Session 3

I. ACT Math Test Format

Section	Questions	Time
Math Section 2	60 Questions – All multiple choice. Calculator allowed on all problems	60 minutes
Science Section 4	6 Passages, 40 Questions – All multiple choice.	35 minutes

No penalty for guessing

II. General Breakdown of Questions

A. Math: Categories of questions

1. Pre-Algebra (integers, primes, fractions, percents, order of operations)
2. Algebra I (equations, inequalities, ratios, systems of equations, functions, factoring)
3. Algebra 2, Data Analysis (exponents, roots, imaginary numbers, quadratics, exponential functions, logarithms, medians, weighted averages, probability, counting problems)
4. Geometry (angles, triangles, circles, area, volume, slope, distance, midpoints, graphing lines, similarity, trig ratios)
5. Advanced Math (matrices, graphing trig functions, basic trig identities, unit circle values)

B. Science: Categories of passages

1. Interpretation of Data (tables, graphs, interpolation, extrapolation, changes in values)
2. Scientific Investigation (methods, design, control, compare and contrast, hypothesis testing, modifications)
3. Evaluation of Models (implications, assumptions, similarities and differences, contradictions, strengths and weaknesses, predictions)

III. ACT Practice Websites

http://www.act.org/content/act/en/products-and-services/the-act/test-preparation/math-practice-test-questions.html?page=0&chapter=1
5 sets of practice problems not by topic

http://blog.prepscholar.com/complete-list-of-free-act-math-practice-questions
Lots of questions by topic

https://www.varsitytutors.com/act_math-help
Lots of questions by topic

http://www.crackact.com/act/math/
80 short practice tests (15 problems each) and 2 full practice tests

https://www.sophia.org/preparing-for-college/act-test-preparation/mathematics
Video reviews by topic. Practice Tests require an account

https://www.powerscore.com/sat/help/content_practice_tests.cfm#official-act
Extra official practice tests, both ACT and SAT

http://www.sparknotes.com/testprep/books/act/chapter10.rhtml
Content overview by topic . No practice problems

http://www.mhpracticeplus.com/act.php
4 online non-official practice tests. Limited videos by topic

https://www.mometrix.com/academy/act-math/
Lots of topical math videos

http://www.highschooltestprep.com/act/math/
Online practice tests in order of difficulty

Pre-Algebra, Algebra 1, Algebra 2 Problem Set

1.

A container is $\frac{1}{8}$ full of water. After 10 cups of water are added, the container is $\frac{3}{4}$ full. What is the volume of the container, in cups?

F. $13\frac{1}{3}$
G. $13\frac{1}{2}$
H. 15
J. 16
K. 40

2.

For all positive integers x, what is the greatest commor factor of the 2 numbers $216x$ and $180x$?

F. 6
G. 72
H. x
J. $12x$
K. $36x$

3.

Two enterprising college students decide to start a business. They will make up and deliver helium balloon bouquets for special occasions. It will cost them \$39.99 to buy a machine to fill the balloons with helium. They estimate that it will cost them \$2.00 to buy the balloons, helium, and ribbons needed to make each balloon bouquet. Which of the following expressions could be used to model the total cost for producing b balloon bouquets?

A. $\$ 2.00b + \$39.99$
B. $\$37.99b$
C. $\$39.99b + \$ 2.00$
D. $\$41.99b$
E. $\$79.98b$

4.

This month, Kami sold 70 figurines in 2 sizes. The large figurines sold for \$12 each, and the small figurines sold for \$8 each. The amount of money he received from the sales of the large figurines was equal to the amount of money he received from the sales of the small figurines. How many large figurines did Kami sell this month?

A. 20
B. 28
C. 35
D. 42
E. 50

5.

Ms. Lewis plans to drive 900 miles to her vacation destination, driving an average of 50 miles per hour. How many miles per hour faster must she average, while driving, to reduce her total driving time by 3 hours?

