3-4 Graphing Functions

Warm Up

Solve each equation for y.

1. $2x + y = 3$ $y = -2x + 3$

2. $-x + 3y = -6$ $y = \frac{1}{3}x - 2$

3. $4x - 2y = 8$ $y = 2x - 4$

4. Generate ordered pairs for $y = \frac{1}{2}x + 1$

using $x = -4, -2, 0, 2$ and 4.

$(-4, -1), (-2, 0), (0, 1), (2, 2), (4, 3)$

Objectives

Graph functions given a limited domain.

Graph functions given a domain of all real numbers.

3-4 Graphing Functions

Scientists can use a function to make conclusions about the rising sea level.

Sea level is rising at an approximate rate of 2.5 millimeters per year. If this rate continues, the function $y = 2.5x$ can describe how many millimeters y sea level will rise in the next x years.

One way to understand functions such as the one above is to graph them. You can graph a function by finding ordered pairs that satisfy the function.

Example 1A: Graphing Solutions Given a Domain

Graph the function for the given domain.

$x - 3y = -6$; D: {–3, 0, 3, 6}

Step 1 Solve for y since you are given values of the domain, or x.

$$x - 3y = -6$$

$$-x \qquad -x$$

Subtract x from both sides.

$$-3y = -x - 6$$

$$\frac{-3y}{-3} = \frac{-x - 6}{-3}$$

Since y is multiplied by –3, divide both sides by –3.

$$y = \frac{1}{3}x + 2$$

Simplify.

Example 1A Continued

Graph the function for the given domain.

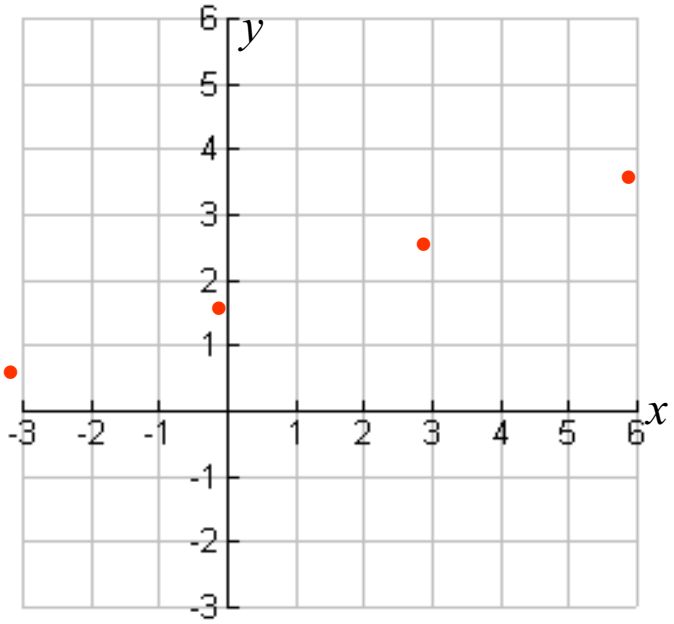
Step 2 Substitute the given value of the domain for x and find values of y.

x	$y = \frac{1}{3}x + 2$	(x, y)
-3	$y = \frac{1}{3}(-3) + 2 = 1$	$(-3, 1)$
0	$y = \frac{1}{3}(0) + 2 = 2$	$(0, 2)$
3	$y = \frac{1}{3}(3) + 2 = 3$	$(3, 3)$
6	$y = \frac{1}{3}(6) + 2 = 4$	$(6, 4)$

Example 1A Continued

Graph the function for the given domain.

Step 3 Graph the ordered pairs.



Example 1B: Graphing Solutions Given a Domain

Graph the function for the given domain.

$f(x) = x^2 - 3$; D: {–2, –1, 0, 1, 2}

Step 1 Use the given values of the domain to find values of $f(x)$.

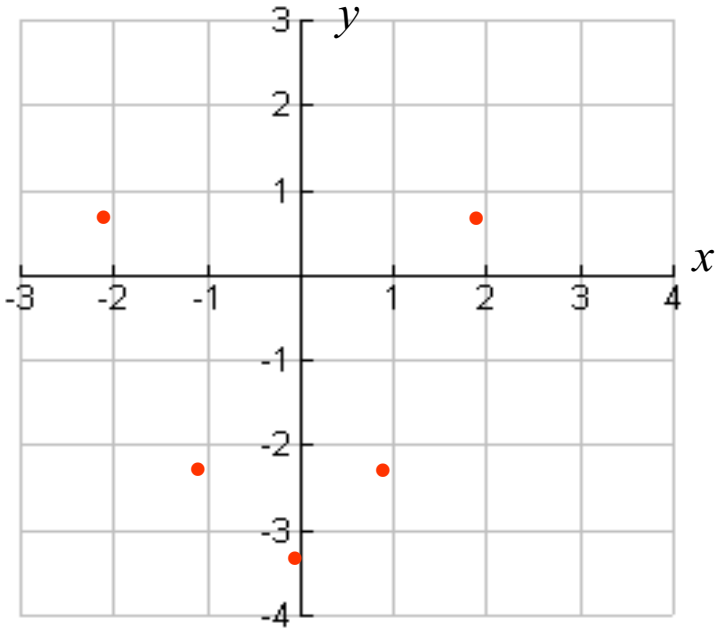
x	$f(x) = x^2 - 3$	$(x, f(x))$
–2	$f(x) = (-2)^2 - 3 = 1$	(–2, 1)
–1	$f(x) = (-1)^2 - 3 = -2$	(–1, –2)
0	$f(x) = 0^2 - 3 = -3$	(0, –3)
1	$f(x) = 1^2 - 3 = -2$	(1, –2)
2	$f(x) = 2^2 - 3 = 1$	(2, 1)

Example 1B Continued

Graph the function for the given domain.

