Unit 6: "Radical and Rational Functions"

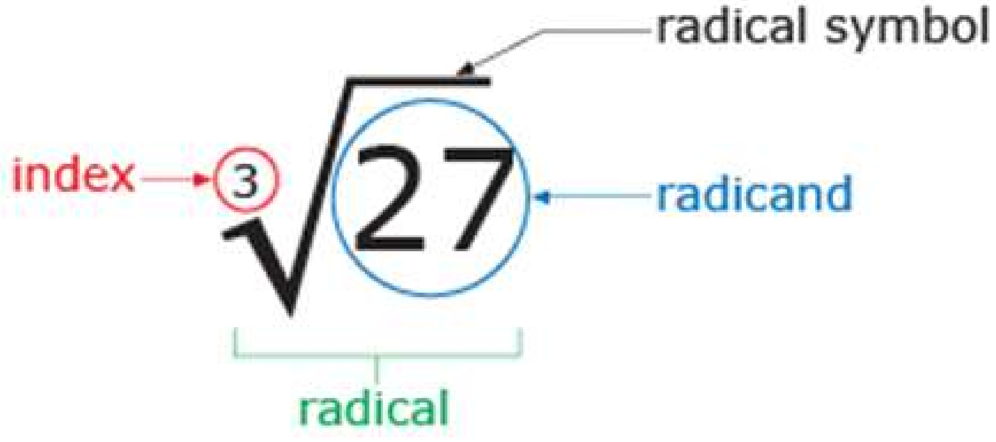
Roots and Radical Expressions

Objective: To evaluate and simplify radical expressions.

n^{th} root – For any real number *a*, *b*, and positive integer *n,* if ***$a^n = b$***, then ***"a is the n^{th} root of b"***.

Example: Since $4^3 = 64$, then "4 is the 3^{rd} root of 64"

❖A ***radical*** is used to indicate a root.



Evaluating Radicals

Find each real number root.

1. $\sqrt{576}$

2. $\sqrt{-64}$

3. $\sqrt[3]{125}$

4. $\sqrt[3]{-1000}$

5. $\sqrt[4]{81}$

6. $\sqrt[5]{-32}$

- **Even roots of a positive number have two real roots – a positive and a negative.**
- **The positive root is called the *"principal root"*.**

Simplifying Radicals

For every number a ≥ 0 and b ≥ 0,

$$\sqrt{} = \sqrt{} \cdot \sqrt{}$$

$\sqrt{50}$

$\sqrt{192}$

$\sqrt{180}$

$\sqrt{51}$

Simplifying Radicals

$$\sqrt{\quad} = \sqrt{\quad} \cdot \sqrt{\quad}$$

$\sqrt[3]{54}$

$\sqrt[3]{40}$

$\sqrt[3]{375}$

$\sqrt[3]{36}$

Simplifying Radicals with Variables

$$\sqrt{\;{}^{4}\;\;{}^{5}}$$

$$\sqrt{\;{}^{7}\;\;{}^{6}\;\;{}^{9}}$$

$$\sqrt{24\;\;{}^{11}\;\;{}^{8}}$$

Simplifying Radicals with Variables

$$\sqrt[3]{\quad 4 \quad 5}$$

$$\sqrt[3]{\quad 7 \quad 6 \quad 9}$$

$$\sqrt[3]{24 \quad 11 \quad 8 \quad}$$

Multiply Two Radicals

$$\sqrt{12} \cdot \sqrt{32}$$

$$5\sqrt{3} \cdot 4\sqrt{6}$$

$$2\sqrt{5\quad^{2}} \cdot 6\sqrt{10\quad^{3}}$$

$$3\sqrt{7\quad^{3}\quad^{5}} \cdot 2\sqrt{21\quad^{3}\quad^{2}}$$

Multiply Two Radicals

$$\sqrt[3]{3} \cdot \sqrt[3]{9}$$

$$2\sqrt[3]{4} \cdot 5\sqrt[3]{6}$$

$$-3\sqrt[3]{25\quad{}^{8}} \cdot 4\sqrt[3]{4\quad{}^{4}\quad{}^{3}}$$

End of Day 1

P 366 #13 – 16, #39 – 46
P 371 #17 - 22

Solving Radical Equations

Objective: To solve radical equations and identify extraneous solutions.

radical equation – is an equation that has a variable in a radicand or has a variable with a rational exponent.

Examples: $\sqrt{} - \quad = \qquad {}^{-} + \quad =$

Non-example: $\sqrt{} + \quad =$

Solving Radical Equations

$$\sqrt{} - 2 = 9$$

Solving Radical Equations

$$2 + \sqrt{3 \quad - 4} = 6$$

Solving Radical Equations

$$10 - \sqrt{2\quad + 1} = 5$$

Solving Radical Equations

$$2\sqrt{5 \quad + 1} - 6 \quad = 0$$

Solving Radical Equations

$$\sqrt[3]{4\quad} + 7 = 5$$

Solving Radical Equations

$$\sqrt[3]{2\quad - 9} = -3$$

Solving Radical Equations

$$\sqrt[3]{4\quad + 5} - 6 = 4$$

Solving Radical Equations

$$\frac{3}{2} + 7 = 34$$

Solving Radical Equations

$$(4\quad - 5)^{\frac{3}{2}} = 16$$

Solving Radical Equations

$$3(\quad + 1)^{\frac{3}{4}} - 7 = 17$$

Solving Radical Equations*

- When solving by taking an even root of both sides you must include a ±.

$$(\quad - 1)^{\frac{2}{3}} = 25$$

Solving Radical Equations*

$$(3 \quad + 8)^{\frac{4}{5}} + 3 = 19$$

Solving Radical Equations*

extraneous solution – is **a solution of a derived equation** that is **NOT a solution of the original equation**.

$$\sqrt{\ } = \quad -2$$

Solving Radical Equations*

$$\sqrt{5\quad - 1} + 3 =$$

Solving Radical Equations*

$$-\ 3+\sqrt{\quad\ +2}=2$$

End of Day 2

$$\sqrt{\quad + 1} + \sqrt{2} = \sqrt{5 \quad + 3}$$

$$\sqrt{\quad + \sqrt{2}} = 2$$

$$\sqrt{\sqrt{\quad + 25}} = \sqrt{\quad + 5}$$

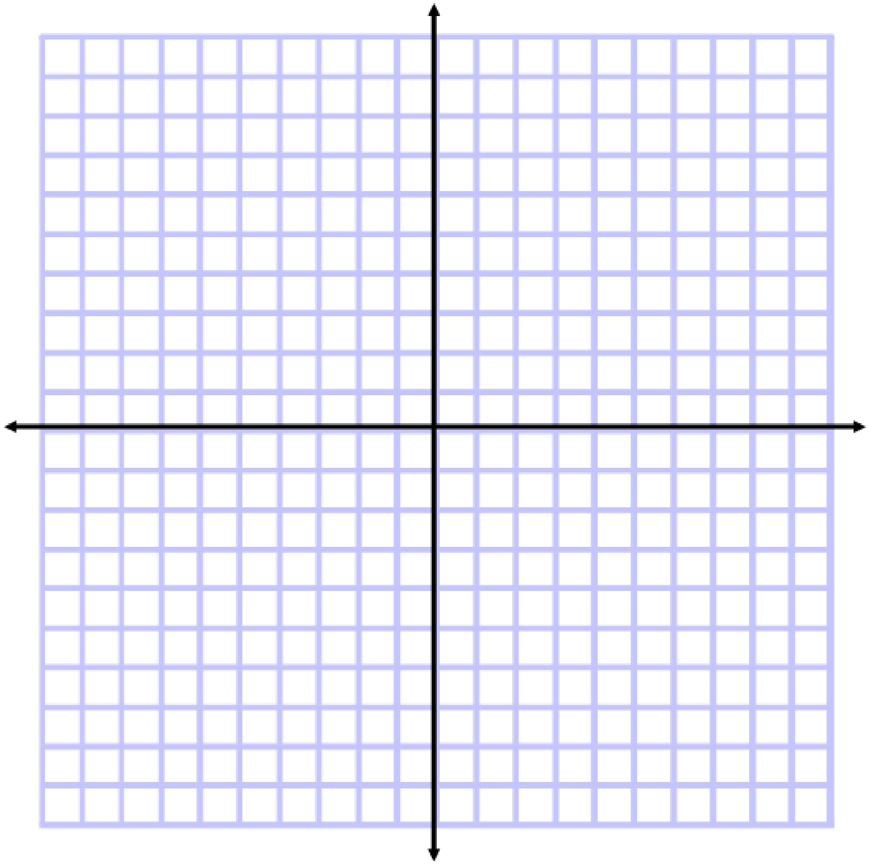
Graphing Radical Functions

Objective: To graph square root and cube root functions and their transformations.

