total cost for rental
0	$40
1	$52
2	$64
3	$76

Reviewing ideas from earlier grades

5. Find the value of each expression:

$6 \cdot \text{-}11 =$

$\text{-}2 \times \text{-}15 =$

$\text{-}12 \div \text{-}4 =$

$11 - 12 =$

$\text{-}8 \times 9 =$

$36 \div \text{-}12 =$

$\text{-}5 - 6 =$

$\text{-}20 + \text{-}10 =$

$\text{-}30 \cdot 3 =$

$\text{-}7 + 40 =$

6. Consider what you know about multiplication of signed numbers. Then complete the following statements and answer the following question.

a. The product of two positive numbers is…

b. The product of a positive and a negative number is…

c. What is the relationship between the product $a \cdot b$ and the product $a \cdot -b$?

Lesson 6: More input-output relationships

LESSON 6: OPENER

Consider the following statement:

"If you take Cassandra's age, double it, and then add nine, you get her mother's age."

1. Give an answer for Cassandra's age and her mother's age that meets this condition.

2. Give a different set of ages for Cassandra and her mother that meets this condition.

3. Give one more set of ages for Cassandra and her mother that meets this condition.

4. Organize your information into an input-output table.

LESSON 6: CORE ACTIVITY

Four students in Cassandra's class made conjectures about her age and her mother's age based on the clue about Cassandra's age:

"If you take Cassandra's age, double it, and then add nine, you get her mother's age."

Cassandra's age (*C*)	Process	Her mother's age (*M*)
13	2•13 + 9	35
14	2•14 + 9	37
15	2•15 + 9	39
16	2•16 + 9	41

1. Write a rule to represent the relationship between Cassandra's age and her mother's age. Use *C* to represent Cassandra's age and *M* to represent her mother's age.

2. How does the rule you wrote relate to the statement "If you take Cassandra's age, double it, and then add nine, you get her mother's age"?

3. How does the rule relate to the process each student used to find Cassandra's mother's age?

4. Does this rule work for the data that you generated in the Opener? Explain.

5. Find a rule for each input-output table. State your rule in two different ways: (1) as a sentence; and (2) as a rule, using **x** for the input and **y** for the output.

a.

x	Process	y
2		-6
3		-9
0		0
-3		9
10		-30

Rule:

(1)

(2)

b.

x	Process	y
-0.5		7.5
0		8
0.5		8.5
1		9
1.5		9.5

Rule:

(1)

(2)

c.

x	Process	y
0		7
3		4
5		2
-5		12
-1		8

Rule:

(1)

(2)

6. Alex is ordering CDs from an online store. Each CD costs $5, and the shipping fee is $10, no matter how many CDs he orders. For example, if Alex orders 3 CDs, then his payment amount will be $25. Find four more possibilities for the number of CDs ordered, **C**, and the payment, **P**, and fill in the table below. Then write a rule relating the number of CDs ordered, **C**, to the payment amount, **P**.

C	Process	P
3		25

Rule:

7. Use the rule you wrote in question 6 to determine how much Alex will have to pay if he orders 11 CDs. Explain how you used the rule to find the answer.

LESSON 6: CONSOLIDATION ACTIVITY

1. For each table, select the rule that best models the input-output relationship. Explain how you know that the rule you selected is the correct rule.

a.

x	y
1	-1
2	-4
3	-7
4	-10
5	-13

Possible rules:
- ○ $y = x - 2$
- ○ $y = 3x - 4$
- ○ $y = -3x + 2$
- ○ $y = -x^2$
- ○ $y = -x^2 + 12$

b.

x	y
-1	-4
0	-5
1	-4
2	-1
3	4

Possible rules:
- ○ $y = x - 5$
- ○ $y = 2x - 2$
- ○ $y = 4x - 8$
- ○ $y = x^2 - 5$
- ○ $y = -4x^2$

2. Find a rule for each input-output table.

a.

x	Process	y
1		2
2		6
3		10
4		14
5		18
x		

Rule:

b.

x	Process	y
1		6
3		4
5		2
7		0
9		-2
x		

Rule:

c.

x	Process	y
0		-2
1		-1
2		2
3		7
4		14
x		

Rule:

3. Allison has a $20 gift certificate to her favorite clothing store. She wants to buy several pairs of jeans, each of which costs $15. Find five more possibilities for the number of pairs of jeans bought, **J**, and how much Allison will have to pay after the gift certificate, **P**, and fill in the table below. Then write a rule relating the number of pairs of jeans bought, **J**, to the amount Allison will have to pay after using the gift certificate, **P**.

J	Process	P
2		10

Rule:

4. Use the rule you wrote in question 3 to determine how much Allison will have to pay for 6 pairs of jeans after using her $20 gift certificate. Explain how you used the rule to find the answer.

LESSON 6: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Find rules for the tables below. State your rules in two different ways: (1) as a sentence; and (2) as a rule, using **x** for the input and **y** for the output.

a.

x	Process	y
0		-3
2		9
-3		-21
1		3
6		33

Rule:

(1)

(2)

b.

x	Process	y
-2		-8
-1		-1
0		0
3		27
5		125

Rule:

(1)

(2)

c.

x	Process	y
-1		3
1		-1
3		-5
3.5		-6
4		-7

Rule:

(1)

(2)

2. Nathan and his brother Zach love to play video games. Right now, Nathan's score is 11 less than 2 times Zach's score. For example, if Zach has 80 points, then Nathan has 149 points. Find four more possibilities for the brothers' scores and fill in the table below. Then write a rule relating Nathan's score, **N**, to Zach's score, **Z**.
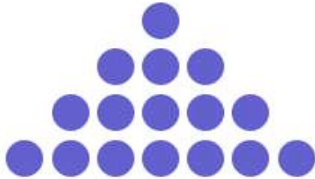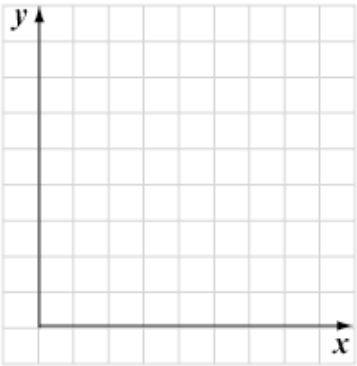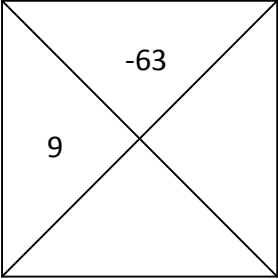
Z	Process	N
80		149

Rule:

3. Use the rule you wrote in question 2 to answer the following questions.

   a. How many points will Nathan have if Zach has 5072 points?

   b. Explain how you used the rule to find Nathan's score.

4. Sean mows lawns in the summer. He charges the same fixed amount for each lawn that he mows. As of today, he has earned $36 after mowing 3 lawns.

   a. Create an input-output table using the headings "Number of lawns mowed" for the input and "Amount of money in dollars" for the output. Fill in the appropriate values for Sean's current situation. Then fill in several other rows of information that fit the situation. For example, what if Sean has mowed 8 lawns? 12 lawns?

   b. Use any patterns you see in your input-output table to find a rule relating the number of lawns mowed and the amount of money in dollars.

   c. Explain how you know that your rule works.

