Mathematics Curriculum Guide

Algebra 2

2017-18

Topic 6: Exponential and Logarithmic Functions

In this unit of exponential and logarithmic functions, students will begin with the characteristics of exponential functions and their graphs, as well as use formulas for compound interest. They will continue with the study logarithmic functions as the inverse function of exponential functions. At that point, students will learn the properties of logarithms to expand and condense logarithmic expressions, and then expand that understanding to exponential and logarithmic equations. Finally, students will apply their skills to modeling data with real-world examples of exponential growth and decay.

Common Misconceptions and Errors:

- **Logarithmic Properties:**
  - Often students will write $\log x - \log y = \frac{\log x}{\log y}$ instead of the correct expression $\log \frac{x}{y}$.
  - Students will also linearize rules and produce such logs as: $\ln(a + b) = \ln a + \ln b$, and $\ln(2x) = 2 \ln x$.
- **Solving Logarithmic Equations:** When solving a logarithmic equation, students forget to check if the answer is in the domain, or if they get two answers and the first one checks, they tend to automatically eliminate the second choice.
  - **Example:** Solve: $\log_2(x - 4) = 3 - \log_2(x + 3)$

    Solution:
    $$\log_2(x - 4) + \log_2(x + 3) = 3$$
    $$\log_2((x - 4)(x + 3)) = 3$$
    $$(x - 4)(x + 3) = 2^3 = 8$$
    $$x^2 - x - 12 = 8$$
    $$x^2 - x - 20 = 0$$
    $$(x - 5)(x + 4) = 0$$
    $$x = 5, -4$$

    The solution $x = 5$ is valid, but the solution $x = -4$ is not. Students often do not check this. This serves as the type of misconception or misbelief that if an algorithm is followed correctly, only correct answers will result.

  - **Example:** Solve: $\log(x^2 - 7) = \log(x - 5)$

    Solution:
    $$\log(x^2 - 7) - \log(x - 5) = 0$$
    $$\log \frac{(x^2-7)}{(x-5)} = 0$$
    $$\frac{(x^2-7)}{(x-5)} = 1$$
    $$(x^2 - 7) = (x - 5)$$
    $$x^2 - x - 2 = 0$$
    $$x = 2, -1$$

    Note: When the solutions are substituted into the original equation, both the left and right side are undefined.

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Transfer Goals

1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.
2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience.
3) Construct viable arguments and critique the reasoning of others using precise mathematical language.

Timeframe: 3 weeks/15 days
Start Date: January 22, 2018
Assessment Dates: Feb. 9, 2018

Standards

F-IF 4 - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-BF 1b - Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

A-SSE 1b- Interpret parts of an expression, such as terms, factors, and coefficients.

A-REI 11- Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of the values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

F-IF 7e - Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, mid-line, and amplitude.

F-IF 8 - Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF 9 - Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-LE 4 - For exponential models, express as a logarithm the solution to ab^{ct}= d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

F-LE 4.1 - Prove simple laws of logarithms.

F-LE 4.2 - Use the definition of logarithms to translate between logarithms in any base.

F-LE 4.3 - Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.

A-CED 2 - Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-BF 4 – Find inverse functions.

Meaning-Making

Understandings

Students will understand that…

- You can represent repeated multiplication with a function in the form of y = ab^x where b is a positive number other than 1.
- The exponential function y = b^x is one-to-one, so it's inverse x = b^y is a function. To express "y as a function of x" for the inverse, write y = $\log_b x$.
- Logarithms and exponents have corresponding properties.
- You can use logarithms to solve exponential equations. You can use exponents to solve logarithmic equations.
- The function y = e^x and y = ln x are inverse functions. Just as before, this means that if a = e^b, then b = ln a, and vice versa.

Essential Questions

Students will keep considering…

- How can you determine the growth rate or decay rate for exponential functions?
- In the equation $y = ab^{x-h} + k$, what are the roles of a, h, and k? Consider both positive and negative values.
- How are the graphs and properties of exponential functions and logarithmic functions related?
- How can you model exponential growth and decay?

