

VMHS Math Circle

IX. The Grand Algebra Review (Part IV: Functions and Graphs)

Brace yourselves, this is going to be long. We're on to functions and graphs now, and this document will basically cover whatever the AMC throws at you concerning this. A couple of problem-solving tactics are brought up here, so you should have the following knowledge to tackle these function questions.

- Completing the square
- Quadratic formula
- Rearranging Variables (Please make sure you know how to do this; it's simple algebra.)

When I talk about functions, I talk about the "f(x)" and the "y equals" and whatever has to do with them. Here, we'll cover various graphs, including lines and parabolas. Other conic sections, like ellipses and hyperbolas, will be included here, but we'll only cover these in the scope of the AMC.

Things like foci and eccentricity are very obscure for the AMC, and stuff about foci will mainly be part of parabolas and only be on the AMC 12. **AMC 10 takers do not need to worry about the more obscure conic sections.** Plus, the AMC has never asked about hyperbolas before. Although this is true, it's better to be safe than sorry. That is, if the AMC asks about hyperbolas, it's better to know how to handle them than not.

Our first category will deal with functions.

Function Manipulations/Compositions

Here, we're going to talk about ways you can manipulate functions or tie them together. First up, let's deal with function operations. **When doing these, think about plugging into x. I'm going to repeat this: plug it into x!** I'm going to show by specific example here because it's a bit tough to explain with words or examples that are too generic.

Function Addition/Subtraction:

Let's say we have a couple of functions:

$$f(x) = 3x$$

$$f(x) = 2x^2 + x.$$

It follows that:

$$f(x + n) = 3(x + n)$$

$$f(x + n) = 2(x + n)^2 + (x + n)$$

for any number n. Similarly,

$$f(x - n) = 3(x - n)$$

$$f(x - n) = 2(x - n)^2 + (x - n).$$

Function Multiplication/Division:

Let's say we have some other functions:

$$f(x) = 5(x + 4)$$

$$f(x) = x^3 + 3x^2 + 3$$

It follows that:

$$f(nx) = 5(nx + 4)$$

$$f(nx) = (nx)^3 + 3(nx)^2 + 3$$

and that

$$f(x/n) = 5(x/n + 4)$$

$$f(x/n) = (x/n)^3 + 3(x/n)^2 + 3$$

for any number n.

Function Compositions/Iterations:

Let's say we have two different functions:

$$g(x) = 5x$$

$$f(x) = x + 5$$

$$f(g(x)) = (5x) + 5$$

$$\text{and } g(f(x)) = 5(x + 5).$$

Note that $g(f(x))$ is the same thing as $(g \circ f)(x)$. Apply this reasoning analogously (similarly) with $f(g(x))$ and virtually every function composition you face. Here, you plug $g(x)$ into $f(x)$, and vice versa.

In a nutshell, $(g + f)(x) = g(x) + f(x)$

$$(g - f)(x) = g(x) - f(x)$$

$$(g * f)(x) = g(x)f(x)$$

$$(g/f)(x) = g(x)/f(x)$$

If we stick with our $f(x)$, to iterate it n times, we'd do $f(\underbrace{f(\dots f(f(x))\dots)}_{n \text{ times}})$.

Simply plug $f(x)$ into itself. Here, $f(f(x)) = (x + 5) + 5$ since we plug $f(x)$ into x .

Note that $f(\underbrace{f(\dots f(f(x))\dots)}_{n \text{ times}})$ is the same thing as $f^n(x)$. The AMC will most likely tell you what $f^n(x)$ means since it does not want to confuse this with exponents.

Inverses:

To find the inverse of a function, or $f^{-1}(x)$, get $f(x)$ by itself, switch x and $f(x)$, and solve for the new $f(x)$. Let's refer to $f(x)$ as y because I don't like pressing the shift key too many times to type in parentheses.

Say we have a function $y = 3x + 2$. Its inverse will be:

$$x = 3y + 2 \rightarrow x - 2 = 3y \rightarrow y = (x - 2)/3.$$

That's basically all there is that you should know. Now, let's bring in some AMC-type examples to get things started.

A function has the property that $f(x + 3) = 3x^2 + 2x + 5$. What is $f(x)$?