A. 5
B. 8
C. 10
D. 15
E. 18

6.

What polynomial must be added to $x^2 - 2x + 6$ so that the sum is $3x^2 + 7x$?

A. $4x^2 + 5x + 6$
B. $3x^2 + 9x + 6$
C. $3x^2 + 9x - 6$
D. $2x^2 + 9x - 6$
E. $2x^2 - 5x + 6$

7.

What values of x are solutions for $x^2 + 2x = 8$?

A. -4 and 2
B. -2 and 0
C. -2 and 4
D. 0 and 2
E. 6 and 8

8.

The diameter, d centimeters, of the metal poles Goodpole Manufacturing produces must satisfy the inequality $|d - 3| \leq 0.001$. What is the maximum diameter, in centimeters, such a metal pole may have?

F. 1.4995
G. 1.5005
H. 2.999
J. 3.000
K. 3.001

9.

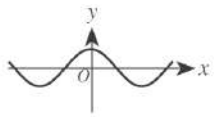
If $f(x) = (3x + 7)^2$, then $f(1) = ?$

A. 10
B. 16
C. 58
D. 79
E. 100

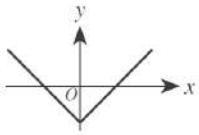
10.

A function f is an *odd* function if and only if $f(-x) = -f(x)$ for every value of x in the domain of f. One of the functions graphed in the standard (x,y) coordinate plane below is an odd function. Which one?

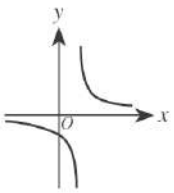
A.



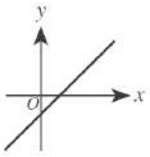
B.



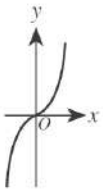
C.



D.



E.



11.

For all x in the domain of the function $\frac{x+1}{x^3-x}$, this function is equivalent to:

F. $\frac{1}{x^2} - \frac{1}{x^3}$

G. $\frac{1}{x^3} - \frac{1}{x}$

H. $\frac{1}{x^2-1}$

J. $\frac{1}{x^2-x}$

K. $\frac{1}{x^3}$

12.

What is the product of the complex numbers $(-3i + 4)$ and $(3i + 4)$?

A. 1
B. 7
C. 25
D. $-7 + 24i$
E. $7 + 24i$

13.

In the real numbers, what is the solution of the equation $8^{2x+1} = 4^{1-x}$?

A. $-\frac{1}{3}$

B. $-\frac{1}{4}$

C. $-\frac{1}{8}$

D. 0

E. $\frac{1}{7}$

14.

On the first day of school, Mr. Vilani gave his third-grade students 5 new words to spell. On each day of school after that, he gave the students 3 new words to spell. In the first 20 days of school, how many new words had he given the students to spell?

A. 28
B. 62
C. 65
D. 68
E. 152

15.

The nth term (t_n) of a certain sequence is defined as $t_n = t_{n-1} + 4$.

If $t_1 = -7$ then $t_{71} =$

A. 273

B. 277

C. 281

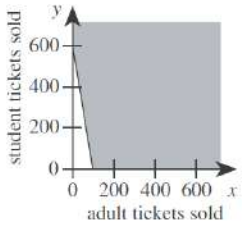
D. 283

E. 287

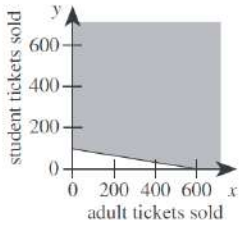
16.

Tickets for the Senior Talent Show at George Washington Carver High School are \$3 for adults and \$2 for students. To cover expenses, a total of \$600 must be collected from ticket sales for the show. One of the following graphs in the standard (x,y) coordinate plane, where x is the number of adult tickets sold and y is the number of student tickets sold, represents all the possible combinations of ticket sales that cover at least \$600 in expenses. Which graph is it?