Acquisition

Knowledge

Students will know…

Vocabulary: asymptote, natural base exponential function, continuously compounded interest, Change of Base Formula, common logarithm, exponential equation, exponential function, exponential growth, logarithm, logarithmic equation, logarithmic function, natural logarithmic function, logarithmic scale

Concepts:

- General form of an exponential function, $y = ab^x$ where $a \neq 0$, with $b > 0$, and $b \neq 1$.
- Exponential Growth and Decay functio, $A(t) = a(1 + r)^t$.
- Formula for continuously compounded interest $A = Pe^{rt}$.
- Transformations of Exponential Functions, $y = ab^{x-h} + k$.
- Families of Logarithmic Functions with transformations, $y = a\ log_b(x - h) + k$.
- Exponents and logarithms are inverse functions.
- Change of Base Formula
- Properties of Logarithms
- Methods for solving exponential equations with common and different bases.
- The natural logarithmic function is the inverse of $x = \ln y$, so you can write it as $y = ln\ x$.

Skills

Students will be skilled at and able to do the following…

- Graph exponential and logarithmic functions and their transformations.
- Determine whether a function represents exponential growth or decay.
- Write an exponential function to model exponential growth or decay.
- Find the amount in a continuously compounded account for given conditions.
- Use exponents to solve logarithmic equations and logarithms to solve exponential equations.
- Show that exponents and logarithms are inverse functions.
- Evaluate expressions to determine the values of logarithms and exponents.
- Simplify or expand logarithms.
- Use the Change of Base Formula and properties of logarithms to evaluate expressions
- Extend using linear models to find exponential and logarithmic and functions.
- Solve exponential equations and natural logarithmic equations.

Topic 6: Exponential and Logarithmic Functions

Transfer is a student's ability to independently apply understanding in a novel or unfamiliar situation. In mathematics, this requires that students use reasoning and strategy, not merely plug in numbers in a familiar-looking exercise, via a memorized algorithm.

Transfer goals highlight the effective uses of understanding, knowledge, and skills we seek in the long run – that is, what we want students to be able to do when they confront new challenges, both in and outside school, beyond the current lessons and unit. These goals were developed so all students can apply their learning to mathematical or real-world problems while simultaneously engaging in the Standards for Mathematical Practices. In the mathematics classroom, assessment opportunities should reflect student progress towards meeting the transfer goals.

With this in mind, the revised **PUSD transfer goals** are:

1) **Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.**
2) **Effectively communicate orally, in writing, and by using models (e.g., concrete, representational, abstract) for a given purpose and audience.**
3) **Construct viable arguments and critique the reasoning of others using precise mathematical language.**

Multiple measures will be used to evaluate student acquisition, meaning-making and transfer. Formative and summative assessments play an important role in determining the extent to which students achieve the desired results in stage one.

Formative Assessment	Summative Assessment
Aligning Assessment to Stage One	
• What constitutes evidence of understanding for this lesson? • Through what other evidence during the lesson (e.g. response to questions, observations, journals, etc.) will students demonstrate achievement of the desired results? • How will students reflect upon, self-assess, and set goals for their future learning?	• What evidence must be collected and assessed, given the desired results defined in stage one? • What is evidence of understanding (as opposed to recall)? • Through what task(s) will students demonstrate the desired understandings?
Opportunities	
• Discussions and student presentations • Checking for understanding (using response boards) • Ticket out the door, Cornell note summary, and error analysis • *Performance Tasks* within a Unit • Teacher-created assessments/quizzes	• Unit assessments • Teacher-created quizzes and/or mid-unit assessments • *Illustrative Mathematics* tasks (https://www.illustrativemathematics.org/) • Performance tasks

Topic 6: Exponential and Logarithmic Functions

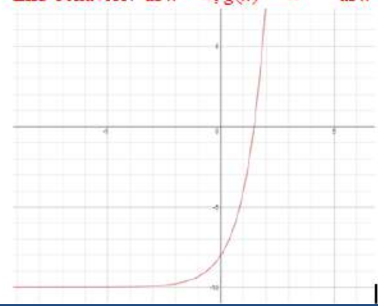
The following pages address how a given skill may be assessed. Assessment guidelines, examples and possible question types have been provided to assist teachers in developing formative and summative assessments that reflect the rigor of the standards. *These exact examples cannot be used for instruction or assessment, but can be modified by teachers.*