All right, so we know that $x + 3 - 3 = x$, of course. Simply plug in $(x - 3)$ into $f(x + 3)$ to get $f(x)$.

$$3(x - 3)^2 + 2(x - 3) + 5 \rightarrow 3(x^2 - 6x + 9) + 2x - 6 + 5 \rightarrow 3x^2 - 18x + 27 + 2x - 6 + 5 \rightarrow 3x^2 - 16x + 26.$$

Let f be a function for which $f(x/3) = x^2 + x + 1$. Find the sum of all values of z for which $f(3z) = 7$.

We know that $3(x/3) = x$. In turn, $3(x) = 3x$. Hence, we need to plug $9x$ into $f(x/3)$ to get $f(3x)$. Doing so gets us

$$f(3x) = 81x^2 + 9x + 1.$$

Substitute "z" into x , plug in 7 into $f(3z)$, and we get:

$$7 = 81z^2 + 9z + 1 \rightarrow 0 = 81z^2 + 9z - 6 \rightarrow 0 = 27z^2 + 3z - 2.$$

Note that the z is there not to confuse you, but to just tell you that there are values that you plug into $f(3x)$ to get an output of 7. All words aside, we use the quadratic formula to solve.

$[-3 + \sqrt{9 + 216}]/54$ and $[-3 - \sqrt{9 + 216}]/54$ are our two solutions. Add them up to get:

$-6/54 = -1/9$. You can also use Vieta's for $27z^2 + 3z - 2$ to get the same answer without using the quadratic formula.

Practice problems with function compositions and iterations will come later after we talk about certain graphs or at the end. As for now, let's get to another aspect of functions.

Domain, Range, and Asymptotes

To start off, **domain is simply the set of all possible x-values that can be inputted into a function. Range is the set of all possible y-values of a function.** If an x-value cannot be plugged in, it is not part of the domain. Here's a table of some domains/ranges of certain functions. AMC 12-type functions like trig/logarithmic functions will be covered in a Pre-Calculus review that will come later.

Here, "k" will act as a constant.

$f(x) =$	Domain	Range
kx , with odd powers of x	All real numbers	All real numbers
kx , with even powers of x	All real numbers	$x \geq \min[f(x)]$ if k is positive, $x \leq \max[f(x)]$ if k is negative.
k/x	$x \neq 0$ or whatever makes the denominator 0	$y \neq 0$ or anywhere the horizontal asymptote/s is/are, unless the graph crosses a horizontal asymptote
$k\sqrt{x}$	$x \geq 0$ or whatever makes what is in the square root sign 0	$x \geq \min[f(x)]$ if k is positive, $x \leq \max[f(x)]$ if k is negative
k^x	All real numbers	$x \geq 0$ if k is positive, $x \leq 0$ if k is negative

Remember that range varies with y-intercept or vertical shifts!

We move on to **asymptotes**. These are basically the **bounding lines of a function**. In a vast majority of cases, a graph cannot cross an asymptote. Instead, the graph gets closer and closer to the asymptote without touching it. Let's say we have a funny looking function $f(x)$ in the form of:

$$\frac{x^2 + 3x + 5}{x^2 + 3x + 2}$$

What would be our horizontal asymptote(s)? Vertical asymptotes?

To find vertical asymptotes, we'd try to find the zeroes of the denominator. These turn out to be:

$$x^2 + 3x + 2 = 0 \rightarrow (x + 2)(x + 1) = 0$$

$$x = -1 \text{ or } -2.$$

The lines $x = -1$ and $x = -2$ are our vertical asymptotes. Now, what about horizontal asymptotes? These, on the other hand, turn out to be what $f(x)$ is when x gets infinitely large. Basically, just plug in a million into x . We see that the x^2 's will make this million extremely large, so much that lower powers of x don't even matter. Hence, it suffices to simply divide the leading coefficients of the numerator and denominator to get the horizontal asymptote or what $f(x)$ is when x approaches infinity. Our horizontal asymptote is $y = 1/1 \rightarrow y = 1$.

Concerning horizontal asymptotes, what happens if the degree (highest power of x) of the numerator is different from the degree of the denominator? Here are two cases.