5. Sean wants to go to a two-day music festival at the end of the summer. Tickets for the festival cost $190. How many lawns will Sean need to mow this summer to earn enough money to buy a concert ticket? Show how you found your answer.

LESSON 6: STAYING SHARP

1. The steps of a magic number puzzle are:
   - Pick a starting number.
   - Multiply by -3.
   - Add 7.

   Follow the steps to complete this table.

Starting number	Process	Ending number
1		
2		
3		
4		

2. Write the rule that represents the relationship between the starting number and the ending number from the magic number puzzle in question 1.

3. Let n represent the row number in this pattern. Write a rule that tells you the number of dots, d, in row n. (Hint: Your rule should begin with "$d =$".)

Row 1
Row 2
Row 3
Row 4

4. Use the rule below to fill in the table, then make a graph.

   $y = 2x - 5$

x	y
3	
4	
5	
6	

5. Simplify each expression:

   $x \cdot 10 =$

   $5 \times (-2) =$

   $-2 - (-7) =$

   $6x \cdot x =$

   $-15 \div 3 =$

   $15 + (-18) =$

   $(2x) \cdot (-2x) =$

   $36 \div -6 =$

   $-8 + 10 =$

   $15 - 7 =$

6. Complete the Square Box Problem.

   -63

   9

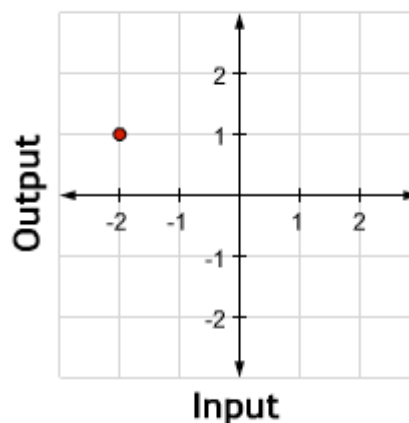# Lesson 7: Representing relationships with graphs

LESSON 7: OPENER

1. This graph shows data from an input-output table. Create the input-output table from the graph.

Input	Output
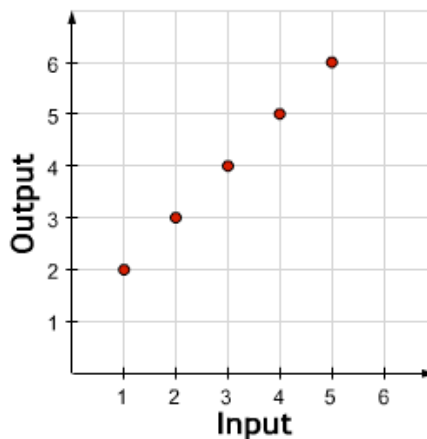
2. Make a graph of the data shown in this input-output table. The first point is plotted for you.

Input	Output
-2	1
-1	-1
0	-2
1	-1
2	1

LESSON 7: CORE ACTIVITY

Objective: To find the missing hiker who is stranded in the middle of the forest.

Instructions:

- Decide which partner will be the "Pilot" of the helicopter and which will be the "Dispatcher." It is important that your partner NOT see your paper while you work on this activity. You might want to sit back-to-back.

- Dispatcher: Mark an intersection point on your grid. Do not tell your partner the coordinates of this point! It represents the location of the missing hiker.

- Pilot: Try to guess the location of the hiker by giving a coordinate pair. If your pair is not correct, the Dispatcher will give you a clue that tells you which direction to move to get to the hiker. The direction should be a combination of east or west and north or south, or a single direction. Always give east or west first before giving north or south.

- A turn consists of a guess from the Pilot followed by a clue from the Dispatcher about which direction to move. Important: Each partner should plot all of the guessed pairs and record the guesses and the clues so that he or she can keep track of the progress toward finding the hiker.

- The Dispatcher and Pilot take turns until the Pilot finds the point at which the hiker is located. When the Pilot locates the hiker, the Dispatcher and Pilot compare their papers to be sure that they both plotted the guesses correctly and that the Pilot's final guess matches the hiker's location on the Dispatcher's grid. Then, switch roles and play again.

- **Wild card**: At any time during a game (and ONLY ONE TIME), the Pilot may ask the Dispatcher if the hiker is in a particular quadrant of the coordinate plane. For example, the Pilot may ask, "Is the hiker in quadrant II?" The Dispatcher will answer with either "yes" or "no," but he or she can give no other information in response to this question. The Pilot may NOT ask "What quadrant is the hiker in?"

When you have completed four games, discuss these reflection questions with your partner. Record your responses in the space provided.

1. You are the Pilot and guess (-3,-2). The Dispatcher gives you the clue "Go east." What does that tell you about the point at which the hiker is located?

2. You are the Pilot and guess (5,-4). The Dispatcher gives you the clue "Go south." What does that tell you about the point at which the hiker is located?

3. When you are the Pilot, where is a good place for your first guess? Why?

4. When during a game did it make sense to use the wild card?

5. Did you develop a strategy to find the location of the hiker in the fewest number of guesses? If so, what is it? What makes this a good strategy?

LESSON 7: CONSOLIDATION ACTIVITY

Each of these graphs describes the relationship between two variables having to do with airplanes. Even though there are no numbers on the graphs, you can use the positions of the points to describe characteristics of the airplanes. In your answers to the following questions, be sure to give **specific** evidence based on the graphs. Then check your answers with your partner. If you and your partner disagree, see if you can figure out whose answer makes the most sense.
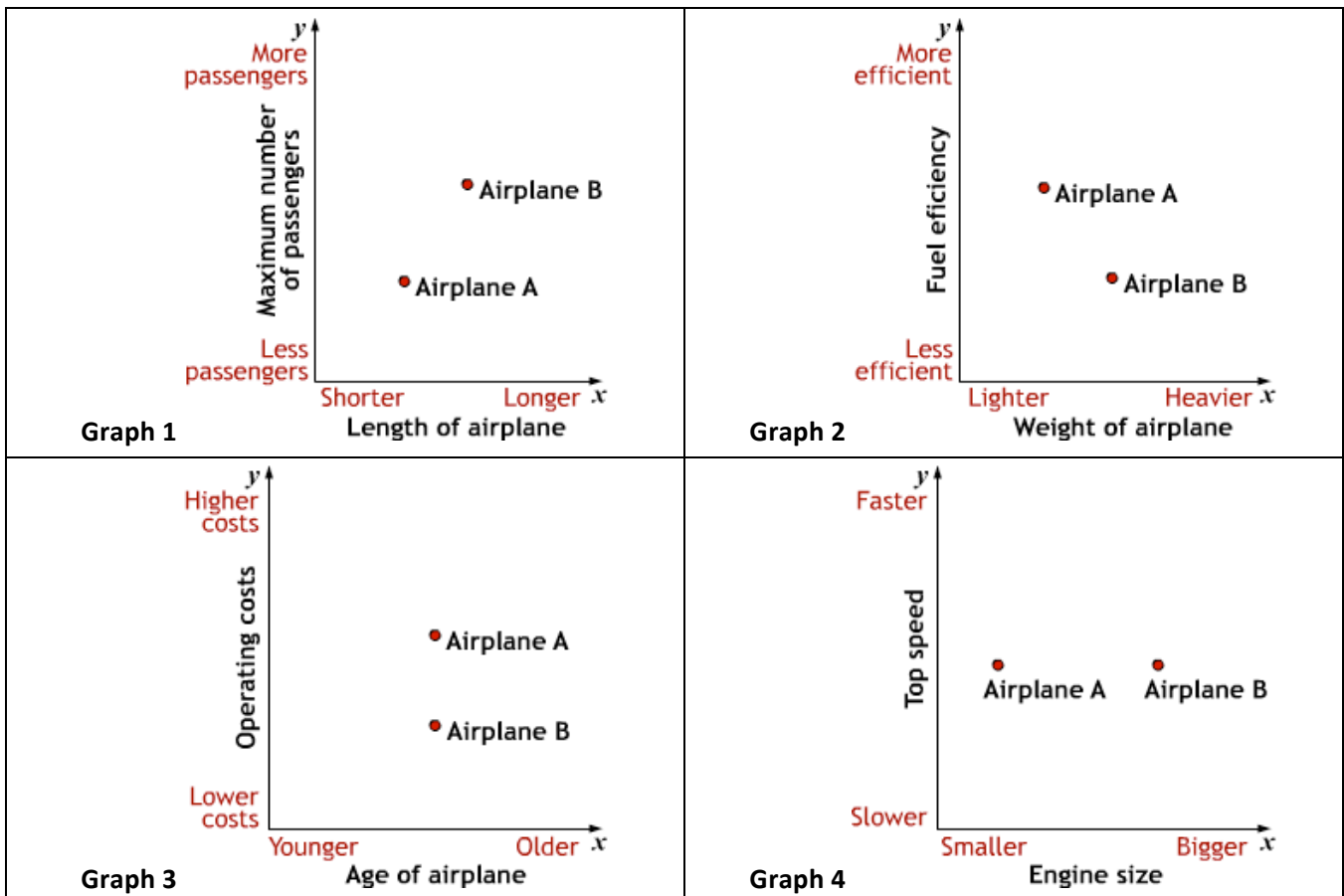
1. **Reading the graphs**