$f(x) = x^2 - 3$; D: {–2, –1, 0, 1, 2}

Step 2 Graph the ordered pairs.



Check It Out! Example 1a

Graph the function for the given domain.

$-2x + y = 3$; D: {–5, –3, 1, 4}

Step 1 Solve for y since you are given values of the domain, or x.

$$-2x + y = 3$$

$+2x \qquad +2x$ *Add 2x to both sides.*

$$y = 2x + 3$$

Check It Out! Example 1a Continued

Graph the function for the given domain.

$-2x + y = 3$; D: {–5, –3, 1, 4}

Step 2 Substitute the given values of the domain for x and find values of y.

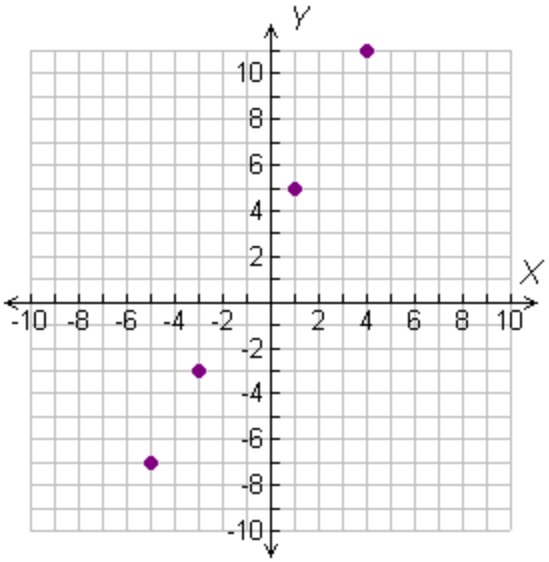
x	$y = 2x + 3$	(x, y)
–5	$y = 2(-5) + 3 = -7$	(–5, –7)
–3	$y = 2(-3) + 3 = -3$	(–3, –3)
1	$y = 2(1) + 3 = 5$	(1, 5)
4	$y = 2(4) + 3 = 11$	(4, 11)

Check It Out! Example 1a Continued

Graph the function for the given domain.

$-2x + y = 3$; D: {–5, –3, 1, 4}

Step 3 Graph the ordered pairs.



Check It Out! Example 1b

Graph the function for the given domain.

$f(x) = x^2 + 2$; D: {–3, –1, 0, 1, 3}

Step 1 Use the given values of the domain to find the values of $f(x)$.

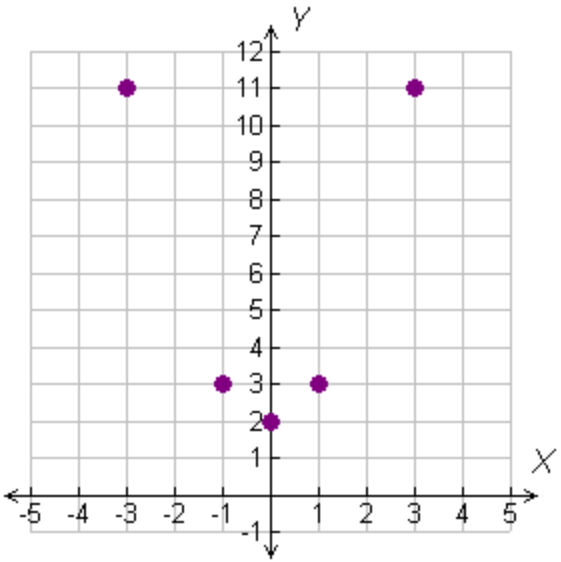
x	$f(x) = x^2 + 2$	$(x, f(x))$
–3	$f(x) = (-3^2) + 2 = 11$	(–3, 11)
–1	$f(x) = (-1^2) + 2 = 3$	(–1, 3)
0	$f(x) = 0^2 + 2 = 2$	(0, 2)
1	$f(x) = 1^2 + 2 = 3$	(1, 3)
3	$f(x) = 3^2 + 2 = 11$	(3, 11)

Check It Out! Example 1b

Graph the function for the given domain.

$f(x) = x^2 + 2$; D: {–3, –1, 0, 1, 3}

Step 2 Graph the ordered pairs.



If the domain of a function is all real numbers, any number can be used as an input value. This process will produce an infinite number of ordered pairs that satisfy the function. Therefore, arrowheads are drawn at both "ends" of a smooth line or curve to represent the infinite number of ordered pairs. If a domain is not given, assume that the domain is all real numbers.

