Mathematics Curriculum Guide

High School Algebra 1

2017-18

Topic 10: Quadratic Functions & Equations

In this unit, students will explore concepts related to the quadratic formula. To begin the unit, students will learn to model a real-world problem using factoring. They will solve problems by analyzing a rectangular figure, writing an equation involving the area and the figure, rewriting the equation in standard quadratic form, and then solving by factoring to get the figure's dimensions. Then students will analyze the graph of a quadratic function of the form $y = ax^2 + bx + c$. Students will use their knowledge of the symmetry of a parabola to determine the vertex. They will construct viable arguments and use algebraic reasoning to answer questions that arise while graphing the function. Finally, students will learn why the quadratic formula works, based on completing the square. They will use the quadratic formula to solve quadratic equations and use the discriminant to find the number of solutions of a quadratic equation.

Common Misconceptions and/or Errors:

- **Factoring:** When factoring quadratic expressions, students might not figure the factors correctly. For the expression $x^2 + 10x - 24$, students might choose 6 and 4 as factors of -24. These two numbers have a sum of 10, but they do not have a product of -24. $(x + 6)(x + 4) = x^2 + 10x + 24$ Or, students might choose 8 and -3 as factors of -24. These two numbers have a product of -24, but they do not have a sum of 10. $(x + 8)(x - 3) = x^2 + 5x - 24$
- **Completing the Square:** When completing the square to solve a quadratic equation, students might forget to add the value that completes the square to both sides. To complete the square on the right side of $x^2 + 10x = 24$, students must remember to add 25 to both sides to get the perfect square trinomial. $x^2 + 10x + 25 = 24 + 25$

  Quadratic Formula: Students may forget or confuse parts of the Quadratic Formula. For example, they may compute using b in place of $-b$ or divide by a instead of $2a$. They may also make a mistake when subtracting $4ac$ as part of the radical if either a or c is negative. To counter these tendencies, students should write out the formula with the substituted values, checking each sign, and simplify step by step without attempting to do any two steps at once.

Topic 10: Quadratic Functions & Equations

Transfer Goals

1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.
2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience.
3) Construct viable arguments and critique the reasoning of others using precise mathematical language.

Timeframe: 4 weeks/19 days
Start Date: May 2, 2018
Assessment Dates: May 26 & 30, 2018

Standards

A-CED.A.1 Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. (Quadratic functions).

A-CED A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

A-CED A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

F-IF. B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF. B.5 Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes.

Meaning-Making

Understandings

Students will understand that…

- The family of quadratic functions models certain situations where the rate of change is not constant.
- Quadratic functions are graphed by a symmetric curve with a highest or lowest point (called the vertex) corresponding to a maximum and or minimum value. The graph of a quadratic function is a U-shaped curve called a parabola. If > 0 , then the parabola opens upward and if $a < 0$, then the parabola opens downward.
- In the standard form of the quadratic function$y = ax^2 + bx + c$, where $a \neq 0$, the value of b translates the position of the axis of symmetry. The axis of symmetry intersects the vertex and cuts the parabola into two matching halves. The axis of symmetry for the graph of the quadratic function $y = ax^2 + bx + c$ is $x = \frac{-b}{2a}$. The x-coordinate of the vertex of the graph is $\frac{-b}{2a}$.
- Quadratic equations can be solved by a variety of methods, including graphing, finding the square root, and the quadratic formula. Some quadratic equations can be solved by using the Zero-Product Property. Sometimes it is useful to write a quadratic equation in standard form before solving.
- Any quadratic equation can be solved by first writing it in the form $m^2 = n$.
- In many cases the negative solution of a quadratic equation will not be a reasonable solution to the original problem. The discriminant of a quadratic equation can be used to determine the number of solutions an equation has.
- Linear, quadratic, or exponential functions can be used to model various sets of data.
- Graphing and testing data can show which type of function best models the data.

Essential Questions

Students will keep considering…

- How is a quadratic function different than a linear function or an exponential function?
- How does the structure of the quadratic function affect its graph?
- What are the ways to solve quadratic equations, and how do you choose which function might be more efficient?
- When applying quadratic equations to real-life situations how do you determine which solutions are reasonable?
- How are completing the square and the quadratic formula connected?
- What can the discriminant tell us about the number of solutions to a quadratic equation and how is this information useful?

Acquisition

Knowledge

Students will know…

Vocabulary: Vertex, Axis of symmetry, maximum, minimum, quadratic equation, standard form of quadratic equation, discriminate, quadratic formula, roots, parent function, vertical motion, roots, zeroes, zero product property

Concepts/Procedures for:

- That the y-intercept of $y = ax^2 + bx + c$ is $(0, c)$.
- The vertical motion model: $h = -16t^2 + c$ and $h = -16t^2 + vt + c$.
- A quadratic equation can have two, one, or no real-number solutions.
- X-intercepts of a function, roots of the equation and zeros of the function are other terms used for the number of solutions of a quadratic function.
- The formula for the discriminant. If $b^2 - 4ac > 0$, then there are two real -number solutions. If $b^2 - 4ac = 0$, then there is one real-number solution. If $b^2 - 4ac < 0$, then there are no real-number solutions.

Skills

Students will be skilled at and able to do the following…

- Graph a quadratic function using the coordinates of the vertex, a table of values, and axis of symmetry.
- Identify the zeros of a quadratic function given its graph.
- Identify and apply the most efficient method to solve a quadratic equation: square roots, factoring, the quadratic formula, or completing the square and explain their choice of method.
- Explain the steps in deriving the quadratic formula by completing the square, verbally and in writing.
- Solve quadratic equations using the quadratic formula.
- Determine whether data can be modeled using a linear, exponential, or quadratic function.

Topic 10: Quadratic Functions & Equations

Transfer is a student's ability to independently apply understanding in a novel or unfamiliar situation. In mathematics, this requires that students use reasoning and strategy, not merely plug in numbers in a familiar-looking exercise, via a memorized algorithm.

Transfer goals highlight the effective uses of understanding, knowledge, and skills we seek in the long run – that is, what we want students to be able to do when they confront new challenges, both in and outside school, beyond the current lessons and unit. These goals were developed so all students can apply their learning to mathematical or real-world problems while simultaneously engaging in the Standards for Mathematical Practices. In the mathematics classroom, assessment opportunities should reflect student progress towards meeting the transfer goals.

With this in mind, the revised **PUSD transfer goals** are:

1) **Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.**
2) **Effectively communicate orally, in writing, and by using models (e.g., concrete, representational, abstract) for a given purpose and audience.**
3) **Construct viable arguments and critique the reasoning of others using precise mathematical language.**

Multiple measures will be used to evaluate student acquisition, meaning-making and transfer. Formative and summative assessments play an important role in determining the extent to which students achieve the desired results in stage one.