Square Root Function

Graph: $y = \sqrt{}$

x	y



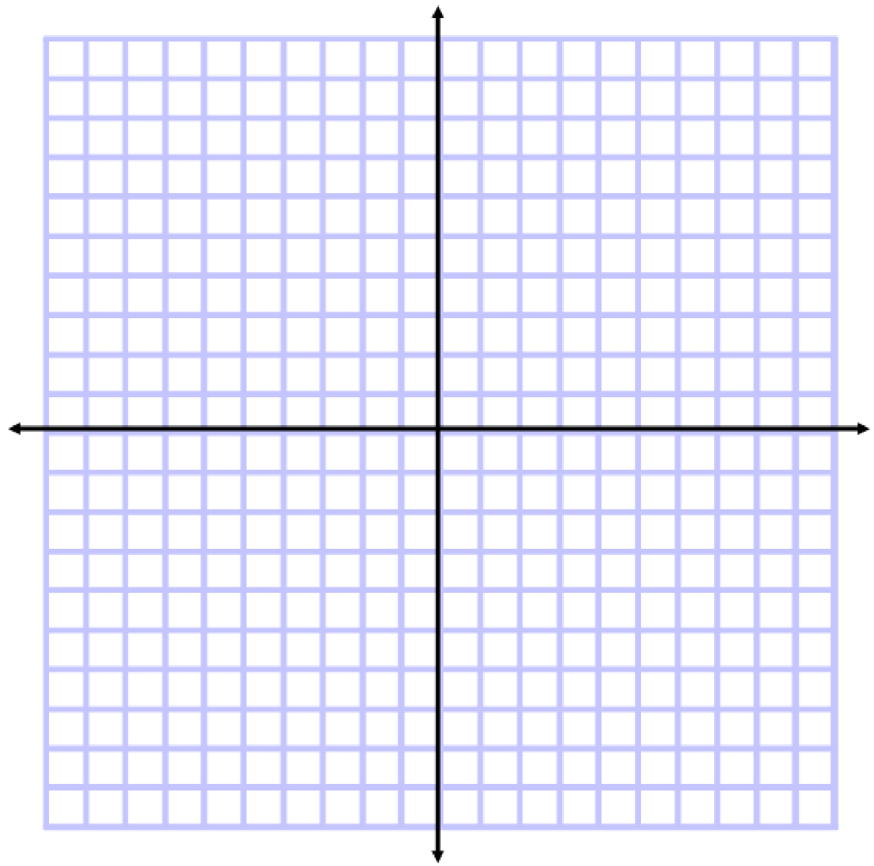
Square Root Function

Graph: $y = \sqrt{\quad} + 2$

Graph: $y = \sqrt{\quad} - 1$

Graph: $y = \sqrt{\quad + 6}$

Graph: $y = \sqrt{\quad - 2}$

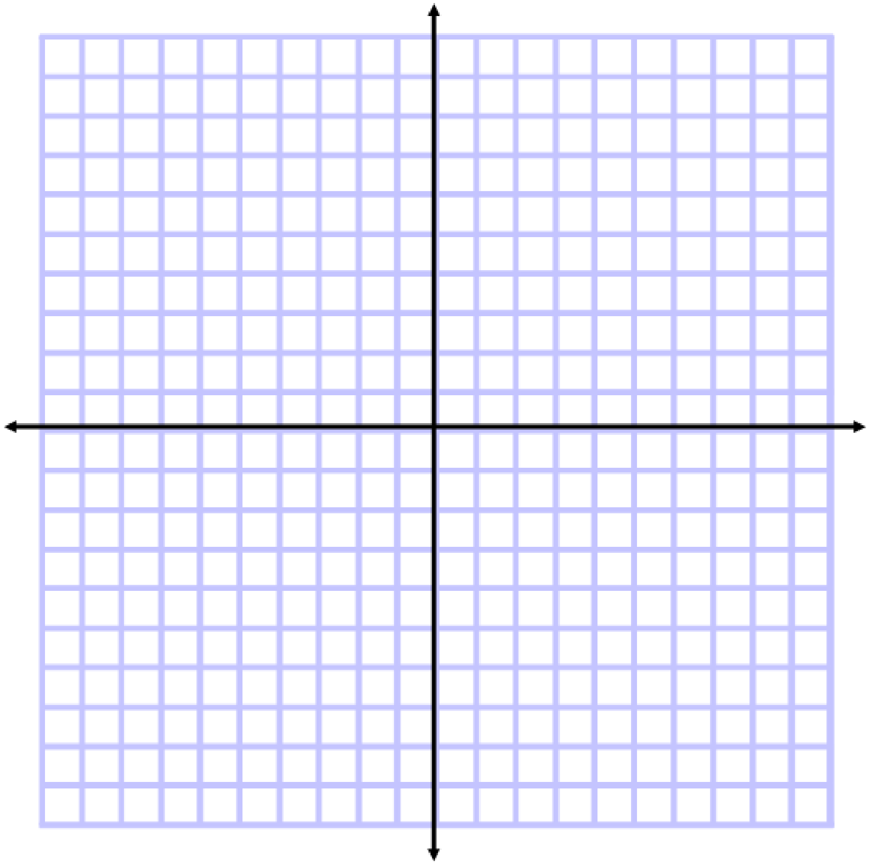


Square Root Function

Graph: $y = \sqrt{\quad + 3} - 2$

Graph: $y = \sqrt{\quad - 1} + 4$

Graph: $y = \sqrt{\quad + 2} + 3$



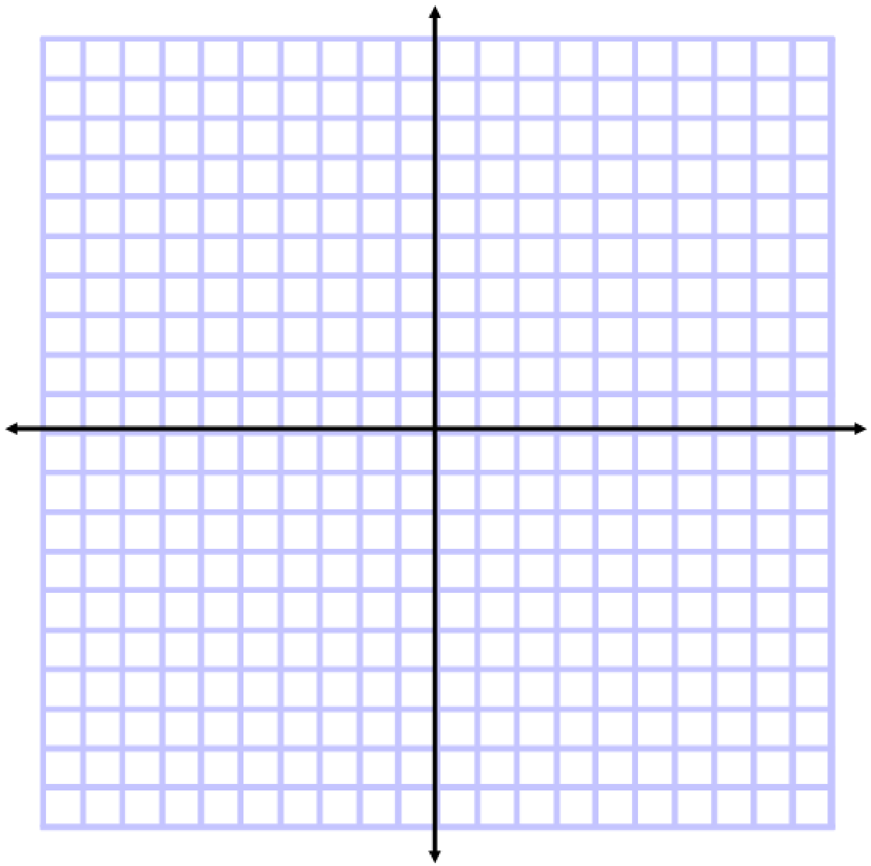
Square Root Function

Graph: $y = -\sqrt{\quad}$

Graph: $y = 2\sqrt{\quad}$