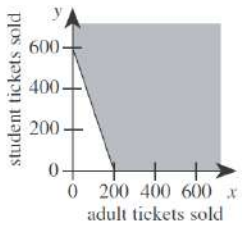
F.



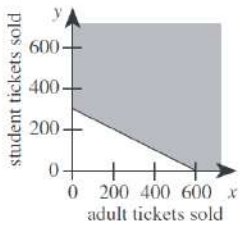
J.



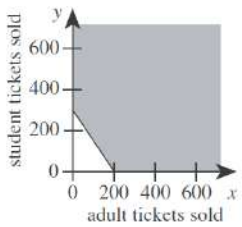
G.



K.



H.



17. Compositions of functions

Consider the functions $f(x) = \sqrt{x}$ and $g(x) = 7x + b$. In the standard (x,y) coordinate plane, $y = f(g(x))$ passes through $(4,6)$. What is the value of b ?

A. 8
B. -8
C. -25
D. -26
E. $4 - 7\sqrt{6}$

18. Graphing Quadratic functions, identifying the vertex

The equation $y = x^2$ is graphed in the standard (x,y) coordinate plane. In which of the following equations is the graph of the parabola shifted 4 units to the left and 2 units up?

A. $y = (x - 4)^2 + 2$

B. $y = (x - 4)^2 - 2$

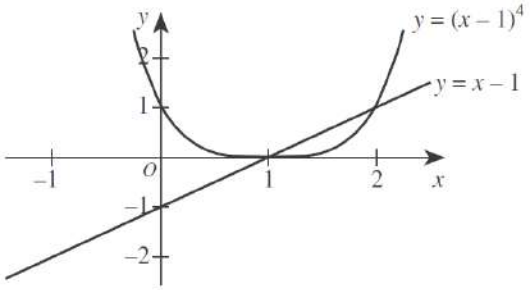
C. $y = (x - 2)^2 + 4$

D. $y = (x + 4)^2 + 2$

E. $y = (x + 4)^2 - 2$

19. Graphs of systems of equations

The graphs of the equations $y = x - 1$ and $y = (x - 1)^4$ are shown in the standard (x,y) coordinate plane below. What real values of x, if any, satisfy the inequality $(x - 1)^4 < (x - 1)$?



A. No real values
B. $x < 0$ and $x > 1$
C. $x < 1$ and $x > 2$
D. $0 < x < 1$
E. $1 < x < 2$

Pre-Algebra, Algebra 1, Algebra 2 Practice

1.

Given $f = cd^3$, $f = 450$, and $d = 10$, what is c ?

F. 0.45
G. 4.5
H. 15
J. 45
K. 150

2.

A number is increased by 25% and the resulting number is then decreased by 20%. The final number is what percent of the original number?

A. 90%
B. 95%
C. 100%
D. 105%
E. 120%

3.

Students studying motion observed a cart rolling at a constant rate along a straight line. The table below gives the distance, d feet, the cart was from a reference point at 1-second intervals from $t = 0$ seconds to $t = 5$ seconds.

t	0	1	2	3	4	5
d	14	20	26	32	38	44

Which of the following equations represents this relationship between d and t ?

A. $d = t + 14$
B. $d = 6t + 8$
C. $d = 6t + 14$
D. $d = 14t + 6$
E. $d = 34t$

4.

An artist makes a profit of $(500p - p^2)$ dollars from selling p paintings. What is the fewest number of paintings the artist can sell to make a profit of at least $60,000 ?

F. 100
G. 150
H. 200
J. 300
K. 600

5.

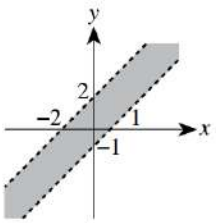
What are the real solutions to the equation $|x|^2 + 2|x| - 3 = 0$?

