

5.2

Verifying Trigonometric Identities



What You Should Learn

- Verify trigonometric identities.



Verifying Trigonometric Identities



Verifying Trigonometric Identities

In this section, you will study techniques for verifying trigonometric identities. In the next section, you will study techniques for solving trigonometric equations.

The key to both verifying identities *and* solving equations is your ability to use the fundamental identities and the rules of algebra to rewrite trigonometric expressions.

Remember that a *conditional equation* is an equation that is true for only some of the values in its domain.



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For example, the conditional equation

$$\sin x = 0$$

Conditional equation

is true only for

$$x = n\pi$$

where n is an integer. When you find these values, you are *solving* the equation.



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On the other hand, **an equation that is true for all real values in the domain of the variable is an *identity*.**

For example, the familiar equation

$$\sin^2 x = 1 - \cos^2 x$$

Identity

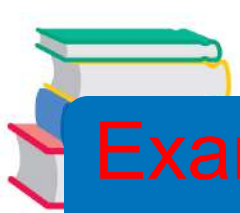
is true for all real numbers x . So, it is an identity.



Verifying Trigonometric Identities

Guidelines for Verifying Trigonometric Identities

1. Work with one side of the equation at a time. It is often better to work with the more complicated side first.
2. Look for opportunities to factor an expression, add fractions, square a binomial, or create a monomial denominator.
3. Look for opportunities to use the fundamental identities. Note which functions are in the final expression you want. Sines and cosines pair up well, as do secants and tangents, and cosecants and cotangents.
4. When the preceding guidelines do not help, try converting all terms to sines and cosines.
5. Always try *something*. Even making an attempt that leads to a dead end provides insight.



Example 1 – Verifying a Trigonometric Identity

Verify the identity.

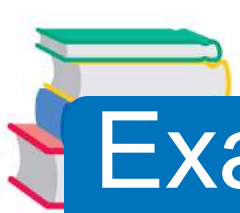
$$\frac{\sec^2 \theta - 1}{\sec^2 \theta} = \sin^2 \theta$$

Solution:

Because the left side is more complicated, start with it.

$$\frac{\sec^2 \theta - 1}{\sec^2 \theta} = \frac{(\tan^2 \theta + 1) - 1}{\sec^2 \theta}$$

Pythagorean identity



Example 1 – Solution

cont'd

$$= \frac{\tan^2 \theta}{\sec^2 \theta}$$

Simplify.

$$= \tan^2 \theta (\cos^2 \theta)$$

Reciprocal identity

$$= \frac{\sin^2 \theta}{\cancel{\cos^2 \theta}} (\cancel{\cos^2 \theta})$$

Quotient identity

$$= \sin^2 \theta$$

Simplify.