Session 1: Problem Set 1: Pre-Algebra and Algebra 1 Solutions

1.

Johanna picked 3 pounds of strawberries at a "pick-your-own" patch. At this particular patch, the cost is $1.50 for the pail and $3.99 per pound of strawberries picked. If a linear equation is created to represent the situation and written in the form $y = mx + b$, which piece of the equation would the value 13.47 in this scenario most likely represent?

Ⓐ b
Ⓑ m
Ⓒ x
Ⓓ y

Difficulty: Medium

Category: Heart of Algebra / Linear Equations

Strategic Advice: When a real-world scenario is modeled using a linear equation, b is a flat fee or starting amount, m is a unit rate, x represents the number of units, and y represents a total amount.

Getting to the Answer: Write the equation in words first, adding the variables as you go. The total cost, y, is equal to the cost per pound, m, multiplied by the number of pounds, x, and added to the cost of the pail, b. You can eliminate A and B because b is 1.50 (the cost of the pail) and m is 3.99 (the cost per pound). You can also eliminate C because Johanna picks 3 pounds, so x is 3. Choice (D) is correct because the total cost of picking 3 pounds is 3.99(3) + 1.50 = 13.47. This means 13.47 most likely represents the total cost, y.

2.

If $y = 12 - x$ and $\frac{3y}{4} + 11 = \frac{-x}{2}$, what is the value of $\frac{x}{5} + \frac{y}{4}$?

Ⓐ -1
Ⓑ $-\frac{5}{4}$
Ⓒ $-\frac{9}{4}$
Ⓓ 33

Difficulty: Hard

Category: Heart of Algebra / Systems of Linear Equations

Strategic Advice: Don't let all the fractions intimidate you. There are two equations and two variables, so solve this system the same way you would solve any other system of equations.

Getting to the Answer: The first equation is already solved perfectly for y, so use substitution. To make the second equation easier to work with, multiply it by 4 to clear the fractions (even though you may have noticed the tempting 4 in the denominator of the desired expression).

$$\begin{aligned} 4\left(\frac{3y}{4} + 11 = \frac{-x}{2}\right) & \\ 3y + 44 &= -2x \\ 3(12 - x) + 44 &= -2x \\ 36 - 3x + 44 &= -2x \\ 80 - 3x &= -2x \\ 80 &= x \end{aligned}$$

Next, substitute 80 for x into the first equation and solve for y.

$y = 12 - 80$

$y = -68$

Finally, substitute the values you found into the expression in the question $\frac{x}{5} + \frac{y}{4}$:

$$\frac{80}{5} + \frac{(-68)}{4} = 16 + (-17) = -1$$

3. Calculator

Rasha volunteers at a charity that helps feed the homeless. He collects donations and then uses the money to buy food for care packages. This week, he collected $145. Each care package will include canned vegetables and bags of rice in the ratio 3:1. The cans cost $0.89 each, and the bags of rice cost $3.49 each. Using the given ratio, what is the maximum number of complete vegetable/rice care packages Rasha can make?

Difficulty: Medium

Category: Heart of Algebra / Inequalities

Strategic Advice: When a question asks about a maximum (or minimum) amount, it usually means you need to create and solve an inequality.

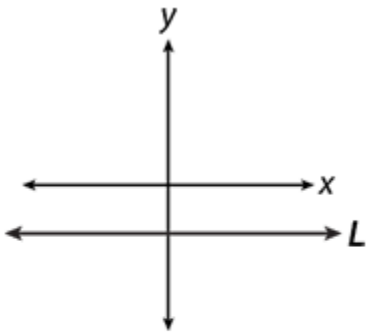
Getting to the Answer: Write the inequality in words first. The cost of 3 cans of vegetables plus the cost of 1 bag of rice, all multiplied by the number of care packages Rasha makes, must be less than or equal to the amount of money he collected, $145. Because you are not asked to differentiate between cans and rice, they can be represented by the same variable. Just don't forget to multiply the cost of 1 can by 3 first ($0.89 × 3 = $2.67). Let p represent the number of care packages:

$(2.67 + 3.49)p \leq 145$

$6.16p \leq 145$

$p \leq 23.54$

4.



Line L shown in the graph could be the graph of which equation?

Ⓐ $x + y = -2$
Ⓑ $x + y = 0$
Ⓒ $x + y - 2 = x$
Ⓓ $x + y + 2 = x$

Difficulty: Medium

Category: Heart of Algebra / Linear Equations

Strategic Advice: Horizontal and vertical lines have special forms. A horizontal line has a slope of 0 and an equation that always looks like $y = b$, where b is a constant. A vertical line has an undefined slope and always looks like $x = b$.

Getting to the Answer: Line L shown in the graph is horizontal, so you are looking for an equation that once simplified (or written in $y = mx + b$ form) looks like $y = b$. In other words, all the x-terms must cancel out. In addition, because the line is below the x-axis, b must be a negative number. Check each answer choice to see if it takes on the desired form. Choice A → $y = -x - 2$, so eliminate it. Choice B → $y = -x$, so eliminate it. Choice C → $y = 2$, which is in the correct form, but 2 is positive and the graph would be above the x-axis, so eliminate it. This means (D) must be correct—subtracting x and 2 from both sides of the equation results in $y = -2$, which could be the equation of line L.

5.

At the grocery store, Gigi buys apples, a magazine, and a gallon of milk. The apples are priced per pound. In her state, there is no sales tax on food. If the total cost of her items is given by the function $C(p) = 1.89p + 1.07(3.99) + 4.49$, then the term $1.07(3.99)$ most likely represents

Ⓐ the cost of one gallon of milk.
Ⓑ the per-pound cost of the apples.
Ⓒ the cost of the magazine, including tax.
Ⓓ the cost of the magazine, not including tax.

Difficulty: Medium

Category: Heart of Algebra / Linear Equations

Strategic Advice: In a real-world scenario, a onetime fee does not depend on the variable and is therefore a constant. A unit rate, however, is always multiplied by the independent variable.

Getting to the Answer: The total cost consists of the gallon of milk (a constant), the per-pound cost of the apples (which depends on the number of pounds), and the cost of the magazine (which is the only taxed item). The constant in the equation is 4.49 and is, therefore, the cost of the gallon of milk; 1.89 is being multiplied by p (the number of pounds), so \$1.89 must be the per-pound cost of the apples. That leaves the remaining term, $1.07(3.99)$, which must be the cost of the magazine (\$3.99) plus a 7% sales tax.

6.

A picture framing shop sells ready-made frames and also does custom framing using different kinds and widths of wood or metal. The shop has a three-day sale. During the sale, for an 11-inch × 14-inch frame, a ready-made frame costs \$12 and a custom frame costs \$30. Over the course of the three days, the shop sells ninety-two 11 × 14 frames and collects \$1,788. Solving which system of equations would yield the number of 11 × 14 ready-made frames r and the number of 11 × 14 custom frames c that the shop sold during the three-day sale?

Ⓐ $\begin{cases} r + c = 92 \\ 12r + 30c = \dfrac{1{,}788}{3} \end{cases}$

Ⓑ $\begin{cases} r + c = 1{,}788 \\ 12r + 30c = 92 \times 3 \end{cases}$

Ⓒ $\begin{cases} r + c = 1{,}788 \\ 12r + 30c = 92 \end{cases}$

Ⓓ $\begin{cases} r + c = 92 \\ 12r + 30c = 1{,}788 \end{cases}$

Difficulty: Medium

Category: Heart of Algebra / Systems of Linear Equations

Strategic Advice: One equation should represent the total *number* of frames, while the other equation should represent the *revenue* from the frames.

Getting to the Answer: The number of custom frames c plus the number of ready-made frames r equals the total number of frames sold, 92. Therefore, one equation is $c + r = 92$. This means you can eliminate B and C. Now write the revenue equation: revenue per custom frame ($30c$) plus revenue per ready-made frame ($12r$) equals the total amount collected (1,788). The revenue equation is $30c + 12r = 1{,}788$. Don't let A fool you. The question says nothing about the revenue *per day* of the sale, so there is no reason to divide by 3. Choice (D) is correct.

7. Calculator

City	Cost per Square Foot
Detroit	$62.45
Atlanta	$74.19
New York City	$288.58
San Francisco	$420.99

In real estate, location is often the number one determinant of home prices. The table above shows the average price per square foot of houses in four cities. Assuming an average home size of 1,500 to 2,000 square feet, which inequality represents how much more in dollars a house in New York City would cost than in Detroit?

Ⓐ $x \geq 226.13$
Ⓑ $62.45 \leq x \leq 288.58$
Ⓒ $93{,}675 \leq x \leq 432{,}870$
Ⓓ $339{,}195 \leq x \leq 452{,}260$

Difficulty: Medium

Category: Heart of Algebra / Inequalities

Strategic Advice: The best way to answer this question is to pretend you are a homebuyer. How much more per square foot would your house cost in New York than Detroit? If the house was 1,500 square feet, how much more would this be? If the house was 2,000 square feet, how much more would this be?

Getting to the Answer: Based on the data in the table, a house would cost $288.58 – $62.45 = $226.13 more per square foot in New York than in Detroit. If the house was 1,500 square feet, it would cost 1,500(226.13) = $339,195 more. If the house was 2,000 square feet, it would cost 2,000(226.13) = $452,260 more. So, the house would cost somewhere between $339,195 and $452,260 more, which can be expressed as the compound inequality $339{,}195 \leq x \leq 452{,}260$.