 a. Which plane is longer?

  Evidence for your answer:

 b. Which plane is more fuel efficient?

  Evidence for your answer:

 c. Which plane is older?

  Evidence for your answer:

 d. Which plane costs more to operate?

  Evidence for your answer:

 e. Which plane can fly faster?

  Evidence for your answer:

 f. Which plane has a bigger engine?

  Evidence for your answer:

2. **The stories of the graphs**

 a. Graph 1 describes the relationship between the length of two planes and the maximum number of passengers each plane can carry. Look at the axes on Graph 1 and the position of the points on the graph. What overall conclusion can be drawn from Graph 1 when considering the pattern of the plotted points?

  Share your interpretation with your partner. When you and your partner agree on a story for Graph 1, write it on your whiteboard.

 b. What overall conclusion can be drawn from Graph 2 when considering the pattern of the plotted points?

 c. What overall conclusion can be drawn from Graph 3 when considering the pattern of the plotted points?

 d. What overall conclusion can be drawn from Graph 4 when considering the pattern of the plotted points?

3. **True/false**

    Use the airplane graphs to determine whether the following statements are true or false. Circle the correct answer. If you circle **True**, explain how you know the statement is true. If you circle **False**, revise the statement so that it is true.

    a.   The heavier airplane is more fuel efficient.   True   False

    b.   The longer airplane carries more passengers.   True   False

    c.   The airplane with the bigger engine can fly faster.   True   False

    d.   Examine the airplane graphs. Write a **false** statement that compares Airplane A and Airplane B. Then show the statement to your partner and ask your partner to rewrite the statement so that it is correct.

         Your statement:

         Your partner's corrected statement:

    e.   Now write your own **true** statement about one of the graphs. Then ask your partner to review your statement. If your partner does not agree, they will provide you some instructions so that you can correct your statement. If your partner agrees that your statement is correct, he or she should sign the sheet below:

         Your statement:

         Partner instruction:

         I reviewed my partner's statement and agree that it correctly describes the graph.
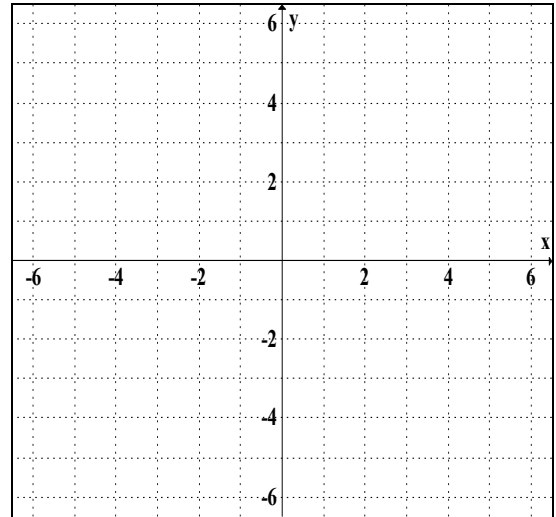
         Partner signature _____ Date _____

LESSON 7: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Plot each of the following points on the coordinate plane. Label each plotted point with its ordered pair.

    a. (-2,5)

    b. (1,3)

    c. (2,-3)

    d. (-4,-1)

2. On the same coordinate grid used for question 1, add a point in quadrant III that has an **x**-coordinate larger than the **y**-coordinate. Label the point with its ordered pair.

3. On the same coordinate grid used for question 1, add a point in quadrant IV with a **y**-coordinate that is the opposite of the **x**-coordinate. Label the point with its ordered pair.

4. This graph shows Tina's new cell phone plan. Use the graph to decide whether each of the following statements is true or false. Provide evidence for your answer.
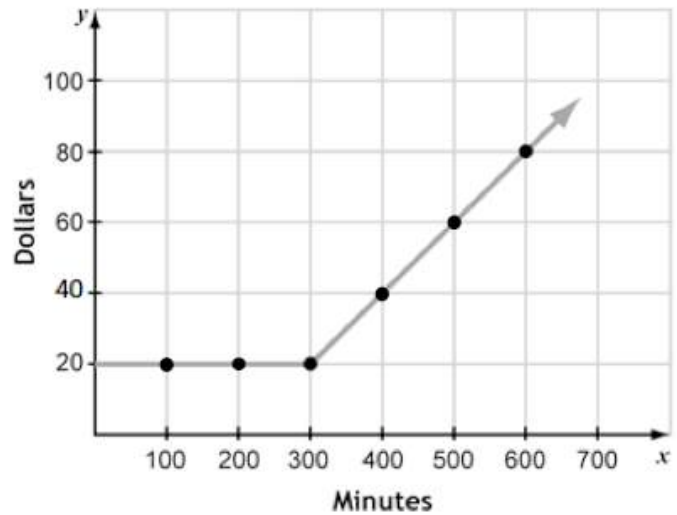
    a. If Tina talks fewer than 100 minutes a month, she does not have to pay anything.

    True / False

    Evidence:

    b. The plan has a $20 minimum monthly charge.

    True / False

    Evidence:

c. If Tina talks 500 minutes in one month, she will pay $100.

True / False

Evidence:

d. The plan includes up to 300 minutes at no additional charge.

True / False

Evidence:

e. The plan charges the same amount for 600 minutes as for 500 minutes.

True / False

Evidence:

f. The plan charges twice as much for 600 minutes as for 300 minutes.

True / False

Evidence:

5. A person's **arm span** is the distance measured from fingertip to fingertip when the person's arms are outstretched. Similarly, a bird's wingspan is the distance from the end of one wing to the end of the other wing when the bird's wings are outstretched.