Graph: $y = \frac{1}{2}\sqrt{\quad}$

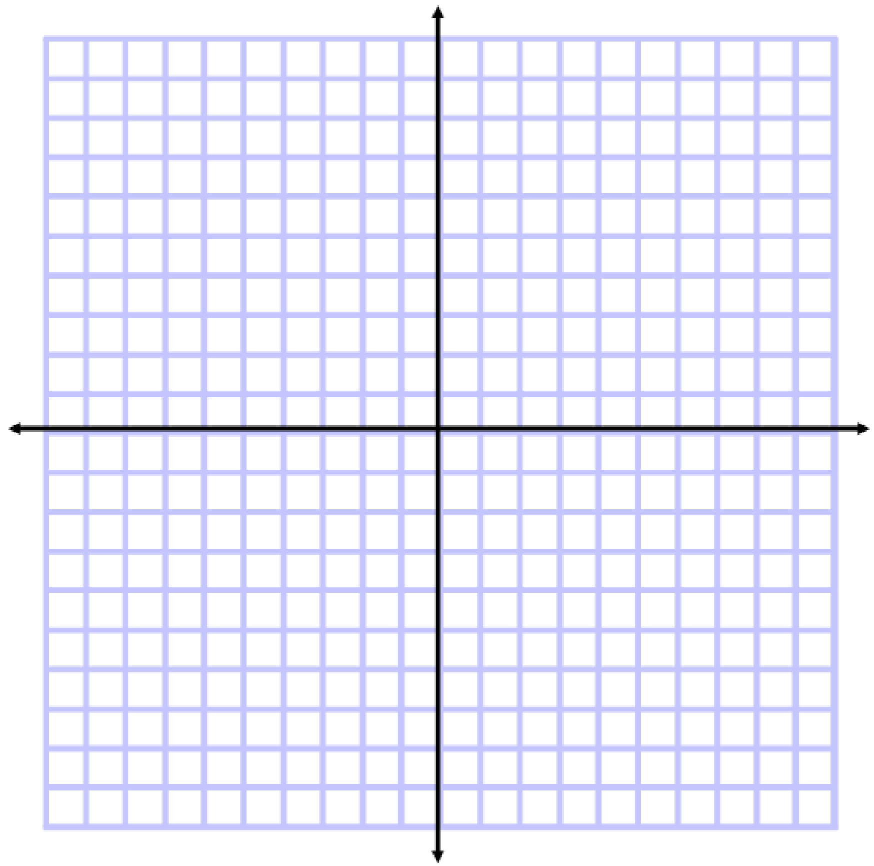
Graph: $y = -3\sqrt{\quad + 1}$



Cube Root Function

Graph: $y = \sqrt[3]{\ }$

x	y



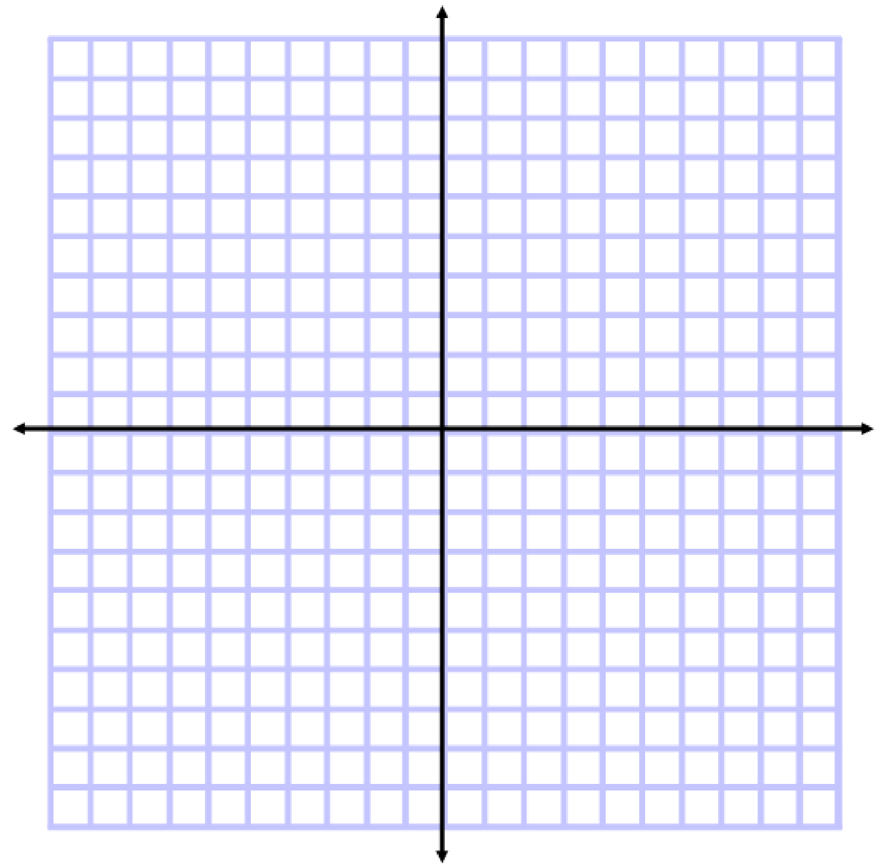
Cube Root Function

Graph: $y = \sqrt[3]{\quad} + 5$

Graph: $y = \sqrt[3]{\quad} - 3$

Graph: $y = \sqrt[3]{\quad + 4}$

Graph: $y = \sqrt[3]{\quad - 2}$

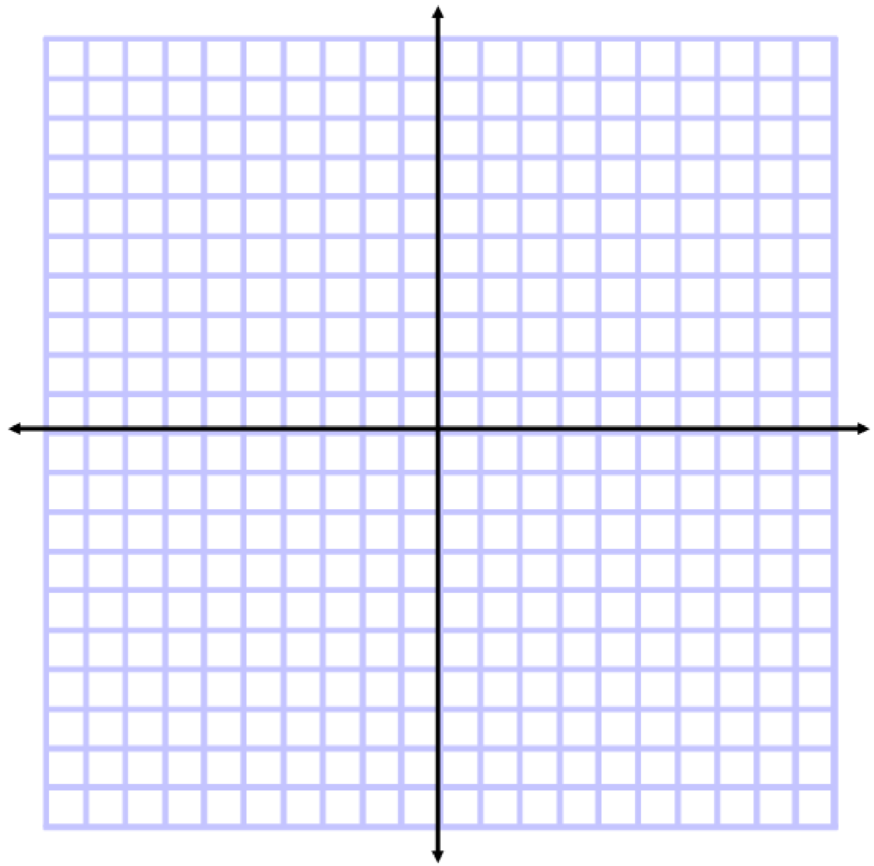


Cube Root Function

Graph: $y = \sqrt[3]{\quad + 1} - 2$

Graph: $y = \sqrt[3]{\quad - 2} + 3$

Graph: $y = \sqrt[3]{\quad - 1} - 5$