8. Calculator

A student is doing a scale drawing of a woolly mammoth on a piece of poster board for her presentation on the last ice age. She was surprised to find that the woolly mammoth, reaching a height of only about 10 feet, 6 inches, was actually smaller than today's African elephant. Even more surprising is the fact that the woolly mammoth's tusks averaged 11.5 feet in length. If the student draws the mammoth 14 inches tall on her poster, approximately how many inches long should she make the tusks?

Ⓐ 12.78
Ⓑ 15.0
Ⓒ 15.33
Ⓓ 16.1

Difficulty: Medium

Category: Problem Solving and Data Analysis / Rates, Ratios, Proportions, and Percentages

Strategic Advice: Pay careful attention to the units. You need to convert all of the dimensions to inches and then find the scale factor.

Getting to the Answer: There are 12 inches in one foot, so the height of the woolly mammoth was 10 × 12 = 120 + 6 = 126 inches. The tusk length was 11.5 feet, or 11.5 × 12 = 138 inches. The student plans to draw the mammoth 14 inches tall, so find the scale factor of the two heights by writing them as a fraction $\frac{14}{126} = \frac{1}{9}$. This means the scale factor is $\frac{1}{9}$.

Multiply this by the length of the real mammoth's tusks to find the scaled length: $138 \times \frac{1}{9} = \frac{138}{9} = 15\frac{1}{3}$. This means the student should make the tusks 15.33 inches long.

9. Calculator

In an effort to decrease reliance on fossil fuels, some energy producers have started to utilize renewable resources. One such power plant uses solar panels to create solar energy during the day and then shifts to natural gas at night or when there is cloud cover. One particularly bright morning, the company increases the amount of its power typically generated by solar panels by 60%. During a cloudy spell, it decreases the amount by 30%, and then when the sun comes back out, it increases the amount again by 75% before shutting the panels down for the night. What is the net percent increase of this company's reliance on solar panels during that day?

Ⓐ 75%
Ⓑ 96%
Ⓒ 105%
Ⓓ 165%

Difficulty: Hard

Category: Problem Solving and Data Analysis / Rates, Ratios, Proportions, and Percentages

Strategic Advice: Start by determining what the question is asking. You need to find the net percent change in the power reliance on solar panels over the course of a day. To do this, you need to know how much the solar panels were relied on at the beginning of the day and how much at the end (neither of which is given).

Getting to the Answer: Whenever you aren't given a concrete starting point, pick one yourself. The best number to use when dealing with percents is 100. First, find how much power was derived from the solar panels after the first increase: $100 \times 0.6 = 60$. So, the company increased power from the solar panels to $100 + 60 = 160$. Next, find the amount after the decrease: $160 \times 0.3 = 48$, so the solar panels then provided $160 - 48 = 112$ units of power. Finally, find the amount after the last increase: $112 \times 0.75 = 84$, so the plant ended the day at $112 + 84 = 196$, which is $196 - 100 = 96$ more than it started the day with. To find the percent change, use the formula $\text{Percent change} = \frac{\text{amount of change}}{\text{original amount}}$ to get $\frac{96}{100} = 0.96 = 96\%$.

10. Calculator

Water is vital to human health. An average person should consume approximately 2.5 ounces of water per hour. However, because of the salt in it, seawater actually dehydrates the human body and should not be consumed. This is why boats must carry a supply of fresh water when embarking on long trips. Suppose a sailboat is traveling at an average speed of 4 nautical miles per hour with 3 people on board and the trip is 232 nautical miles. What is the minimum number of ounces of water the boat should stock before leaving?

Ⓐ 69.6
Ⓑ 145
Ⓒ 435
Ⓓ 1,113.6

Difficulty: Easy

Category: Problem Solving and Data Analysis / Rates, Ratios, Proportions, and Percentages

Strategic Advice: This is a question about rates, so pay careful attention to the units. As you read the question, decide if and when you will need to convert units.

Getting to the Answer: First determine how long it will take the boat to complete the trip. Set up and solve a proportion:

$$\frac{4 \text{ nautical miles}}{1 \text{ hour}} = \frac{232 \text{ nautical miles}}{x \text{ hours}}$$
$$4x = 232$$
$$x = 58$$

The question asks for the total number of ounces of water needed. The recommended rate of consumption is given in ounces per hour, and you now know the number of hours that it will take the boat to complete the trip. Multiply the number of total hours by the number of ounces needed per hour: $58 \times 2.5 = 145$. Be careful—this isn't the answer. Remember, there are 3 people on board. This is the amount 1 person needs to consume during the trip, so multiply by 3 to get $145 \times 3 = 435$ ounces.

11. Calculator

If $M = 3x^2 + 9x - 4$ and $N = 5x^2 - 12$, what is $2(M - N)$?

Ⓐ $-2x^2 + 9x + 8$
Ⓑ $-4x^2 + 18x - 32$
Ⓒ $-4x^2 + 18x + 16$
Ⓓ $8x^2 + 9x - 16$

Difficulty: Medium

Category: Passport to Advanced Math / Exponents

Strategic Advice: Adding polynomials is typically safer than subtracting them, because you may forget to distribute the negative sign when subtracting more than one term.

Getting to the Answer: To find $M - N$, multiply each term of N by -1 and then add the two polynomials by combining like terms.

$-1N = -5x^2 + 12$

$M + (-N) = 3x^2 + 9x - 4 - 5x^2 + 12$

$= -2x^2 + 9x + 8$

Don't forget to multiply the resulting expression by 2 to get $2(-2x^2 + 9x + 8) = -4x^2 + 18x + 16$.

12.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

When a spring is pressed tightly between two objects, it remains still. When one or both of those objects is disturbed, the spring starts to move. The equation above can be used to find the time period T in which a mass m, attached to a spring, makes a single oscillation (going all the way down and then back up). The variable k is a constant. Which of the following equations could be used to find the mass of the object?

Ⓐ $m = \frac{2\pi k}{T^2}$

Ⓑ $m = \frac{kT^2}{4\pi^2}$

Ⓒ $m = \frac{T^2}{4\pi^2 k}$

Ⓓ $m = \sqrt{\frac{T}{2\pi k}}$

Difficulty: Medium

Category: Passport to Advanced Math / Exponents

Strategic Advice: Don't spend too much time reading the scientific explanation of the equation. Focus on the question at the very end—it's just asking you to solve the equation for m.

Getting to the Answer: First, square both sides of the equation to get m out from under the radical. Then, divide both sides by $4\pi^2$. Finally, multiply both sides by k.

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$T^2 = (2\pi)^2\left(\frac{m}{k}\right)$$
$$T^2 = 4\pi^2\left(\frac{m}{k}\right)$$
$$\frac{T^2}{4\pi^2} = \frac{m}{k}$$
$$\frac{kT^2}{4\pi^2} = m$$

Session 1: Practice 1: Pre-Algebra and Algebra 1 Solutions

1.

A publishing company ships books to schools, some of which are hardback textbooks and some are paperback workbooks. Each shipping box can hold a maximum of 20 textbooks or 64 workbooks. Each textbook takes up 192 cubic inches of space, and each workbook takes up 60 cubic inches of space. One box can hold a maximum of 3,840 cubic inches. The shipping department is packing a box containing both types of books. Which of the following systems of inequalities can the department use to determine how many textbooks, t, and workbooks, w, can be shipped in one box?

Ⓐ
$t \leq 20$
$w \leq 64$
$60t + 192w \leq 3{,}840$

Ⓑ
$t \geq 20$
$w \geq 64$
$192t + 60w \geq 3{,}840$

Ⓒ
$t \leq 20$
$w \leq 64$
$192t + 60w \leq 3{,}840$

Ⓓ
$t \leq 192$
$w \leq 60$
$20t + 64w \leq 3{,}840$

Difficulty: Easy

Category: Heart of Algebra / Inequalities

Strategic Advice: Use the Kaplan Method for Translating English into Math. The clue "holds a maximum" means it can hold exactly that much or less, so use the symbol ≤. This means you can eliminate B.

Getting to the Answer: The box can hold a maximum of 20 textbooks, so the first inequality is $t \leq 20$. The box can hold a maximum of 64 workbooks, so the second inequality is $w \leq 64$ (eliminate D). The third inequality deals with the size of each book. The box can fit t textbooks multiplied by the size of the textbook, 192 cubic inches, and w workbooks multiplied by the size of the workbook, 60 cubic inches. The box can fit a maximum of 3,840 cubic inches total, so the last inequality is $192t + 60w \leq 3{,}840$.

2.

Which of the following does not represent a linear relationship?

Ⓐ

x	−1	−4	−7	−10	−13
y	8	6	4	2	0

Ⓑ

x	−3	−1	1	3	5
y	5	3	1	−1	−3

Ⓒ

x	1	2	3	4	5
y	−5	−5	−5	−5	−5

Ⓓ

x	−2	−1	0	1	2
y	4	1	0	1	4

Difficulty: Medium

Category: Heart of Algebra / Linear Equations

Strategic Advice: Check to see whether the change in the y-values compared with the change in the x-values is constant for each pair of values.

Getting to the Answer: The table in (D) does not represent a linear relationship because the x-values change by +1 each time, while the y-values change by −3, then −1, then +1, then +3. A linear relationship has a constant rate of change, which means it is either always increasing or always decreasing by the same amount. This data clearly changes direction and is therefore not linear.