- If the degree of the top is lower than the degree of the bottom, there is no horizontal asymptote.
- If the degree of the top is higher than the degree of the bottom, do polynomial long division and ignore the remainder. Whatever line you get will be the slant asymptote.

With all this aside, let's get to a practice problem. The AMC doesn't typically ask about asymptotes, but it leans more heavily towards domain and range.

State the domain and range of the function $f(x) = \sqrt{\sqrt{\sqrt{\sqrt{x-2}}}}$.

Yup. The AMC likes to do wacky things with various functions. Despite this, we want to find things that make our problem-solving process easier. What helps to find the domain is to find whatever makes whatever is inside the left-most radical equal to zero. To clear this up, let's work with our $x - 2$. Setting this equal to zero, we know that $x - 2 = 0 \rightarrow x = 2$. $\sqrt{\sqrt{\sqrt{\sqrt{0}}}} = \sqrt{0}$, which is 0. Hence, we're good with assuming that 2 is the lowest possible x-value that we can possibly put into this function.

As for the range, we have to find the y-values that can possibly work. The range for square root functions is the set of y-values that are greater than or equal to the minimum of $f(x)$.

We can experiment a bit. We know that positive square root functions can't ever be negative. Since this one has no vertical shift, 0 is our minimum. If we plug in 2.1, we get a positive number for $f(x)$ because the square root of a positive real number can always be a positive real number. This reasoning applies analogously (similarly) to all numbers greater than 2, so we know that our range is $x \geq 0$.

Our domain is $x \geq 2$, and our range is $x \geq 0$.

More specific skills aren't necessarily tested by the AMC. What the test will do is choose more generic skills and tie in a problem-solving aspect that students must get through. The point is, there's a very slim chance that the AMC will ever ask about specifically finding asymptotes. Rather, it will have its own problem solving twist that doesn't involve specific skills of finding asymptotes.

Lines

No punches pulled, these are the simple $y = mx + b$ lines that you encountered in algebra I. However, there are some things about lines you should know, for knowing these things will help your problem-solving experience.

Slope:

It's easy to know that the slope of a line is the change in y over the change in x. However, you should recognize that the slope of a line passing through the origin is simply y/x for any point (x, y) on that line.

Point-Slope, Slope-Intercept, and Standard Form:

This will likely serve as review for you.

Point slope is denoted as:

$$(y - y_1) = m(x - x_1),$$

where (x_1, y_1) are known x and y and m is the slope.

Slope-intercept is denoted as:

$$y = mx + b,$$

where m is the slope, and b is the y-intercept.

Standard form is denoted as:

$$Ax + By = C,$$

where A, B, and C are coefficients.

To find the x-intercept, set y equal to zero and solve for x. To find the y-intercept, set x equal to zero and solve for y. To find the x-intercept, set y equal to zero and solve for x.

Line Perpendiculars:

A fixed point P's perpendicular to a line L is a line segment with endpoints P and L such that its slope is the negative reciprocal of line L's slope.

If a particle that moves along the line is at the foot of the perpendicular from P, its distance from that point is at a minimum. Any other point on that line is more distant from P.

Intersections of lines:

To find where two lines intersect, simply solve the system of equations generated by the equations of those two lines. For example, if you tried to find where the lines:

$$\begin{aligned}y &= x + 1 \\ y &= -2x + 5\end{aligned}$$

intersect, just solve the system of equations to get the (x, y) coordinate.

Here are some problems dealing solely with lines.

Jamal runs on a linear race track. This track is mapped on the coordinate plane as the line $y = -2x + 5$. If a landmark is located at $(0, 5/2)$ in this mapping, at what point on the linear race track is the distance between Jamal and that landmark at a minimum?