Typically, a person's arm span is very close to the person's height. For example, if a person is 68 inches tall, his or her arm span will be close to 68 inches. But because everyone is different, a person's actual arm span will often be a little longer or a little shorter than his or her height.

This graph shows the relationship between the height and arm span of three people and two other kinds of animals. Using the clues provided, match each point with the correct person or animal. Put your answers in the table. Provide evidence for each answer.

Arm Span and Height
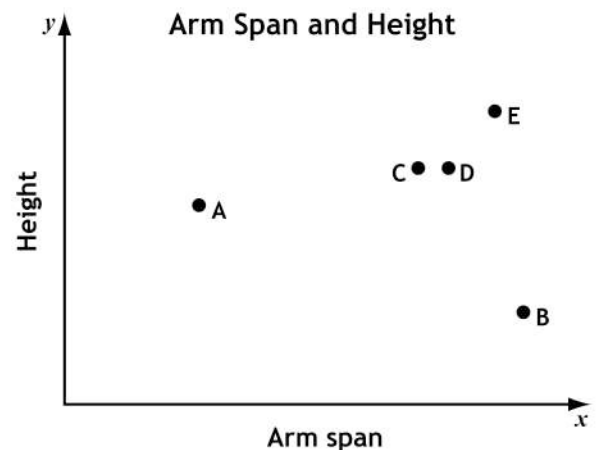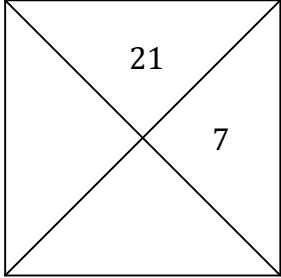
Clues:

- Yao Ming is a very tall basketball player who is about 7'5" tall. His arm span is said to be 7'4" long.

- Michael Phelps and Candace Parker are both Olympic gold-medal winners. They are both 6'4" tall, and both have arm spans that are longer than their height. However, Michael Phelps has an arm span that is longer than Candace Parker's arm span.

- Members of the largest species of kangaroo are often about 5 feet tall. These kangaroos have very short forelimbs (arms).

- Bald eagles are typically 29-42 inches long and have a wingspan of about 8 feet (96 inches). (Use the eagle's length as its "height" and its wingspan as its "arm span" when analyzing the graph.)

Person or animal	Point on graph	Evidence for answer
Yao Ming		
Candace Parker		
Michael Phelps		
Kangaroo		
Eagle		

LESSON 7: STAYING SHARP

Practicing skills & concepts

1. Complete the Square Box Problem.

21

7

2. Complete the following input-output table using the rule **Output** = (**Input**)^2 – 2.

Input	Output
1	
2	
3	7
	14
	47
9	

Preparing for upcoming lessons

3. Find the next three terms in the following sequence:

22, 21, 18, 13, 6, …

Explain your reasoning:

4. Draw a graph representing this situation:

"The Math Team practices weekly. There are 5 members at the first practice. Each week, there are 3 more people at practice, until 4 weeks later, there are 17 people at practice."

Reviewing ideas from earlier grades

5. Find the value of each expression.

-48 ÷ −6 =

-6 + 11 =

6 + (-8) =

-7 - 8 =

8 - (-8) =

6 × -12 =

-9 - (-6) =

25 ÷ -5 =

14•(-2) =

-45 + -15 =

6. Consider what you know about division of signed numbers. Then complete the following statements and answer the following question:

a. When dividing a positive number by a positive number, the quotient is…

b. When dividing a negative number by a positive number, the quotient is…

c. What is the relationship between the quotient $a \div b$ and the quotient $-a \div b$?

Lesson 8: Revisiting expressions

LESON 8: OPENER

Consider the expression 2*ab*.

1. Evaluate the expression for *a* = 2 and *b* = 5.

2. Evaluate the expression for *a* = 2 and *b* = -5

3. Find another set of values for *a* and *b* that gives the same result as in question 2.

LESSON 8: CORE ACTIVITY

1. Think about possible values for the expression $\dfrac{1}{d}$.

    Decide whether each number given is or is not a possible value for the expression. Write "Yes" if the given number is a possible value for the expression. Write "No" if the given number is NOT a possible value for the expression.

8	2	-1	-6	0	1/2	-1.25

2. **Expressions Game**

    Objective: To earn as many points as possible by determining whether it is possible for a given algebraic expression to have a particular value, then justifying your conclusion by either substituting a number into the expression that gives the desired value OR explaining why it is impossible for the expression to have that value.

    Materials:

    - Expression and value cards
    - Score sheets (one for each player)
    - Calculator

    Instructions:

    - Cut out the expression and value cards that your teacher will provide. Keep each type of card in a separate pile, face down.
    - Decide who will be Player 1 and Player 2 for Game 1. Player 1 goes first by turning over one expression card and one value card. Both players record the expression and value in the designated columns on their score sheets.
    - Both players determine whether the expression/value combination is possible and decide how they will justify their answer.
    - Player 1 answers either "Possible" and demonstrates this by substituting a specific number into the expression that gives the indicated value, or "Impossible" and explains why. For example, "-*b* is a positive number when *b* is equal to
      -2". Player 1 records the information on the score sheet.
    - Player 2 can challenge Player 1's conclusion, the number substituted, or the substitution process. If Player 1 is correct, then he or she gets 5 points. ***The conclusion and substitution must be done correctly to get any points.*** A correct answer with an incorrect substitution gets 0 points. Use a calculator to check the computation if necessary.
    - Player 2 then turns over new expression and value cards, and play continues as described above.
    - Player 1 and Player 2 take turns going first until judgments have been made about eight (8) expression/value combinations.
    - At the end of the game, players total their points. The player with the highest total score wins.

3. When you and your partner have finished playing two games, answer the following questions:

    a. What strategy did you use to figure out whether an expression/value combination was possible?

    b. Which expression/value combinations were challenging to figure out? What made them challenging?

    c. What expression/value combinations did you think were impossible that turned out to be possible? Why did you think that they were impossible at first? Give an example for each that shows that they are possible.

    d. Which expression/value combinations were impossible? What made them impossible?

LESSON 8: ONLINE ASSESSMENT

Today you will take an online assessment.

LESSON 8: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Evaluate each expression when x = -2 and y = 4.

   a. $2(x + y)$

   b. $2x + y$

   c. $|x + y|$

   d. $|x - y|$

2. For each table, plot the ordered pairs on the coordinate plane provided.

   a. Note: In this table, the x- and y-coordinates are all non-negative, so the grid contains only quadrant I.
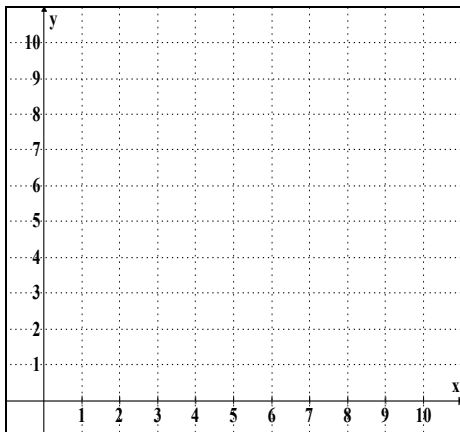
   b. Note: In this table, there is a mixture of positive and negative x- and y-coordinates, so the grid contains all four quadrants.