Unit Skills	SBAC Targets (DOK)	Selected Standards	Examples
• Graph exponential and logarithmic functions and their transformations. • Determine whether a function represents exponential growth or decay. • Write an exponential function to model exponential growth or decay. • Find the amount in a continuously compounded account for given conditions. • Use exponents to solve logarithmic equations and logarithms to solve exponential equations.	Create equations that describe numbers or relationships. (1,2) Represent and solve equations graphically. (1,2) Interpret functions that arise in applications in terms of a context. (1,2) Analyze functions using different representations. (1,2) Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace. (2,3)	**A-CED 2** - Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **F-IF 8** - Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **F-IF 4** - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **F-BF 1b** - Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. **A-REI 11**- Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of the values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	During a 1-year period, a population of tropical insects grew according to the model $P = P_0(1.46)^t$, where P is the population, P_0 is the initial population, and t is time in years. Which equation can be used to model the approximate weekly growth rate? (Assume 52 weeks in a year.) Ⓐ $P = P_0(1.0073)^{52t}$ Ⓑ $P = P_0(1.0088)^{52t}$ Ⓒ $P = P_0(1.0281)^{52t}$ Ⓓ $P = P_0(1.0371)^{52t}$ The population of country A was 40 million in the year 2000 and has grown continually in the years following. The population P, in millions, of the country t years after 2000 can be modeled by the function $P(t) = 40e^{0.027t}$, where $t \geq 0$. Based on the model, the solution to the equation $50 = 40e^{0.027t}$ gives the number of years it will take for the population of country A to reach 50 million. What is the solution to the equation expressed as a logarithm? Ⓐ $0.027\ln(1.25)$ Ⓑ $\frac{\ln(1.25)}{0.027}$ Ⓒ $\ln\left(\frac{1.25}{0.027}\right)$ Ⓓ $\ln\left(\frac{0.027}{1.25}\right)$

Topic 6: Exponential and Logarithmic Functions

Unit Skills	SBAC Targets (DOK)	Selected Standards	Examples (continued)
• Graph exponential and logarithmic functions and their transformations. • Determine whether a function represents exponential growth or decay. • Write an exponential function to model exponential growth or decay. • Find the amount in a continuously compounded account for given conditions. • Use exponents to solve logarithmic equations and logarithms to solve exponential equations.	Create equations that describe numbers or relationships. (1,2) Represent and solve equations graphically. (1,2) Interpret functions that arise in applications in terms of a context. (1,2) Analyze functions using different representations. (1,2) Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace. (2,3)	**A-CED 2** - Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **F-IF 8** - Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **F-IF 4** - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **F-BF 1b** - Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. **A-REI 11**- Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of the values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	The population of country A was 40 million in the year 2000 and has grown continually in the years following. The population P, in millions, of the country t years after 2000 can be modeled by the function $P(t) = 40e^{0.027t}$, where $t \geq 0$. **25. Part A** Based on the model, what was the average rate of change, in millions of people per year, of the population of country A from 2000 to 2005? Give your answer to the nearest hundredth. **Part C** Based on the model, in which years will the population of country A be greater than 55 million? Select **all** that apply. Ⓐ 2004 Ⓑ 2007 Ⓒ 2010 Ⓓ 2013 Ⓔ 2016 Ⓕ 2019 **Part D** For another country, countr[illegible] 2000 can be modeled by the function $M(t) = 35e^{-0.042t}$, where $t \geq 0$. Based on the models, what year will be the first year in which the population of country B will be greater than the population of country A? Ⓐ 2009 Ⓑ 2012 Ⓒ 2021 Ⓓ The population of country B will not exceed the population of country A. **Part A** A bank offers a savings account that accrues simple interest annually based on an initial deposit of $500. If $S(t)$ represents the money in the account at the end of t years and $S(5) = 575$, write a function that could be used to determine the amount of money in the account over time. Show your work or explain your reasoning. Enter your equation and your reasoning in the space provided. **Part B** Another bank offers a savings account that accrues compound interest annually at a rate of 3%. What is the initial amount needed in this account so that it will have the same amount of money at the end of 10 years as the account in Part A at the end of 10 years? Show your work or explain your reasoning. Enter your answer and your reasoning in the space provided.