Graphing Functions Using a Domain of All Real Numbers	
Step 1	Use the function to generate ordered pairs by choosing several values for x.
Step 2	Plot enough points to see a pattern for the graph.
Step 3	Connect the points with a line or smooth curve.

Example 2A: Graphing Functions

Graph the function $-3x + 2 = y$.

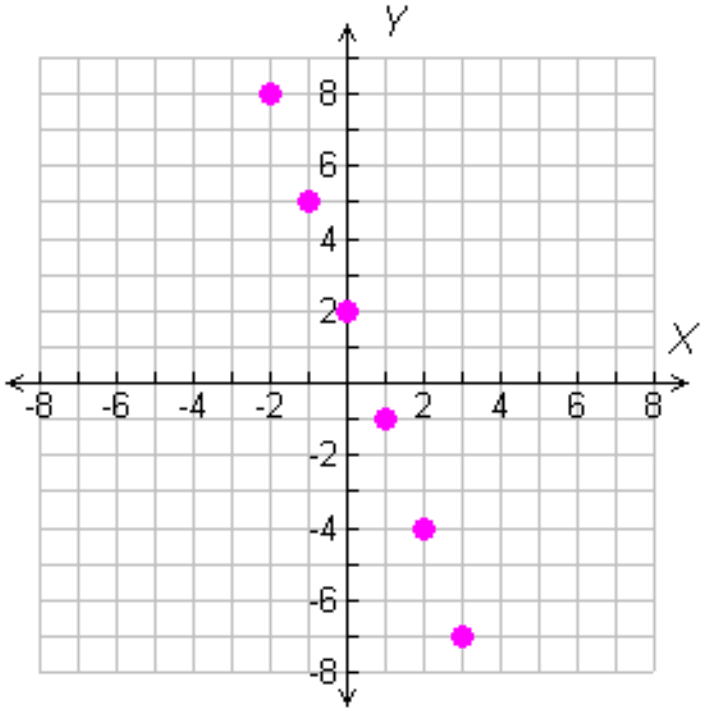
Step 1 Choose several values of x and generate ordered pairs.

x	$-3x + 2 = y$	(x, y)
-2	$-3(-2) + 2 = 8$	$(-2, 8)$
-1	$-3(-1) + 2 = 5$	$(-1, 5)$
0	$-3(0) + 2 = 2$	$(0, 2)$
1	$-3(1) + 2 = -1$	$(1, -1)$
2	$-3(2) + 2 = -4$	$(2, -4)$
3	$-3(3) + 2 = -7$	$(3, -7)$

Example 2A Continued

Graph the function $-3x + 2 = y$.

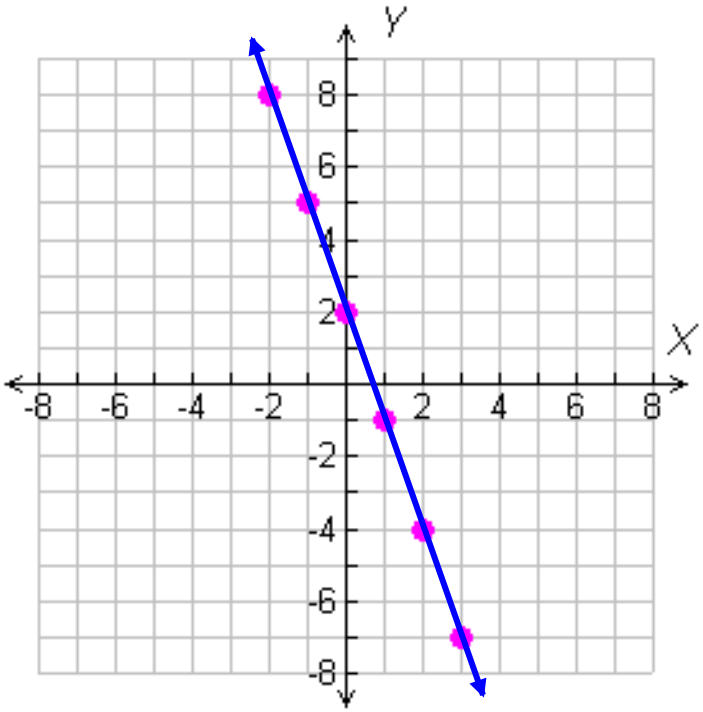
Step 2 Plot enough points to see a pattern.



Example 2A Continued

Graph the function $-3x + 2 = y$.

Step 3 The ordered pairs appear to form a line. Draw a line through all the points to show all the ordered pairs that satisfy the function. Draw arrowheads on both "ends" of the line.



Example 2B: Graphing Functions

Graph the function $g(x) = |x| + 2$.