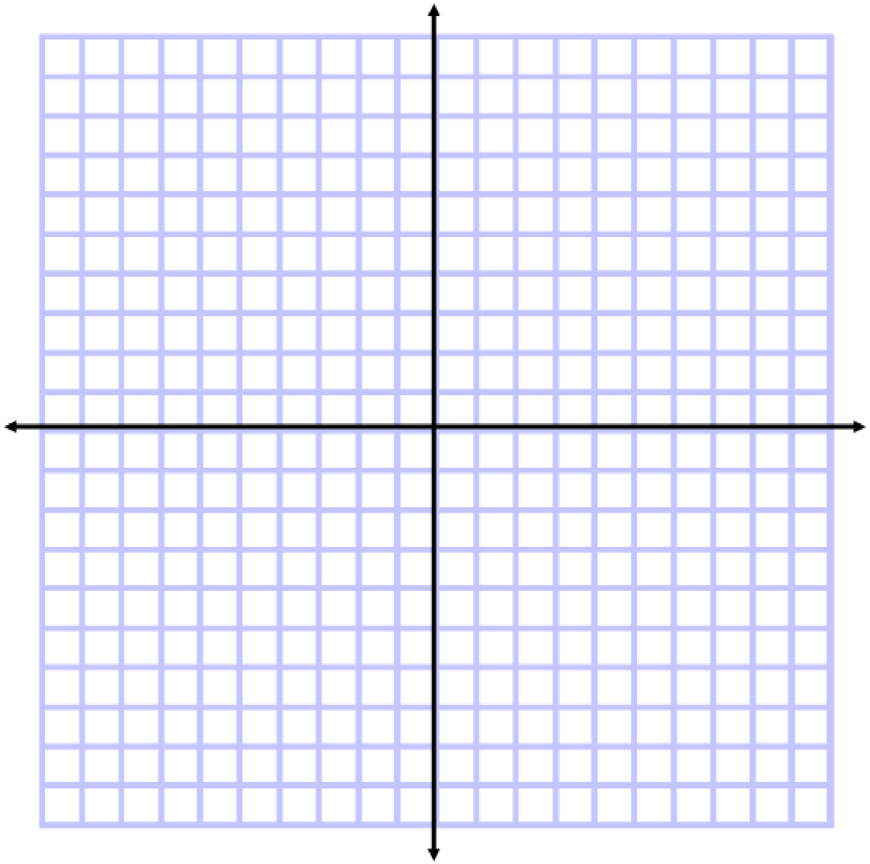


Cube Root Function

Graph: $y = -\sqrt[3]{}$

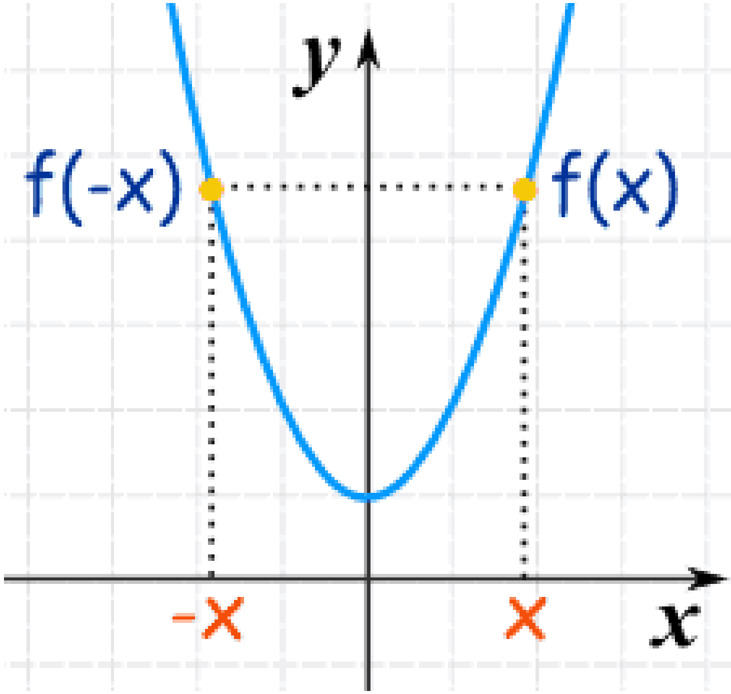
Graph: $y = 2\sqrt[3]{}$

Graph: $y = \frac{1}{2}\sqrt[3]{} - 5$



Even and Odd Functions

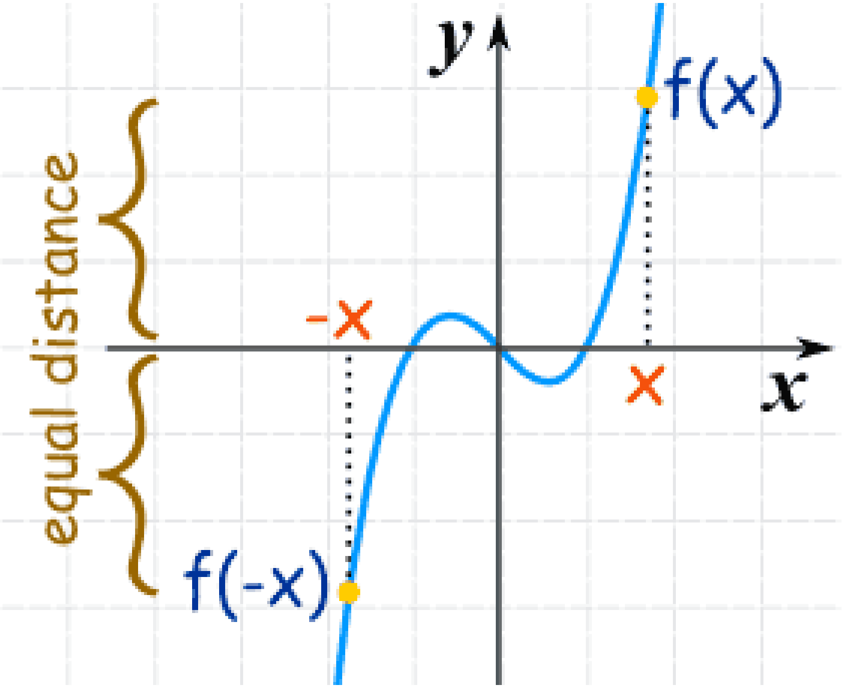
- A function is ***even*** when $\mathbf{f(x) = f(\text{-}x)}$
- Graph is symmetric about the **y-axis**.



$f(x) = x^2+1$

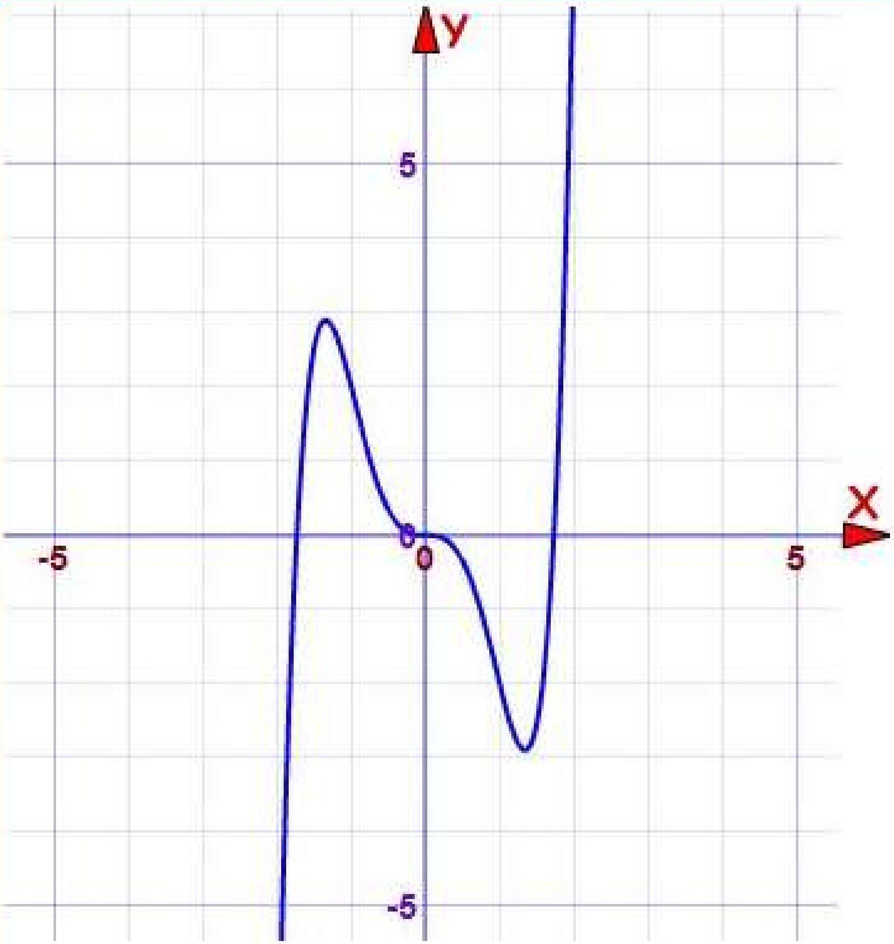
Even and Odd Functions

- A function is ***odd*** when **-f(x) = f(-x)**
- Graph is symmetric about the **origin (180° Rot)**.