x	y
0	0
1	2
1	5
3	3
4	1
5	6
6	3
6	7
9	0

x	y
-4	6
-3	5
-2	4
-1	3
0	2
1	1
2	0
3	-1
4	-2
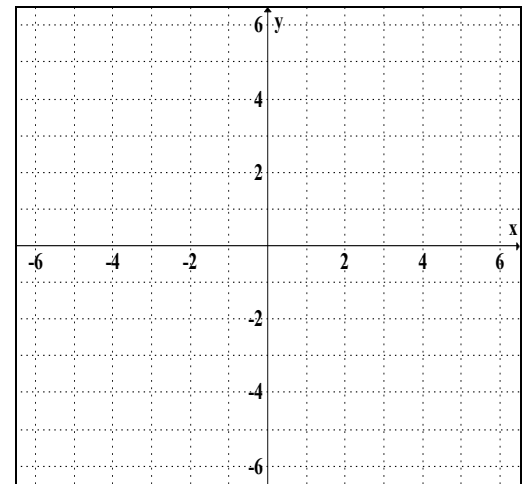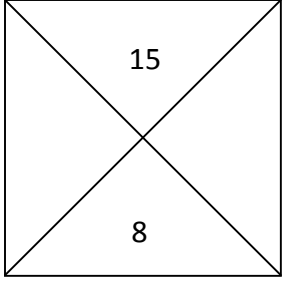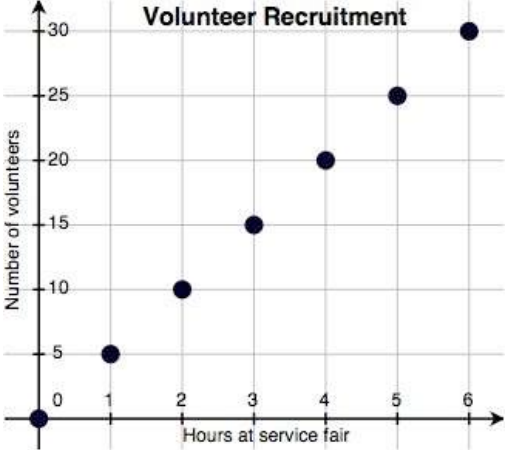
c.

x	y
-8	1
-6	0
-5	-0.5
-2	-2
0	-3
2.5	-4.5
5	-5.5
7	-6.5
10	-8
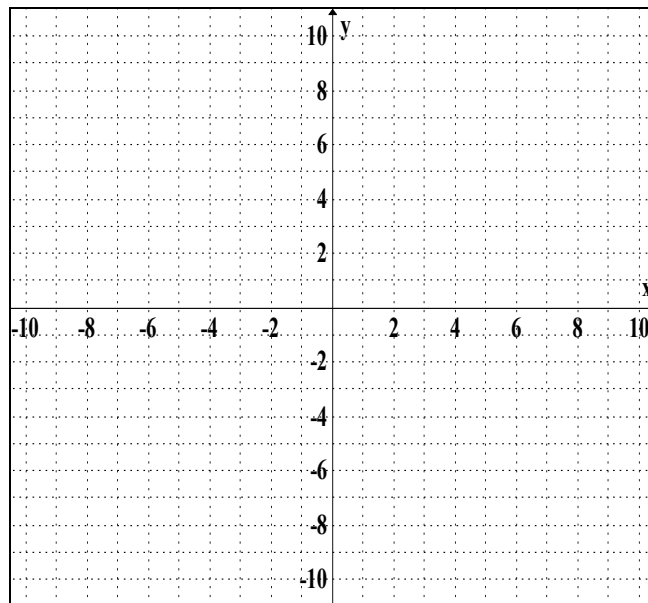
LESSON 8: STAYING SHARP

<table>
<tr><td rowspan="3">Practicing skills & concepts</td><td>

1. Complete the Square Box Problem.

15

8

</td><td>

2. What is the rule for this input-output table?

Input	Output
-2	-5
-1	-1
0	3
1	7
2	11

Answer with supporting work:

</td></tr>
</table>

Preparing for upcoming lessons

3. Find the next three terms in this sequence:

3, 11, 19, 27, 35, …

Explain your reasoning:

4. Describe the relationship represented by this graph by writing a complete sentence.

Reviewing ideas from earlier grades

5. Find the value of each expression:

$-8 - (-7) =$

$0 \div (-6) =$

$6 \div (-1) =$

$-11 \times -5 =$

$2 - 14 =$

$-10 + 4 =$

$-5 \cdot 8 =$

$-7 + -5 =$

$-6 \div (-6) =$

$55 + (-25) =$

6. Consider what you know about multiplication of signed numbers. Then complete the following statements and answer the following question:

a. The product of a negative number and a positive number is…

b. The product of two negative numbers is…

c. What is the relationship between the product $-a \cdot b$ and the product $-a \cdot -b$?

Lesson 9: Making graphs

LESSON 9: OPENER
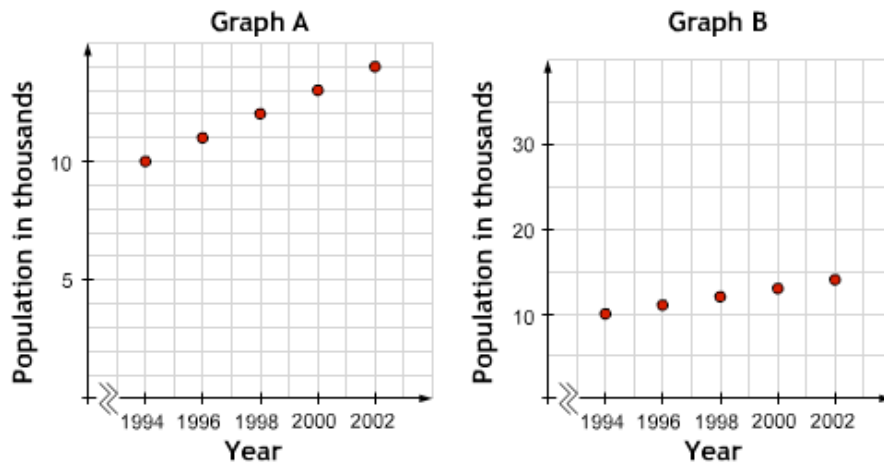
Graph A

Graph B

1. These two graphs show the population growth in a certain town between 1994 and 2002. The local chamber of commerce created Graph A. An environmental group that wants to discourage a manufacturing plant from building in the town created Graph B.

    Tom thinks that the population is growing quickly based on Graph A. Marty thinks the population is growing slowly based on Graph B. Frances thinks both graphs show the same thing. Who is right?

LESSON 9: CORE ACTIVITY

Part I. For each problem:

- *By yourself:* Read the information. Then make the graph.

- *With your partner:* When both of you have finished making a graph, compare them. If your graphs are not the same, see if you can figure out whose graph is correct.