We want to find a point on $y = -2x + 5$ such that that point and the point $(0, 5/2)$ will form a line perpendicular to $y = -2x + 5$. The foot of this perpendicular will result in the minimum distance from the landmark. Here, the slope we'll be looking for is $1/2$. Using point-slope, we have:

$$\begin{aligned}y - 5/2 &= (1/2)(x - 0) \\ y &= x/2 + 5/2.\end{aligned}$$

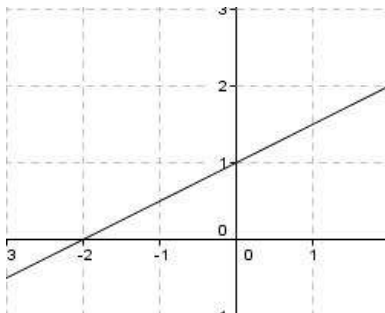
Now, we look for the point where the lines $y = -2x + 5$ and $y = x/2 - 5/2$ intersect because the foot of the perpendicular must lie on $y = -2x + 5$. Solving this system yields:

$$\begin{aligned}y &= -2x + 5 \\ y &= x/2 + 5/2 \\ x/2 + 5/2 &= -2x + 5 \\ 5x/2 &= 5/2 \rightarrow x = 1 \\ y &= -2(1) + 5 = 3\end{aligned}$$

$(1, 3)$.

Note that if Jamal passes this point, we will be further away from the landmark. This fact is here just for reference for future problems.

The line shown below is reflected along $y = x$. If this new line is expressed as $y = mx + b$, what will be mb ?



A reflection along $y = x$ means that our new line is the inverse of the old line. From what we are given, the y-intercept is 1, and the slope is $1/2$. Thus, the line depicted in the graph is $y = x/2 + 1$. The inverse of this line is $x = y/2 + 1 \rightarrow x - 1 = y/2 \rightarrow 2(x - 1) = y \rightarrow y = 2x - 2$.
Here, $m = 2$, and $b = -2$. $mb = 2(-2) = -4$.

Now that we've talked about lines, let's move on to some other graphs.

Parabolas

This section closely ties in with the quadratics found in part I of this review of algebra. Again, there are two forms you should know about the equations of parabolas:

Vertex Form:

$$y = a(x - h)^2 + k.$$

Where "h" is the x-coordinate of the vertex and "k" is the y-coordinate of the vertex. In other words, our vertex is (h, k).

Standard Form:

$$y = ax^2 + bx + c.$$

Where a, b, and c are coefficients.

In the scope of the AMC 10, all there is to know is what vertex form means and how one gets standard form from vertex form (algebra and simplifying, of course). Remember in vertex form that "h" is the x-coordinate of the vertex and "k" is the y-coordinate of the vertex!

Remember to factor and complete the square when converting standard form into vertex form. Also, try not to get your numbers messed up. Here is an example to make things clearer.

Convert the parabola $y = 3x^2 + 4x + 6$ into vertex form. What is the vertex of this parabola?

To do this, we factor out a 3, which yields:

$$y = 3(x^2 + 4x/3 + 2).$$

Now, we focus only on the x^2 and x . To complete the square, we must add **whatever is inside the parentheses** by $4/9$ because we must make $(b/2)^2 = c$ to complete the square. Plus, we need to add to both sides of the equation to keep equality. However, **we must multiply "c" by whatever is in front of the parentheses** because equality gets screwed up if we don't. Doing so, we have:

$$y + 4/3 = 3(x^2 + 4x/3 + 4/9 + 2)$$

Simplifying, we get:

$$y + 4/3 = 3((x + 2/3)^2 + 2)$$

$$\rightarrow y = 3(x + 2/3)^2 + 14/3.$$

Looking at the form $y = a(x - h)^2 + k$, $a = 3$, $h = -2/3$, and $k = 14/3$. Our vertex is (h, k) $\rightarrow (-2/3, 14/3)$.

However, there is a more comprehensive aspect of parabolas that AMC takers should be familiar with. Thankfully, AMC 10 takers need not worry about this part of the review, although there may be those who've taken Algebra 2 and have covered this. Quick note, I'll just throw out the bold, red label **AMC 12 only** to AMC 12 only material like I'm about to do now.