$f(x) = x^3 - x$

Even and Odd Functions



The function shown in the graph is:

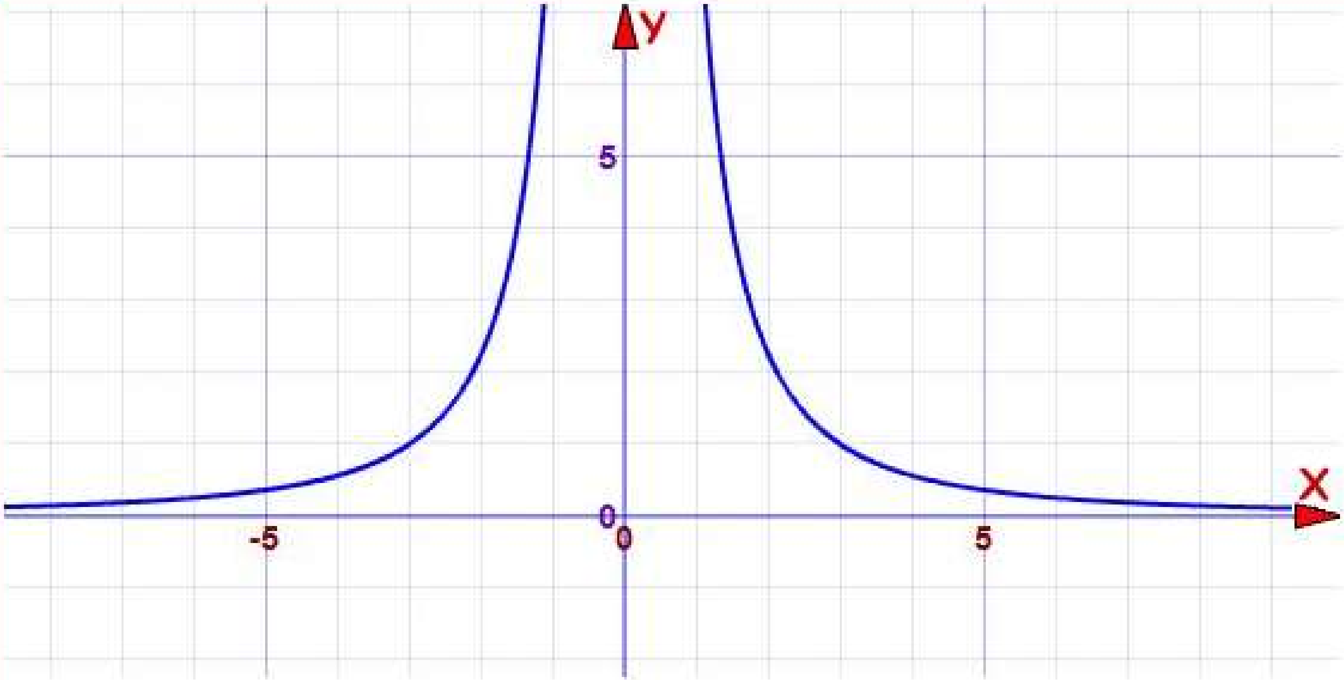
A Even

B Odd

C Neither even nor odd

D Both even and odd

Even and Odd Functions



The function shown in the graph is:

A Even

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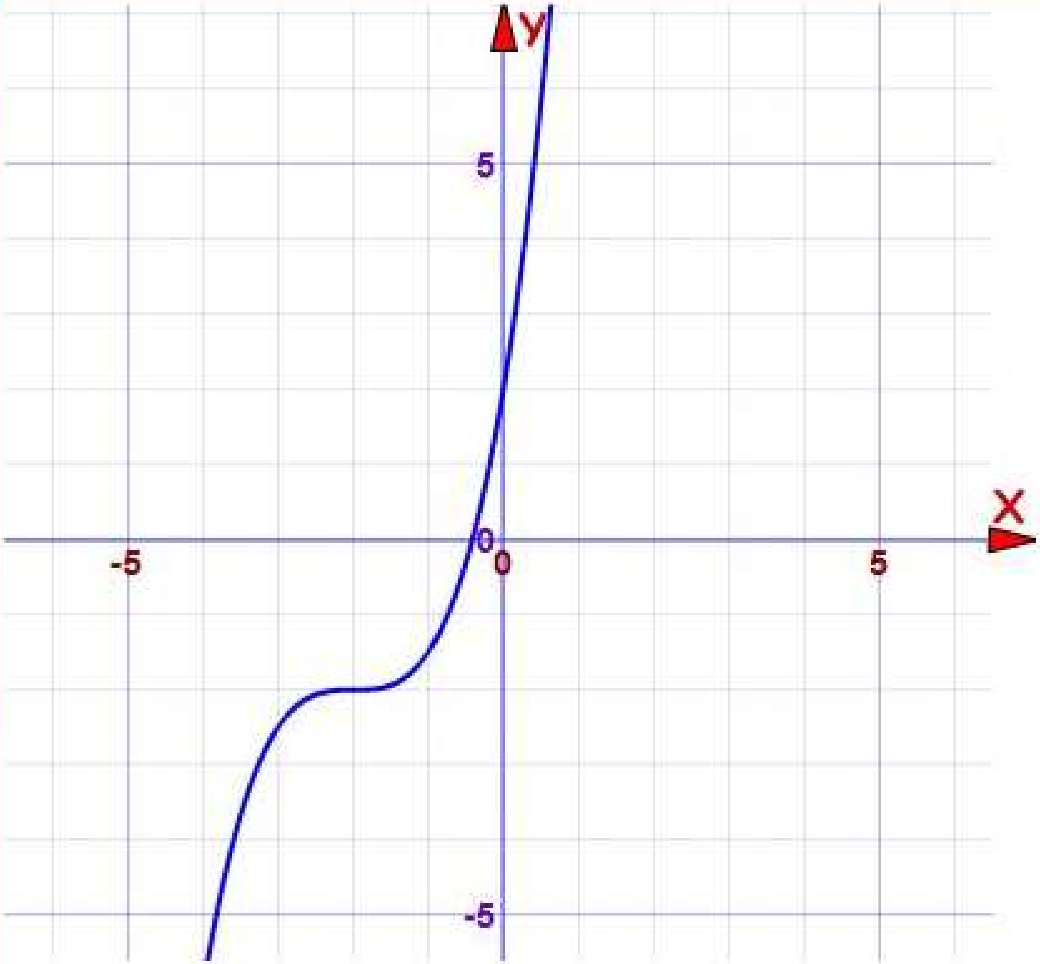
Even and Odd Functions



The function shown in the graph is:

A Even

B Odd

C Neither even nor odd

D Both even and odd

Even and Odd Functions

Which one of the following functions is even?

A $f(x) = x^4 + x^3$

B $g(x) = x^4 + x^2$

C $h(x) = x^5 + x^3$

D $k(x) = x^3 + x$

Which one of the following functions is odd?

A $f(x) = 3x^4 - 4x^3$

B $g(x) = 5x^4 + 3x^2$

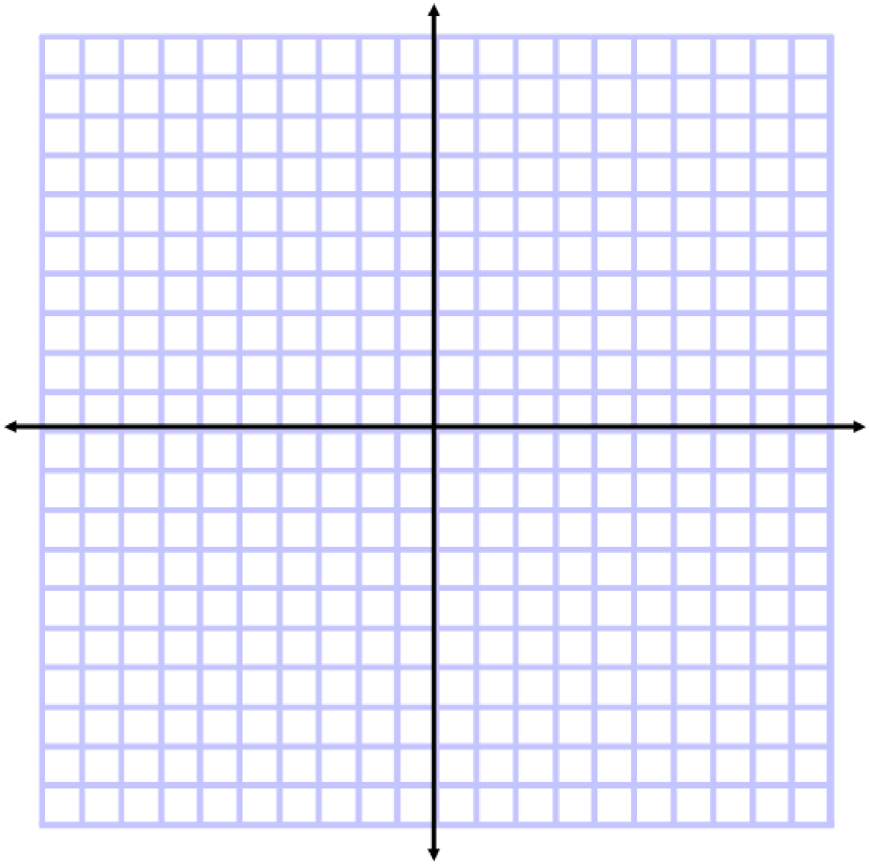
C $h(x) = 6x^5 - x^3$

D $k(x) = x^6 + 8x^2$

Cube Root Function

Graph: y = $\sqrt[3]{\quad + 4}$

Graph: y = $\sqrt[3]{\quad - 3}$



Simplify

$$\sqrt{8\ \ ^{3}} \cdot \sqrt{6\ \ ^{5}}$$

Solve

$$\sqrt[3]{\ \ + 5} = 6$$

$$\sqrt{\ \ - 3} = x - 5$$

Unit 6 "Radical and Rational Functions"