3.

$$\begin{cases} Ax - 2y = 18 \\ Bx + 6y = 26 \end{cases}$$

If the graphs of the lines in the system of equations above intersect at (4, −1), what is the value of $\frac{B}{A}$?

Ⓐ -3
Ⓑ $-\frac{1}{3}$
Ⓒ $\frac{1}{2}$
Ⓓ 2

Difficulty: Medium

Category: Heart of Algebra / Systems of Linear Equations

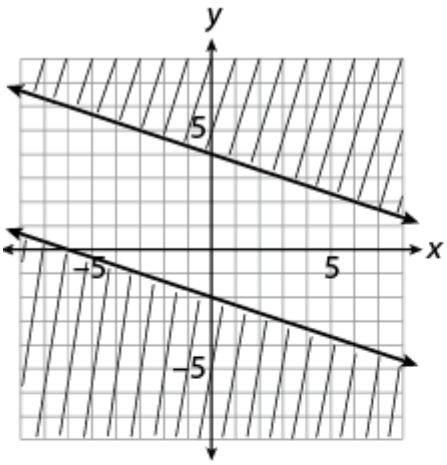
Strategic Advice: If the graphs intersect at (4, −1), then the solution to the system is $x = 4$ and $y = -1$. This means you can substitute these values into both equations and go from there.

Getting to the Answer: Substitute the values of x and y into each equation and solve for A and B. Then, divide B by A.

$Ax - 2y = 18$	$Bx + 6y = 26$
$A(4) - 2(-1) = 18$	$B(4) + 6(-1) = 26$
$4A + 2 = 18$	$4B - 6 = 26$
$4A = 16$	$4B = 32$
$A = 4$	$B = 8$

Therefore, $\frac{B}{A} = \frac{8}{4} = 2$.

4.



Which of the following systems of inequalities could be represented by the graph shown?

Ⓐ $\begin{cases} 3x - y \geq 4 \\ 3x - y \leq -2 \end{cases}$

Ⓑ $\begin{cases} 3x + y \geq 4 \\ 3x + y \leq -2 \end{cases}$

Ⓒ $\begin{cases} x - 3y \geq 12 \\ x - 3y \leq -6 \end{cases}$

Ⓓ $\begin{cases} x + 3y \geq 12 \\ x + 3y \leq -6 \end{cases}$

Difficulty: Hard

Category: Heart of Algebra / Inequalities

Strategic Advice: The system in the graph shows shading on opposite sides of two parallel lines, which means there is no solution to the system. This means you're looking for two equations with the same slope, different *y*-intercepts, and different inequality symbols.

Getting to the Answer: The equations are all given in standard form ($Ax + By = C$). It would be very time-intensive to convert all four systems to slope-intercept form, so look for ways to eliminate choices more quickly. Each line in the graph falls 1 unit and runs 3 units, so you need to find two lines that have a slope of $-\frac{1}{3}$. Use the trick $m = \frac{-A}{B}$ to quickly determine the slopes. The lines in A have a slope of $\frac{-3}{-1} = 3$, so eliminate A; the lines in B have a slope of $\frac{-3}{1} = -3$, so eliminate B; the lines in C have a slope of $\frac{-1}{-3} = \frac{1}{3}$, so eliminate C (pay attention to the sign). This means (D) must be correct. The lines in (D) have a slope of $\frac{-1}{3} = -\frac{1}{3}$, which matches the graph. You don't have to check the shading because none of the other slopes were a match.

5.

Equation 1	
x	*y*
5	–8
4	–5
3	–2
2	1

Equation 2	
x	*y*
–8	3
–6	4
–4	5
–2	6

The tables above represent data points for two linear equations. If the two equations form a system, what is the *x*-coordinate of the solution to that system?

Difficulty: Medium

Category: Heart of Algebra / Systems of Linear Equations

Strategic Advice: The solution to the system is the point that both tables will have in common, but the tables, as given, do not share any points. You could use the data to write the equation of each line and then solve the system, but this would use up valuable time on Test Day. Instead, whenever data is presented in a table, look for patterns that can be extended.

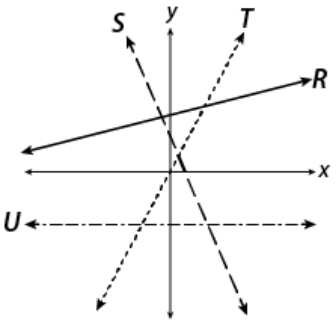
Getting to the Answer: In the table on the left, the *x*-values decrease by 1 each time and the *y*-values increase by 3. In the table on the right, the *x*-values increase by 2 each time and the *y*-values increase by 1. Use these patterns to continue the tables:

Equation 1	
x	*y*
5	–8
4	–5
3	–2
2	1
1	4
0	**7**

Equation 2	
x	*y*
–8	3
–6	4
–4	5
–2	6
0	**7**
2	8

The point (0, 7) satisfies both equations, so the *x*-coordinate of the solution to the system is 0.

6.



Which of the following lists correctly orders the lines in the figure above according to their slopes, from least to greatest?

Ⓐ *R*, *T*, *S*, *U*
Ⓑ *S*, *U*, *R*, *T*
Ⓒ *S*, *R*, *U*, *T*
Ⓓ *U*, *S*, *R*, *T*

Difficulty: Easy

Category: Heart of Algebra / Linear Equations

Strategic Advice: A horizontal line has a slope of 0. Lines that slant downward from left to right have a negative slope, and lines that slant upward from left to right have a positive slope.

Getting to the Answer: Only one line has a negative slope, *S*, so it should come first in the list. This means you can eliminate A and D. Next comes the horizontal line with a slope of 0, which is line *U*. You can now eliminate C, which means (B) must be correct. To confirm (which isn't absolutely necessary), there are two lines with positive slopes: *R* and *T*. Line *T* has a steeper slant than line *R*, which means line *T* has a greater slope; therefore, the correct ordering is *S*, *U*, *R*, *T*.

7. Calculator

Price per Pencil	Projected Number of Units Sold
$0.20	150,000
$0.25	135,000
$0.30	120,000
$0.35	105,000
$0.40	90,000
$0.45	75,000

Generally, the price of an item is a good indicator of how many units of that item will be sold. The lower the price, the more units will be sold. A marketing department develops a table showing various price points and the projected number of units sold at that price point. Which of the following represents the linear relationship shown in the table, where x is the price and y is the number of units sold?

Ⓐ $y = 0.03x + 150{,}000$
Ⓑ $y = 300{,}000x + 75{,}000$
Ⓒ $y = -300{,}000x + 90{,}000$
Ⓓ $y = -300{,}000x + 210{,}000$

Difficulty: Medium

Category: Heart of Algebra / Linear Equations

Strategic Advice: Take a quick peek at the answer choices. The equations are given in slope-intercept form, so start by finding the slope.

Getting to the Answer: Find the slope by substituting two pairs of values from the table (try to pick easy ones, if possible) into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$. Keep in mind that the projected number of units sold depends on the price, so the price is the independent variable (x) and the projected number is the dependent variable (y). Using the points (0.2, 150,000) and (0.4, 90,000), the slope is:

$$m = \frac{150{,}000 - 90{,}000}{0.2 - 0.4}$$
$$= \frac{60{,}000}{-0.2}$$
$$= -300{,}000$$

This means you can eliminate A and B because the slope is not correct. Don't let B fool you—the projected number of units sold goes down as the price goes up, so there is an inverse relationship, which means the slope must be negative. To choose between C and D, you could find the y-intercept of the line, but this is a fairly time-intensive process. Instead, choose any pair of values from the table, such as (0.2, 150,000), and substitute into C and D only. Choice (D) is correct because 150,000 = −300,000(0.2) + 210,000 is a true statement.

8. Calculator

A mailing supply store sells small shipping boxes in packs of 8 or 20. If the store has 61 packs in stock totaling 800 small shipping boxes, how many packs have 20 boxes in them, assuming all the packs are full?

Ⓐ 26
Ⓑ 32
Ⓒ 35
Ⓓ 40

Difficulty: Medium

Category: Heart of Algebra / Systems of Linear Equations

Strategic Advice: Create a system of linear equations where e represents the number of packs with 8 boxes and t represents the number of packs with 20 boxes. Before selecting your final answer, make sure you answered the right question (the number of packs that have 20 boxes).

Getting to the Answer: The first equation should represent the total number of packs, $e + t = 61$. The second equation should represent the total number of boxes. Because e represents packs with 8 boxes and t represents packs with 20 boxes, the second equation should be $8e + 20t = 800$. Now, solve the system using substitution. Solve the first equation to find that $e = 61 - t$. Then, substitute the result into the second equation:

$8e + 20t = 800$

$8(61 - t) + 20t = 800$

$488 - 8t + 20t = 800$

$488 + 12t = 800$

$12t = 312$

$t = 26$

We assigned the variable t to the number of packs with 20 boxes, so 26 packs have 20 boxes. This is what the question asks for, so you don't need to find the value of e.

9. Calculator

The American political system is largely a twoparty system. In fact, only six candidates who were not associated with either the Republican or the Democratic Party have been elected governor in any state since 1990. In one such election, the ratio of votes received for the Independent candidate to the Democratic candidate to the Republican candidate was approximately 19:18:13. If 510,000 votes were cast in the election, how many more votes were cast for the Independent candidate than for the Republican candidate?