F. ± 1
G. ± 3
H. 1 and 3
J. -1 and -3
K. ± 1 and ± 3

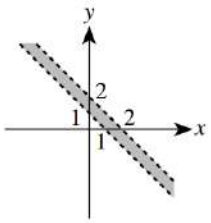
6.

Which of the following is the graph of the region $1 < x + y < 2$ in the standard (x,y) coordinate plane?

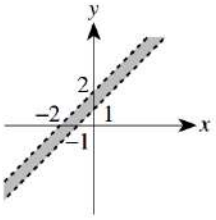
F.



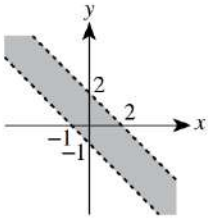
J.



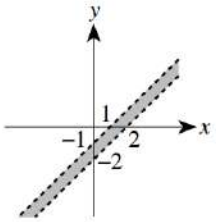
G.



K.

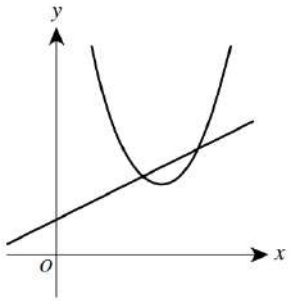


H.



7.

Which of the following describes a true relationship between the functions $f(x) = (x - 3)^2 + 2$ and $g(x) = \frac{1}{2}x + 1$ graphed below in the standard (x,y) coordinate plane?



F. $f(x) = g(x)$ for exactly 2 values of x
G. $f(x) = g(x)$ for exactly 1 value of x
H. $f(x) < g(x)$ for all x
J. $f(x) > g(x)$ for all x
K. $f(x)$ is the inverse of $g(x)$

8.

Given $f(x) = x - \frac{1}{x}$ and $g(x) = \frac{1}{x}$, what is $f\left(g\left(\frac{1}{2}\right)\right)$?

F. -3

G. $-\frac{3}{2}$

H. $-\frac{2}{3}$

J. 0

K. $\frac{3}{2}$

9.

The square below is divided into 3 rows of equal area. In the top row, the region labeled A has the same area as the region labeled B. In the middle row, the 3 regions have equal areas. In the bottom row, the 4 regions have equal areas. What fraction of the square's area is in a region labeled A ?

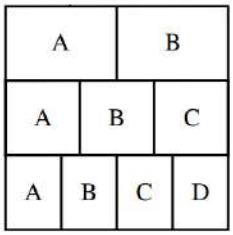
F. $\frac{1}{9}$

G. $\frac{3}{9}$

H. $\frac{6}{9}$

J. $\frac{13}{12}$

K. $\frac{13}{36}$



10.

For every positive 2-digit number, x, with tens digit t and units digit u, let y be the 2-digit number formed by reversing the digits of x. Which of the following expressions is equivalent to $x - y$?

F. $9(t - u)$
G. $9(u - t)$
H. $9t - u$
J. $9u - t$
K. 0

11.

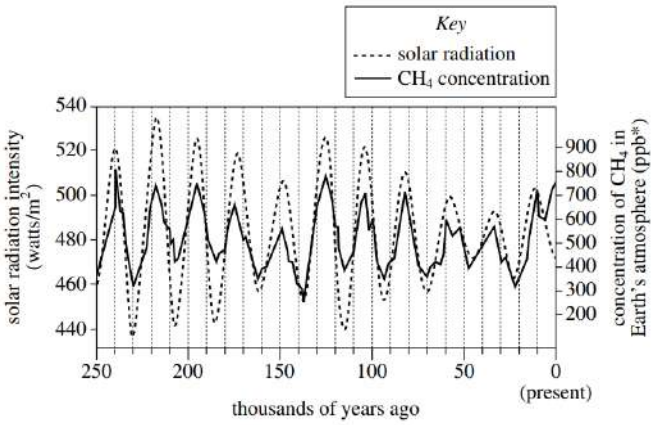
If $x:y = 5:2$ and $y:z = 3:2$, what is the ratio of $x:z$?