1. Finish the scaling on the coordinate grid and then plot the ordered pairs.

X	y
-5	-25
-4	-20
-3	-15
-2	-10
-1	- 5
0	0
1	5
2	10
3	15
4	20
5	25
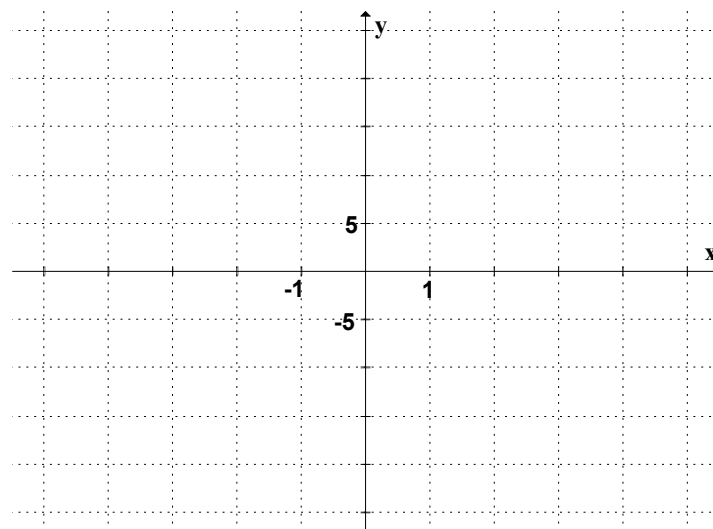
2. Plot the ordered pairs on the grid. Label and scale your axes. Remember, the scales can be different for the **x**- and **y**-axes, but each axis must have a consistent scale.

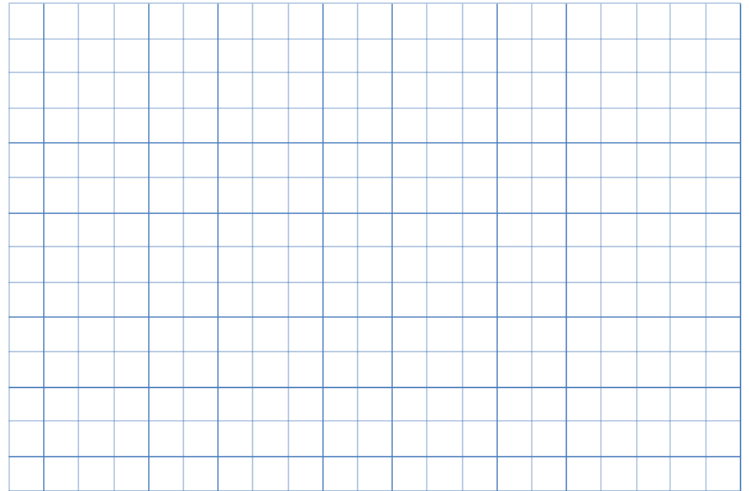| x	y
0	55
1	54
2	53
5	50
10	45
20	35
50	5
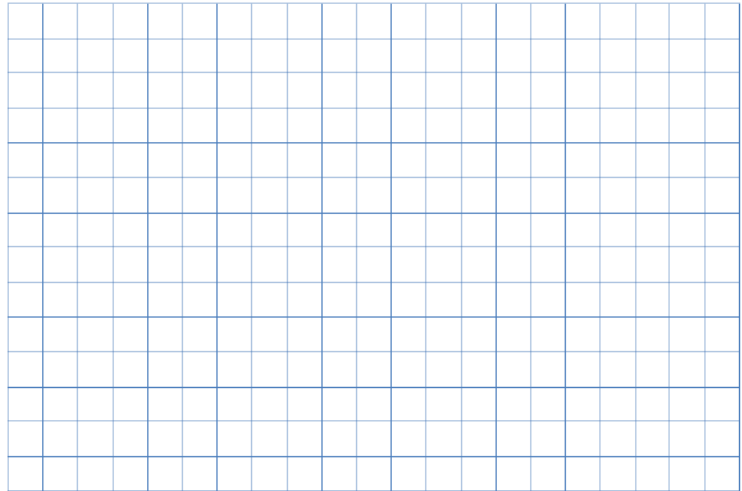
3. The previous problems in this activity did not have a context, or a "real-world" setting. In this problem, consider the situation of the distance traveled by a car moving at a steady speed of 55 miles per hour. Plot the data points on the grid. Label and scale the axes. In addition to containing the variables **x** and **y**, your axis labels should include a description of what **x** and **y** represent in this problem.

x Time (hours)	y Distance (miles)
0	55
1	110
2	165
3	220
4	275
5	330
6	385

4. An electronics store is having a big clearance sale. The table represents the number of televisions, **y,** remaining in stock **x** hours after the beginning of the sale. Create a graph of the data. Be sure to label your axes.

Hours since sale started x	Number of televisions remaining y
1	90
3	67
4	52
5	39
6	31
8	17
9	9

5. Think about how you created the graphs in questions 3 and 4.

   a. How did you determine the scale to use on the **x**- and **y**-axes?

   b. How did you estimate the locations of points between gridlines?

Part II. Examine the following tables and graphs. Each graph has a mistake. Find the mistake and create a correct graph. Then, compare answers with your partner. If you disagree, discuss the reasons and evidence for your answers to determine the correct answer.

6. **Table 1**   a. What is wrong with Graph 1?

x	y
1	1
4	7
6	11
7	13

   b. Use the data in Table 1 to draw a correct graph.
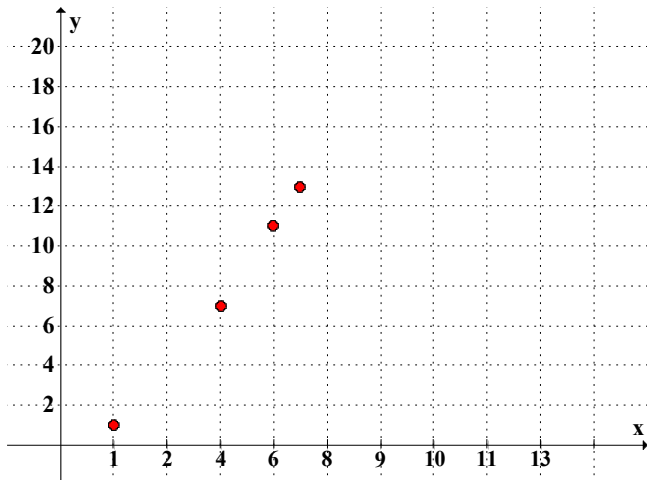
Graph 1   **Corrected graph**
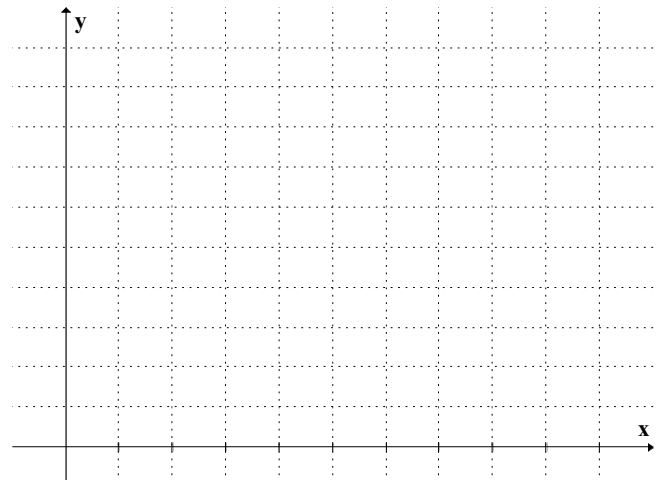
7. **Table 2**