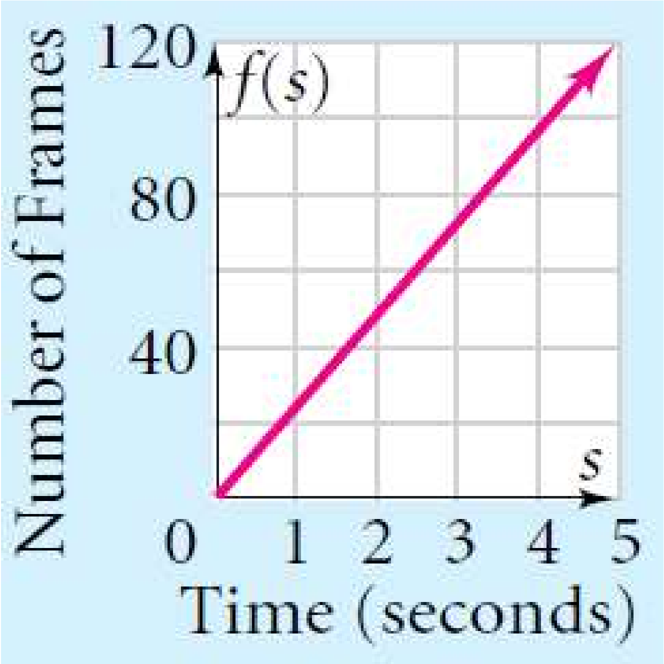
Title: Inverse Variation

Introduction to Rational Functions

Objective: To identify and use inverse variation and combined variation.

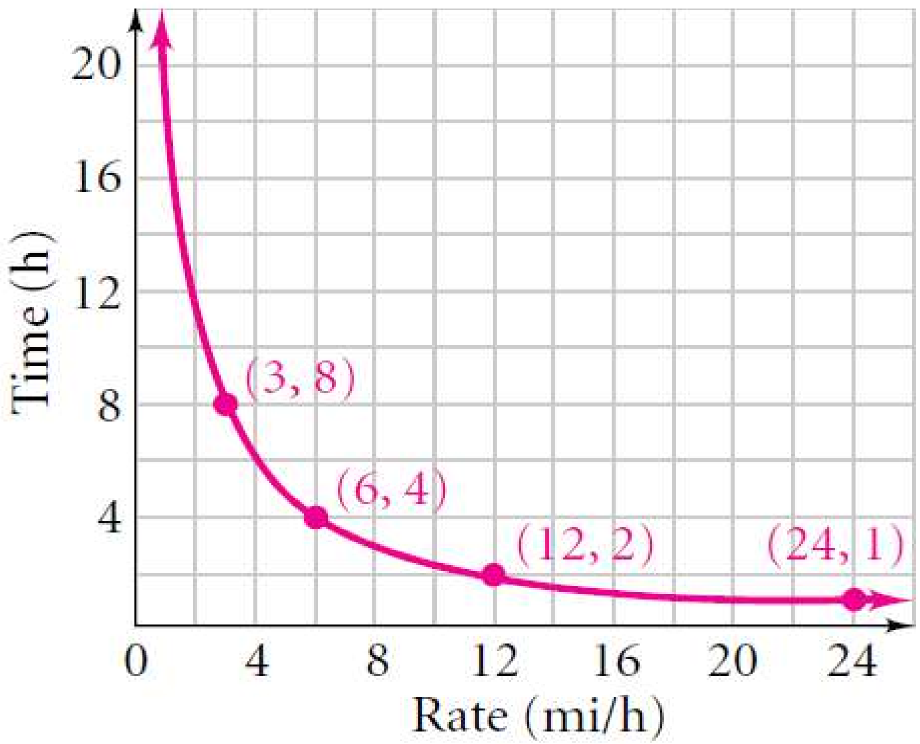
Direct Variation: a function of the form , $y = kx$
where k is a nonzero *constant of variation*.

s number of seconds	$f(s)$ number of frames
1	24
2	48
3	72
4	96
5	120



Inverse Variation: a function of the form $y = \frac{k}{x}$, where *k* is a nonzero *constant of variation*.

Rate (mi/h)	Time (h)
3	8
6	4
12	2
24	1



Example

Suppose that ***x and y vary directly***, and x = 2 when y = 5.
Write the function that models the **direct variation**.

Example

Suppose that ***x and y vary inversely***, and x = 2 when y = 5.
Write the function that models the **inverse variation**.

Example

Suppose that x and y vary inversely, and $x = 0.7$ when $y = 1.4$. Write the function that models the inverse variation.

Example

Suppose that x and y vary directly, and $x = 0.7$ when $y = 1.4$. Write the function that models the direct variation.

Example

Is the relationship between the variables in the table **Direct Variation**, **Inverse Variation**, or **Neither**? If the relationship is Direct or Inverse Variation, then write an equation to model.

a.

x	0.5	2	6
y	1.5	6	18

b.

x	0.2	0.6	1.2
y	12	4	2

c.

x	1	2	3
y	2	1	0.5

<u>Example</u>

Is the relationship between the variables in the table **Direct Variation**, **Inverse Variation**, or **Neither**? If the relationship is Direct or Inverse Variation, then write an equation to model.

a.

x	0.8	0.6	0.4
y	0.9	1.2	1.8

b.

x	2	4	6
y	3.2	1.6	1.1

c.

x	1.2	1.4	1.6
y	18	21	24

Example

The pressure P of a sample of gas at a constant temperature varies inversely as the volume V. Use the data in the table to write a equation that models the relationship. Use your equation to estimate the pressure when the volume is 11 in^3.

Pressure ($lb/in.^2$)	**Volume ($in.^3$)**
3	32
5	19.2
8	12

Combined Variation: combines direct and inverse variations in more complicated relationships.

Examples of Combined Variations

Combined Variation	Equation Form
y varies directly with the square of x.	
y varies inversely with the cube of x.	
z varies jointly with x and y.	
z varies jointly with x and y and inversely with w.	
z varies directly with x and inversely with the product of w and y.	

Example:

Physics Newton's Law of Universal Gravitation is modeled by the formula $F = \frac{Gm_1m_2}{d^2}$. F is the gravitational force between two objects with masses m_1 and m_2, and d is the distance between the objects. G is the gravitational constant. Describe Newton's law as a combined variation.

Example:

Suppose z varies directly as x and inversely as the square of y. When x = 35 and y = 7, the value of z is 50. Write a function that models the relationship then find z when x = 5 and y = 10.

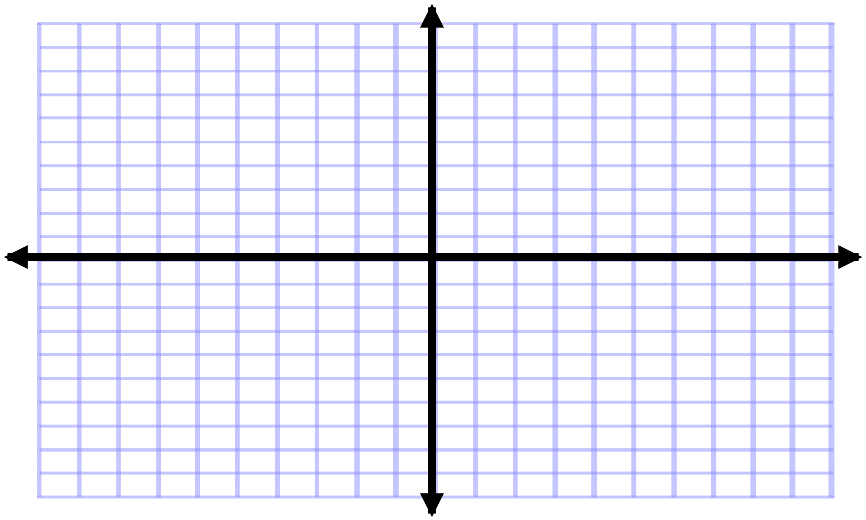
End of Day 3

P 412 36 – 45 all
P 481 13 – 15, 21 – 27

Title: Graphing Inverse Variations

Objectives: To learn to graph inverse variations & translations of inverse variation.

