Mathematics Curriculum Guide

Algebra 2

2017-18

Topic 9: Periodic Functions & Trigonometry

Transfer Goals

1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.
2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience.
3) Construct viable arguments and critique the reasoning of others using precise mathematical language.

Timeframe: 20 days
Start Date: March 26, 2018
Assessment Dates: April 27, 2018

Standards

F-IF 4 – For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F-IF 7 – Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F-IF 7e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-TF 1 – Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF 2 – Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **2.1 CA** Graph all 6 basic trigonometric functions

F-TF 5 – Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Meaning-Making

Understandings

Students will understand that…

- Periodic behavior is behavior that repeats over intervals of constant length.
- The measure of an angle in standard position is the input for two important functions. The outputs are the coordinates (called cosine and sine) of the point on the terminal side of the angle that is 1 unit from the origin.
- An angle with a full circle rotation measures 2π radians. An angle with a semicircle rotation measures π radians.
- As the terminal side of an angle rotates about the origin (beginning at 0°), its sine value on the unit circle increases from 0 to 1, decreases from 1 to -1, and then increases back to 0.
- As the terminal side of an angle rotates about the origin (beginning at 0°), its cosine value on the unit circle decreases from 1 to -1, and then increases back to 1.
- The tangent function has infinitely many points of discontinuity with a vertical asymptote at each point. Its range is all real numbers. Its period is π, half that of both the sine and cosine functions.
- You can translate periodic functions in the same way that you translate other functions.
- Cosine, sine, and tangent have reciprocals. Cosine and secant are reciprocals as are sine and cosecant. Tangent and cotangent are also reciprocals.
- The amplitude of a periodic function is half the difference of the maximum and minimum values of the function.

Essential Questions

Students will keep considering…

- How can you model periodic behavior?
- What function has as its graph a sine curve with amplitude 4, period π, and a minimum at the origin?
- If you know the value of sin θ, how can you find cos θ, tan θ, csc θ, sec θ, and cot θ?

Acquisition

Knowledge

Students will know…

Vocabulary – periodic function, cycle, period, amplitude, midline, standard position, initial side, terminal side, coterminal angles, unit circle, cosine of θ, sine of θ, central angle, intercepted arc, radian, sine function, sine curve, phase shift, tangent of ϑ, tangent function, cosecant, secant, cotangent

- Procedures and formulas for :
  - Finding amplitude and midline of a periodic function.
  - Measuring and sketching angles in standard position.
  - Finding the cosine and sine of an angle.
- Key Concepts for :
  - Proportion Relating Radians and Degrees
  - Converting Between Radians and Degrees
  - Length of an Intercepted Arc
  - Families of Sine and Cosine Functions
  - Tangent of an Angle
  - Reciprocal Functions
- Properties of:
  - Sine, Cosine, and Tangent Functions

Skills

Students will be skilled at and able to do the following…

- Identify properties of trig functions and explore periodic behavior.
- Graph periodic functions accurately.
- Apply techniques for analyzing graphs of rational functions to analyze graphs of trigonometric functions and other periodic functions.
- Find coordinates of points on the unit circle.
- Find the measure of angles in standard position, the quadrants in which they lie, and the positive and negative coterminal angles for given angles.
- Find the values of the cosine and sine of an angle.
- Convert between and use radians and degrees.
- Find the length of an arc of a circle.
- Write the formulas of trigonometric functions.
- Find cycles, amplitudes, periods, minimums, and maximums of trigonometric functions.
- Find the value of the reciprocal trigonometric functions based on the corresponding trigonometric functions.

Topic 9: Periodic Functions & Trigonometry

Transfer is a student's ability to independently apply understanding in a novel or unfamiliar situation. In mathematics, this requires that students use reasoning and strategy, not merely plug in numbers in a familiar-looking exercise, via a memorized algorithm.

Transfer goals highlight the effective uses of understanding, knowledge, and skills we seek in the long run – that is, what we want students to be able to do when they confront new challenges, both in and outside school, beyond the current lessons and unit. These goals were developed so all students can apply their learning to mathematical or real-world problems while simultaneously engaging in the Standards for Mathematical Practices. In the mathematics classroom, assessment opportunities should reflect student progress towards meeting the transfer goals.

With this in mind, the revised **PUSD transfer goals** are:

1) **Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.**
2) **Effectively communicate orally, in writing, and by using models (e.g., concrete, representational, abstract) for a given purpose and audience.**
3) **Construct viable arguments and critique the reasoning of others using precise mathematical language.**

Multiple measures will be used to evaluate student acquisition, meaning-making and transfer. Formative and summative assessments play an important role in determining the extent to which students achieve the desired results in stage one.