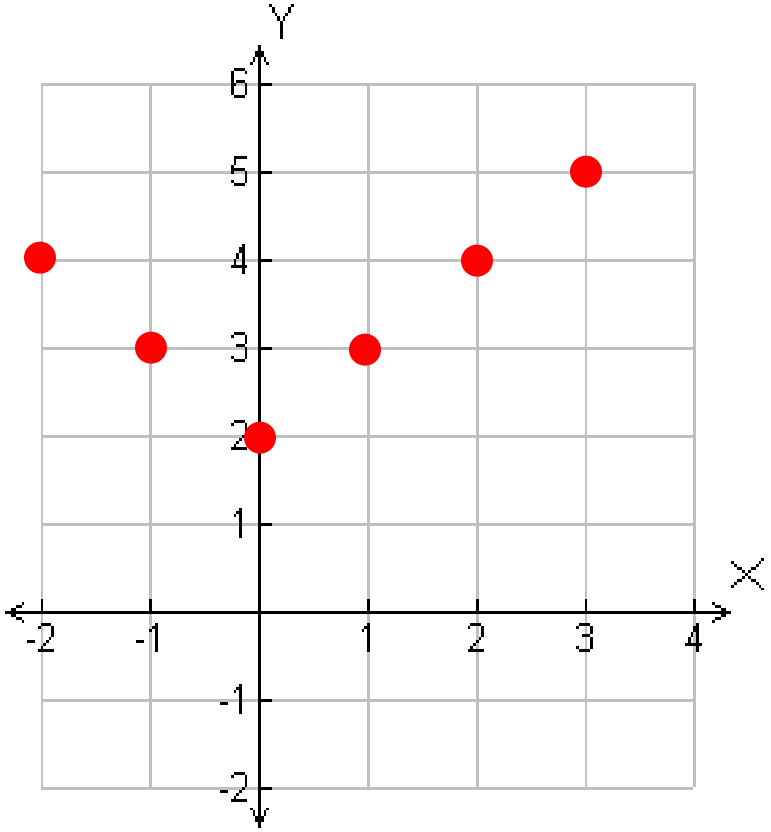
Step 1 Choose several values of x and generate ordered pairs.

x	$g(x) = \|x\| + 2$	$(x, g(x))$
–2	$g(x) = \|-2\| + 2 = 4$	(–2, 4)
–1	$g(x) = \|-1\| + 2 = 3$	(–1, 3)
0	$g(x) = \|0\| + 2 = 2$	(0, 2)
1	$g(x) = \|1\| + 2 = 3$	(1, 3)
2	$g(x) = \|2\| + 2 = 4$	(2, 4)
3	$g(x) = \|3\| + 2 = 5$	(3, 5)

Example 2B Continued

Graph the function $g(x) = |x| + 2$.

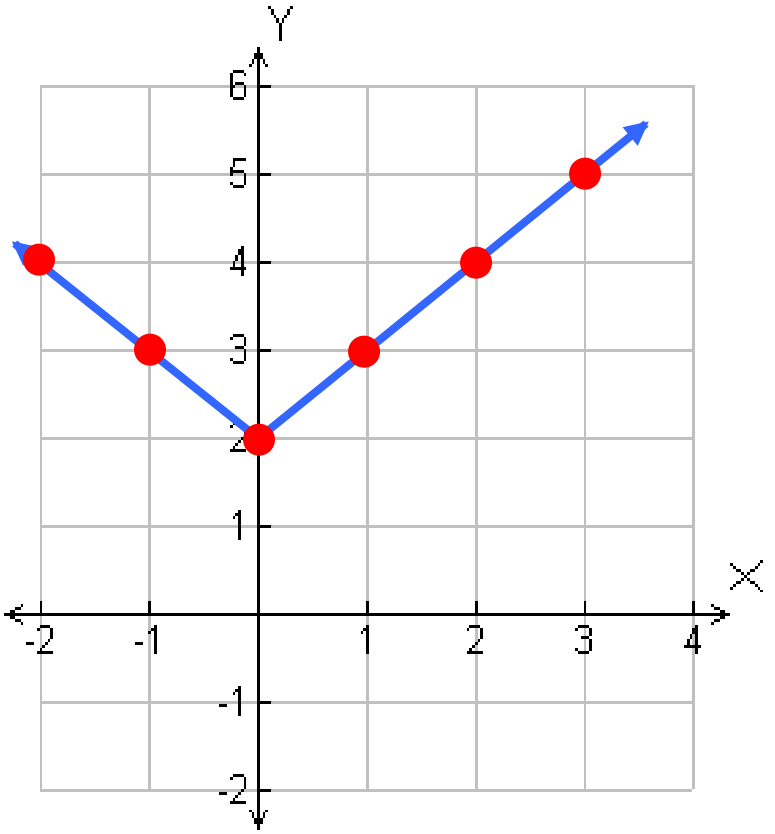
Step 2 Plot enough points to see a pattern.



Example 2B Continued

Graph the function $g(x) = |x| + 2$.

Step 3 The ordered pairs appear to form a v-shaped graph. Draw lines through all the points to show all the ordered pairs that satisfy the function. Draw arrowheads on the "ends" of the "V".



Example 2B Continued

Graph the function $g(x) = |x| + 2$.

Check If the graph is correct, any point on it will satisfy the function. Choose an ordered pair on the graph that was not in your table. (4, 6) is on the graph. Check whether it satisfies $g(x) = |x| + 2$.