AMC 12 Only

Focus, directrix, and latus rectum:

- The definition of a parabola is all points equidistant from the focus and directrix. It's obvious to say that the distance of the focus from the parabola's vertex and the distance of the directrix from the vertex are equal. Thus, the vertex is halfway between the focus and directrix.
 - Given a parabola $y = a(x - h)^2 + k$, the focus is $a = 1/4c \rightarrow c = 1/4a$ away from the vertex where the parabola is facing, and the directrix is $1/4a$ away from the vertex in the opposite direction. For example, given the parabola $y = x^2$, the focus is at $(0, 1/4)$, and the directrix is at $(0, -1/4)$.
- The latus rectum of a parabola is the chord parallel to the directrix that passes through the focus with endpoints A and B on the parabola. The focus must be the midpoint of the latus rectum due to the definition of the parabola. From this, we find that the focus must lie on the axis of symmetry of the parabola (basically the axis that cuts the parabola into two equal halves).
- When dealing with slanted parabolas, however, the reasoning still applies with focus and directrix: the focus and directrix are still equidistant from all points on the parabola. Additionally, the focus still lies on the axis of symmetry, though the latter is slanted. This will be clear with the practice problems that will come at the end of this entire review. Let's start with an easier one for now.

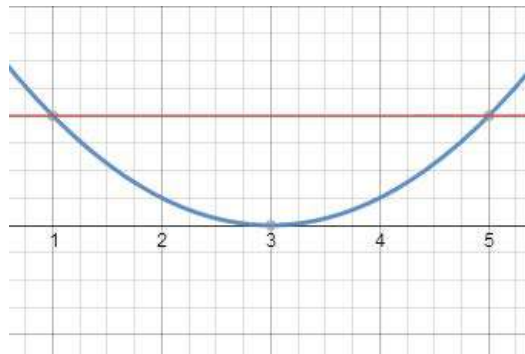
A parabola has vertex V and focus F located at $(3, 1)$. FV is perpendicular to the x-axis. If the latus rectum of this parabola has length 4, what is the equation of this parabola?

If FV is perpendicular to the x-axis, this parabola opens straight up. Luckily, these points have the same y-coordinate because of this. Now, since the latus rectum of this parabola has length 4, we need to find points on this parabola that are 4 apart.

F must be located at the midpoint of the latus rectum. Therefore, the endpoints of the latus rectum are $(1, 1)$ and $(5, 1)$, enough to make the latus rectum 4 units long.

From the definition of the parabola, the distances of a point from the focus and the directrix must be equal. Hence, our directrix is $y = 1 - 2 \rightarrow y = -1$, subtracting 2 since $(1, 1)$ was 2 away from the focus. Combining our finding that the parabola opens straight up with the fact that the vertex must be halfway between the focus and directrix, the coordinates of our vertex are $(3, 0)$. Since our focus was 1 unit away from the vertex, $1/4a = 1 \rightarrow a = 1/4$.

Our equation is thus $y = (1/4)(x - 3)^2$. Here's a sketch of this parabola for the sake of verification.



We can see that the focus is at $(3, 1)$, and the latus rectum is 4 units long. This parabola opens straight up, too.

I've thrown this red line in here to denote the end of AMC 12-exclusive material. There isn't necessarily any AMC 10-exclusive material because the AMC 10 has a smaller scope of material to cover. All AMC 10 skills are within the scope of what AMC 12 takers must learn; hence, no AMC 10-exclusive material. Now onto some more types of graphs. (Goodness gracious. This section is going to be long.)

Circles

Circles are pretty simple. There is a problem-solving trick that's pretty important when it comes to circles (and perhaps all other conics), but I'll get to that later. Let's bring in what the equation of a circle is.

Equation of a Circle (as if I wasn't redundant enough):

$$(x - h)^2 + (y - k)^2 = r^2$$

Where "h" is the x-coordinate of the center and "k" is the y-coordinate of the center. This circle's center is (h, k). "r" is the radius of the circle.

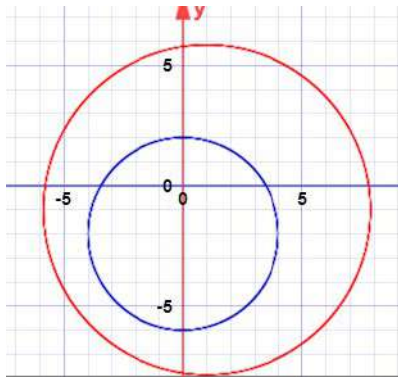
One thing very important to recognize about a circle is that one should be ready to complete the square in order to get the explicit equation for a circle. Doing so will help immensely. To clear things up, let's bring in an example.