Formative Assessment	**Summative Assessment**
Aligning Assessment to Stage One	
• What constitutes evidence of understanding for this lesson? • Through what other evidence during the lesson (e.g. response to questions, observations, journals, etc.) will students demonstrate achievement of the desired results? • How will students reflect upon, self-assess, and set goals for their future learning?	• What evidence must be collected and assessed, given the desired results defined in stage one? • What is evidence of understanding (as opposed to recall)? • Through what task(s) will students demonstrate the desired understandings?
Opportunities	
• Discussions and student presentations • Checking for understanding (using response boards) • Ticket out the door, Cornell note summary, and error analysis • *Performance Tasks* within a Unit • Teacher-created assessments/quizzes	• Unit assessments • Teacher-created quizzes and/or mid-unit assessments • *Illustrative Mathematics* tasks (https://www.illustrativemathematics.org/) • Performance tasks

Topic 10: Quadratic Functions & Equations

The following pages address how a given skill may be assessed. Assessment guidelines, examples and possible question types have been provided to assist teachers in developing formative and summative assessments that reflect the rigor of the standards. *These exact examples cannot be used for instruction or assessment, but can be modified by teachers.*

Unit Skills	SBAC Targets (DOK)	Standards	Examples
• Graph a quadratic function using the coordinates of the vertex, a table of values, and axis of symmetry. • Identify the zeros of a quadratic function given its graph. • Identify and apply the most efficient method to solve a quadratic equation: square roots, factoring, the quadratic formula, or completing the square and explain their choice of method. • Explain the steps in deriving the quadratic formula by completing the square, verbally and in writing. • Solve quadratic equations using the quadratic formula. • Determine whether data can be modeled using a linear, exponential, or quadratic function.	Create equations that describe numbers or relationships. (1,2) Understand solving equations as a process of reasoning and explain the reasoning. (1,2) Solve equations and inequalities in one variable. (1,2) Interpret functions that arise in applications in terms of a context. (1,2) Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace. (2,3)	**A-CED.A.1** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. (Quadratic functions). **A-CED A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A-CED A.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. **A-CED A.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. **F-IF. B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **F-IF. B.5** Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes.	Use the information provided to answer Part A and Part B for question 32. Consider the function $f(x) = 2x^2 + 6x - 8$. **32. Part A** What is the vertex form of $f(x)$? Ⓐ $f(x) = 2(x - 3)^2 - 4$ Ⓑ $f(x) = 2(x + 3)^2 - 4$ Ⓒ $f(x) = 2(x - 1.5)^2 - 12.5$ Ⓓ $f(x) = 2(x + 1.5)^2 - 12.5$ **Part B** What is a factored form of $f(x)$? Ⓐ $f(x) = (2x + 1)(x - 8)$ Ⓑ $f(x) = (2x - 1)(x + 8)$ Ⓒ $f(x) = 2(x + 4)(x - 1)$ Ⓓ $f(x) = 2(x - 4)(x + 1)$ In the xy-coordinate plane, the graph of the equation $y = 3x^2 - 12x - 36$ has zeros at $x = a$ and $x = b$, where $a < b$. The graph has a minimum at $(c, -48)$. What are the values of a, b, and c? Ⓐ $a = 2,\ b = 4,\ c = 2$ Ⓑ $a = -2,\ b = 6,\ c = 2$ Ⓒ $a = -3,\ b = 3,\ c = 0$ Ⓓ $a = 3,\ b = 6,\ c = 2$

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Unit Skills	SBAC Targets (DOK)	Standards	Examples
• Graph a quadratic function using the coordinates of the vertex, a table of values, and axis of symmetry. • Identify the zeros of a quadratic function given its graph. • Identify and apply the most efficient method to solve a quadratic equation: square roots, factoring, the quadratic formula, or completing the square and explain their choice of method. • Explain the steps in deriving the quadratic formula by completing the square, verbally and in writing. • Solve quadratic equations using the quadratic formula. • Determine whether data can be modeled using a linear, exponential, or quadratic function.	Create equations that describe numbers or relationships. (1,2) Understand solving equations as a process of reasoning and explain the reasoning. (1,2) Solve equations and inequalities in one variable. (1,2) Interpret functions that arise in applications in terms of a context. (1,2) Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace. (2,3)	**A-CED.A.1** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. (Quadratic functions). **A-CED A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A-CED A.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. **A-CED A.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. **F-IF. B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **F-IF. B.5** Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes.	The expression $3x^2 - 33x - 180$ can be factored into the form $a(x + b)(x + c)$, where a, b, and c are constants, to reveal the zeros of the function defined by the expression. What are the zeros of the function defined by $3x^2 - 33x - 180$? Select **all** that apply. Ⓐ −15 Ⓑ −10 Ⓒ −6 Ⓓ −4 Ⓔ 4 Ⓕ 6 Ⓖ 10 Ⓗ 15 **4.** Which factorization can be used to reveal the zeros of the function $f(n) = -12n^2 - 11n + 15$? Ⓐ $f(n) = -n(12n + 11) + 15$ Ⓑ $f(n) = (-4n + 3)(3n + 5)$ Ⓒ $f(n) = -(4n + 3)(3n + 5)$ Ⓓ $f(n) = (4n + 3)(-3n + 5)$ **13.** Find the equation that is equivalent to the quadratic equation shown. $x^2 - 6x - 27 = 0$ Ⓐ $x(x - 3) = 27$ Ⓑ $(x - 6)^2 = 63$ Ⓒ $(x - 3)^2 = 36$ Ⓓ $(x - 3)^2 = 28$

Topic 10: Quadratic Functions & Equations

Transfer Goals
1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution. 2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience. 3) Construct viable arguments and critique the reasoning of others using precise mathematical language.

Essential Questions	Standards
Essential Questions: • How is a quadratic function different than a linear function or an exponential function? • How does the structure of the quadratic function affect its graph? • What are the ways to solve quadratic equations, and how do you choose which function might be more efficient? • When applying quadratic equations to real-life situations how do you determine which solutions are reasonable? • How are completing the square and the quadratic formula connected? • What can the discriminant tell us about the number of solutions to a quadratic equation and how is this information useful?	**Standards:** A-CED 1, A-CED 2, A-CED 3, A-CED 4, F-IF 4, F-IF 5 **Timeframe:** 4 weeks/19 days **Start Date:** May 2, 2018 **Assessment Dates:** May 25-26, 2018

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Resources
2 Days	**Lesson 9-4: Factoring to Solve Quadratic Equations** (pp. 568-572) SMP 1,2,3,4,7 **A-REI 4**, **A-SSE 3**, **A-CED 1**, **F-IF 8a**	**Focus Question(s):** • How can you use the zero product property to solve quadratic equations? • When can you use it? **Inquiry Question:** p. 568 Solve It!	• Some quadratic equations can be solved by using the Zero-Product Property. • Sometimes it is useful to write a quadratic equation in standard form before solving.	**Vocabulary**: quadratic equation, standard form of a quadratic equation, Zero-Product Property **Students will know…** • How to use the Zero-Product Property. • That the Zero-Product Property can be applied when a quadratic equation can be factored over the set of integers.	• Solve a quadratic function by factoring. • Identify the most efficient method to solve a quadratic equation: square roots, factoring, and explain their choice of method.	**Notes:** • Provide examples similar to problems 6, 28, 34, 36 pp. 570-571. **Thinking Maps:** Use a Flow Map to give the steps for using the Zero-Product Property to find the solutions of a quadratic equation. **CC Problems:** #5,6,7, 34, 35, 36, 37, 42 **STEM:** #4, 28