$g(x) = \|x\| + 2$	
6	$\|4\| + 2$
6	$4 + 2$
6	6 ✓

Substitute the values for x and y into the function. Simplify.

The ordered pair (4, 6) satisfies the function.

Check It Out! Example 2a

Graph the function $f(x) = 3x - 2$.

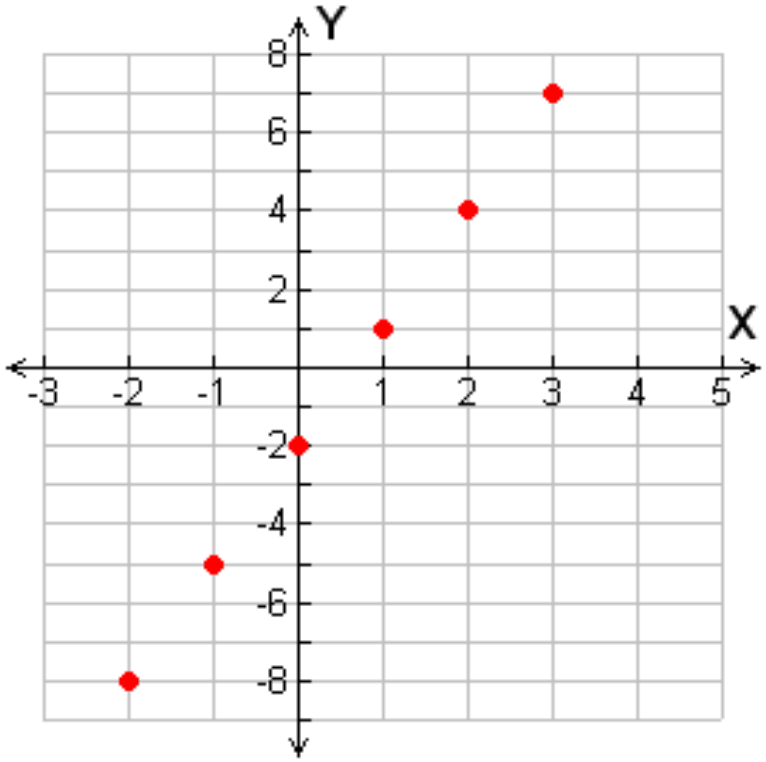
Step 1 Choose several values of x and generate ordered pairs.

x	$f(x) = 3x - 2$	$(x, f(x))$
-2	$f(x) = 3(-2) - 2 = -8$	$(-2, -8)$
-1	$f(x) = 3(-1) - 2 = -5$	$(-1, -5)$
0	$f(x) = 3(0) - 2 = -2$	$(0, -2)$
1	$f(x) = 3(1) - 2 = 1$	$(1, 1)$
2	$f(x) = 3(2) - 2 = 4$	$(2, 4)$
3	$f(x) = 3(3) - 2 = 7$	$(3, 7)$

Check It Out! Example 2a Continued

Graph the function $f(x) = 3x - 2$.

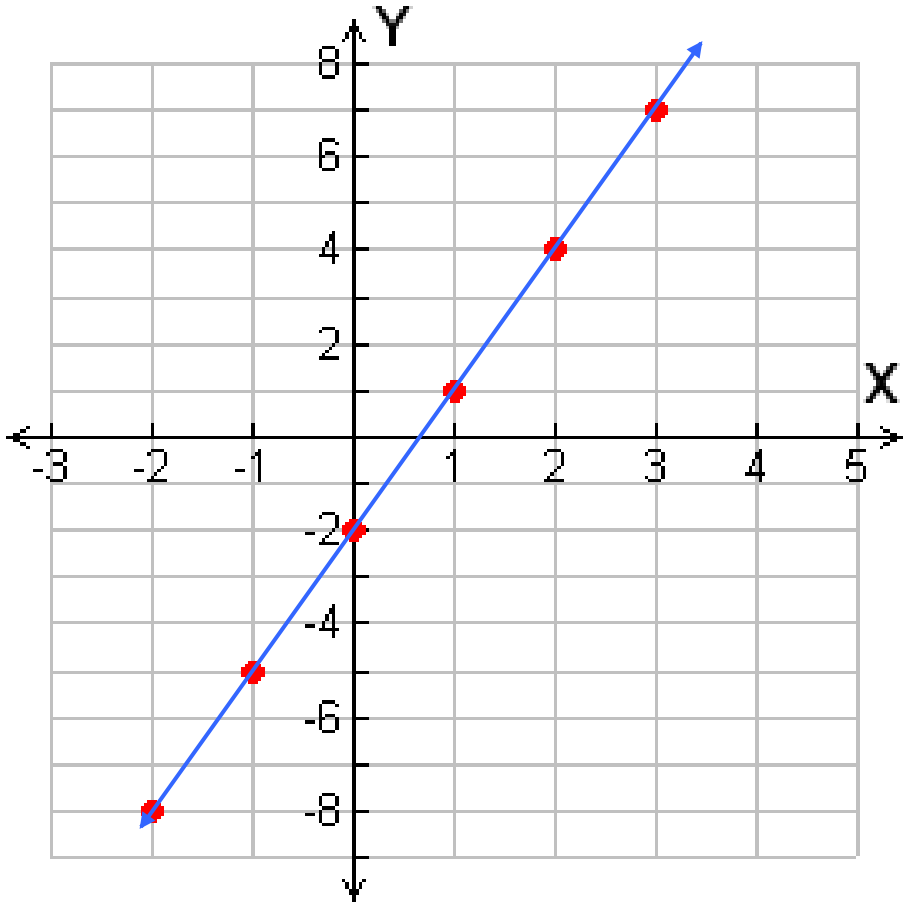
Step 2 Plot enough points to see a pattern.



