3

Exponential and Logarithmic Functions

3.3 Properties of Logarithms

What You Should Learn

- Rewrite logarithms with different bases.
- Use properties of logarithms to evaluate or rewrite logarithmic expressions.
- Use properties of logarithms to expand or condense logarithmic expressions.
- Use logarithmic functions to model and solve real-life problems.

Change of Base

Change of Base

Most calculators have only two types of log keys, one for common logarithms (base 10) and one for natural logarithms (base *e*).

Although common logs and natural logs are the most frequently used, you may occasionally need to evaluate logarithms to other bases. To do this, you can use the following **change-of-base formula.**

Change of Base

Change-of-Base Formula

Let a, b, and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. Then $\log_a x$ can be converted to a different base using any of the following formulas.

Base b	*Base 10*	*Base e*
$\log_a x = \dfrac{\log_b x}{\log_b a}$	$\log_a x = \dfrac{\log_{10} x}{\log_{10} a}$	$\log_a x = \dfrac{\ln x}{\ln a}$

We will usually use the base 10 formula so that we can enter it into our calculator.

Example 1 – *Changing Bases Using Common Logarithms*

a. $\log_4 25 = \dfrac{\log_{10} 25}{\log_{10} 4}$

$\log_a x = \dfrac{\log_{10} x}{\log_{10} a}$

$\approx \dfrac{1.39794}{0.60206}$

Use a Calculator.

≈ 2.23

Simplify.

b. $\log_2 12 = \dfrac{\log_{10} 12}{\log_{10} 2}$

$\approx \dfrac{1.07918}{0.30103}$

≈ 3.58

Properties of Logarithms

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Properties of Logarithms

Let a be a positive real number such that $a \neq 1$, and let n be a real number. If u and v are positive real numbers, then the following properties are true.

	Logarithm with Base a	*Natural Logarithm*
1. Product Property:	$\log_a(uv) = \log_a u + \log_a v$	$\ln(uv) = \ln u + \ln v$
2. Quotient Property:	$\log_a \frac{u}{v} = \log_a u - \log_a v$	$\ln \frac{u}{v} = \ln u - \ln v$
3. Power Property:	$\log_a u^n = n \log_a u$	$\ln u^n = n \ln u$

Example 3 – *Using Properties of Logarithms*

Write each logarithm in terms of ln 2 and ln 3.

a. ln 6 **b.** $\ln \frac{2}{27}$

Solution:

a. $\ln 6 = \ln(2 \cdot 3)$ Rewrite 6 as 2 • 3.

$= \ln 2 + \ln 3$ Product Property

b. $\ln \frac{2}{27} = \ln 2 - \ln 27$ Quotient Property

$= \ln 2 - \ln 3^3$ Rewrite 27 as 3^3

$= \ln 2 - 3 \ln 3$ Power Property

Rewriting Logarithmic Expressions

Rewriting Logarithmic Expressions

The properties of logarithms are useful for rewriting logarithmic expressions in forms that simplify the operations of algebra. This is true because they convert complicated products, quotients, and exponential forms into simpler sums, differences, and products, respectively.

Example 5 – *Expanding Logarithmic Expressions*

Use the properties of logarithms to expand each expression.

a. $\log_4 5x^3y$

b. $\ln \dfrac{\sqrt{3x - 5}}{7}$

Solution:

a. $\log_4 5x^3y = \log_4 5 + \log_4 x^3 + \log_4 y$ — Product Property

$= \log_4 5 + 3 \log_4 x + \log_4 y$ — Power Property

Example 5 – *Solution*

cont'd

b. $\ln \frac{\sqrt{3x - 5}}{7} = \ln \frac{(3x - 5)^{1/2}}{7}$ — Rewrite radical using rational exponent.

$= \ln(3x - 5)^{1/2} - \ln 7$ — Quotient Property

$= \frac{1}{2}\ln(3x - 5) - \ln 7$ — Power Property

Rewriting Logarithmic Expressions

In Example 5, the properties of logarithms were used to *expand* logarithmic expressions.

In Example 6, this procedure is reversed and the properties of logarithms are used to *condense* logarithmic expressions.

Example 6 – *Condensing Logarithmic Expressions*

Use the properties of logarithms to condense each expression.

a. $\frac{1}{2}\log_{10}x + 3\log_{10}(x + 1)$

b. $2\ln(x + 2) - \ln x$

c. $\frac{1}{3}[\log_2 x + \log_2(x - 4)]$

Example 6 – *Solution*

a. $\frac{1}{2}\log_{10}x + 3\log_{10}(x+1) = \log_{10}x^{1/2} + \log_{10}(x+1)^3$

Power Property

$$= \log_{10}\left[\sqrt{x}(x+1)^3\right]$$

Product Property

b. $2\ln(x+2) - \ln x = \ln(x+2)^2 - \ln x$

Power Property

$$= \ln\frac{(x+2)^2}{x}$$

Quotient Property

Example 6 – *Solution*

cont'd

c. $\frac{1}{3}[\log_2 x + \log_2(x - 4)] = \frac{1}{3}\{\log_2[x(x - 4)]\}$ Product Property

$= \log_2[x(x - 4)]^{1/3}$ Power Property

$= \log_2 \sqrt[3]{x(x - 4)}$ Rewrite with a radical.