Topic 6: Exponential and Logarithmic Functions

Unit Skills	SBAC Targets (DOK)	Selected Standards	Examples
• Show that exponents and logarithms are inverse functions. • Evaluate expressions to determine the values of logarithms and exponents. • Simplify or expand logarithms. • Use the Change of Base Formula and properties of logarithms to evaluate expressions. • Extend using linear models to find exponential and logarithmic and functions. • Solve exponential equations and natural logarithmic equations.	Create equations that describe numbers or relationships. (1,2) Represent and solve equations graphically. (1,2) Interpret functions that arise in applications in terms of a context. (1,2) Analyze functions using different representations. (1,2) Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace. (2,3)	**F-BF 4** – Find inverse functions. **F-LE 4** - For exponential models, express as a logarithm the solution to ab^{ct} = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. **F-LE 4.1** - Prove simple laws of logarithms. **F-LE 4.2** - Use the definition of logarithms to translate between logarithms in any base. **F-IF 7e** - Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, mid-line, and amplitude. **A-SSE 1b**- Interpret parts of an expression, such as terms, factors, and coefficients.	**F.IF.C.7e Item 1** Given the function $g(x) = 2(3)^x - 10$, identify any asymptotes, x- and y-intercepts, and identify the end behavior. Use the information to graph the function. Answer: x-intercept $x = 1.465$, y-intercept $y = -8$, asymptote $y = -10$ End behavior: as $x \to \infty$, $g(x) \to \infty$ as $x \to -\infty$, $g(x) \to -10$  Solve the equation $27^x = 9^{x-3}$ for x. Enter your answer in the box. **F.IF.C.9 Item 1** (with connections to standard F.IF.B.5) Select all that are true about the functions $f(x) = \log_3(x)$ and $g(x) = 3^x$. A. $f(x)$ and $g(x)$ are inverses. B. $f(x)$ and $g(x)$ have the same asymptote. C. $f(x)$ passes through the point (1,0) and $g(x)$ passes through the point (0,1). D. $f(x)$ and $g(x)$ have the same domain and range. E. The domain of $f(x)$ is the range of $g(x)$. Answer: A, C, E

Topic 6: Exponential and Logarithmic Functions

Transfer Goals	
1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution. 2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience. 3) Construct viable arguments and critique the reasoning of others using precise mathematical language.	
Essential Questions: • How can you determine the growth rate or decay rate for exponential functions? • In the equation $y = ab^{x-h} + k$, what are the roles of a, h, and k? Consider both positive and negative values. • How are the graphs and properties of exponential functions and logarithmic functions related? • How can you model exponential growth and decay?	**Standards:** F-IF 4, F-IF 7e, F-IF 8, F-IF 9, F-LE 4, F-LE 4.1, F-LE 4.2, F-LE 4.3, A-CED 2, F-BF 1b, F-BF 4, A-SSE 1b, A-REI 11 **Timeframe:** 3 weeks/15 days **Start Date:** January 22, 2018 **Assessment Dates:** February 9, 2018