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
1 Day	**Lesson 10-2: Simplifying Radicals** (pp. 619-625) SMP 1,2,3,4,7 **A-REI 2**	**Focus Question(s):** • How do you simplify radicals? **Inquiry Question:** p. 619 Solve It!	• Radical expressions can be simplified using multiplication and division properties of square roots.	**Vocabulary:** radical expression, perfect squares **Students will know…** • That the Multiplication Property of Square roots can be used to simplify radicals by removing perfect square factors from the radicand.	• Simplify radicals by removing perfect-square factors from the radicand.	**Note:** • The purpose of this section is to have students review how to simplify radicals that they will apply when solving quadratic equations using the quadratic formula and using square roots. • See attachment for a sample lesson. **Thinking Maps:** Use a Flow Map to show the process for simplifying within radicals. **CC Problems:** #7, 8, 9, 48, 49, 54, 55, 57, 73 **STEM:** #34
1 Day	**Lesson 9-3: Solving Quadratic Equations Using Square Roots** (pp. 561-566) SMP 1,2,3,4,7 **A-REI 4b, A-APR 3, A-CED 1, A-CED 4**	**Focus Question(s):** • How do you solve a quadratic equation using square roots? • Can you always use it? Why or why not? • What example can you give of an equation where it will work and will not work? **Inquiry Question:** p. 561 Solve It!	• Quadratic equations can be solved by a variety of methods, including factoring and using square roots. • In many cases the negative solution of a quadratic equation will not be a reasonable solution to the original problem.	**Vocabulary:** quadratic equation, standard form of a quadratic equation, perfect squares, square roots **Students will know…** • That a quadratic equation of the form $x^2 = k$ or $m^2 = n$ can be solved by finding the square roots of each side. • How to recognize when a quadratic equation has no real solutions when using square roots to solve quadratic equations.	• Write quadratic equations in standard form to the form $x^2 = k$. • Solve quadratic equations by finding square roots.	**Notes:** • Provide examples similar to problems 1-19 and 37-39, 43-48, and 50-52 on pp. 563-565. • Teacher will need to include problems like $4x^2 - 32 = 0$. **CC Problems:** #5,6,7, 40, 41, 51, 52, 55, 57 **STEM:** #50

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
2 Days	**Lesson 9-5: Completing the Square** (pp. 576-581) SMP 1,2,3,4 **A-REI 4a, A-REI 1, A-REI 4b**	**Focus Question(s):** • What is the relationship between completing the square of a quadratic equation and solving a quadratic equation using square roots? **Inquiry Question:** p. 576 Solve It!	• Quadratic equations can be solved by a variety of methods, including factoring, using square roots, using the quadratic formula, and completing the square. • Any quadratic equation can be solved by first writing it in the form $x^2 = k$ or $m^2 = n$. • Any quadratic equation can be solved by completing the square.	**Vocabulary:** quadratic equation, perfect squares, square roots, completing the square, perfect-square trinomials **Students will know…** • How to transform a quadratic expression to a perfect-square trinomial. • The process of completing the square. • The relationship between completing the square of a quadratic equation and solving a quadratic equation using square roots.	• Solve a quadratic equation by completing the square. • Identify the most efficient method to solve a quadratic equation: square roots, factoring, or completing the square and explain their choice of method.	**Note:** • Provide examples similar to problems 1-2 on pg. 577 and problems 7-18 on pg. 579. **CC Problems:** #5, 6, 32, 33, 43, 44, 45, 49
1 Day	**Performance Task** Photo Frame					**NOTE:** • See attachment for this performance task.
2 Days	**Lesson 9-1: Quadratic Graphs and their Properties** (pp. 546-552) SMP 1,2,3,4,5,6 **F-IF 7a**, **F-IF 4**, **F-IF 5**, **F-BF 3**	**Focus Question(s):** • What are the characteristics of a quadratic function that are necessary to draw its graph? • How do the values of g and c affect the graph of a quadratic function? **Inquiry Question:** p. 546 Solve It!	• The family of quadratic functions models certain situations where the rate of change is not constant. • Quadratic functions are graphed by a symmetric curve with a highest or lowest point corresponding to a maximum and or minimum value.	**Vocabulary:** quadratic function, standard form of a quadratic function, quadratic parent function, parabola, axis of symmetry, vertex, maximum, minimum, opens up, opens down **Students will know…** • Standard form: $y = ax^2 + bx + c$ where $a \neq 0$. • That the y-intercept of $y = ax^2 + bx + c$ is $(0, c)$. • The graph of a quadratic function is a U-shaped curve called a parabola. If > 0, then the parabola opens upward and if $a < 0$, then the parabola opens downward. • The highest and lowest point of the parabola is its vertex. The axis of symmetry intersects the vertex and cuts the parabola into two matching halves.	• Graph a quadratic function using the coordinates of the vertex, a table of values, and axis of symmetry.	**Note:** • Provide examples similar to problems 5, 6, 28, 33 on pp. 549-550. • [Graphs of Quadratic Functions – Dominoes Activity](http://map.mathshell.org) (http://map.mathshell.org Lesson: Graphs of Quadratic Functions) **CC Problems:** #5,6, 28, 33, 38, 48 **STEM:** #46, 49

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
3 Days	**Lesson 9-2: Quadratic Functions** (pp. 553-558) Day 1: Lesson on graphing quadratic function Day 2: Lesson on graphing quadratic function/Review Day 3: Teacher Generated Quiz 2 SMP 1,2,3,4,7 **F-IF 7**, **A-CED 2**, **F-IF 4**, **F-IF 8**, **F-IF 9**	**Focus Question(s):** • How would you graph a quadratic function that is not factorable? • How do you determine the axis of symmetry and the vertex of a quadratic function? **Inquiry Question:** p. 553 Solve It!	• *In the quadratic function* $y = ax^2 + bx + c$ *the value of* b *translates the position of the axis of symmetry.* • *The axis of symmetry for the graph of the quadratic function* $y = ax^2 + bx + c$ *is* $x = \frac{-b}{2a}$. • *The x-coordinate of the vertex of the graph is* $x = \frac{-b}{2a}$.	**Vocabulary:** quadratic function, standard form of a quadratic function, quadratic parent function, parabola, axis of symmetry, vertex, maximum, minimum, opens up, opens down **Students will know…** • How to graph a quadratic function $y = ax^2 + bx + c$ that is not factorable. • How to find the axis of symmetry of quadratic functions. • How to find the vertex of quadratic functions.	• Graph a quadratic function using the coordinates of the vertex, a table of values, and axis of symmetry.	**NOTES:** • Provide examples similar to problem 1 on pg. 554 and problems 7-25 on pp. 556-557. • On review, include problems from sections 9.1 to 9.2 (part 1). • On teacher generated quiz 2, include problems from sections 9.1 to 9.2 (part 1). **CC Problems:** #5,6, 26, 27, 30, 31, 33, 34, 36 **STEM:** #46, 49
2 Days	**Vertical Motion** (p. 555 Problem 2) Day 1: Lesson on objects that are dropped Day 2: Lesson on objects that are thrown SMP 1,2,3,4,6,7 **A-REI 4**, **A-SSE 3**, **A-CED 1**, **F-IF 8**	**Focus Question(s):** • How would you use quadratic equations to model the height of falling objects? **Inquiry Question:** p. 555 Problem 2	• A quadratic equation can be used to model the path of a falling object.	**Vocabulary:** vertical motion **Students will know…** • An object that is dropped can be modeled using $h = -16t^2 + s$ or $h = -16t^2 + c$ • An object that is thrown can be modeled using $h = -16t^2 + vt + s$ or $h = -16t^2 + vt + c$.	• Use vertical motion model to represent objects that are dropped and objects that are thrown. • Solve vertical motion problems by factoring, using square roots, and using the quadratic formula.	**NOTES:** • On day 1 of lesson, include examples similar to problem 5 on pg. 549. • On day 2 of lesson, include examples similar to problem 2 on pg. 555. • Please see the following attached pages: Notes-Vertical Motion Model Day 1 and Notes-Vertical Motion Model Day 2
3 Days	**Unit 10 Review** (Use Textbook Resources and/or Teacher Created Items)					
2 Days	**Topic 10 Assessment** (Created and provided by PUSD)					