Check It Out! Example 2a Continued

Graph the function $f(x) = 3x - 2$.

Step 3 The ordered pairs appear to form a line. Draw a line through all the points to show all the ordered pairs that satisfy the function. Draw arrowheads on both "ends" of the line.



Check It Out! Example 2b

Graph the function $y = |x - 1|$.

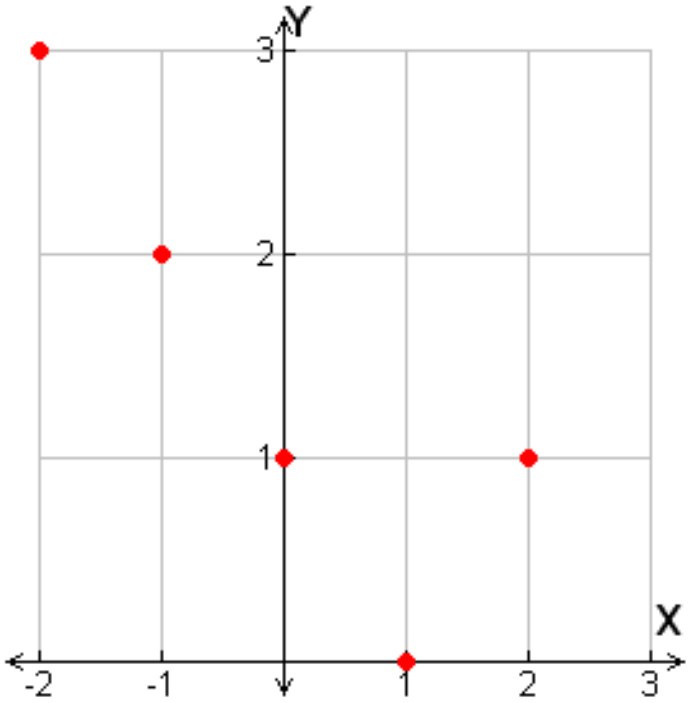
Step 1 Choose several values of x and generate ordered pairs.

x	$y = \|x - 1\|$	(x, y)
-2	$y = \|-2 - 1\| = 3$	$(-2, 3)$
-1	$y = \|-1 - 1\| = 2$	$(-1, 2)$
0	$y = \|0 - 1\| = 1$	$(0, 1)$
1	$y = \|1 - 1\| = 0$	$(1, 0)$
2	$y = \|2 - 1\| = 1$	$(2, 1)$

Check It Out! Example 2b Continued

Graph the function $y = |x - 1|$.

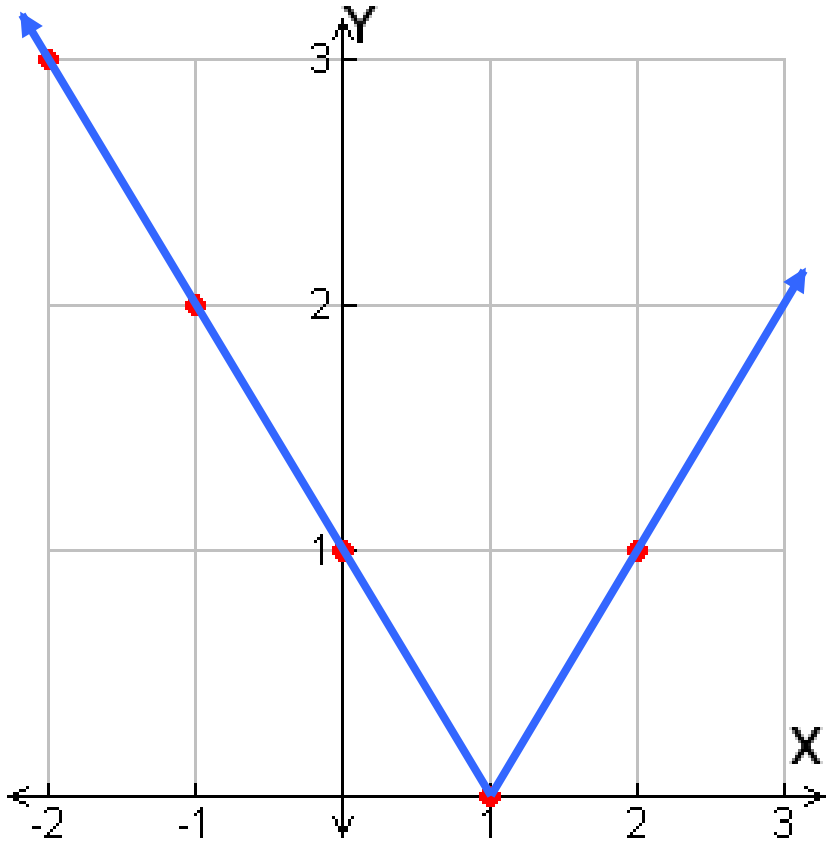
Step 2 Plot enough points to see a pattern.