Example 1: Graph **y = −**



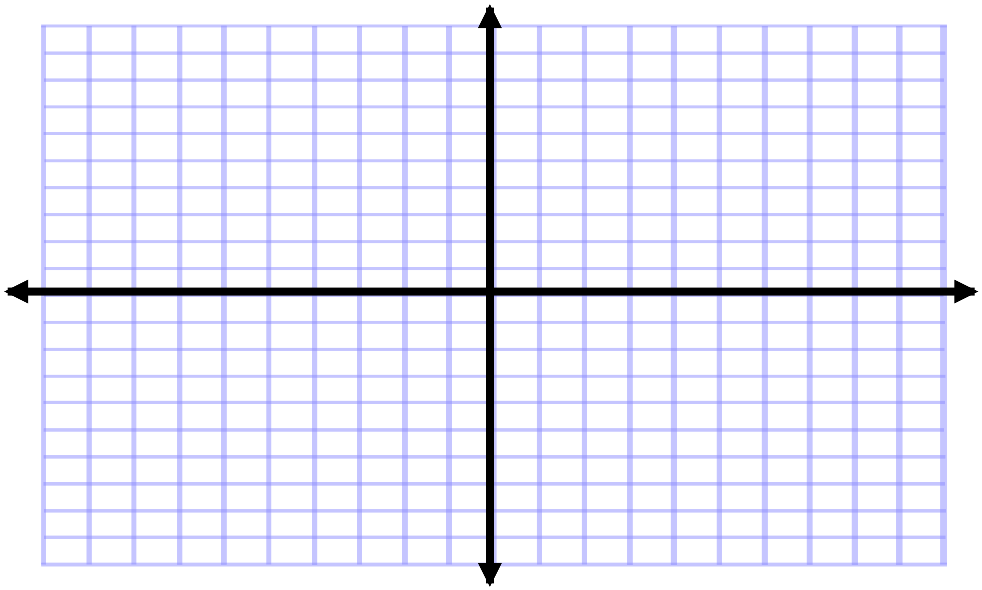
The graph has two parts. Each part is called a **branch.**

The *x*-axis is a horizontal asymptote.

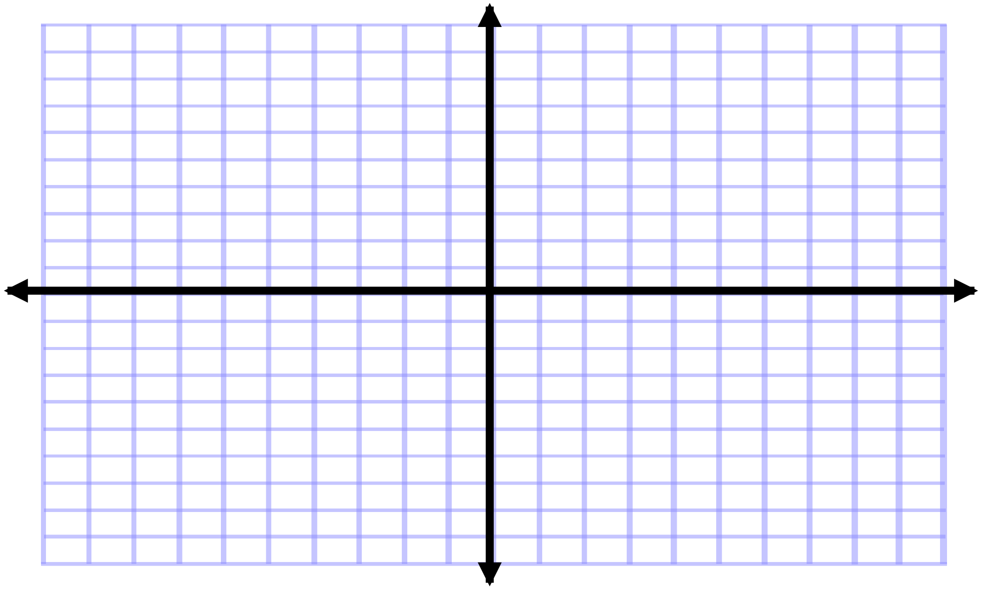
The *y*-axis is a vertical asymptote.

Asymptote: line that a graph approaches as x or y increases in value.

Example 2: Graph **y =** ——



Example 3: Graph **y =** $\frac{-}{}$



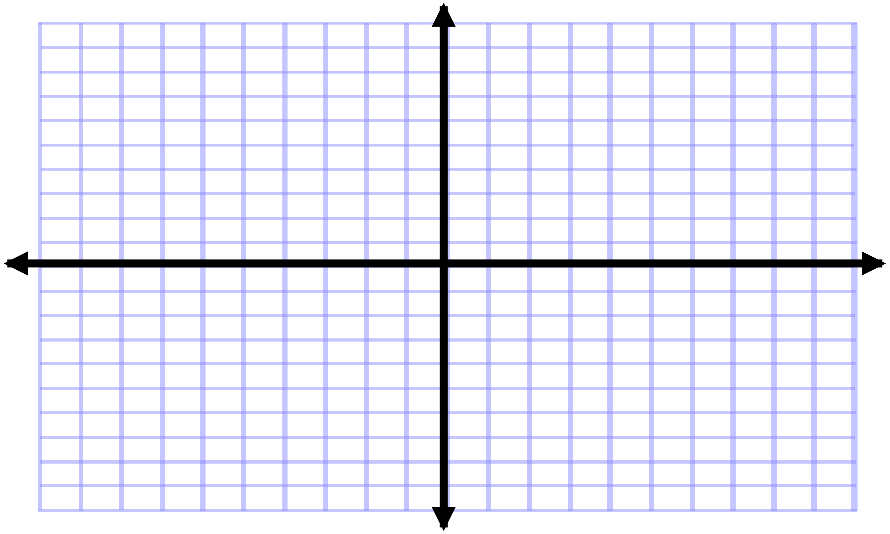
Translations of Inverse Variations

The graph of $y = \frac{k}{x - b} + c$ is a translation of $y = \frac{k}{x}$ by b units horizontally and c units vertically.

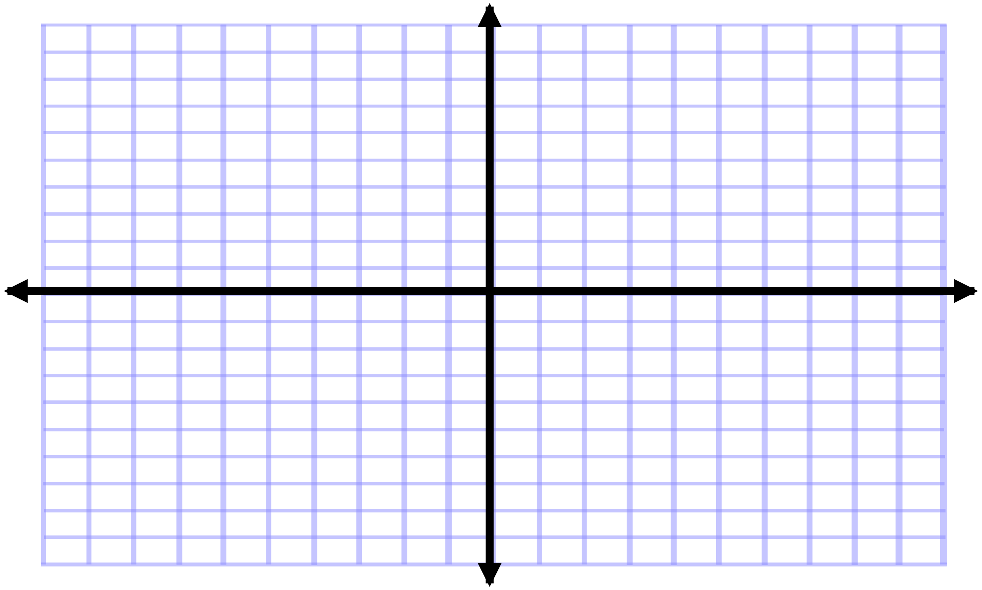
The vertical asymptote is $x = b$.

The horizontal asymptote is $y = c$.

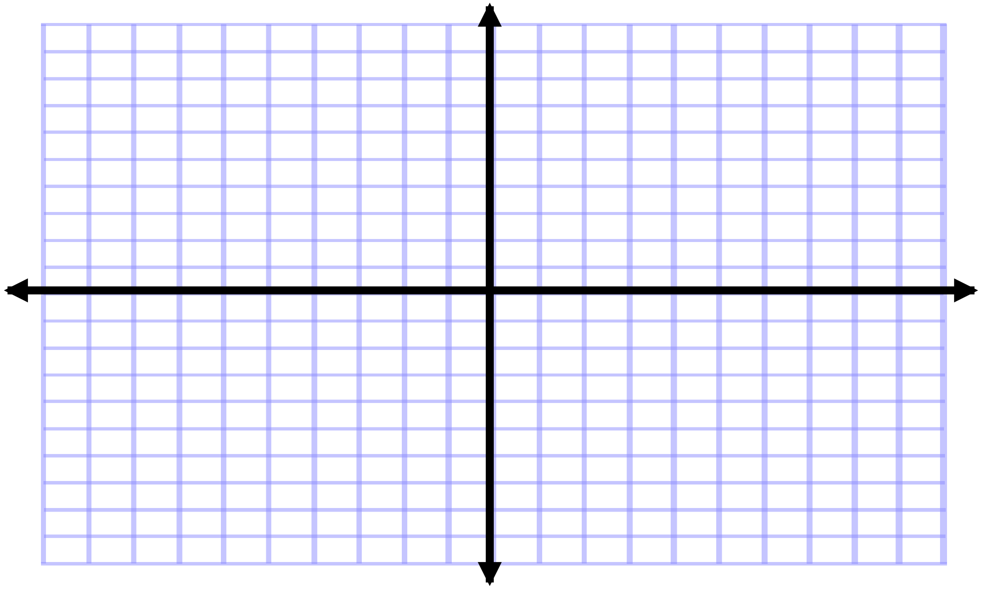
$$y = \frac{1}{x - 2} - 3$$



Example 6: Graph $y = \frac{\quad}{-} + 2$



Example 7: Graph $y = \frac{-}{+} - 2$



Writing the Equation of a Translation

Write an equation for the translation of $y = \frac{5}{x}$ that has asymptotes at $x = -2$ and $y = 3$.