Ⓐ 6,000
Ⓑ 10,200
Ⓒ 61,200
Ⓓ 193,800

Difficulty: Easy

Category: Problem Solving and Data Analysis / Rates, Ratios, Proportions, and Percentages

Strategic Advice: Don't let the three-way ratio confuse you. You can answer this question just like any other ratio question. Before selecting your answer, make sure you answered the right question (how many *more* votes for the Independent candidate than for the Republican candidate).

Getting to the Answer: Set up an equation using parts: 19 parts of the vote were cast for the Independent candidate, 18 parts for the Democrat, and 13 parts for the Republican. You don't know how big a part is, so call it x. Now, write and solve an equation:

$19x + 18x + 13x = 510{,}000$

$50x = 510{,}000$

$x = 10{,}200$

This means each part is equal to 10,200 votes. Now, you could multiply 19 by this number and 13 by this number, and then subtract. Or, you could recognize that the Independent received 19 – 13 = 6 more parts of the vote than the Republican, or 6(10,200) = 61,200 more votes.

10. Calculator

Mount Fuji in Japan was first climbed by a monk in 663 ad and subsequently became a Japanese religious site for hundreds of years. It is now a popular tourist site. When ascending the mountain, tourists drive part of the distance and climb the rest of the way. Suppose a tourist drove to an elevation of 2,390 meters and from that point climbed to the top of the mountain, and then descended back to the car taking the same route. If it took her a total of 7 hours to climb up and back down, and she climbed at an average rate of 264 vertical meters per hour going up and twice that going down, approximately how tall is Mount Fuji?

Ⓐ 1,386 meters
Ⓑ 2,772 meters
Ⓒ 3,776 meters
Ⓓ 5,172 meters

Difficulty: Medium

Category: Problem Solving and Data Analysis / Rates, Ratios, Proportions, and Percentages

Strategic Advice: There are a few ways to answer this question, but the quickest way is to average the tourist's climb rate over the 7 hours.

Getting to the Answer: The distance going up and back down is the same (because she uses the same route), so find the average of the tourist's speed over both the ascent and the descent. She climbed 264 meters per hour going up and twice that, 528 meters per hour, going back down, so her average climb rate was 264 + 528 = 792 ÷ 2 = 396 meters per hour. It took her 7 hours. Use the formula

Distance = rate × time to find the distance:

Distance = 396 × 7

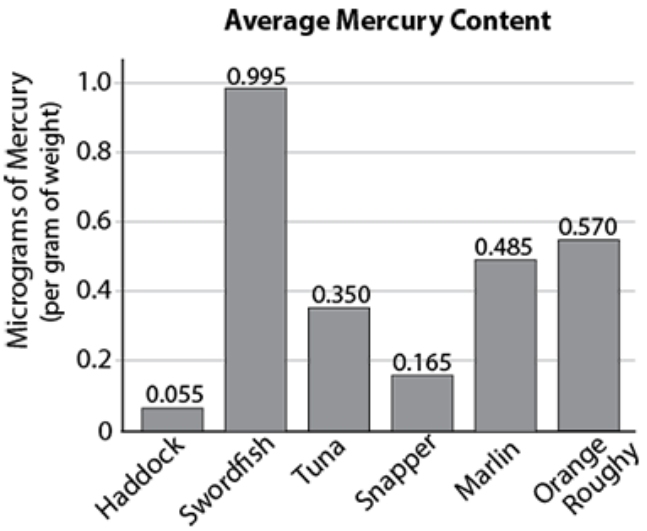
Distance = 2,772

But remember, this amount represents both up and down the mountain, so divide by 2 to find that the vertical distance between the point where she started climbing and the top of the mountain is 1,386 meters. Be careful—this is not the answer! The question asks how tall Mount Fuji is, so don't forget to add the vertical distance she drove, 2,390 meters, to get 2,390 + 1,386 = 3,776 meters.

Session 1: Problem Set 2: Algebra 2 and Statistics **Solutions**

1. Calculator

Mercury is a naturally occurring metal that can be harmful to humans. The current recommendation is for humans to take in no more than 0.1 microgram for every kilogram of their weight per day. Fish generally carry high levels of mercury, although certain fish have higher mercury content than others. Fish, however, are healthy sources of many other nutrients, so nutritionists recommend keeping them in the human diet. The figure below shows the average mercury content of several types of fish.



If a person weighs 82 kilograms, how many grams of snapper can he safely consume per day? Round your answer to the nearest gram.

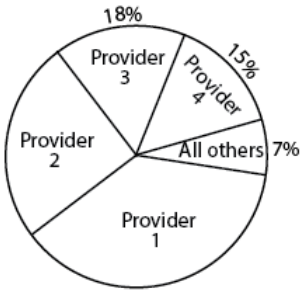
Difficulty: Medium

Category: Problem Solving and Data Analysis / Statistics and Probability

Strategic Advice: The key to answering a question like this is in reading the labels on the graph carefully. You do not need to convert grams to micrograms or vice versa. They are simply the units for mercury content.

Getting to the Answer: Start by determining how many micrograms of mercury a person who weighs 82 kilograms can consume: $82 \times 0.1 = 8.2$ micrograms. Next, find *snapper* on the bar graph and determine how many micrograms it contains (per gram of weight): 0.165. Divide the number of micrograms the person can consume, 8.2, by the number in each gram of snapper to arrive at $8.2 \div 0.165 = 49.697$, or about 50 grams.

2. Calculator



A company's market share is the percent of consumers who utilize the services or buy the products of that company. The pie chart above shows the different market shares of cable providers in a certain region. If the ratio of the market shares of Provider 1 to Provider 2 is 3:2, what is Provider 1's market share?

Ⓐ 24%
Ⓑ 30%
Ⓒ 36%
Ⓓ 42%

Difficulty: Easy

Category: Problem Solving and Data Analysis / Rates, Ratios, Proportions, and Percentages

Strategic Advice: Break the question into steps. Before you can use the ratio, you need to find the percent of the market that utilizes Provider 1 and Provider 2.

Getting to the Answer: The ratio given in the question is 3:2, so write this as 3 parts (Provider 1) and 2 parts (Provider 2). You don't know how big a part is, so call it x. This means that $3x + 2x$ = the percent of consumers that utilize Provider 1 and Provider 2 for cable services, which is 100% – all the other providers: 100 – (18 + 15 + 7) = 100 – 40 = 60.

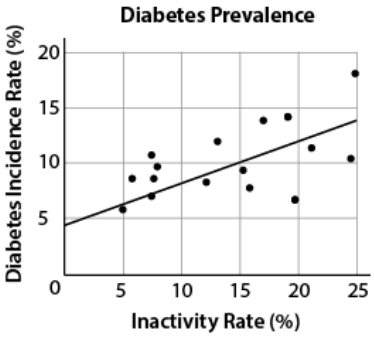
$3x + 2x = 60$

$5x = 60$

$x = 12$

Each part has a value of 12 and three parts use Provider 1, so Provider 1's market share is $3 \times 12 = 36\%$.

3.



Increased physical activity has been linked to a lower incidence rate of diabetes. The scatterplot above shows the relationship between the percent of people in a certain country whose daily activity qualifies them as "inactive" and the incidence rate of diabetes in that country. The line of best fit for the data is also shown. Which of the following best represents the meaning of the y-intercept of the line of best fit in the context of this question?

Ⓐ The predicted incidence rate of diabetes when the entire country is considered active
Ⓑ The predicted incidence rate of diabetes when the entire country is considered inactive
Ⓒ The predicted percent of people who will be active when the incidence rate of diabetes is 0%
Ⓓ The predicted percent of people who will be inactive when the incidence rate of diabetes is 0%

Difficulty: Medium

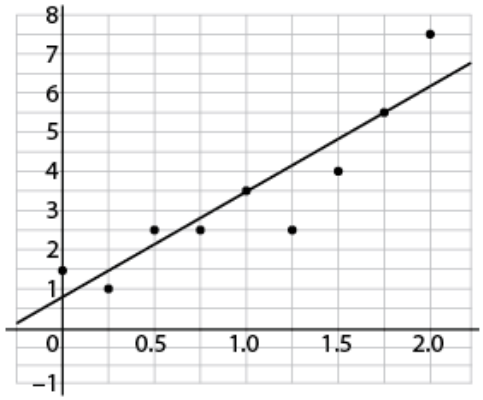
Category: Problem Solving and Data Analysis / Scatterplots

Strategic Advice: You don't need to know the *y*-intercept to answer the question, so don't waste valuable time trying to find it. Instead, use the labels on the axes to determine the meaning of the *y*-intercept.

Getting to the Answer: The *y*-intercept is the point at which $x = 0$. In this real-world scenario, the percent of people who are considered inactive is graphed on the *x*-axis, so the *y*-intercept occurs when 0% of the country is inactive. Now, look carefully at the way the answer choices are worded. Choice (A) is correct because it says the entire country is *active*, which is the same as saying 0% are inactive. Choice B says the entire country is *inactive*, which means 100% is inactive (not 0% inactive).

4.

Use the data in the scatterplot and the line of best fit shown to answer the following question



Which of the following values most accurately reflects the average rate of change of the data based on the line of best fit?

Ⓐ $\frac{3}{8}$

Ⓑ $\frac{3}{4}$

Ⓒ $\frac{4}{3}$

Ⓓ $\boxed{\frac{8}{3}}$

Difficulty: Medium

Category: Problem Solving and Data Analysis / Scatterplots

Strategic Advice: The average rate of change is the same as the slope of the line of best fit. Look for two points that lie on the line (or as close as possible to the line) and then find the slope using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$. Pay careful attention to how the axes are labeled to make sure you write the points accurately.