Check It Out! Example 2b Continued

Graph the function $y = |x - 1|$.

Step 3 The ordered pairs appear to form a v-shaped graph. Draw lines through the points to show all the ordered pairs that satisfy the function. Draw arrowheads on both "ends" of the "V".



Check It Out! Example 2b Continued

Graph the function $y = |x - 1|$.

Check If the graph is correct, any point on the graph will satisfy the function. Choose an ordered pair on the graph that is not in your table. (3, 2) is on the graph. Check whether it satisfies $y = |x - 1|$.

y	$\|x - 1\|$
2	$\|3 - 1\|$
2	$\|2\|$
2	2 ✓

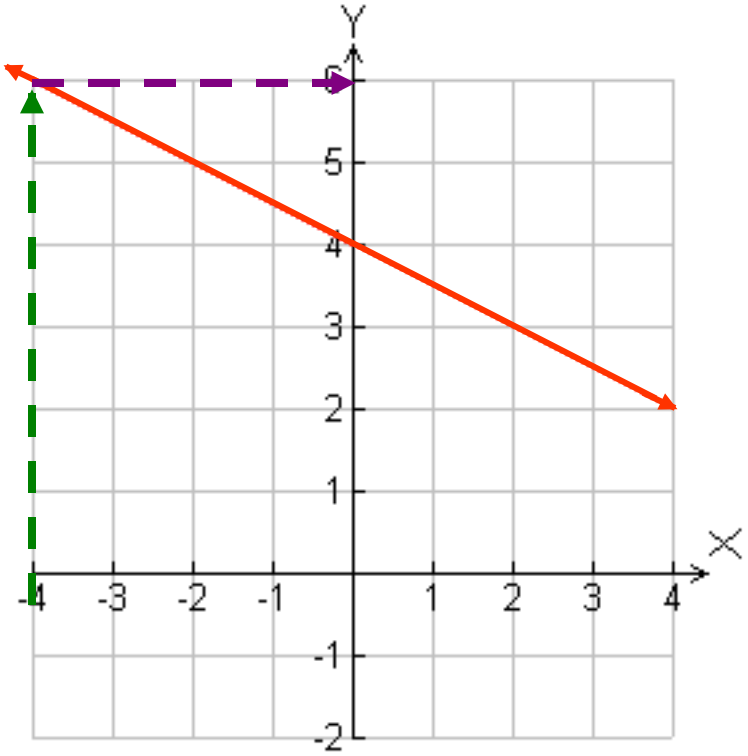
Substitute the values for x and y into the function. Simplify.

The ordered pair (3, 2) satisfies the function.

Example 3: Finding Values Using Graphs

Use a graph of the function $f(x) = -\frac{1}{2}x + 4$ to find the value of $f(x)$ when $x = -4$. Check your answer.

Locate –4 on the *x*-axis. Move up to the graph of the function. Then move right to the *y*-axis to find the corresponding value of *y*.



$f(-4) = 6$

Example 3 Continued

Use a graph of the function $f(x) = -\frac{1}{2}x + 4$ to find the value of $f(x)$ when $x = -4$. Check your answer.

$f(-4) = 6$

Check Use substitution.

$f(x) = -\frac{1}{2}x + 4$

6	$-\frac{1}{2}(-4) + 4$
6	$2 + 4$
6	6 ✓

Substitute the values for x and y into the function.

Simplify.

The ordered pair (–4, 6) satisfies the function.

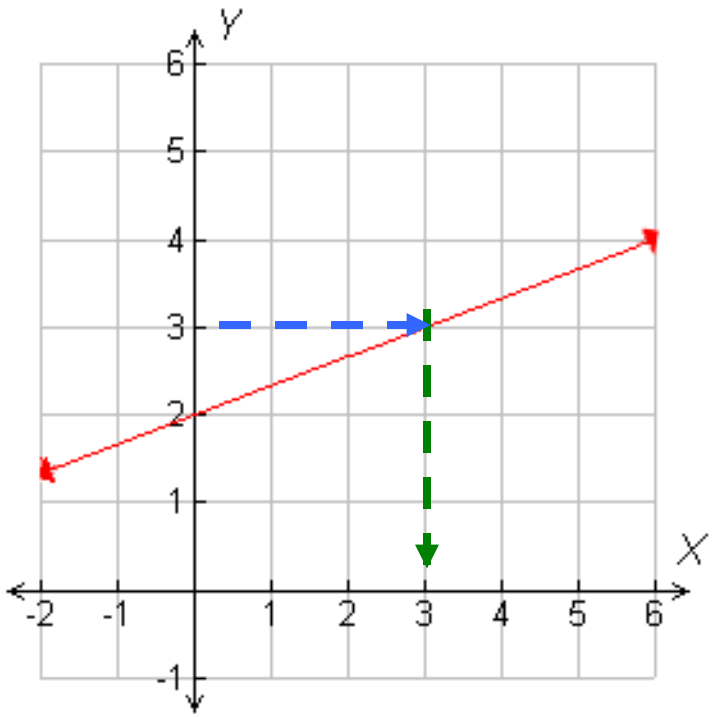
Check It Out! Example 3

Use the graph of $f(x) = \frac{1}{3}x + 2$ to find the value of x when $f(x) = 3$.