A. 3:1
B. 3:5
C. 5:3
D. 8:4
E. 15:4

Science: Interpretation of Data

Passage III

Greenhouse gases such as methane (CH_4) warm Earth's climate. Figure 1 shows the concentration of CH_4 in Earth's atmosphere and the solar radiation intensity at Earth's surface for tropical Europe and Asia over the past 250,000 years. As the figure shows, the CH_4 concentration and the solar radiation intensity have increased and decreased at the same times over most of this period. Figure 2 shows the same types of data for the same region over the past 11,000 years. This figure is consistent with the hypothesis that the greenhouse gases from human activities may have begun warming Earth's climate thousands of years earlier than once thought.



*ppb = parts per billion

Figure 1

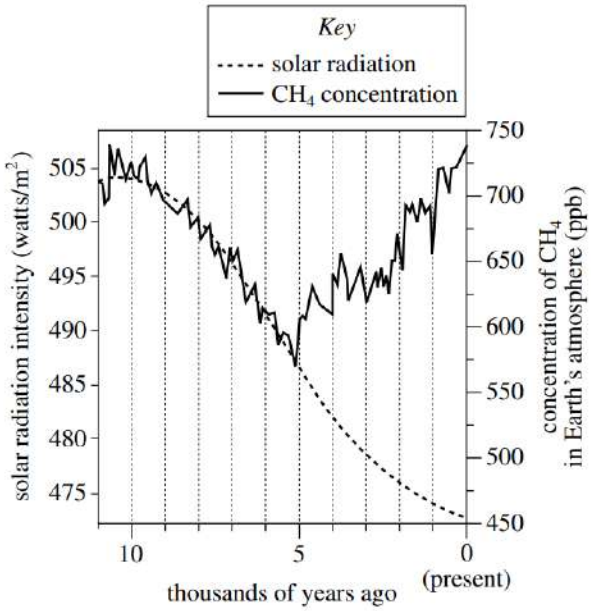


Figure 2

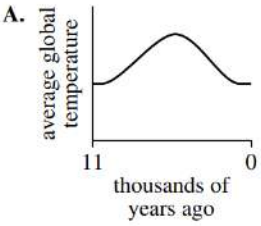
15. According to Figure 2, the solar radiation intensity 8,000 years ago was closest to which of the following?

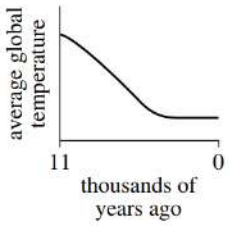
A. 490 watts/m^2
B. 495 watts/m^2
C. 500 watts/m^2
D. 505 watts/m^2

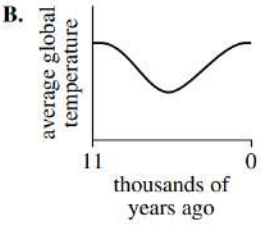
16. According to Figure 2, if the trend in the CH_4 concentration had continued to match the trend in the solar radiation intensity, the CH_4 concentration at present would most likely be:

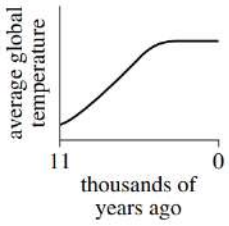
F. less than 550 ppb.
G. between 550 ppb and 600 ppb.
H. between 600 ppb and 650 ppb.
J. greater than 650 ppb.

17. Suppose that whenever the CH_4 concentration increases, a corresponding, immediate increase in average global temperature occurs, and that whenever the CH_4 concentration decreases, a corresponding, immediate decrease in average global temperature occurs. Based on Figure 2, which of the following graphs best represents a plot of average global temperature over the past 11,000 years?

A.


C.


B.


D.


18. Based on Figure 1, the average solar radiation intensity over the past 250,000 years was closest to which of the following?