Two circles have the equations $x^2 + y^2 + 4y - 12 = 0$ and $x^2 - 2x + y^2 + 2y = 45$. What is the area outside the smaller circle but inside the bigger circle?

We need to complete the square. It follows that:

$$\begin{aligned} x^2 + y^2 + 4y - 12 = 0 &\rightarrow x^2 + y^2 + 4y = 12 \rightarrow x^2 + y^2 + 4y + 4 = 12 + 4 \rightarrow x^2 + (y + 2)^2 = 16 \\ x^2 - 2x + y^2 + 2y = 45 &\rightarrow x^2 - 2x + 2 + y^2 + 2y + 2 = 45 + 2 + 2 \rightarrow (x - 1)^2 + (y + 1)^2 = 49 \end{aligned}$$

Graphing, we have:



This way, we know that the smaller circle actually lies within the larger. The smaller circle has radius 4, and the larger circle has radius 7. We subtract the area of the smaller circle from the area of the larger.

$$7^2\pi - 4^2\pi = 33\pi.$$

That's basically it for circles, since all there is to know is how the equation of the circle would look and how one can manipulate it in his/her own favor. We move on to the last section, which is AMC 12-exclusive.

Ellipses and Hyperbolas

AMC 12 Only

Ellipses and hyperbolas form an extremely obscure topic in the realm of the AMC. Only the AMC 12 will ask about ellipses/hyperbolas, although, like I said earlier, the AMC 12 has never asked about hyperbolas (in the form that I'm about to show). Clearly, hyperbolas involving inverse variation (the old $y = k/x$) were part of the AMC, but in terms of conic sections, hyperbolas are part of no man's land. However, the AMC 12 has asked about ellipses. What I'll do here is juxtapose ellipses and hyperbolas for a bit.

Ellipse:

Basically an oval, the definition of an ellipse is a set of points that exist such that the sum of the distances from all points to both foci of the ellipse is constant.

The equation of an ellipse is:

$$(y - k)^2/a^2 + (x - h)^2/b^2 = 1.$$

With (h, k) being the center again, with "a" and "b" interchangeable depending on which is bigger. The foci of an ellipse lie along the major axis (longer line segment cutting an ellipse in half) and are "c" apart from the center. To find c, utilize the below relation:

$$a^2 - b^2 = c^2.$$

Hyperbola:

Basically two U-shapes facing in opposite directions, the definition of a hyperbola is the set of points that exist such that the ratio of the distances of said points to both the foci and directrix is constant.

The equation of a hyperbola is:

$$\text{Opens up/down: } (y - k)^2/a^2 - (x - h)^2/b^2 = 1.$$

$$\text{Opens left/right: } (x - h)^2/b^2 - (y - k)^2/a^2 = 1.$$

With (h, k) being the center, but this time the center of something we call a "central rectangle." If we extend the diagonals of this central rectangle infinitely, we form the bounding lines of the hyperbola.

The foci of a hyperbola are "c" away from the center of the central rectangle, lying on the major axis (longer line segment cutting the central rectangle in half) of the central rectangle. To find c, utilize the below relation (cough Pythagorean theorem cough):

$$a^2 + b^2 = c^2$$

The vertices of a hyperbola are "a" away from the center of the central rectangle, lying on the major axis.

Stuff to know for both:

The bigger quantity in the denominator is "a." The length of the major axis of an ellipse and the central rectangle of a hyperbola is $2a$. The length of the minor axis is $2b$.

The coefficients of the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ determine what graph you get, depending on what $B^2 - 4AC$ is equal to. If it equals 0, then you'll get a parabola. If it's less than 0, then you'll get an ellipse. If it's greater than 0, you'll get a hyperbola. You might get lines or points, but use

your intuition to distinguish between lines, points, ellipses, and all that. This entire equation is pretty obscure material for the AMC, anyway. It's best not to stress over it too much.

Though ellipses and hyperbolas are obscure material, there is one thing you should definitely know about an ellipse: finding possible tangent lines. This involves some plugging in, substituting, and usage of the quadratic formula. The technique of "zeroing out the discriminant" rings again. Why do we want this discriminant to be equal to zero? Because we want only one solution and only one point of intersection. Here is a sample problem to illustrate what I'm talking about.