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Resources
1 day	**Opening Activity:** Introduction to the Common Core Performance Task p. 433					
1 day	**Lesson 7.1: Exploring Exponential Models** SMP: 1,2,3,4,5,6 (pp. 434-441) **F-IF 7e**, **F-IF 8**, **A-CED 2** *Prep for Performance Task* *(Apply What You Have Learned) p. 441 (Lesson 7.1)*	**Focus Question(s):** • How can you determine the growth rate or decay rate for an exponential function given two consecutive y-values? **Inquiry Question(s):** Pg. 440 #30	• Repeated multiplication can be represented with a function in the form of $y = ab^x$ where *b* is a positive number other than 1. • An exponential function is a function with the general form $y = ab^x$, a≠0, with b>0, and b≠1. In an exponential function, the base *b* is a constant. The exponent *x* is the independent variable with domain the set of real numbers.	**Vocabulary:** exponential function, exponential growth, exponential decay, asymptote, growth factor, decay factor • General form of an exponential function, $y = ab^x$, where a≠0, with b>0, and b≠1. • Exponential Growth and Decay function, $A(t) = a(1 + r)^t$	• Graph an exponential function • Determine whether a function represents exponential growth or decay, and find the y-intercept • **Write an exponential function to model exponential growth or decay**	**Thinking Map:** Circle Map to record how to determine growth or decay. **Common Core Problems:** #7,8,9,26,29,30, 32,33,42,50 **STEM:** #31, 43

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
2 days	**Lesson 7.2: Properties of Exponential Functions** SMP: 1,2,3,4,5, 7 (pp. 443-450) **F-IF 7e**, **F-IF 8**, **A-CED 2**	**Focus Question(s):** • In the equation $y = ab^{(x-h)} + k$, what are the roles of *a, h,* and *k*? Consider both positive and negative values. **Inquiry Question(s):** Pg. 448 #33	• The factor *a* in $y = ab^x$ can stretch or compress, and possibly reflect the graph of the parent function $y = b^x$. • The function $y = ab^x$, *a>0, b>1,* models exponential growth. $y = ab^x$ models exponential decay if *0<b<1.*	**Vocabulary:** natural base exponential function, continuously compounded interest • Parent Function, $y = ab^x$ • Transformations of Exponential Functions, $(y = ab^{(x-h)} + k\ )$ • Formula for continuously compounded interest, $A(t) = P \cdot e^{rt}$	• Graph exponential functions that have base *e*. • Graph a function as a transformation of its parent function. • **Find the amount in a continuously compounded account for given conditions.**	**Thinking Map:** Tree Map for the Properties of Exponential Functions. **Common Core Problems:** #5,6,22,31,33, 40, 42, **STEM:** #36, 37, 41, 43
1 day	**Lesson 7.3: Logarithmic Functions as Inverses** SMP: 1,2,3,4,5 (pp. 451-458) **F-BF 4**, **F-IF 7e**, **F-IF 8**, **F-IF 9**, **A-CED 2**, **A-SSE 1b**	**Focus Question(s):** • How can you use the properties of exponents to evaluate a logarithm? • How can you use the graph of an exponential function to graph its inverse? **Inquiry Question(s):** Pg. 451 Solve It! Pg. 457 #60 Pg. 456 #10	• The exponential function $y = b^x$ is one-to-one, so its inverse $x = b^y$ is a function. To express "*y* as a function of x" for the inverse, write $y = log_b x$. • An exponential function is a function with the general form $y = ab^x$, a≠0, with b>0, and b≠1. In an exponential function, the base *b* is a constant. The exponent *x* is the independent variable with domain the set of real numbers. • Logarithms are exponents. In fact, $log_b a = c$ if and only if $b^c = a$.	**Vocabulary:** logarithm, logarithmic function, common logarithm, logarithmic scale • For b>0, b≠1, $log_b a = c$ if and only if $b^c = a$. • Families of Logarithmic Functions with transformations, $y = alog_b(x-h) + k$	• Write exponential equations in logarithmic form • Write and evaluate logarithms • Use a logarithmic scale • Graph logarithmic functions • Find the inverse of a logarithmic function • Find the domain and range of a logarithmic function • Translate $y = log_b x$	Be sure to cover Example 3 on page 453 and reconnect to 7.5 later. **Thinking Map:** Flow Map to sequence graphing the inverse. **Common Core Problems:** #9,10,11,44,60, 86 **STEM:** #32-35, 45