Algebra 1 **Unit 10: Simplifying Radical Worksheet** Name:

Lesson 10-2 Date:

Directions:

How do you simplify radicals?	1. $\sqrt{49}$	2. $\sqrt{81}$	3. $\sqrt{625}$
	4. $\sqrt{45}$	5. $\sqrt{72}$	6. $\sqrt{150}$
	7. $\sqrt{189}$	8. $\sqrt{300}$	9. $\sqrt{108}$
10. Compare and contrast: number 1 and number 4 from above.			
How do you simplify a radical with a coefficient?	11. $-3\sqrt{63}$	12. $7\sqrt{32}$	13. $-7\sqrt{28}$
	14. $-\sqrt{24}$	15. $9\sqrt{169}$	16. $4\sqrt{242}$

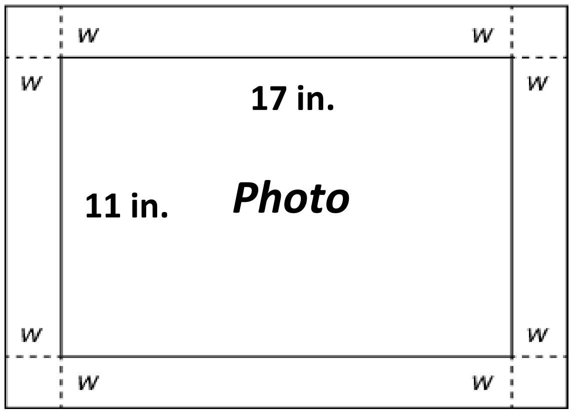
How do you simplify a radical when it is the numerator of a fraction?	17. $\frac{\sqrt{36}}{4}$	18. $\frac{\sqrt{75}}{3}$	19. $\frac{-2\sqrt{45}}{3}$

20. Jose and Alvin were having a disagreement. Jose stated that the expressions $\frac{3}{4}\sqrt{28}$ and $\frac{3\sqrt{28}}{4}$ are equivalent. Alvin disagreed. Who is right? Why? Explain your reasoning.

When an operation is inside the radical, how do the order of operations apply?	21. $\sqrt{36-25}$	22. $\sqrt{60-12}$	23. $\sqrt{100-25}$
	24. $\sqrt{45+5}$	25. $\sqrt{64+8}$	26. $\sqrt{49+3}$
	27. Ismael simplified # 21 below. What is his mistake? $\sqrt{36-25}$ $\sqrt{36}-\sqrt{25}$ (6) - (5) 1		

Practice SBAC Question Date: ______________________ Period: _______

Photography You are constructing a frame for a rectangular photo that measures 11 inches by 17 inches as shown in the diagram below. You want the frame to be the same width all the way around and the total area of the frame and photo to be 315 in^2. How wide should the frame be?



Part A Write an equation that can be used to determine the width, w, of the photo frame.

Part B Determine the width, in inches, of the photo frame.

CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Graphs of Quadratics (Day 2)

Mathematics Assessment Resource Service

University of Nottingham & UC Berkeley

Beta Version

For more details, visit: http://map.mathshell.org

Graphs of Quadratics (Day 2)

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to understand what the different algebraic forms of a quadratic function reveal about the properties of its graphical representation. In particular, the lesson will help you identify and help students who have the following difficulties:

- Understanding how the factored form of the function can identify a graph's roots.
- Understanding how to identify a graph's vertex (maximum or minimum point).
- Understanding how the standard form of the function can identify a graph's intercept.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

A-SSE: Write expressions in equivalent forms to solve problems.

F-IF: Analyze functions using different representations.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*:

1. Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

INTRODUCTION

The lesson is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work, and create questions for students to answer in order to improve their solutions.
- After a whole-class interactive introduction, students work in pairs on a collaborative discussion task in which they match quadratic graphs to their algebraic representation. As they do this, they begin to link different algebraic forms of a quadratic function to particular properties of its graph.
- After a whole-class discussion, students return to their original assessment tasks and try to improve their own responses.

MATERIALS REQUIRED

- Each individual student will need two copies of the assessment task *Quadratic Functions* and a mini-whiteboard, pen, and eraser, or graph paper.
- Each pair of students will need *Domino Cards 1* and *Domino Cards 2*, cut horizontally into ten 'dominoes'.

TIME NEEDED

15 minutes before the lesson, and 80-minute lesson (or two 40-minute lessons.) Timings are approximate and will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Quadratic Functions* (15 minutes)

Have the students do this task in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *Quadratic Functions*.

Briefly introduce the task and help the class to understand the problem and its context.

Read through the task and try to answer it as carefully as you can.

Show all your work so that I can understand your reasoning.

It is important that as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to be able to answer questions such as these confidently. This is their goal.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding, and their different problem solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write a list of your own questions, based on your students' work, using the ideas that follow. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the end of the lesson.

Common issues	Suggested questions and prompts
Q1. Student has difficulty getting started	• You are given two pieces of information. Which form of a quadratic equation can you match this information to?
Q2. Student makes incorrect assumptions about what the different forms of the equation reveal about the properties of its parabola	• What does an equation in standard form tell you about the graph? Explain. • What does an equation in factored form tell you about the graph? Explain.
Q2. Student uses an inefficient method For example: For each quadratic function, the student figures out the coordinates of several points by substituting x-values into the equation.	• Your method is quite difficult work. Think about the information each equation tells you about its graph. Think about the information each graph tells you about its equation.
Student makes a technical error For example: The student makes an error when manipulating an equation.	• Check your answer.
Student correctly answers all the questions **The student needs an extension task.**	• Q2. Can you think of any more coordinates for the key features of the Graphs 1, 2,3, and 4? Explain your answers. • Another quadratic has the same coordinates for the minimum, but the y-intercept is (0,14). What is the equation of this curve? [$y = 2x^2 - 12x + 14$]

SUGGESTED LESSON OUTLINE

If you have a short lesson, or you find the lesson is progressing at a slower pace than anticipated, then we suggest you end the lesson after the first collaborative activity and continue in a second lesson.