F. 400 watts/m^2
G. 440 watts/m^2
H. 480 watts/m^2
J. 520 watts/m^2

19. One *solar radiation cycle* is the time between a maximum in the solar radiation intensity and the next maximum in the solar radiation intensity. According to Figure 1, the average length of a solar radiation cycle during the past 250,000 years was:

A. less than 15,000 years.
B. between 15,000 years and 35,000 years.
C. between 35,000 years and 55,000 years.
D. greater than 55,000 years.

20. Which of the following statements best describes the primary effect of CH_4 on Earth's climate?

F. CH_4 gives off visible light to space, cooling Earth's climate.
G. CH_4 gives off ultraviolet radiation to space, warming Earth's climate.
H. CH_4 absorbs heat as it enters Earth's atmosphere from space, cooling Earth's climate.
J. CH_4 absorbs heat that comes up from Earth's surface, warming Earth's climate.

Science: Scientific Investigation

Passage I

Researchers studied how diet and the ability to smell food can affect the life span of normal fruit flies (Strain N) and fruit flies unable to detect many odors (Strain X).

Study 1

Three tubes (Tubes 1–3), each with *15% sugar yeast* (SY) *medium* (a diet with 15% sugar and 15% killed yeast), were prepared. Then, 200 virgin female Strain N fruit flies less than 24 hr old were added to each tube. No additional substance was added to Tube 1. Additional odors from live yeast were added to Tube 2, and live yeast was added to Tube 3. The percent of fruit flies alive was determined every 5 days for 75 days (see Figure 1).

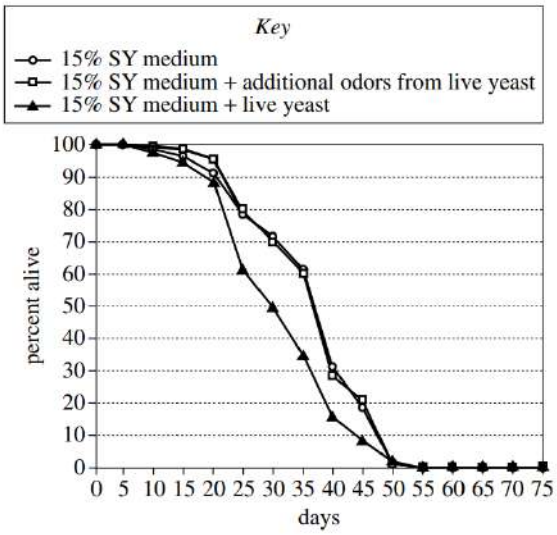


Figure 1

Study 2

Three tubes (Tubes 4–6), each with 5% SY medium (a diet with 5% sugar and 5% killed yeast), were prepared. Then, 200 virgin female Strain N fruit flies less than 24 hr old were added to each tube. No additional substance was added to Tube 4. Additional odors from live yeast were added to Tube 5, and live yeast was added to Tube 6. The percent of fruit flies alive was determined every 5 days for 75 days (see Figure 2).

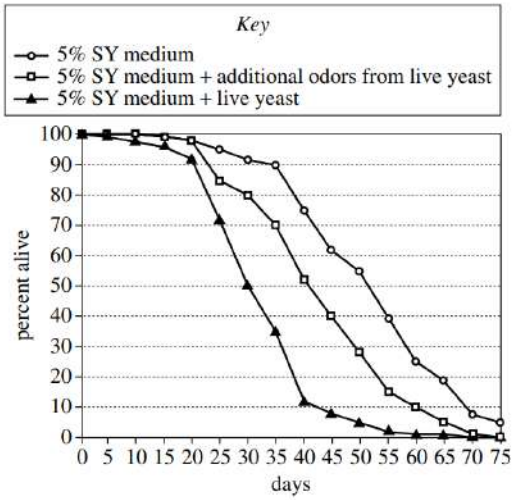


Figure 2

Study 3

Strain N fruit flies were modified to produce Strain X fruit flies. Strain X fruit flies lack *Or83b* (a protein required to detect a wide range of odors); therefore, they cannot detect many odors. The average life span was determined for virgin female Strain N and virgin female Strain X fruit flies fed with various SY media (see Table 1).