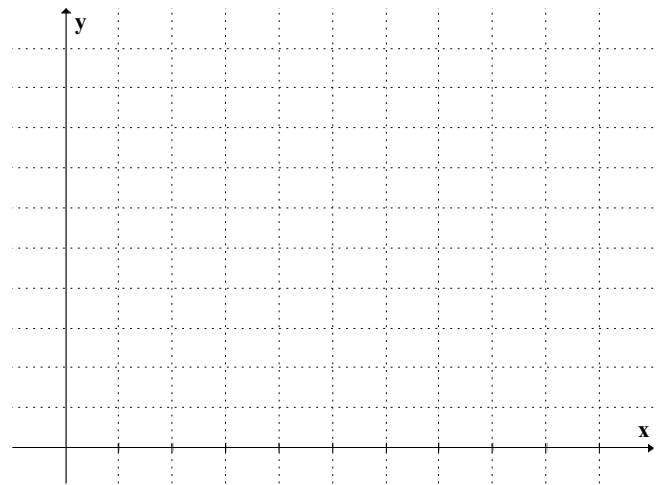| x	y
0	0
4	2
5	2.5
8	4
9	4.5

a. What is wrong with Graph 2?

b. Use the data in Table 2 to draw a correct graph.
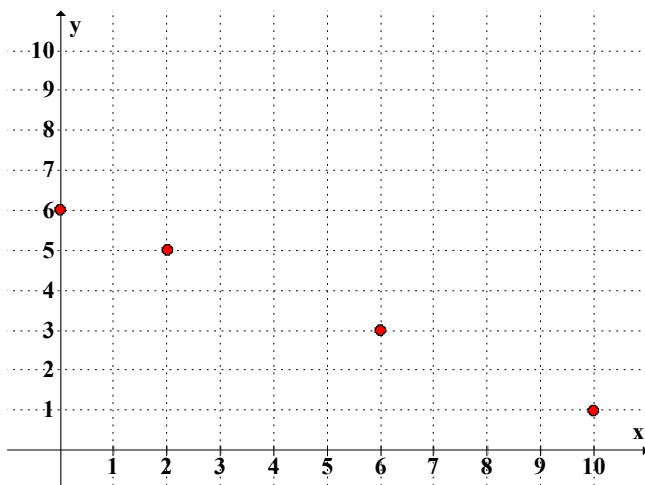
Graph 2
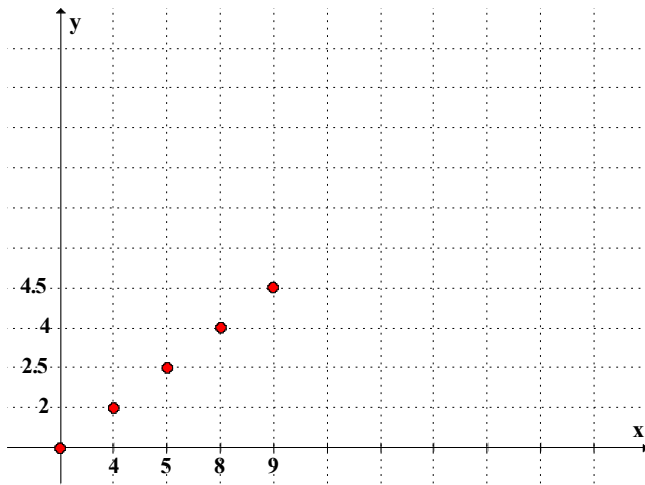
Corrected graph

8. **Table 3**

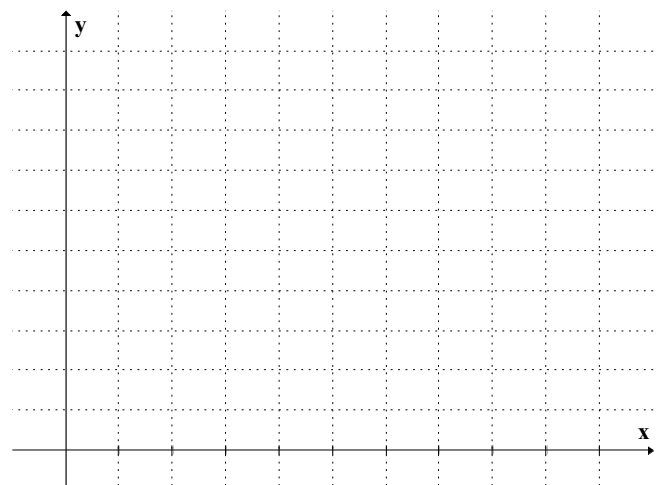| x	y
1	10
3	6
5	2
6	0

a. What is wrong with Graph 3?

b. Use the data in Table 3 to draw a correct graph.

Graph 3

Correct graph

LESSON 9: REVIEW ONLINE ASSESSMENT

You will work with your class to review the online assessment questions.

Problems we did well on:	Skills and/or concepts that are addressed in the problems we did well on:
Problems we did not do well on:	Skills and/or concepts that are addressed in the problems we did not do well on:

Addressing areas of incomplete understanding

Use this page and notebook paper to take notes and re-work particular online assessment problems that your class identifies.

Problem #_____	Work for problem:
Problem #_____	Work for problem:
Problem #_____	Work for problem:

LESSON 9: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

Next class period, you will take an end-of-unit assessment. One good study skill to prepare for tests is to review the important skills and ideas you have learned. Use this list to help you review these skills and concepts, especially by reviewing related course materials.

Important skills and ideas you have learned so far in the unit **Getting started**:

- How to use norms and routines to help you collaborate and learn
- How to apply and share problem-solving strategies and reflect on your solutions
- How to work with variables, patterns, and solutions
- How examining and reflecting on your perceptions of your math ability can help you learn
- How understanding that intelligence is malleable (not fixed) can help you learn
- How working on challenging problems affects the brain
- How to compute with signed numbers
- How to use variables to represent unknowns
- How to create and evaluate algebraic expressions
- How to apply the distributive property
- How to use variables to generalize input-output relationships
- How to plot points on a graph
- How to make graphs from tables
- How to interpret the meaning of points on a graph
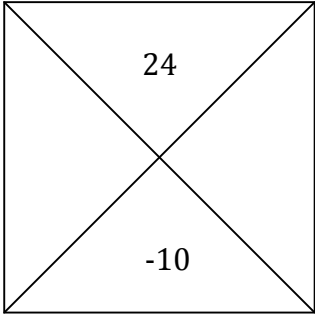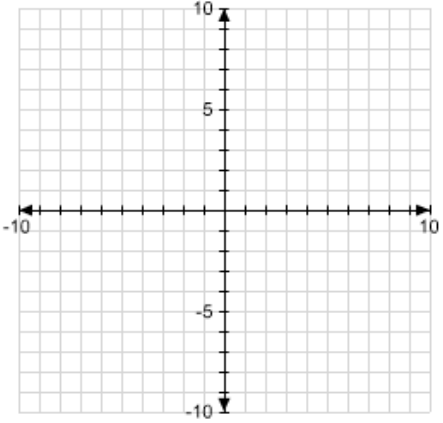
Homework Assignment

Part I: Study for the end-of-unit assessment by reviewing the key topic ideas listed above.

Part II: Complete the online *More practice* for this topic. Note the skills and concepts on which you need more review. Refer to related activities and animations to help you study.