Whole-class interactive introduction: key features of quadratics (10 minutes)

Give each student either a mini-whiteboard, pen and eraser, or graph paper.

Introduce the lesson with:

Today, we are going to look at the key features of a quadratic function.

On your mini-whiteboards, draw the x*-and* y*-axis and sketch two parabolas that look quite different from each other.*

Allow students to work for a few minutes and then ask them to show you their whiteboards.

Be selective as to which student you ask to explain his or her graphs. Look for two sets of parabolas in particular:

- one of which has a maximum point, the other a minimum;
- one of which one has two roots, the other one or none;
- that are not parabolas.

What makes your two graphs different?

What are the common features of your graphs?

Elicit responses from the class and try to keep your own interventions to a minimum. Encourage students to use mathematical terms such as roots, y-intercepts, turning points, maximum, minimum.

As students suggest key features, write them as a list on the board under the heading 'Key Features of a Graph of a Quadratic'.

Ask about the vertex:

Is the vertex in your graph a maximum or minimum?

Can the parabola of a quadratic function have more than one vertex/no vertex?

If all students have drawn graphs with minimums, ask students to draw one with a maximum.

Ask about roots:

How many roots does each of your graphs have?
Where are these roots on your parabola?

Does anyone have a graph with a different number of roots?

How many roots can a quadratic have?

If all students have drawn graphs with two roots, ask a student to draw one with one or no roots.

Ask about y-intercepts:

Has anyone drawn a graph with different y*-intercepts?*

Do all quadratic curves have a y*-intercept?*

Can a quadratic have more than one y*-intercept?*

Write on the board these two equations of quadratic functions:

Standard Form:	Factored Form:
1. $y = x^2 - 10x + 24$	2. $y = (x - 4)(x - 6)$

Here are the equations of two quadratic functions.

Without performing any algebraic manipulations, write the coordinates of a key feature of each of their graphs.

For each equation, select a different key feature.

Explain to students they should use key features from the list on the board.

For example, students may answer:

Equation 1. The *y*-intercept is at the point (0,24). The graph has a minimum, because the coefficient of *x* is positive.

Equation 2. The graph has a minimum and has roots at (4,0) and (6,0).

What do the equations have in common? [They are different representations of the same function.]

Whole-class introduction to Dominos (10 minutes)

Organize the class into pairs. Give each pair of students cut-up ‘dominos’ A, E, and H from *Domino Cards 1* and *Domino Cards 2*.

Explain to the class that they are about to match graphs of quadratics with their equations, in the same way that two dominoes are matched. If students are unsure how to play dominos, spend a couple of minutes explaining the game.

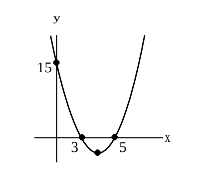
The graph on one ‘domino’ is linked to its equations, which is on another ‘domino’.

Place Card H on your desk. Figure out, which of the two remaining cards should be placed to the right of card H and which should be placed to its left.

A

$y = x^2 + 2x - 35$

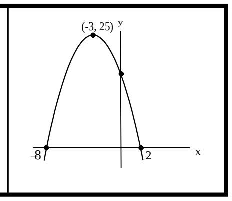
....



H

$y = x^2 - 8x + 15$

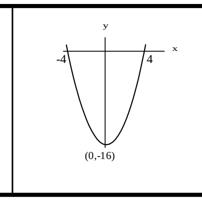
$y = (x-3)(x-5)$



E

$y = -x^2 - 6x + 16$

$y = -(x+8)(x-2)$



Encourage students to explain **why** each form of the equation matches the curve:

Dwaine, explain to me how you matched the cards.

Alex, please repeat Dwaine's explanation in your own words.

Which form of the function makes it easy to determine the coordinates of the roots/ y-intercept/ whether the parabola has a maximum or a minimum?

Are the two different forms of the function equivalent? How can you tell?

Check student understanding of coordinates on the parabola, using card H to answer the following:

What are the coordinates of the vertex of the parabola on Card H? Does this represent a minimum or a maximum? [(−3,25); maximum]

What are the coordinates of the y-intercept in the equation on Card H? How did you determine this? [(0,16)-From equation on Card E]

At this stage, students may find it helpful to write what each form of the function reveals about the key features of its graph.

If you think students need further work on understanding the relationship between a graph and its equations, then ask students to make up two different algebraic functions, the first in standard form and the second in factored form. Students are then to take these equations to a neighboring pair and ask them to explain to each other what each equation reveals about its graph.

Collaborative work: matching the dominos (15 minutes)

Give each pair of students all the remaining cut up *Domino Cards*.

Explain to students that the aim is to produce a closed loop of dominos, with the last graph connecting to the equations on 'domino' A. Students may find it easier to begin by laying the dominos out in a long column or row rather than in a loop.

You may want to use Slide P-1 of the projector resource to display the following instructions.

Take turns at matching pairs of dominos that you think belong together.

Each time you do this, explain your thinking clearly and carefully to your partner.

It is important that you both understand the matches. If you don't agree or understand ask your partner to explain their reasoning. You are both responsible for each other's learning.

On some cards an equation or part of an equation is missing. Do not worry about this, as you can carry out this task without this information.

You have two tasks during small-group work: to make a note of student approaches to the task, and support student problem solving.

Make a note of student approaches to the task

Notice how students make a start on the task, where they get stuck, and how they respond if they do come to a halt. You can use this information to focus a whole-class discussion towards the end of the lesson.

Support student problem solving

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. If several students in the class are struggling with the same issue, write a relevant question on the board. You might also ask a student who has performed well on a particular part of the task to help a struggling student.

The following questions and prompts may be helpful:

Which form of the function makes it easy to determine the coordinates of the roots /y-intercept/turning point of the parabola?

How many roots does this function have? How do you know? How are these shown on the graph?

Sharing work (5 minutes)

As students finish matching the cards, ask one student from each group to visit another group's desk.

If you are staying at your desk, be ready to explain the reasons for your group's matches.

If you are visiting another group, write your card matches on a piece of paper. Go to another group's desk and check to see which matches are different from your own.

If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.

When you return to your own desk, you need to consider as a pair whether to make any changes to your own work.

You may want to use Slide P-2 of the projector resource to display these instructions.

Collaborative work: completing the equations (15 minutes)

Now you have matched all the domino cards, I would like you to use the information on the graphs to fill in the missing equations and parts of equations.

You shouldn't need to do any algebraic manipulation!

Support the students as in the first collaborative activity.

For students who are struggling ask:

This equation is in standard form but the final number is missing. Looking at its graph, what is the value for y when x is zero? How can you use this to complete the standard form equation?

You need to add the factored form equation. Looking at its graph, what is the value for x when y is zero? How can you use this to complete the factored form equation?