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
2 days	**Lesson 7.4: Properties of Logarithms** SMP: 1,2,3 (pp. 462-468) **F-LE 4**	**Focus Question(s):** • How can you derive the Power Property of Logarithms from the Power Property of Exponents? • How can you use the inverse relationship between exponential and logarithmic functions to understand the properties of logs? **Inquiry Question(s):** Pg. 466 #47 or 48	• Logarithms and exponents have corresponding properties. • An exponential function is a function with the general form $y = ab^x$, a≠0, with b>0, and b≠1. In an exponential function, the base *b* is a constant. The exponent *x* is the independent variable with domain the set of real numbers.	**Vocabulary:** Change of Base Formula • Properties of Logarithms (product, quotient, power) • Change of Base Formula $log_b m = \frac{log_c m}{log_c b}$, for any positive numbers *m, b,* and *c,* with *b≠1* and *c≠1*.	• Simplify logarithms • Expand logarithms • Use the Change of Base Formula • Use a Logarithmic Scale • Use the properties of logarithms to evaluate expressions	**Thinking Map:** Tree Map for the Properties of Logarithms. **Common Core Problems:** #6,7,8,45,47,48, 49, 82,83 **STEM:** #38, 46, 73-74
2 days	**Lesson 7.5: Exponential and Logarithmic Equations** SMP: 1,3,4,5,7 (pp. 469-476) **F-LE 4**, **A-REI 11** *Prep for Performance Task (Apply What You Have Learned) p. 476 (Lesson 7.5)*	**Focus Question(s):** • How is the relationship between exponents and logarithms used to solve problems? • What methods can be used to solve an exponential equation? • What methods can be used to solve a logarithmic equation? **Inquiry Question(s):** Pg. 469 Solve It!	• Logarithms can be used to solve exponential equations. Exponents can be used to solve logarithmic equations. • An exponential function is a function with the general form $y = ab^x$, a≠0, with b>0, and b≠1. In an exponential function, the base *b* is a constant. The exponent *x* is the independent variable with domain the set of real numbers. • The exponential function $y = b^x$ and the logarithmic function $y = log_b x$ are inverse functions.	**Vocabulary:** exponential equation, logarithmic equation • Methods for solving exponential equations with common and different bases	• Solve an exponential equation with common and different bases. • Solve an exponential equation with a graph or table. • Use properties of exponential and logarithmic functions to solve equations and systems.	Be sure to cover Example 3 on page 453 and reconnect to lesson 7.3. **Thinking Map:** Flow Map to sequence solving procedures. **Common Core Problems:** #5, 6, 46, 47, 49 ,58 **STEM:** #48, 62-63, 82, 83

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
2 days	**Lesson 7.6: Natural Logarithms** SMP: 1,3,4,5 (pp. 462-468) **F-LE 4**	**Focus Question(s):** • How can you use the relationship between $y = e^x$ and $y = ln\ x$ to solve exponential and logarithmic equations? **Inquiry Question(s):** Pg. 481 #40	• The function $y = e^x$ and $y = ln\ x$ are inverse functions. Just as before, this means that if $a = e^b$, then $b = ln\ a$, and vice versa. • The exponential function $y = b^x$ and the logarithmic function $y = log_b x$ are inverse functions.	**Vocabulary:** natural logarithmic function • If $y = e^x$, then $x = log_e y$ $= ln\ y$. The natural logarithmic function is the inverse of $x = ln\ y$, so you can write it as $y = ln\ x$.	• Simplify a natural logarithmic expression. • Solve a natural logarithmic equation. • Solve an exponential equation.	**Thinking Map:** Double-bubble to compare and contrast natural logs to regular logs. **Common Core Problems:** #9, 10, 40, 52, 53, 63 **STEM:** #38-39, 41, 57, 58-59, 64, 65
1 day	**Topic 6 Performance Task** Textbook p. 486 *Pull it all Together* Have students work collaboratively to reflect on *Completing the Performance Task* and *On Your Own*					
2 days	**Review Topic 6 Concepts & Skills** Use Textbook Resources and/or Teacher Created Items					
1 day	**Topic 6 Assessment** (Created and provided by PUSD)					

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