Getting to the Answer: Using the points (1, 3.5) and (1.75, 5.5), the average rate of change is $\frac{5.5 - 3.5}{1.75 - 1} = \frac{2}{0.75} = 2.\overline{6}$ or $\frac{8}{3}$.

5.

Which of the following functions has a domain of $x \geq 2$?

Ⓐ $f(x) = -x^2 + 2$

Ⓑ $\boxed{g(x) = -\sqrt{x - 2}}$

Ⓒ $h(x) = -\sqrt{x} + 2$

Ⓓ $k(x) = -|x - 2|$

Difficulty: Medium

Category: Passport to Advanced Math / Functions

Strategic Advice: The domain of a function is the set of x-values (inputs) for which the function is defined. Of all the parent functions, the only ones that have a *restricted* domain (a domain that is not all real numbers) are the square root function (because the square root of a negative number is imaginary) and the rational function (because you cannot divide by 0).

Getting to the Answer: The domain in the question is restricted to numbers greater than or equal to 2, so you can immediately eliminate A and D—the domain of a quadratic function and an absolute function is all real numbers. To choose between (B) and C, you can draw a quick sketch or think about how transformations affect the domain of each function. The domain of the parent function $f(x) = \sqrt{x}$ is $x \geq 0$ (nonnegative numbers). In (B), the parent function is reflected vertically across the

horizontal axis (which doesn't change the domain) and then shifted to the right 2 (making the domain $x \geq 2$), so (B) is correct. Note that in C, the function is reflected across the horizontal axis and then shifted *up* 2 units, which adds 2 to the *range* of the function, not the domain.

6.

$$g(x) = \begin{cases} x^2 - 1, \text{ if } x \leq 0 \\ \frac{x^2}{3} + 1, \text{ if } 0 < x \leq 3 \\ 5x + 3, \text{ if } x > 3 \end{cases}$$

For the piecewise defined function $g(x)$ shown above, what is the value of $g(2)$?

Difficulty: Medium

Category: Passport to Advanced Math / Functions

Strategic Advice: Piecewise defined functions look intimidating, but they are usually very simple functions—they're just written in pieces. Your job is to figure out which piece of the function you need to use to answer the question.

Getting to the Answer: The right-hand side of each piece of the function tells you what part of the domain (which x-values) goes with that particular function. In this function, only values of x that are less than zero go with the top function, values of x between 0 and 3 go with the middle function, and values of x that are greater than 3 go with the bottom function. Because 2 is between 0 and 3, plug it into the middle function and simplify:

$$g(2) = \frac{(2)^2}{3} + 1$$
$$= \frac{4}{3} + 1$$
$$= \frac{4}{3} + \frac{3}{3} = \frac{7}{3}$$

7.

$$\frac{x}{x-1} - \frac{2}{x} = \frac{1}{x-1}$$

What is one possible solution to the rational equation shown above?

Category: Passport to Advanced Math / Exponents

Strategic Advice: Solving a rational equation takes patience and a good deal of algebraic manipulation. You'll need to find a common denominator and multiply both sides of the equation by that denominator. The next steps will depend on what kind of equation results from the previous steps.

Getting to the Answer: Start by multiplying both sides of the equation (all three terms) by the common denominator, which is $x(x + 1)$. Try to write neatly, especially when canceling terms, so you don't lose track of anything.

$$x(x+1)\left(\frac{x}{x+1}\right) - x(x-1)\left(\frac{2}{x}\right) = x(x+1)\left(\frac{1}{x+1}\right)$$

$$x(x) - 2(x-1) = x(1)$$

$$x^2 - 2x + 2 = x$$

The resulting equation is quadratic, so set it equal to zero and either try to factor it or use the quadratic formula to solve it.

$$x^2 - 2x + 2 = x$$
$$x^2 - 3x + 2 = 0$$
$$(x-1)(x-2) = 0$$

$$x - 1 = 0$$
$$x = 1$$
or
$$x - 2 = 0$$
$$x = 2$$

Be careful here—whenever there is a variable in the denominator of an equation, you must check to make sure that the solutions do not result in division by zero. The solution $x = 1$ *does* result in division by 0, so it is invalid. That means the only correct solution is $x = 2$.

8.

$$\left(36x^4y^7\right)^{\frac{1}{2}}$$

Which of the following is equivalent to the expression given above?

Ⓐ $\dfrac{36x^4y^7}{2}$

Ⓑ $6xy^2\sqrt{y}$

Ⓒ $6x^2y^3\sqrt{y}$

Ⓓ $\left(36x^4y^7\right)^{-2}$

Difficulty: Medium

Category: Passport to Advanced Math / Exponents

Strategic Advice: To make the expression look more familiar, rewrite the fraction exponent as a radical. Then, find the largest perfect squares of each factor and take their square roots (which allows you to bring them outside the radical).

Getting to the Answer:

$$\left(36x^4y^7\right)^{\frac{1}{2}}$$
$$= \sqrt{36x^4y^7}$$
$$= \sqrt{(6^2)(x^2)^2(y^3)^2y}$$
$$= 6x^2y^3\sqrt{y}$$

You could also use prime factorization and look for pairs of factors that are the same in order to bring them outside the radical.

9.

Given the function $f(x) = \frac{1}{4}x - 2$, what domain value corresponds to a range value of $-\frac{5}{3}$?

Ⓐ $-\frac{29}{12}$

Ⓑ $\frac{4}{3}$

Ⓒ $\frac{7}{3}$

Ⓓ $\frac{29}{12}$

Difficulty: Medium

Category: Passport to Advanced Math / Functions

Strategic Advice: Don't answer this question too quickly—you may be tempted to substitute $-\frac{5}{3}$ for x, but $-\frac{5}{3}$ is the output (range), not the input (domain).

Getting to the Answer: The given range value is an output value, so substitute $-\frac{5}{3}$ for $f(x)$ and use inverse operations to solve for x, which gives you the corresponding domain value. Start by multiplying the equation by the greatest common multiple of 3 and 4, which is 12, in order to clear the fractions.

$$-\frac{5}{3} = \frac{1}{4}x - 2$$
$$(12)\left(-\frac{5}{3}\right) = (12)\frac{1}{4}x - 2(12)$$
$$-20 = 3x - 24$$
$$4 = 3x$$
$$\frac{4}{3} = x$$

10.

$$\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

In electronic circuits, resistors are often paired to manage the flow of the electrical current. To find the total resistance of a pair of parallel resistors, electricians use the formula shown above, where R_1 is the resistance of the first resistor and R_2 is the resistance of the second resistor. Which of the following is another way to represent this formula?

Ⓐ $\frac{R_1 R_2}{R_1 + R_2}$

Ⓑ $\frac{R_1 + R_2}{R_1 R_2}$

Ⓒ $\frac{1}{R_2} - \frac{1}{R_1}$

Ⓓ $R_1 + R_2$

Difficulty: Hard

Category: Passport to Advanced Math / Exponents

Strategic Advice: Simplifying a complex rational expression requires planning and patience. Here, you need to write the denominator of the big expression as a single fraction and then you can simply "flip it" to adjust for the "1 over."

Getting to the Answer: Start by writing $\frac{1}{R_1}+\frac{1}{R_2}$ as a single term. To do this, find the common denominator and write each piece of the sum in terms of that denominator. The common denominator is $R_1\ R_2$.

$$\begin{aligned}\frac{1}{R_1}+\frac{1}{R_2} &= \frac{R_2}{R_2}\left(\frac{1}{R_1}\right)+\frac{R_1}{R_1}\left(\frac{1}{R_2}\right)\\ &= \frac{R_2}{R_1R_2}+\frac{R_1}{R_1R_2}\\ &= \frac{R_1+R_2}{R_1R_2}\end{aligned}$$

But remember, this fraction was the denominator under 1, so you need to write the reciprocal (flip it); the correct expression is $\frac{R_1R_2}{R_1+R_2}$.

11.

Which of the following are solutions to the quadratic equation $(x-2)^2=\frac{16}{25}$?

Ⓐ $x=\pm\sqrt{\frac{4}{5}}$

Ⓑ $x=-\frac{4}{5}, x=\frac{4}{5}$

Ⓒ $x=\frac{6}{5}, x=\frac{14}{5}$

Ⓓ $x=\frac{14}{5}, x=-\frac{14}{5}$

Difficulty: Medium

Category: Passport to Advanced Math / Quadratics

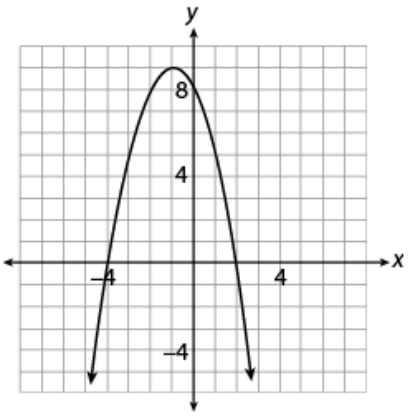
Strategic Advice: Taking the square root is the inverse operation of squaring, and both sides of the equation are already perfect squares, so take their square roots. Then solve the resulting equations. Remember, there will be two equations to solve.