Sharing work (5 minutes)

When students have completed the task, ask the student who has not already visited another pair to check their answers those of another pair of students. Students are to share their reasoning as they did earlier in the lesson unit.

Extension work

If a pair of students successfully completes the task then they could create their own dominos using the reverse side of the existing ones. To do this students will need to use algebraic manipulation to figure out all three forms of the function. Once students have written on all the dominos they should give them to another pair to match up. This is a demanding task so you may want to limit the number of dominos students use.

Whole-class discussion: overcoming misconceptions (10 minutes)

Organize a discussion about what has been learned. The intention is to focus on the relationships between the different representations of quadratic functions, not checking that everyone gets the right answers.

Ella, where did you place this card? How did you decide?

Ben, can you put that into your own words?

What are the missing equations for this graph? How did you work them out?

Did anyone use a different method?

Improving individual solutions to the assessment task (10 minutes)

Return to the students their original assessment *Quadratic Functions*, as well as a second blank copy of the task.

Look at your original responses and think about what you have learned this lesson.

Using what you have learned, try to improve your work.

If you have not added questions to individual pieces of work, then write your list of questions on the board. Students should select from this list only the questions they think are appropriate to their own work.

If you find you are running out of time, then you could set this task in the next lesson or for homework.

SOLUTIONS

Assessment task: Quadratic Functions

1. a. A matches 3, because it has two positive roots and a positive y-intercept.

   B matches 4, because it has one positive and one negative root.

   C matches 1, because it is the only function with no roots.

   D matches 2 because it is the only function with a maximum value.

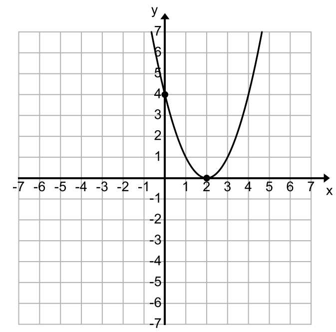
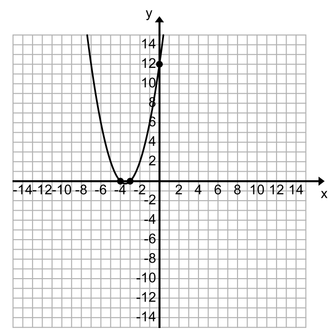
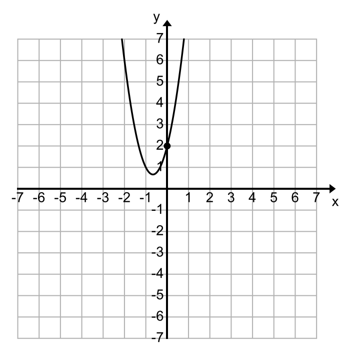
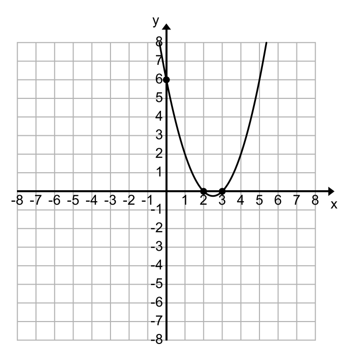
   b. P (6,8); Q (−8,0); R(4,0); S(0, −48).

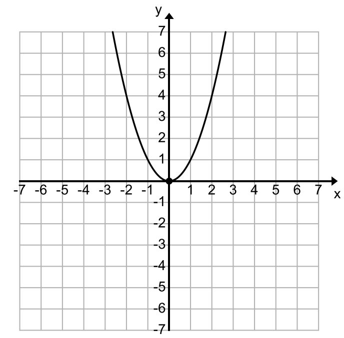
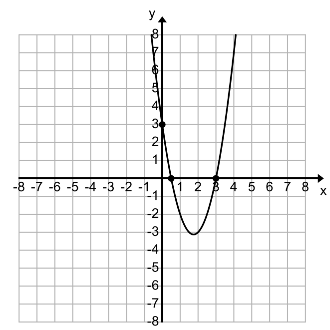
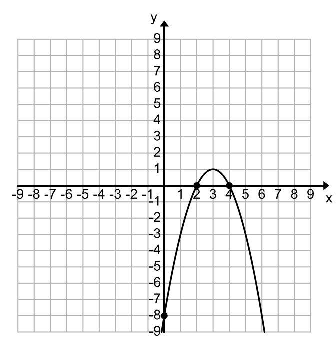
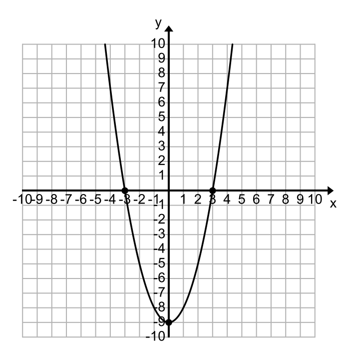
2. a. $y = (x - 3)^2 - 4$ or $y = x^2 - 6x + 5$

   b. $y = (x - 5)(x - 1)$. The function crosses the x-axis at (5,0) and (1,0).

Collaborative work: *Matching the dominos (Answer Key)*

Cards should be placed in this order:

A $y = x^2 - 3x - 10$ $y =$	1 
H $y = x^2 - 4x + 4$ $y =$	7 
E $y = x^2 + 7x + 12$ $y =$	5 
F $y = 3x^2 + 4x + 2$ No Roots	6 

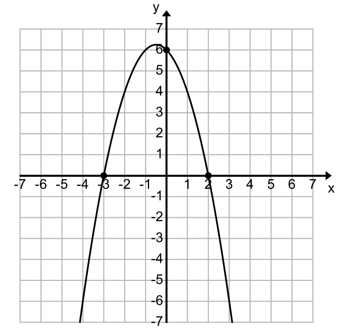
B $y = x^2 - 5x + 6$ $y =$	2 
G $y = x^2$ $y =$	7 
J $y = 2x^2 - 7x + 3$ $y =$	10 
C $y = -x^2 + 6x - 8$ $y =$	3 

D

$$y = x^2 - 9$$

$y =$ ....

4

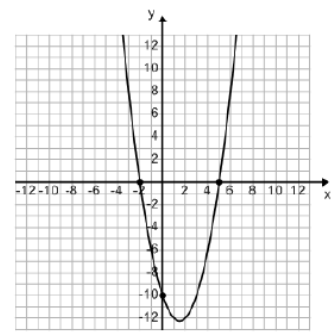


I

$$y = -x^2 - x + 6$$

$y =$ ....