Part III: Complete *Lesson 9: Staying Sharp.*

LESSON 9: STAYING SHARP

Practicing skills & concepts

1. Complete the Square Box Problem.

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-10

2. Write a rule for this input-output table.

Input	Output
2	7
6	15
9	21
10	23

Answer with evidence:

Preparing for upcoming lessons

3. Find the next three terms in this sequence:

1, 4, 9, 16, 25, …

Explain your reasoning:

4. Graph at least four points from the rule $y = 2x + 1$.

Reviewing ideas from earlier grades

5. Find the value of each expression:

$-6 + -7 =$

$-70 \div (-10) =$

$7 \times -9 =$

$-2 \bullet (-11) =$

$-7 - (-4) =$

$-4 + 4 =$

$3 - 11 =$

$-6 \div 2 =$

$100 \div (-25) =$

$-30 - 15 =$

6. Consider what you know about the division of signed numbers. Then complete the following statements and answer the following question:

a. When dividing a positive number by a negative number, the quotient is…

b. When dividing a negative number by a negative number, the quotient is…

c. What is the relationship between the quotient $a \div -b$ and the quotient $-a \div -b$?

Lesson 10: Checking for understanding

LESSON 10: OPENER

Today you will take your first end-of-unit assessment in this course. The test is designed to help you see how well you understand the big ideas and skills you have studied so far. The test provides information to both you and your teacher about the ideas and skills you still need to work on.
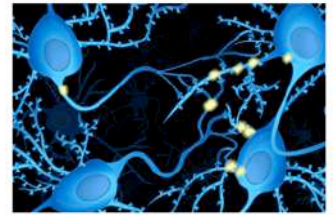
Remember: You have the ability, through working hard and challenging yourself, to become smarter! Look at the assessment as an opportunity to demonstrate what you know and as an opportunity to challenge yourself and "grow your brain."

How well prepared do you feel for this assessment? What are some important things to remember as you take this test? Write a short paragraph in which you respond to these prompts.

LESSON 10: END-OF-UNIT ASSESSMENT

Today you will take an end-of-unit assessment.

LESSON 10: CONSOLIDATION ACTIVITY

Matthew plays on his school's varsity baseball team. Matthew often does warm-up drills in which he throws the ball up in the air and catches it when it comes down. Coach Simpson recently videotaped Matthew's drills. By analyzing the tape in very slow motion, Matthew has collected some data about one particular throw. He hopes it can help him fine-tune his timing when catching the ball.

Elapsed time from beginning of throw in seconds	Ball height in feet
0.0	7
0.3	12
0.6	16
0.9	16
1.2	12
1.5	7

Matthew enters the data from his throw in his graphing calculator. Then he makes a scatterplot on the calculator. The window he uses and the graph he sees are shown.

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1
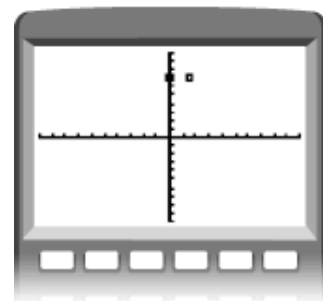
"What a strange throw," Matthew thinks. "This can't be right!" What should Matthew do to get a better view of his data?

LESOSN 10: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

1. When a driver gets caught speeding in a certain town, the fine is determined by the graph shown here. Complete the following statements by filling in the blanks.
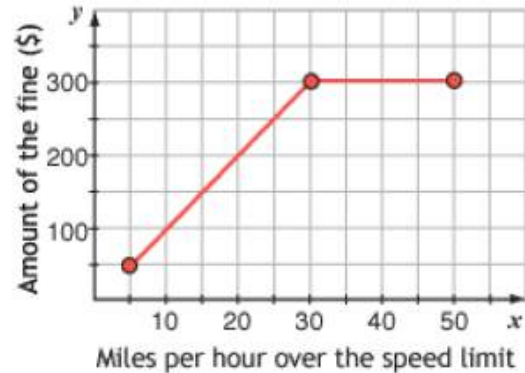
   a. Going 20 miles per hour over the speed limit will result in a fine of $ _____ .

   b. Going 40 miles per hour over the speed limit will result in a fine of $ _____ .

   c. If a driver received a fine of $150, then that driver must have exceeded the speed limit by _____ miles per hour.

2. Rectangular tables are arranged in a row with chairs around each row, as shown in the picture.

   1 table — 6 chairs

   2 tables — 10 chairs
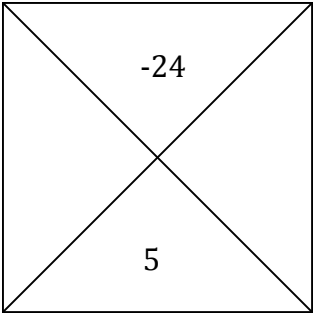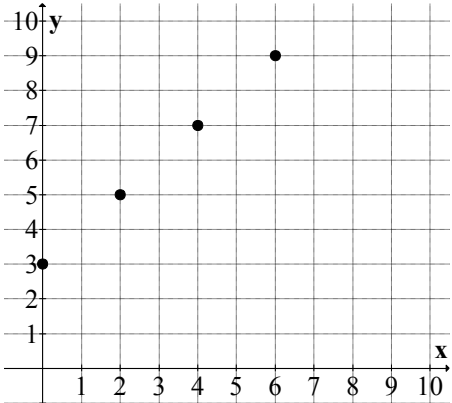
   3 tables — 14 chairs
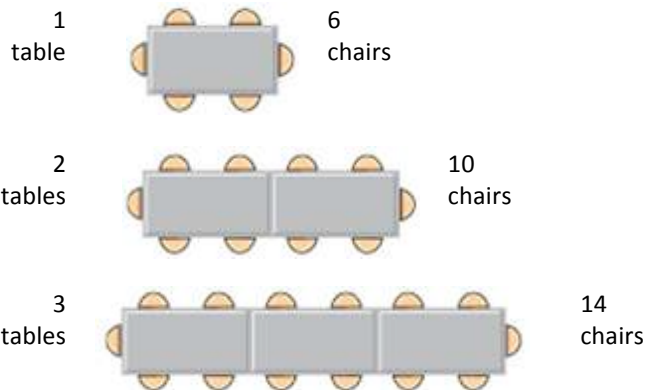
   There is a relationship between the number of tables in a row and the number of chairs that can fit in the row. Answer the following questions. Provide evidence for your answers.

   If the pattern continues:

   a. How many chairs will fit around a row of 5 tables?

   b. How many chairs will fit around a row of 11 tables?

   c. How many tables are needed to seat 30 chairs?

LESSON 10: STAYING SHARP

Practicing skills & concepts

1. Complete the Square Box Problem.

-24

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2. What is the rule for the following table?

x	y
0	2
3	11
5	17
10	32

Answer with evidence:

Preparing for upcoming lessons

3. In this sequence, term number 1 has a value of 6, term number 2 has a value of 12, term number 3 has a value of 22, and so on:

6, 12, 22, 36, 54, …

If n represents the term number, demonstrate that the rule $2n^2 + 4$ will generate the sequence.

4. Describe the relationship represented by this graph by writing a function rule relating y and x. (Hint: It may help you to make a table.)

Reviewing ideas from earlier grades

5. Find the value of each expression.

$5 + (-1)$

$13 - (-7)$

$2 \cdot (-3)$

$24 \div (-4)$

6. Complete each statement.

a. The product of two numbers is positive when…

b. The sum of two numbers is positive when…

c. You can rewrite subtraction as…

d. When you add two numbers in the opposite order, you get…