Formative Assessment	Summative Assessment
Aligning Assessment to Stage One	
• What constitutes evidence of understanding for this lesson? • Through what other evidence during the lesson (e.g. response to questions, observations, journals, etc.) will students demonstrate achievement of the desired results? • How will students reflect upon, self-assess, and set goals for their future learning?	• What evidence must be collected and assessed, given the desired results defined in stage one? • What is evidence of understanding (as opposed to recall)? • Through what task(s) will students demonstrate the desired understandings?
Opportunities	
• Discussions and student presentations • Checking for understanding (using response boards) • Ticket out the door, Cornell note summary, and error analysis • *Performance Tasks* within a Unit • Teacher-created assessments/quizzes	• Unit assessments • Teacher-created quizzes and/or mid-unit assessments • *Illustrative Mathematics* tasks (https://www.illustrativemathematics.org/) • Performance tasks

Topic 9: Periodic Functions & Trigonometry

Transfer Goals
1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution. 2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience. 3) Construct viable arguments and critique the reasoning of others using precise mathematical language.

Essential Questions:

- How can you model periodic behavior?
- What function has as its graph a sine curve with amplitude 4, period π, and a minimum at the origin?
- If you know the value of sin θ, how can you find cos θ, tan θ, csc θ, sec θ, and cot θ?

Standards: F-IF 4, F-IF 7, F-IF 7e, F-TF 1, F-TF 2, F-TF 2.1, F-TF 5

Timeframe: 20 days
Start Date: March 26, 2018
Assessment Dates: April 27, 2018
(extra days have been added to this pacing to allow for the SBAC prep and testing)

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Resources
5 days	**Five additional days are within the pacing to allow for SBAC Review and Testing that is scheduled for April 23 – 27.**					
1 day	**Lesson 13.1: Exploring Periodic Data** SMP: 1,2,3,4,6 (pp. 828-834) **F-IF 4, F-TF 5** *Prep for Performance Task p. 834*	**Focus Question** • How does the graph of a function help you determine whether it is periodic? **Inquiry Question** Solve It, pg. 828	• Periodic behavior is behavior that repeats over intervals of constant length. • The amplitude of a periodic function is half the difference of the maximum and minimum values of the function.	**Vocabulary:** periodic function, cycle, period, amplitude, midline **Concepts:** • Procedures for finding the amplitude and midline of a function, as well as identifying cycles and periods.	• Identify cycles and periods of periodic function. • Find the amplitude of periodic functions. • Apply techniques for analyzing graphs of rational functions to analyze graphs of trigonometric functions and other periodic functions.	**Recommended Practice:** 13-1 Practice Form G #14 **Thinking Map:** Use a Double-Bubble Map to compare and contrast identifying cycles and periods with periodic functions. **CC Problems:** #3, 4, 5, 23, 24, 26, 36

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
1 day	**Lesson 13.2: Angles and the Unit Circle** SMP: 1,2,3,5 (pp. 836-842) **F-TF 2**	**Focus Question** • How are angles in standard position and coordinated points in the unit circle related to the coordinated grid? **Inquiry Question** Solve It, pg. 836	• The measure of an angle in standard position is the input for two important functions. The outputs are the coordinates (called cosine and sine) of the point on the terminal side of the angle that is 1 unit from the origin. • Two angles in standard position are coterminal angles if they have the same terminal side. • The measure of an angle is positive when the rotation from the initial side to the terminal side is in the counterclockwise direction. The measure is negative when the rotation is clockwise. • Sine is the y-coordinate where the terminal side of an angle intersects the unit circle. The cosine is the x-coordinate.	**Vocabulary:** standard position, initial side, terminal side, coterminal angles, unit circle, cosine of θ, sine of θ **Concepts:** • Procedures for measuring and sketching angles in standard position • Procedures for finding the cosine and sine of an angle • The unit circle definitions of the sine and cosine functions reveal that these are periodic functions	• Find the length of legs of right triangles. • Find the measure of angles in standard position and the quadrants in which they lie. • Find coordinates of points on the unit circle. • Find the positive and negative coterminal angles for given angles.	**Thinking Map:** Create a Bridge Map to show the relationships between angles in standard position and coterminal angles. **CC Problems:** #5, 6, 39, 40-44, 51, 52, 58, 59
1 day	**Lesson 13.3: Radian Measure** SMP: 1,2,3,4,6 (pp. 844-850) **F-TF 1**	**Focus Question** • How can radian measures of angles be used to find the length of an arc of a circle? **Inquiry Question** #35, pg. 849	• An angle with a full circle rotation measures 2π radians. An angle with a semicircle rotation measures π radians. • To convert degrees to radians multiply by $\frac{\pi\ radians}{180°}$. To convert radians to degrees, multiply by $\frac{180°}{\pi\ radians}$. • For a circle of radius *r* and a central angle of measure θ (in radians), the length of the intercepted arc is $s = r\theta$.	**Vocabulary:** central angle, intercepted arc, radian **Concepts:** • Proportion Relating Radians and Degrees • Converting Between Radians and Degrees • Length of an Intercepted Arc	• Convert the measure of angles in degrees to radians and from radians to degrees • Find the length of an arc of a circle • Apply that "unit" of radian measure is the "unit" of a circle of radius 1.	**Recommended Practice:** 13-3 Think About a Plan (Green Notebook) **Thinking Map:** Create Flow Maps to show converting between radians and degrees and vice versa. **CC Problems:** #4, 5, 35, 45, 46, 48, 53 **STEM:** #34