Table 1

Strain	SY medium: % sugar	SY medium: % killed yeast	Average life span (days)
Strain N	3	3	50.1
	5	5	50.1
	7.5	7.5	43.9
	10	10	44.8
	15	15	41.6
Strain X	3	3	61.6
	5	5	62.5
	7.5	7.5	58.9
	10	10	58.6
	15	15	55.6

1. In which of Studies 1 and 2 did some of the fruit flies live for more than 75 days, and what diet were those fruit flies fed?

A. Study 1; 5% SY medium
B. Study 1; 15% SY medium
C. Study 2; 5% SY medium
D. Study 2; 15% SY medium

2. During Studies 1 and 2, why did the size of the fruit fly population in each tube decrease rather than increase?

F. The birthrate was 0, because the initial population contained only males.
G. The birthrate was 0, because the initial population contained only virgin females.
H. The death rate was 0, because the initial population contained only males.
J. The death rate was 0, because the initial population contained only virgin females.

3. Study 1 differed from Study 2 in which of the following ways?

   A. Female fruit flies were tested in Study 1, whereas male fruit flies were tested in Study 2.
   B. Male fruit flies were tested in Study 1, whereas female fruit flies were tested in Study 2.
   C. The SY medium tested in Study 1 contained a lower percent of sugar than did the SY medium tested in Study 2.
   D. The SY medium tested in Study 1 contained a higher percent of sugar than did the SY medium tested in Study 2.

4. Suppose that an additional trial in Study 3 had been performed using a 12% SY medium (a diet with 12% sugar and 12% killed yeast). The average life span of the Strain X fruit flies in this trial would most likely have been:

   F. less than 55.6 days.
   G. between 55.6 days and 58.6 days.
   H. between 58.6 days and 61.6 days.
   J. greater than 61.6 days.

5. The researchers had predicted that decreasing a fruit fly's ability to detect odors would increase its life span. Are the results of Study 3 consistent with this prediction?

   A. No; for each SY medium tested, the average life span of Strain X fruit flies was longer than the average life span of Strain N fruit flies.
   B. No; for each SY medium tested, the average life span of Strain N fruit flies was longer than the average life span of Strain X fruit flies.
   C. Yes; for each SY medium tested, the average life span of Strain X fruit flies was longer than the average life span of Strain N fruit flies.
   D. Yes; for each SY medium tested, the average life span of Strain N fruit flies was longer than the average life span of Strain X fruit flies.

6. Suppose the researchers wanted to determine whether a defect in the ability to detect odors would change the life span of fruit flies fed 15% SY medium when live yeast is added to the diet or when additional odors from live yeast are added to the diet. Which of the following experiments should be performed?

   F. Repeat Study 1 except with Strain X fruit flies
   G. Repeat Study 1 except with Strain N fruit flies
   H. Repeat Study 2 except with Strain X fruit flies
   J. Repeat Study 2 except with Strain N fruit flies

7. The results for which 2 tubes should be compared to determine how a reduced calorie diet affects life span in the absence of live yeast and additional odors from live yeast?

   A. Tube 1 and Tube 4
   B. Tube 1 and Tube 2
   C. Tube 2 and Tube 5
   D. Tube 5 and Tube 6

Science: Evaluation of Models

Passage II

In the fall, monarch butterflies (*Danaus plexippus*) in eastern North America migrate to Mexico, where they overwinter in high-altitude forests of *oyamel fir* (an evergreen conifer). The butterflies store (accumulate) body lipids to use as a source of energy at a later time. Consider the following 3 hypotheses pertaining to when the butterflies store lipids and when the energy from the stored lipids is used, with respect to migration and overwintering.