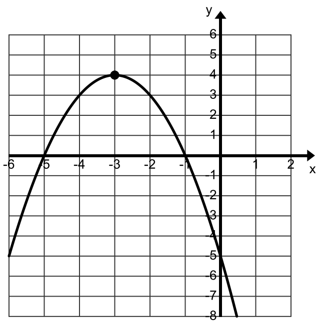
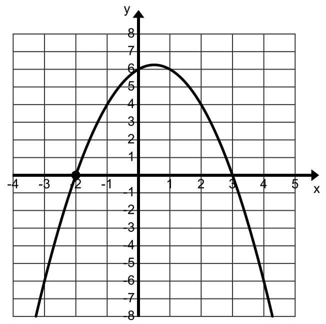
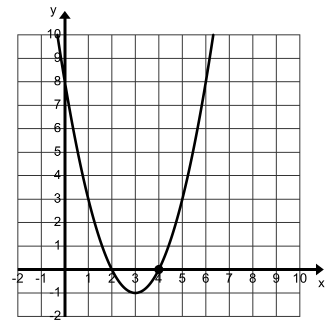
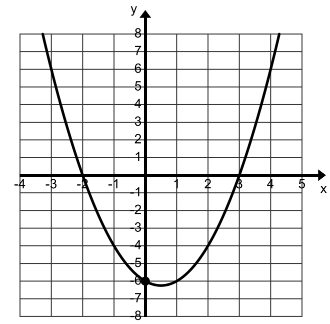
9



Quadratic Functions

The following are 4 equations of quadratic functions and 4 sketches of the graphs of quadratic functions.

A. $y = x^2 - 6x + 8$	B. $y = (x-3)(x+2)$	C. $y = -x^2 - 6x - 5$	D. $y = -(x+2)(x-3)$

1. 	2. 	3. 	4. 

1. Match the equation to its graph and explain your decision.

   Equation A ***matches*** Graph ____ because ____.

   Equation B ***matches*** Graph ____ because ____.

   Equation C ***matches*** Graph ____ because ____.

   Equation D ***matches*** Graph ____ because ____.

____

2. Write the coordinates of the points seen on each graph. Then tell whether each point represents a vertex, a root, or a y-intercept.

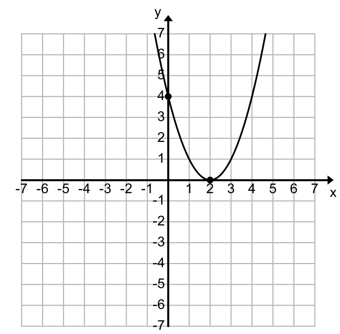
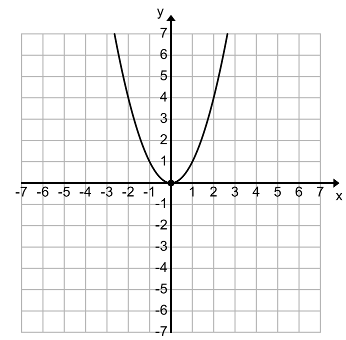
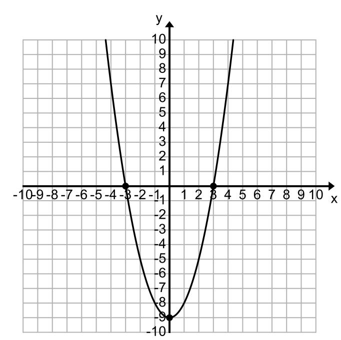
   Graph 1: (....,....) ______________ Graph 2: (....,....) ______________

   Graph 3: (....,....) ______________ Graph 4: (....,....) ______________

3. Sketch the graph of a quadratic function has a y intercept at (0,5) and a minimum at (2, 1).

4. Sketch a graph that represents a quadratic function with 0, 1, and 2 roots.

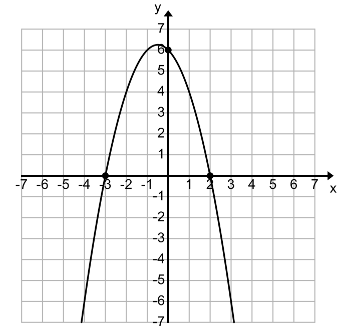
Domino Cards:

A $y = x^2 - 3x - 10$ $y =$	
B $y = x^2 - 5x + 6$ $y =$	
C $y = -x^2 + 6x - 8$ $y =$	

D

$y = x^2 - 9$

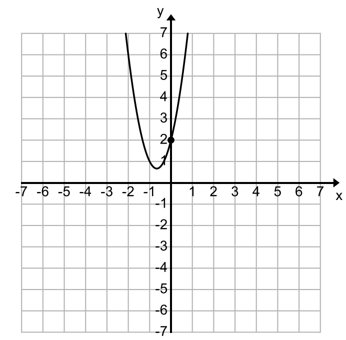
$y =$ ....



E

$y = x^2 + 7x + 12$

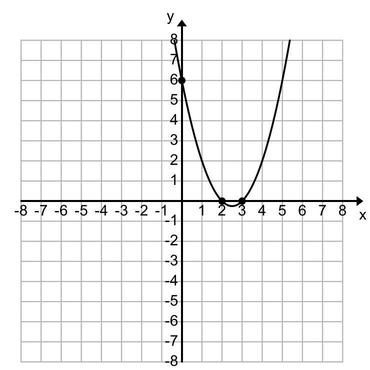
$y =$ ....



F

$y = 3x^2 + 4x + 2$

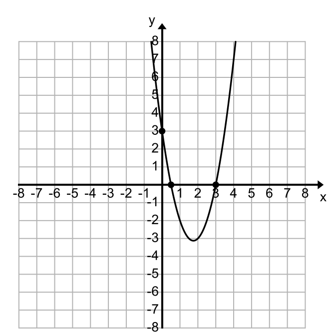
No Roots

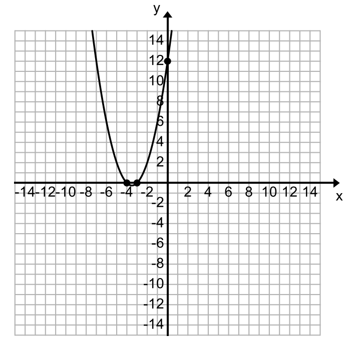
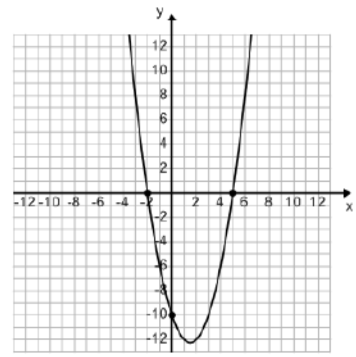
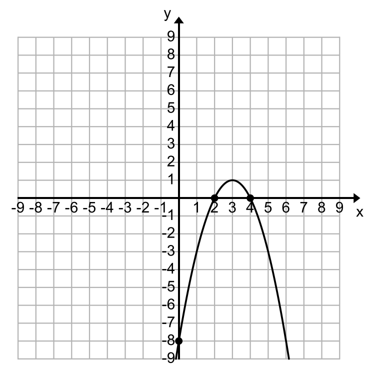


G

$y = x^2$

$y =$ ....



H $y = x^2 - 4x + 4$ $y =$	
I $y = -x^2 - x + 6$ $y =$	
J $y = 2x^2 - 7x + 3$ $y =$	

Matching Dominos

- Take turns at matching pairs of dominos that you think belong together.
- Each time you do this, explain your thinking clearly and carefully to your partner.
- It is important that you both understand the matches. If you don't agree or understand, ask your partner to explain their reasoning. You are both responsible for each other's learning.
- On some cards an equation or part of an equation is missing. Do not worry about this, as you can carry out this task without this information.