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
1 day	**Lesson 14.1: Trigonometric Identities** SMP: 1,2,3,4 (pp. 904-910) **F-TF 8**	**Focus Question(s):** • How is simplifying an expression different from verifying an identity? **Inquiry Question(s):** Pg. 904 Solve It!	• The interrelationships among the six basic trig functions make it possible to write trig expressions in various equivalent forms, some of which can be significantly easier to work with than others, in mathematical applications. • Known identities can be used to verify other identities. • Trig identities can be used to simplify trig expressions.	**Vocabulary:** trigonometric identity **Concepts:** • Reciprocal Identities • Tangent Identity • Cotangent Identity • Pythagorean Identities	• Verify trigonometric identities.	**Thinking Map:** Create a Double-Bubble to compare and contrast how simplifying an expression is different from verifying an identity. **CC Problems:** #5,6, 28, 62 **STEM:** #67
1 day	**Lesson 14.2: Solving Trigonometric Equations Using Inverses** SMP: 1,2,3,4,5 (pp. 911-918) **F-TF 6**	**Focus Question(s):** • What is the advantage of defining the inverse trigonometric functions for sine, cosine, and tangent? **Inquiry Question(s):** Pg. 911 Solve It!	• Some trigonometric equations can be solved using an inverse trigonometric function to find one solution. Then periodicity can be used to find all solutions. • The values of inverse trigonometric functions are measures of angles. The unit circle can be used to find the values in either radians or degrees.	**Vocabulary:** trigonometric identity, inverse **Concepts:** • Inverse Trigonometric Functions	• Evaluate inverse trigonometric functions. • Solve trigonometric equations.	**CC Problems:** #5,6, 42, 44, 61 **STEM:** #43, 66
2 days	**Lesson 14.3: Right Triangles and Trigonometric Ratios** SMP: 1,2,3,4,5 (pp. 919-926) **G-SRT 6**	**Focus Question(s):** • How are the ratios of the lengths of pairs of sides in a right triangle related to a given acute angle in a right triangle? **Inquiry Question(s):** Pg. 919 Solve It!	• If the domains of the trigonometric functions are restricted to angle measures between 0° and 90°, the function values are the trigonometric ratios associated with the acute angles of a right triangle. • In right triangle trigonometry, the value of one trigonometric ratio determines the values of the others.	**Vocabulary:** trigonometric ratios **Concepts:** • Trigonometric Ratios for a Circle • Trigonometric Ratios for a Right Triangle	• Find lengths of sides in a right triangle. • Find measures of angles in a right triangle.	**Thinking Map:** Create a Double-Bubble to compare and contrast ratios for a circle and a right triangle. **CC Problems:** #5,6, 34, 44, 45, 46, 51 **STEM:** #25