Getting to the Answer:

$$\begin{aligned}(x-2)^2 &= \frac{16}{25}\\ \sqrt{(x-2)^2} &= \sqrt{\frac{16}{25}}\\ x-2 &= \pm\frac{\sqrt{16}}{\sqrt{25}}\\ x &= 2\pm\frac{4}{5}\end{aligned}$$

Now, simplify each equation: $x=2-\frac{4}{5}=\frac{10}{5}-\frac{4}{5}=\frac{6}{5}$ and $x=2+\frac{4}{5}=\frac{10}{5}+\frac{4}{5}=\frac{14}{5}$.

12.



The graph of the function $f(x) = -x^2 - 2x + 8$ is shown in the figure above. For what values of x does $f(x) = 5$?

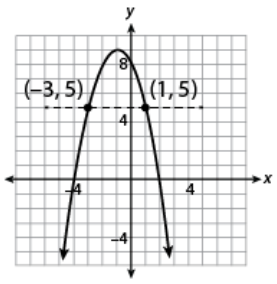
Ⓐ –4 and 2
Ⓑ –3 and 1
Ⓒ –1 and 9
Ⓓ 5 and 8

Difficulty: Medium

Category: Passport to Advanced Math / Quadratics

Strategic Advice: This question is very straightforward if you understand the language of functions. Although you could set the second equation equal to 0 and solve for x, the solution can be found simply by looking at the graph.

Getting to the Answer: The statement $f(x) = 5$ means to find the x-values on the graph when y is 5. To do this, draw a horizontal line across the graph at $y = 5$ and read the x-coordinates of the points where the line intersects the parabola.



The function $f(x) = -x^2 - 2x + 8$ has x-values of –3 and 1 when $y = 5$.

13.

If the equation $\frac{2}{9}x^2 + \frac{8}{3}x - 7 = 3$ has solutions x_1 and x_2, what is the product of x_1 and x_2?

Ⓐ –45
Ⓑ –15
Ⓒ –5
Ⓓ 3

Difficulty: Hard

Category: Passport to Advanced Math / Quadratics

Strategic Advice: This is a quadratic equation, so you need one side to equal 0 and then, best-case scenario, you'll be able to factor. If not, you can rely on the quadratic formula.

Getting to the Answer: First, subtract 3 from both sides of the equation. Then multiply everything by 9 to clear the fractions.

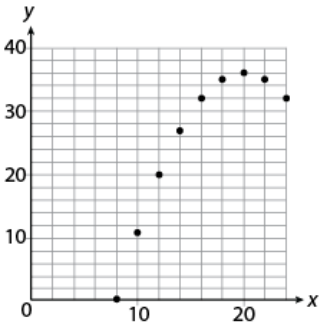
$$\frac{2}{9}x^2+\frac{8}{3}x-7=3$$
$$\frac{2}{9}x^2+\frac{8}{3}x-10=0$$
$$9\left(\frac{2}{9}x^2+\frac{8}{3}x-10=0\right)$$
$$2x^2+24x-90=0$$

Each number in the equation is divisible by 2, so factor out a 2 and go from there.

$$2x^2+24x-90=0$$
$$2(x^2+12x-45)=0$$
$$2(x+15)(x-3)=0$$

The solutions are −15 and 3, but be careful! The question asks for the product of the solutions, so the correct answer is (−15)(3) = −45.

14. Calculator



If a quadratic equation is used to model the data shown in the scatterplot above, and the model fits the data exactly, which of the following is a solution to the quadratic equation?

Ⓐ 28
Ⓑ 32
Ⓒ 34
Ⓓ 36

Difficulty: Hard

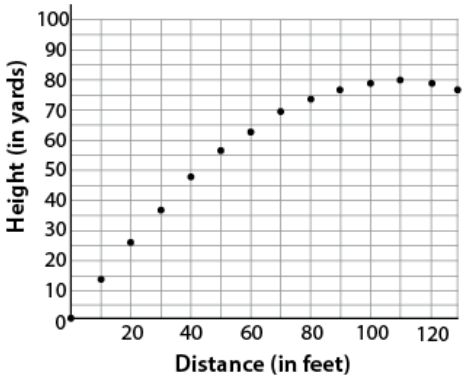
Category: Problem Solving and Data Analysis / Scatterplots

Strategic Advice: This question requires a conceptual understanding of modeling data and properties of quadratic equations. You also need to recall that a *solution* to an equation is the same as the *x*-intercept of the equation's graph.

Getting to the Answer: The graph of a quadratic equation is symmetric with respect to its axis of symmetry. The axis of symmetry occurs at the *x*-value of the vertex, which according to the graph is 20. You can also see from the graph that one of the *x*-intercepts is $x = 8$. This means that 8 is a solution to the quadratic equation. Unfortunately, 8 isn't one of the answer choices. However, because the graph of a quadratic equation is symmetric, the other solution (*x*-intercept) must be the same distance from the vertex as 8 is, which is 20 − 8 = 12 units. Therefore, the other solution to the equation is $x = 20 + 12 = 32$.

Session 1: Practice 2: Algebra 2 and Statistics Solutions

1.



The figure above shows part of the path of a planned roller coaster hill. What is the sum, in feet, of the vertical height and the horizontal distance that the roller coaster will travel while on this particular hill?

Ⓐ 220
Ⓑ 300
Ⓒ 460
Ⓓ 900

Difficulty: Hard

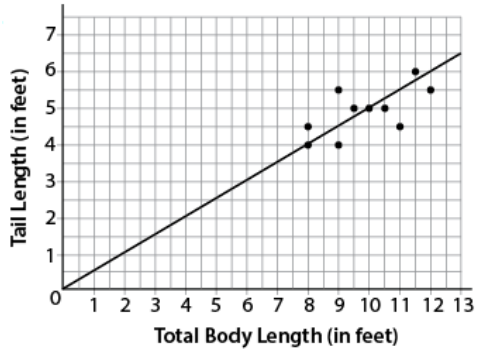
Category: Passport to Advanced Math /

Scatterplots

Strategic Advice: Make sure you read the axis labels and the question carefully. You'll also need to rely on your knowledge of quadratic equations.

Getting to the Answer: The question asks for the sum of the vertical height and the horizontal distance that the roller coaster will travel above ground. The data points follow a parabolic (U-shaped) path, which means you can use properties of quadratic equations to find the solution. The vertical height is fairly straightforward—the vertex of the parabola is located at (110, 80), so the vertical height that the roller coaster reaches is 80 yards (notice the units). To find the horizontal distance, think about symmetry. Because the vertex occurs at a distance of 110 feet, the total horizontal distance that the roller coaster will travel is twice that, or 220 feet. Convert 80 yards to feet and add the result to 220 to arrive at the correct answer, $80 \times 3 = 240$ and $240 + 220 = 460$ feet.

2.



The Florida Department of Wildlife caught and tagged 10 adult female alligators as part of an effort to protect this endangered species. They took various measurements and readings related to body size and health. The total body length is plotted against the tail length in the scatterplot shown above, along with a line of best fit. Which of the following equations best models the data?

Ⓐ $y = 0.5x$
Ⓑ $y = 2x$
Ⓒ $y = 0.4x + 1$
Ⓓ $y = 0.6x - 1$

Difficulty: Easy

Category: Problem Solving and Data Analysis / Scatterplots

Strategic Advice: Don't get bogged down in the contextual information in this question. You're looking for the equation that best matches the line drawn through the data points.

Getting to the Answer: The best-fit line begins at the origin, which means the y-intercept is 0 (the b in the equation $y = mx + b$), so you can eliminate C and D. Now, find the slope of the line. Between (0, 0) and (8, 4), the line rises 4 units and runs 8 units, so the slope is $\frac{4}{8} = \frac{1}{2}$ which is equivalent to 0.5. This means (A) is correct.

3. Calculator

A dendrologist (a botanist who studies trees exclusively) is examining the way in which a certain tree sheds its leaves. He tracks the number of leaves shed each day over the period of a month, starting when the first leaf is shed. He organizes the data in a scatterplot and sees that the data can be modeled using an exponential function. He determines the exponential model to be $f(x) = 6(1.92)^x$, where x is the number of days after the tree began to shed its leaves. What does the value 1.92 in the function tell the dendrologist?

Ⓐ The number of leaves shed almost doubles each day.
Ⓑ The number of leaves shed almost doubles every six days.
Ⓒ The number of leaves left on the tree is reduced by about 92% each day.
Ⓓ The number of leaves left on the tree is reduced by about 92% every six days.

Difficulty: Medium

Category: Problem Solving and Data Analysis / Scatterplots

Strategic Advice: The dendrologist uses an exponential function to model the data. When an exponential equation is written in the form of $f(x) = ab^x$, a is the starting amount and b is the rate of growth or decay.

Getting to the Answer: Read the question carefully. The dendrologist is studying the number of leaves shed, not the number of leaves left on the tree, so you can eliminate C and D. Remember, a is the starting amount, not the unit of time, so it can't represent the number of days, which means you can also eliminate B. Choice (A) is correct because 1.92 is b in the equation, which represents the growth rate, so it tells the dendrologist that the number of leaves shed almost doubles (192% is very close to 200%) each day.

4 and 5. Calculator

Plants are capable of cross-pollinating with related but different plants. This creates a hybrid Sometimes, a hybrid plant is superior to the two different plants from which it was derived. This as "hybrid vigor." Scientists can examine the DNA of a plant to see if it is a hybrid. This can information because if the plant appears superior, it would be beneficial to develop more of these An agricultural scientist examines an orchard that has several types of apple trees and orange which ones are hybrids. Some of the information she collected is shown in the table below.