The ellipse $(x - 2)^2/16 + (y - 3)^2 = 1$ is graphed on the coordinate plane. Given that line L passes through the origin and at least one point on the ellipse, what is the minimum possible absolute value of the slope line L can have?

Let "a" represent the slope of this line. Hence, $y = ax$ due to both the y and x intercepts being 0. Plugging ax into y, we get:

$$(x - 2)^2/16 + (ax - 3)^2 = 1 \rightarrow (x - 2)^2 + 16(ax - 3)^2 = 16$$

$$\rightarrow x^2 - 4x + 4 + 16a^2x^2 - 96ax + 144 = 16 \rightarrow x^2(16a^2 + 1) - x(4 + 96a) + 132 = 0.$$

We want to zero out this discriminant ($b^2 - 4ac$) to get tangent lines. Tangent lines yield the minimum possible slopes of line L. Doing so gets us something a bit brutal to calculate.

$$(4 + 96a)^2 - 4(132)(16a^2 + 1) = 0$$

$$\rightarrow 9216a^2 + 768a + 16 - 8448a^2 - 528 = 0$$

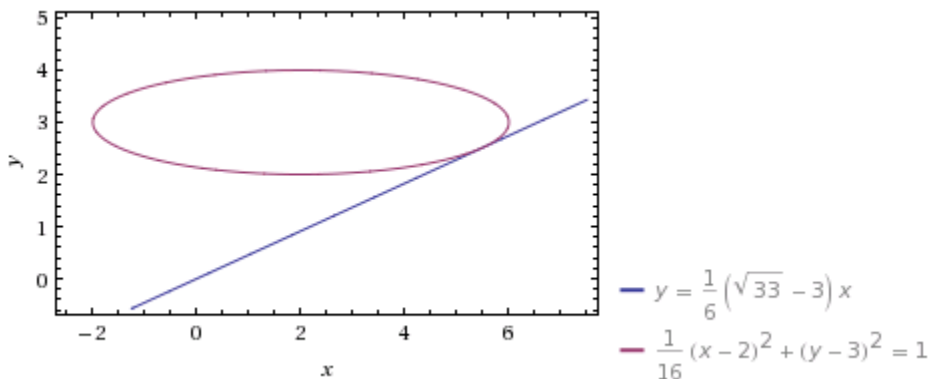
$$\rightarrow 768a^2 + 768a - 512 = 0$$

Thank the lord, 256 is a common factor of this entire expression. This reduces to:

$$3a^2 + 3a - 2 = 0.$$

Solving for a, we use the quadratic formula. This gets us $[-3 \pm \sqrt{9 + 24}]/6 = [-3 \pm \sqrt{33}]/6$. Since the absolute value of $[-3 - \sqrt{33}]/6$ is greater than the absolute value of $[-3 + \sqrt{33}]/6$, $[-3 + \sqrt{33}]/6$ is what we are looking for.

If you don't think this is right, here's a graph plotting the ellipse mentioned here and the line $y = [-3 + \sqrt{33}]x/6$. The latter is a tangent line, as you can see.



TIPS:

- When you try to drop a perpendicular from a point to a line in the coordinate plane, you can get away with using your protractor (choosing a point on the line whose line segment with that point is perpendicular to said line). There's an AMC problem that will come in which you can use this.
- Don't forget that the equation of a circle involves the square of the radius, not the radius itself. If you have the circle $x^2 + y^2 = 9$, know that the radius is 3, not 9!

- Also remember that “a” is half the major axis, and “b” is half the minor axis, for hyperbolas and ellipses.
- Again, make sure you check your work. Functions and graphs can involve some heavy algebra, so make sure you are actively checking your work.
- Say you actually get an equation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. Rest assured, you’re not going to be able to graph it. If it’s an ellipse, and you’re trying to find a tangent line, just plug something in right away and zero out the discriminant. No need to waste time trying to come up with a graph.
- In some cases, if you can’t see something you can graph or get a tough-looking equation, try completing the square, like I mentioned earlier.
- Don’t forget to distribute and keep equality when trying to convert the standard form of a parabola into vertex form.



Gosh, this was long.