Sharing Work

- One student from each group is to visit another group's poster.
- If you are staying at your desk, be ready to explain the reasons for your group's matches.
- If you are visiting another group:
  - Write your card matches on a piece of paper.
  - Go to another group's desk and check to see which matches are different from your own.
  - If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.
  - When you return to your own desk, you need to consider as a pair whether to make any changes to your own work.

Mathematics Assessment Project
CLASSROOM CHALLENGES

This lesson was designed and developed by the
Shell Center Team at the
University of Nottingham
Malcolm Swan, Nichola Clarke, Clare Dawson, Sheila Evans
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and Daniel Pead

It was refined on the basis of reports from teams of observers led by
David Foster, Mary Bouck, and Diane Schaefer based on their
observation of trials in US classrooms along with comments from teachers
and other users.

This project was conceived and directed for MARS: Mathematics
Assessment Resource Service by
Alan Schoenfeld, Hugh Burkhardt, Daniel Pead, and Malcolm Swan
and based at the University of California, Berkeley

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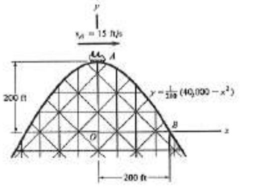


ɡebra I NOTES
rtical Motion Model
ay 1)

Name ________________________

Date _____________ Period _____

Content Objective:

- **Students will demonstrate application of the vertical motion model by solving 1 problem.**

Language Objective:

- **Using academic language, students will verbally explain how they solved a vertical motion problem.**

Standards for Mathematical Practice (SMP#1):

- **Students make sense of problems and persevere in solving them.**

Object is Dropped	**Object is Thrown**
$h = -16t^2 + s$ $h =$ height after object falls $t =$ time $s =$ intial height	$h = -16t^2 + vt + s$ $h =$ height $t =$ time $s =$ intial height $v =$ initial velocity

To solve a quadratic equation using the vertical motion model…

Step 1- Determine if the object is being dropped or thrown.

Step 2- Identify the values of the variables.

Step 3- Substitute the values into the mathematical model.

Step 4- Solve.

Example 1:

An engineering student is in an "egg dropping contest." The goal is to create a container for an egg so it can be dropped from a height of 32 feet without breaking the egg. To the nearest tenth of a second, about how long will it take for the egg's container to hit the ground? Assume there is no air resistance.

Try: An engineering student is in an “egg dropping contest.” The goal is to create a container for an egg so it can be dropped from a height of 48 feet without breaking the egg. To the nearest tenth of a second, about how long will it take for the egg’s container to hit the ground? Assume there is no air resistance.

1) An engineering student is in an “egg dropping contest.” The goal is to create a container for an egg so it can be dropped from a height of 96 feet without breaking the egg. To the nearest tenth of a second, about how long will it take for the egg’s container to hit the ground? Assume there is no air resistance.

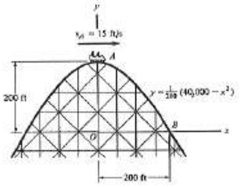
2) An engineering student is in an “egg dropping contest.” The goal is to create a container for an egg so it can be dropped from a height of 192 feet without breaking the egg. To the nearest tenth of a second, about how long will it take for the egg’s container to hit the ground? Assume there is no air resistance.

3) A boulder falls off the top of a cliff during a storm. The cliff is 60 feet high. Find how long it will take for the boulder to hit the road below? Round your answer to the nearest hundredth of a second.

4) You drop keys from a window 30 feet above ground to your friend below. Your friend does not catch them. How long will it take for the keys to reach the ground? Round your answer to the nearest hundredth of a second.

5) An acorn falls 45 feet from the top of a tree. How long will it take for the acorn to reach the ground? Round your answer to the nearest hundredth of a second.

6) How long would it take a free-fall ride at an amusement park to drop 121 feet? Assume there is no air resistance. Round your answer to the nearest hundredth of a second.



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Name _________________________

Date _____________ Period _____

Content Objective:

- **Students will demonstrate application of the vertical motion model by solving 1 problem.**

Language Objective:

- **Using academic language, students will verbally explain how they solved a vertical motion problem.**

Standards for Mathematical Practice (SMP#1):

- **Students make sense of problems and persevere in solving them.**

Object is Dropped	Object is Thrown
$h = -16t^2 + s$ $h =$ height after object falls $t =$ time $s =$ intial height	$h = -16t^2 + vt + s$ $h =$ height $t =$ time $s =$ intial height $v =$ initial velocity

To solve a quadratic equation using the vertical motion model…

Step 1- Determine if the object is being dropped or thrown.

Step 2- Identify the values of the variables.

Step 3- Substitute the values into the mathematical model.

Step 4- Solve.

Example 1:

You are competing in the Field Target Event at a hot-air balloon festival. You throw a marker down from an altitude of 200 feet toward a target. When the marker leaves your hand, its speed is 30 feet per second. How long will it take the marker to hit the target? Round your answer to the nearest hundredths.

Try: You are competing in the Field Target Event at a hot-air balloon festival. You throw a marker down from an altitude of 200 feet toward a target. When the marker leaves your hand, its speed is 50 feet per second. How long will it take the marker to hit the target? Round your answer to the nearest hundredth of a second.

1) You are competing in the Field Target Event at a hot-air balloon festival. You throw a marker down from an altitude of 150 feet toward a target. When the marker leaves your hand, its speed is 25 feet per second. How long will it take the marker to hit the target? Round your answer to the nearest hundredth of a second.

2) You are competing in the Field Target Event at a hot-air balloon festival. You throw a marker down from an altitude of 100 feet toward a target. When the marker leaves your hand, its speed is 10 feet per second. How long will it take the marker to hit the target? Round your answer to the nearest hundredth of a second.

3) You throw a ball downward with an initial speed of 10 feet per second out of a window to a friend 20 feet below. Your friend does not catch the ball. How long will take for the ball to reach the ground? Round your answer to the nearest hundredth of a second.

4) A batter hit a pitched baseball when it is 3 feet off the ground at 80 feet per second. How long will it take the ball to hit the ground in the outfield? Round your answer to the nearest hundredth of a second.

5) If a goalie kicks a soccer ball with an upward velocity of 65 feet per second and his foot meets the ball 3 feet off the ground, approximately how long will it take the ball to hit the ground? Round your answer to the nearest hundredth of a second.

6) You jump off a 15 foot diving board with an initial upward speed of 3 feet per second. How long will it take you to hit the water? Round your answer to the nearest hundredth of a second.

7) A falcon dives toward a pigeon on the ground. When the falcon is at a height of 100 feet the pigeon sees the falcon, which is diving at 220 feet per second. Estimate the time the pigeon has to escape.

Try: You are competing in the Field Target Event at a hot-air balloon festival. You throw a marker down from an altitude of 200 feet toward a target. When the marker leaves your hand, its speed is 50 feet per second. How long will it take the marker to hit the target? Round your answer to the nearest hundredth of a second.