Write an equation for the translation of y = -2/x that has asymptotes at x = 4 and y = -2.

Assignment:

In the Algebra 2 Textbook:

p. 488 #2, 3, 14-24

Classwork: Part 1

Suppose that x and y vary inversely. Write a function that models each inverse variation.

1. $x = 1$ when $y = 11$

2. $x = -13$ when $y = 100$

3. $x = 1$ when $y = 1$

4. $x = 28$ when $y = -2$

5. $x = 1.2$ when $y = 3$

6. $x = 2.5$ when $y = 100$

Classwork: Part 2

Describe the combined variation that is modeled by each formula.

16. $A = \pi r^2$

17. $A = 0.5bh$

18. $h = \frac{2A}{b}$

19. $V = \frac{Bh}{3}$

Classwork: Part 3

Write the function that models each relationship. Find z when $x = 4$ and $y = 9$.

24. z varies directly with x and inversely with y. When $x = 6$ and $y = 2$, $z = 15$.

25. z varies jointly with x and y. When $x = 2$ and $y = 3$, $z = 60$.

26. z varies directly with the square of x and inversely with y. When $x = 2$ and $y = 4$, $z = 3$.

27. z varies inversely with the product of x and y. When $x = 2$ and $y = 4$, $z = 0.5$.

End of Day 5

Unit 6 "Radical and Rational Functions"
Title: Rational Expressions

Objective: To 1) simplify rational expressions, 2) multiply and divide rational expressions, and 3) identify any restrictions on the variables.

A rational expression is in ***simplest form*** when its numerator and denominator have no common factors (other than 1).

Simplest Form

$$\frac{(x+2)(x-4)}{x(x-3)(x+5)}$$

$$\frac{x}{x-1}$$

Not in Simplest Form

$$\frac{4(x-3)}{3(x-3)}$$

$$\frac{(x+2)(x-4)}{(x+5)(x+2)}$$

Restrictions on a Variable: is any value of a variable that makes the denominator of the original expression or the simplified expression equal zero.

Example:

Simplify each expression. State any restrictions on the variables.

$$\frac{14x+20}{2x}$$

Example:

Simplify each expression. State any restrictions on the variables.

$$\frac{6x-15}{4x-10}$$

Example:

Simplify each expression. State any restrictions on the variables.

$$\frac{x^2 + 10x + 25}{x^2 + 9x + 20}$$

Example:

Simplify each expression. State any restrictions on the variables.

$$\frac{2x^2 - 3x - 2}{x^2 - 5x + 6}$$

Example:

Simplify each expression. State any restrictions on the variables.

$$\frac{x^2 + 13x + 40}{x^2 - 2x - 35}$$

Example:

Simplify each expression. State any restrictions on the variables.

$$\frac{-6 - 3x}{x^2 - 6x + 8}$$

End of Day 6

Multiplying and Dividing Rational Expressions

Factor all parts of the expression

You can cancel top to bottom and diagonally

When dividing, flip the second fraction and multiply.

Example:

Multiply. State any restrictions on the variables.

$$\frac{2x^2 + 7x + 3}{x - 4} \cdot \frac{x^2 - 16}{x^2 + 8x + 15}$$

Example:

Multiply. State any restrictions on the variables.

$$\frac{a^2 - 4}{a^2 - 1} \times \frac{a + 1}{a^2 + 2a}$$

Example:

Divide. State any restrictions on the variables.

$$\frac{a^2 + 2a - 15}{a^2 - 16} \div \frac{a + 1}{3a - 12}$$

Example:

Divide. State any restrictions on the variables.

$$\frac{x-3}{2x^2+9x-5} \div \frac{6x-18}{2x^2-15x+7}$$

End of Day 7

Unit 6 "Radical and Rational Functions"
Title: Adding and Subtracting
Rational Expressions

Objective: To find the least common multiples of rational expressions and use them to add & subtract rational expressions.

Adding/Subtracting Fractions - The denominators must be the same!

Find the least common multiple of $4x^2 - 36$ and $6x^2 + 36x + 54$.

a. $3x^2 - 9x - 30$ and $6x + 30$

b. $5x^2 + 15x + 10$ and $2x^2 - 8$

Example

$$\frac{2x}{x^2+5x+4}+\frac{5}{3x+3}$$

Example

$$\frac{1}{x^2+12x+20}+\frac{3x}{8x+16}$$

Example

$$\frac{2x-7}{3x^2+21x+30}+\frac{4x}{3x+15}$$

Example

$$\frac{7x}{5x^2-125}-\frac{4}{3x+15}$$

$$\frac{-2}{3x^2+36x+105}-\frac{4x}{6x+30}$$

$$\frac{x}{3x^2-9x+6}-\frac{2x+1}{3x^2+3x-6}$$

End of Day 8

Complex Fractions

Complex Fraction: a fraction that has a fraction in its numerator or denominator or both.

$$\frac{\frac{1}{x}}{y}$$

$$\frac{3}{1-\frac{1}{2y}}$$

$$\frac{\frac{x-2}{x}-\frac{2}{x+1}}{\frac{3}{x-1}-\frac{1}{x+1}}$$

Example:

Simplify $\dfrac{\dfrac{1}{x} + 3}{\dfrac{5}{y} + 4}$.

Example:

$$\frac{3}{1 - \frac{1}{2y}}$$

Example:

$$\frac{\dfrac{x-2}{x}-\dfrac{2}{x+1}}{\dfrac{3}{x-1}-\dfrac{1}{x+1}}$$

End of Day

Unit 6 "Radical and Rational Functions"
Title: Rational Equations

Objective: To solve equations that contain rational expressions and use rational equations in solving problems.

$$\frac{5}{4x} = \frac{1}{x-2}$$

Steps:

1. **Cross Multiply.**
2. **Solve equation.**
3. **Check Answers for Extraneous Solutions!**

Example

$$\frac{-4}{5x+10}=\frac{3}{x+2}$$

Example

$$\frac{1}{x-3} = \frac{6x}{x^2-9}$$

Example

$$\frac{-2}{x^2 - 2} = \frac{2}{x - 4}$$

$$\frac{1}{2x}+\frac{2}{5x}=\frac{1}{2}$$

Steps:

1. **Find the LCD.**
2. **Multiply each side of equation by the LCD.**
3. **Solve equation.**
4. **Check Answers for Extraneous Solutions!**

Example

$$\frac{3}{5x} - \frac{4}{3} = \frac{2}{3}$$

Example

$$\frac{4}{x} - \frac{3}{x+1} = 1$$

Example

$$\frac{7}{x^2 - 5x} + \frac{2}{x} = \frac{3}{2x - 10}$$

Example

$$\frac{5}{x-1}=\frac{2}{x-6}-\frac{1}{x^{2}-7x+6}$$

$$x + \frac{10}{x-2} = \frac{x^2 + 3x}{x-2}$$

End of Day

Word Problems

Apply Rational Equations

One pump can fill a swimming pool with water in 6 hours. A second pump can fill the same pool in 4 hours. If both pumps are used at the same time, how long would it take to fill the pool?

$$\frac{1}{6} + \frac{1}{4} = \frac{1}{}$$

Apply Rational Equations

Ben can paint a room in 10 hours. Cody can do it in 8 hours. If they work together, how long would it take to paint the room?

$$\frac{1}{10} + \frac{1}{8} = \frac{1}{}$$

Apply Rational Equations

Tim can stuff envelopes three times as fast as his wife Jill. They have to stuff 5000 envelopes for a fund-raiser. Working together, Tim and Jill can complete the job in four hours. How long would it take each of them working alone?

$$\frac{1}{\quad} + \frac{1}{3} = \frac{1}{4}$$

Apply Rational Equations

Jim and Alberto have to paint 6000 square feet of hallway in an office building. Alberto works twice as fast Jim. Working together, they can complete the job in 15 hours. How long would it take each of them working alone?

$$\frac{1}{} + \frac{1}{2} = \frac{1}{15}$$

End of Day