Hypothesis 1

Monarch butterflies require energy from stored lipids for migration and during the overwintering period. The butterflies first store lipids before they begin their migration. During migration, as stored lipids are converted to energy, lipid mass continuously decreases. When the butterflies reach the overwintering sites, ending their migration, they must store lipids again before beginning the overwintering period.

Hypothesis 2

Monarch butterflies require energy from stored lipids for migration but not during the overwintering period. The butterflies store lipids before they begin their migration. During migration, as stored lipids are converted to energy, lipid mass continuously decreases. Because energy from stored lipids is not required during the overwintering period, the butterflies do not store lipids while at the overwintering sites.

Hypothesis 3

Monarch butterflies require energy from stored lipids during the overwintering period but not for migration. The butterflies do not store lipids before they begin their migration. Instead, lipids are stored during migration; therefore, lipid mass continuously increases from the beginning of migration until the end of migration. The butterflies arrive at the overwintering sites with enough lipids to provide themselves with energy during the overwintering period, so they do not store lipids while at the overwintering sites.

8. Which hypothesis, if any, asserts that monarch butterflies store lipids during 2 distinct periods?

   F. Hypothesis 1
   G. Hypothesis 2
   H. Hypothesis 3
   J. None of the hypotheses

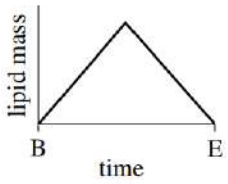
9. Which hypothesis, if any, asserts that monarch butterflies require energy from stored lipids neither for migration nor during the overwintering period?

   A. Hypothesis 1
   B. Hypothesis 2
   C. Hypothesis 3
   D. None of the hypotheses

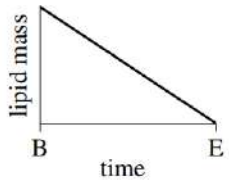
10. Based on Hypothesis 3, which of the following figures best depicts the change in the lipid mass of a monarch butterfly from the beginning of migration to the end of migration?

(Note: In each figure, B represents the beginning of migration and E represents the end of migration.)

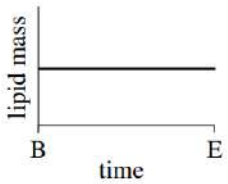
F.



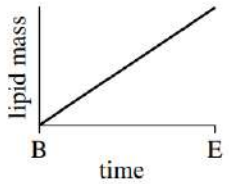
H.



G.



J.



11. Assume that changes in the body mass of a monarch butterfly are caused only by changes in the mass of the butterfly's stored lipids. The statement "The percent of a monarch butterfly's body mass that is made up of lipids is greater at the beginning of migration than at the end of migration" is supported by which of the hypotheses?

A. Hypothesis 1 only
B. Hypothesis 2 only
C. Hypotheses 1 and 2 only
D. Hypotheses 1, 2, and 3

12. To store lipids, monarch butterflies convert sugar from nectar they have consumed into lipids. A supporter of which hypothesis, if any, would be likely to claim that to ensure the butterflies can store lipids for the overwintering period, nectar must be present at the butterflies' overwintering sites?

F. Hypothesis 1
G. Hypothesis 2
H. Hypothesis 3
J. None of the hypotheses

13. Which of the following statements about lipids in monarch butterflies is consistent with all 3 hypotheses?

A. The butterflies' lipid masses do not change during the overwintering period.
B. The butterflies' lipid masses change during migration.
C. The butterflies use energy from stored lipids during the overwintering period.
D. The butterflies use energy from stored lipids for migration.

14. When the monarch butterflies use their stored lipids, the lipids must be broken down to produce energy-rich molecules that can be readily used by cells. Which of the following molecules is produced as a direct result of the breakdown of the lipids?

F. ATP
G. Starch
H. DNA
J. Amino acids