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
2 days	**Lesson 13.4: The Sine Function** SMP: 1,2,3,4,5 (pp. 851-858) **F-IF 4, F-IF 7, F-IF 7e, F-TF 2, F-TF 2.1**	**Focus Question(s):** • How do the coefficients *a* and *b* in *y=a sin bx* change the graph of the parent function *y= sin x*? **Inquiry Question(s):** pg. 857 #37 (Calculator needed)	• As the terminal side of an angle rotates about the origin (beginning at 0°), its sine value on the unit circle increases from 0 to 1, decreases from 1 to -1, and then increases back to 0. • A non-translated sine function can be completely described in terms of tits amplitude and period.	**Vocabulary:** sine function, sine curve **Concepts:** • Properties of Sine Functions	• Graph the parent sine function with a period of 2π and amplitude of 1. • Use the properties of sine function to graph cycles in the intervals from 0 to 2π, the amplitude of |a|, and reflection in the x-axis.	**Thinking Map:** Begin a Tree Map to record key information about Sine, Cosine, and Tangent Functions. **Common Core Problems:** #3, 4, 5, 39, 46 **STEM:** #49-51
1 day	**Lesson 13.5: The Cosine Function** SMP: 1,2,3,4,5 (pp. 861-867) **F-TF 5, F-IF 7e, F-TF 2, F-IF 4** *Prep for Performance Task p. 867*	**Focus Question:** • How does knowing properties of *y = sin ϑ* help you understand the properties of *y = cos ϑ*? **Inquiry Question:** Pg. 861 Solve It!	• As the terminal side of an angle rotates about the origin (beginning at 0°), its cosine value on the unit circle decreases from 1 to -1, and then increases back to 1. • A non-translated cosine function can be completely described in terms of tits amplitude and period.	**Vocabulary:** cosine function **Concepts:** • Properties of Cosine Functions	• Use the properties of *y = sin ϑ* to help understand the properties of *y = cos ϑ*. • Graph and write cosine functions. • Solve trigonometric equations.	**Thinking Maps**: Add to the *Tree Map created in lesson 13.4.* **Common Core Problems:** #5, 6, 35, 36, 42, 45 **STEM:** #42, 44

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
1 day	**Lesson 13.7: Translating Sine and Cosine Functions** SMP: 1,2,3,4,5,7 (pp. 875-882) **F-TF 5, F-IF 7e**	**Focus Question:** • How are translations of trigonometric functions similar to other functions? **Inquiry Question:** Pg. 881 #40	• You can translate periodic functions in the same way that you translate other functions. • All functions in the sine and cosine family of functions can be completely described by their amplitude, period, vertical shift, and horizontal shift (or phase shift).	**Vocabulary:** phase shift **Concepts:** • Families of Sine and Cosine Functions (i.e. parent functions and transformed functions for sine and cosine)	• Graph translations of sine and cosine functions. • Write equations of translations of sine and cosine functions.	**Thinking Maps**: Add to the *Tree Map created in lesson 13.4.* **Common Core Problems:** #4, 5, 43, 47 **STEM:** #40
1 day	**Lesson 13.6: The Tangent Function** SMP: 1,2,3,4,5 (pp. 868-874) **F-IF 7e, F-TF 2, F-TF 5**	**Focus Question:** • How is the tangent of an angle ϑ related to the sine and cosine of the angle ϑ? To the unit circle? **Inquiry Question:** Pg. 868 Solve It!	• The tangent function has infinitely many points of discontinuity with a vertical asymptote at each point. Its range is all real numbers. Its period is π, half that of both the sine and cosine functions. • Suppose $y = a \tan b\vartheta$, with $b > 0$ and ϑ measured in radians. Then $\frac{\pi}{b}$ is the period of the function, one cycle occurs in the interval $-\frac{\pi}{2b}$ to $\frac{\pi}{2b}$, and there are vertical asymptotes at each end of the cycle.	**Vocabulary:** tangent of ϑ, tangent function **Concepts:** • Tangent of an Angle • Properties of Tangent Functions	• Graph tangent functions with period and asymptotes.	**Thinking Maps**: Add to the *Tree Map created in lesson 13.4.* **Common Core Problems:** #5, 6, 7, 36, 37, 38, 50, 51 **STEM:** #47
1 day	**Lesson 13.8: Reciprocal Trigonometric Functions** SMP: 1,2,3,4,5 (pp. 883-890) **F-IF 7e** *Prep for Performance Task p. 834*	**Focus Question:** • How do you evaluate and graph a reciprocal trigonometric function? **Inquiry Question:** Pg. 883 Solve It!	• Cosine, sine, and tangent have reciprocals. Cosine and secant are reciprocals as are sine and cosecant. Tangent and cotangent are also reciprocals. • Reciprocal trigonometric functions can be evaluated using what you know about sine, cosine, and tangent. • The graphs of reciprocal trigonometric functions have asymptotes when their denominators equal 0.	**Vocabulary:** cosecant, secant, cotangent **Concepts:** • Reciprocal functions	• Evaluate and graph reciprocal trigonometric functions.	**Thinking Maps**: Add to the *Tree Map created in lesson 13.4.* **Common Core Problems:** #6, 7, 8, 38, 48, 49, 50, 51, 52, 53, 60, 61, 63, 64

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
1 days	**Review Topic 9 Concepts & Skills** Use Textbook Resources and/or Teacher Created Items					
1 day	**Topic 9 Assessment** (Created and provided by PUSD)					

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