	Apple Trees	Orange Trees	Total
Hybrid			402
Non-hybrid		118	
Totals			628

4. Based on the data, if 45% of the apple trees are not hybrids, how many apple trees are hybrids?

Ⓐ 50
Ⓑ 132
Ⓒ 226
Ⓓ 240

Difficulty: Medium

Category: Problem Solving and Data Analysis / Statistics and Probability

Strategic Advice: Start by filling in any cells in the table that you can, using the information provided in the table itself (kind of like a sudoku puzzle).

Getting to the Answer: Because there are 628 trees total and 402 are hybrids, you know that 628 – 402 = 226 are not hybrids. Then, because 118 orange trees are not hybrids, you know that 226 – 118 = 108 apple trees are not hybrids. Now, you've reached the point at which the table can't help you anymore. So, look at the question. It says that 45% of the apple trees are not hybrids.

Use the formula Percent × whole = part to arrive at the equation $0.45w = 108$. Then, solve for w by dividing: 108 ÷ 0.45 = 240, which tells you there are 240 apple trees in total. This means there are 240 – 108 = 132 apple trees that are hybrids.

5.

The scientist wants to study the orange trees to check for hybrid vigor. If she chooses one orange tree at random, what is the probability that it will be a hybrid?

Ⓐ $\frac{59}{194}$

Ⓑ $\frac{97}{157}$

Ⓒ $\frac{135}{314}$

Ⓓ $\frac{135}{194}$

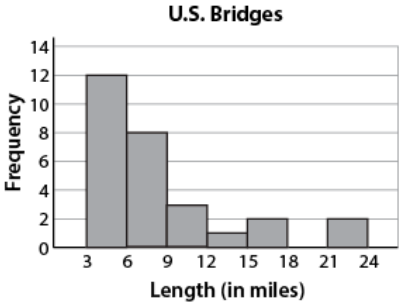
Difficulty: Medium

Category: Problem Solving and Data Analysis / Statistics and Probability

Strategic Advice: Start by completing the rest of the table. Use the information you found in the previous question.

Getting to the Answer: Because there are 402 hybrids in total, there are 402 – 132 = 270 orange trees that are hybrids, which means there are 270 + 118 = 388 orange trees in total. Now, find the probability that if the scientist selects one orange tree, it will be a hybrid. There are 388 orange trees total, and of those, 270 are hybrids, so the probability of picking a hybrid is $\frac{270}{388} = \frac{135}{194}$.

6. Calculator



The Lake Pontchartrain Causeway Bridge in Louisiana is the longest bridge in the United States, at 23.83 miles long. The histogram above shows the distribution of the lengths, in miles, of 28 of the longest bridges in the United States, including Lake Pontchartrain Causeway Bridge. Which of the following could be the median length of the 28 bridges represented in the histogram?

Ⓐ 5.9
Ⓑ 7.9
Ⓒ 9.2
Ⓓ 9.9

Difficulty: Medium

Category: Problem Solving and Data Analysis / Statistics and Probability

Strategic Advice: The *median* of a data set is the middle value when the data points are arranged from least to greatest (or greatest to least). When there is an even number of data points, the median is the average of the two middle values.

Getting to the Answer: The histogram represents the lengths of 28 bridges, so the median length is the average of the 14th and 15th longest bridges. Because the number of bridges that are less than 6 miles long is 12, and the number of bridges that are less than 9 miles long is 12 + 8 + 20, the median length of the 28 bridges must be between 6 and 9 miles (because 14 and 15 lie between 12 and 20). Of the choices given, only (B) matches this criterion.

7. Calculator

In the United States, the original full retirement age was 65. The retirement age has since been pushed to 66 and will soon move to 67, as life expectancies go up. The Social Security Administration periodically conducts studies regarding retirement age. One such study focused on whether or not retiring early lowers a person's life expectancy. The study found a weak positive correlation between retirement age and life expectancy. If data from the study were graphed in a scatterplot, which of the following statements would be true?

Ⓐ The slope of the line of best fit would be a large positive number.
Ⓑ The slope of the line of best fit would be a negative number close to 0.
Ⓒ The data points would follow, but not closely, an increasing line of best fit.
Ⓓ The data points would be closely gathered around an increasing line of best fit.

Difficulty: Medium

Category: Problem Solving and Data Analysis / Scatterplots

Strategic Advice: It's a good idea to get comfortable with the vocabulary used in statistics questions. *Correlation* simply means relationship. The word *weak* refers to the strength of the relationship, which has no effect on slope, but rather on how closely the data points follow the line of best fit.

Getting to the Answer: Be careful not to confuse slope and strength. Simply because a data set shows a weak correlation does not mean the slope will be close to zero. The data can still be gathered around a steep line of best fit. So, you can eliminate A and B. Also, keep in mind that the terms *weak* and *positive* are not related but rather are two independent descriptors of the correlation. So, the fact that the rate of change is positive has nothing to do with the strength of the correlation. In a weak correlation, the data points will follow the line of best fit, but not as closely as in a strong correlation, which means (C) is correct.

8.

Given the polynomial $6x^4 + 2x^2 - 8x - c$, where c is a constant, for what value of c will $\frac{6x^4+2x^2-8x-c}{x+2}$ have no remainder?

Ⓐ −120
Ⓑ −60
Ⓒ 60
Ⓓ 120

Difficulty: Hard

Category: Passport to Advanced Math / Exponents

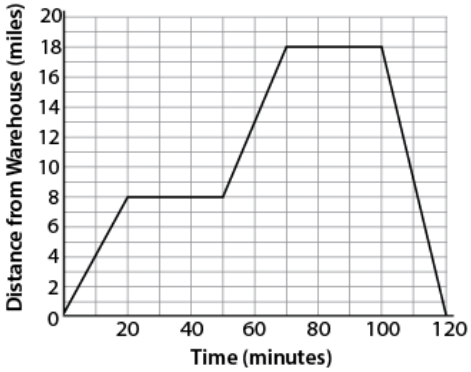
Strategic Advice: Don't bother with trial and error; it will take far too long. Use polynomial long division and your reasoning skills instead.

Getting to the Answer: Use long division to divide the two expressions. Don't forget to fill in 0 as a placeholder for the missing x^3 term.

$$
\begin{array}{r}
6x^3 - 12x^2 + 26x - 60 \\
x+2 \overline{)\,6x^4+0x^3+2x^2-8x-c} \\
\underline{-(6x^4+12x^3)} \\
-12x^3+2x^2-8x-c \\
\underline{-(-12x^3-24x^2)} \\
26x^2-8x-c \\
\underline{-(26x^2+52x)} \\
-60x-c \\
\underline{-(-60x-120)} \\
-c+120
\end{array}
$$

To make sure there is no remainder, c would have to be 120.

9. Calculator



The graph above shows a delivery truck's distance from the company's warehouse over a two-hour period, during which time the delivery people made two deliveries and then returned to the warehouse. Based on the graph, which of the following statements could be true?

Ⓐ Each delivery took 30 minutes to complete, not including driving time.
Ⓑ The location of the second delivery was about 70 miles from the warehouse.
Ⓒ The truck traveled about 18 miles from the time it left the warehouse until it returned.
Ⓓ The second delivery was about 18 miles farther from the warehouse than the first delivery.

Difficulty: Medium

Category: Passport to Advanced Math / Functions

Strategic Advice: Pay careful attention to the axis labels as you read the answer choices. Time is graphed on the *x*-axis, and distance is graphed on the *y*-axis.

Getting to the Answer: Compare each answer choice to the graph, eliminating false statements as you go.

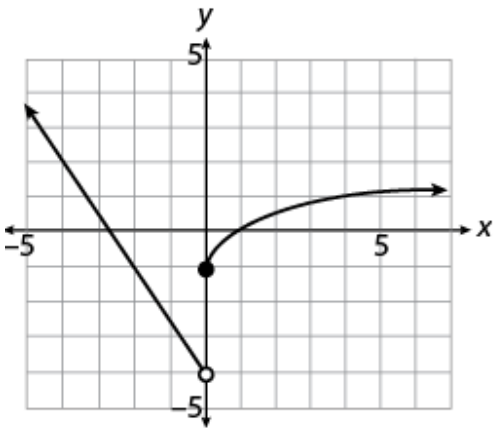
Choice (A): The truck is stopped when it is making a delivery. This means its distance is not changing, and the graph should be flat. Both flat sections of the graph span 30 minutes (20 to 50 and 70 to 100), so each delivery took 30 minutes. Choice (A) is correct. If you're confident in your answer, move on to the next question. If not, you can quickly check the other answer choices to be sure.

Choice (B): The second delivery starts at (70, 18), which means it was about 18 miles away from the warehouse, not 70.

Choice (C): When the truck arrived at the first delivery, it was about 8 miles from the warehouse, and when it was at the second delivery, it was about 18 miles from the warehouse. Then, it had to travel 18 miles back to the warehouse, so it traveled a total of 36 miles, not 18.

Choice (D): The second delivery took place 18 miles from the warehouse, and the first delivery took place 8 miles from the warehouse, which means the second delivery was about 10 miles farther from the warehouse, not 18.

10.



Which of the following piecewise functions could have been used to generate the graph above?

(A) $g(x) = \begin{cases} -\frac{3}{2}x - 4, & \text{if } x < 0 \\ \sqrt{x} - 1, & \text{if } x \geq 0 \end{cases}$

(B) $g(x) = \begin{cases} -\frac{3}{2}x - 4, & \text{if } x < 0 \\ \sqrt{x - 1}, & \text{if } x \geq 0 \end{cases}$

(C) $g(x) = \begin{cases} -\frac{3}{2}x - 4, & \text{if } x < 0 \\ \sqrt{x + 1}, & \text{if } x > 0 \end{cases}$

(D) $g(x) = \begin{cases} -\frac{2}{3}x - 4, & \text{if } x < 0 \\ \sqrt{x} + 1, & \text{if } x \geq 0 \end{cases}$

Difficulty: Hard

Category: Passport to Advanced Math / Functions

Strategic Advice: Graphing piecewise functions can be tricky. Try describing the graph in words first and then find the matching function. Use words like "to the left of" (which translates as *less than*) and "to the right of" (which translates as *greater than*).

Getting to the Answer: First, notice that both pieces of the graph either start or stop at 0, but one has a closed dot and the other has an open dot. This means you can eliminate C right away because the inequality symbol in both equations would lead to open dots on the graph. To choose among the remaining answers, think about parent functions and transformations. To the left of $x = 0$, the graph is a line with a slope of $-\frac{3}{2}$ and a y-intercept of -4, so you can eliminate D because the slope of the line is incorrect. Now, look to the right of $x = 0$—the graph is a square root function that has been moved down 1 unit, so its equation is $y = \sqrt{x} - 1$. This means (A) is correct. (The square root portion of C would have been moved to the left 1 unit rather than down 1.)

11.

$$18 - \frac{(3x)^{\frac{1}{2}}}{2} = 15$$

What value of x satisfies the equation above?

Difficulty: Medium

Category: Passport to Advanced Math / Exponents

Strategic Advice: Solving an equation that has a fractional exponent can be very intimidating, so rewrite that part of the equation using a radical instead. Then, solve the equation the same way you would any other: Isolate the variable using inverse operations, one step at a time.

Getting to the Answer: After rewriting the equation using a radical, start by subtracting 18 from both sides. Next, multiply both sides of the equation by −2. Then, square both sides to remove the radical. Finally, divide both sides by 3.

$$\begin{aligned} 18 - \frac{(3x)^{\frac{1}{2}}}{2} &= 15 \\ 18 - \frac{\sqrt{3x}}{2} &= 15 \\ -\frac{\sqrt{3x}}{2} &= -3 \\ \sqrt{3x} &= 6 \\ 3x &= 36 \\ x &= 12 \end{aligned}$$

12. Calculator

$$\left(5x^4 - \frac{1}{4}x^3 + 3x\right) \div \frac{1}{2}x$$

What is the result of dividing the two expressions above?

(A) $\frac{5}{2}x^3 - \frac{1}{8}x^2 + \frac{3}{2}$

(B) $\frac{5}{2}x^3 - 2x^2 + \frac{3}{2}x$

(C) $10x^3 - \frac{1}{2}x^2 + 6$

(D) $10x^3 - \frac{1}{8}x^2 + 6x$

Difficulty: Medium

Category: Passport to Advanced Math / Exponents

Strategic Advice: Division and factoring are interchangeable, so think of factoring out the x. Then, instead of dividing by $\frac{1}{2}$, you can multiply by its reciprocal, 2. Using these two strategies will make solving a question like this considerably easier.

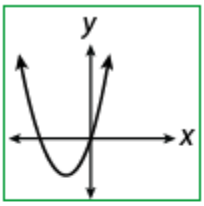
Getting to the Answer: First, divide (factor) out the x by subtracting 1 from each exponent: The result is $\left(5x^4 - \frac{1}{4}x^3 + 3x\right) \div x = 5x^3 - \frac{1}{4}x^2 + 3$. Now, multiply each term by 2 to get this:

$$5x^3 - \frac{1}{4}x^2 + 3 \div \frac{1}{2} = 2\left(5x^3 - \frac{1}{4}x^2 + 3\right)$$
$$= 10x^3 - \frac{1}{2}x^2 + 6$$

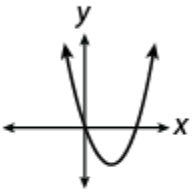
13.

If $a = 0$ and $b < 0$, then which of the following could be the graph of $f(x) = (x - a)(x - b)$?

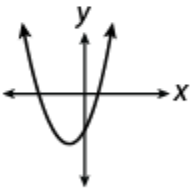
(A)



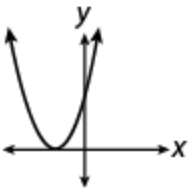
(B)



(C)



(D)



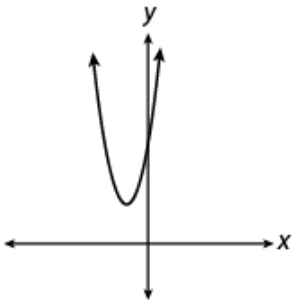
Difficulty: Medium

Category: Passport to Advanced Math / Quadratics

Strategic Advice: If $a = 0$, then one factor must be $(a - 0)$, which means $x = 0$ is a root. This means the graph must cross through the origin, so you can eliminate C and D right away.

Getting to the Answer: Look at the remaining two choices, (A) and B. The question states that $b < 0$. This means b is negative, which means the other x-intercept must fall to the left of the y-axis, so (A) is correct. Because this question is in the calculator section of the test, you could also use the Picking Numbers strategy. Choose a value for b (that is less than 0), such as -2, and graph the equation $y = (x - 0)(x - (-2))$ to see what the graph looks like.

14. Calculator



If the equation of the graph shown above is $y = 2(x + 3)^2 + 10$, what is the y-intercept of the graph?

Category: Passport to Advanced Math / Quadratics

Strategic Advice: Graphically, a y-intercept is in the form $(0, y)$, so the y-intercept of the graph is the value of y when 0 is substituted for x in the equation.

Getting to the Answer: Don't forget to follow the correct order of operations as you simplify the expression.

$y = 2(0 + 3)^2 + 10$

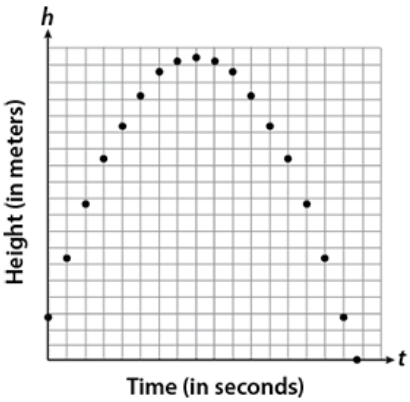
$= 2(3)^2 + 10$

$= 2(9) + 10$

$= 18 + 10$

$= 28$

15.



A physics class is using simulation software to study water bottle rockets before attempting to build one for the National Physics Competition. Their first simulation is of a rocket without a parachute launched from the roof of the gymnasium. The scatterplot shows the approximate path of the rocket. The software program generates the equation $h = -4.9t^2 + 39.2t + 14$ to model the data, where h is the height in meters of the rocket t seconds after it was launched. What does the number 14 most likely represent in this equation?

Ⓐ The number of seconds the rocket was in the air
Ⓑ The height of the gymnasium from which the rocket was launched
Ⓒ The number of seconds that it took the rocket to reach its maximum height
Ⓓ The maximum height of the rocket

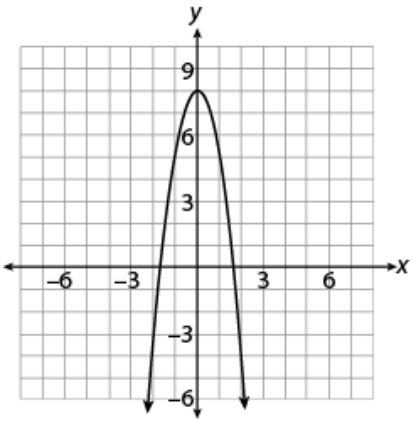
Difficulty: Medium

Category: Passport to Advanced Math / Quadratics

Strategic Advice: When a quadratic equation is written in standard form, $y = ax^2 + bx + c$, the value of c is the y-intercept of the equation's graph. This is because substituting 0 for x results in $y = a(0)^2 + b(0) + c = c$.

Getting to the Answer: In a real-world scenario, the y-intercept represents an initial amount. In this question, height is what is being measured, so the y-intercept represents the initial height of the bottle rocket. Because the rocket was fired from the roof of the gymnasium, the height of the gymnasium must be 14 meters, making (B) correct.

16.



Vadim graphs the equation $y = -3x^2 + 8$, which is shown in the figure above. He realizes, however, that he miscalculated and should have graphed $y = -\frac{1}{3}x^2 + 8$. How will this affect his graph?

Ⓐ It will change the y-intercept.
Ⓑ It will make the parabola open in the opposite direction.
Ⓒ It will make the parabola cross the x-axis closer to the origin.
Ⓓ It will make the parabola cross the x-axis farther from the origin.

Difficulty: Medium

Category: Passport to Advanced Math / Quadratics

Strategic Advice: Don't waste time trying to graph the second equation. Instead, think about the question conceptually. The magnitude of the coefficient of x^2 (not the sign) determines how wide or narrow the graph is.

Getting to the Answer: Changing the coefficient of x^2 from –3 to $-\frac{1}{3}$ will make the graph narrower or wider (in this case, wider), which means the only things that will change are the x-intercepts. This means you can eliminate A and B. To choose between C and (D), recall that fraction coefficients (between 0 and 1) result in wider graphs, so the x-intercepts will spread out and therefore be farther from the origin.