3. Check your answer.

Locate 3 on the y-axis. Move right to the graph of the function. Then move down to the x-axis to find the corresponding value of x.

$f(3) = 3$



Check It Out! Example 3 Continued

Use the graph of $f(x) = \frac{1}{3}x + 2$ to find the value of x when $f(x) =$

3. Check your answer.

f(3) = 3

Check Use substitution.

$$f(x) = \frac{1}{3}x + 2$$

3	$\frac{1}{3}(3) + 2$
3	$1 + 2$
3	3 ✓

Substitute the values for x and y into the function.

Simplify.

The ordered pair (3, 3) satisfies the function.

Graphing Functions

Recall that in real-world situations you may have to limit the domain to make answers reasonable. For example, quantities such as time, distance, and number of people can be represented using only nonnegative values. When both the domain and the range are limited to nonnegative values, the function is graphed only in Quadrant I.

Example 4: Problem-Solving Application

A mouse can run 3.5 meters per second. The function $y = 3.5x$ describes the distance in meters the mouse can run in x seconds. Graph the function. Use the graph to estimate how many meters a mouse can run in 2.5 seconds.

Example 4 Continued

Understand the Problem

The **answer** is a graph that can be used to find the value of y when x is 2.5.

List the **important information:**

- The function $y = 3.5x$ describes how many meters the mouse can run.

Example 4 Continued

Make a Plan

Think: What values should I use to graph this function? Both the number of seconds the mouse runs and the distance the mouse runs cannot be negative. Use only nonnegative values for both the domain and the range. The function will be graphed in Quadrant I.

Example 4 Continued

3 **Solve**

Choose several nonnegative values of x to find values of y.

x	$y = 3.5x$	(x, y)
0	$y = 3.5(0) = 0$	(0, 0)
1	$y = 3.5(1) = 3.5$	(1, 3.5)
2	$y = 3.5(2) = 7$	(2, 7)
3	$y = 3.5(3) = 10.5$	(3, 10.5)

Example 4 Continued

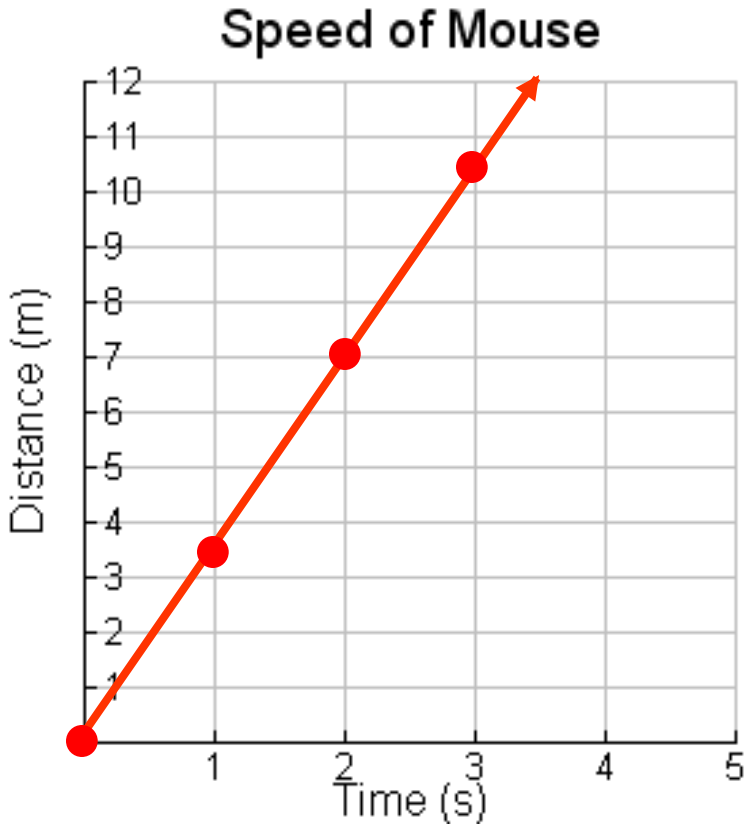
3 **Solve**

Graph the ordered pairs.

Draw a line through the points to show all the ordered pairs that satisfy this function.

Use the graph to estimate the *y*-value when *x* is 2.5.

A mouse can run about 8.75 meters in 2.5 seconds.



Example 4 Continued

Look Back

As time increases, the distance traveled also increases, so the graph is reasonable. When x is between 2 and 3, y is between 7 and 10.5. Since 2.5 is between 2 and 3, it is reasonable to estimate y to be 8.75 when x is 2.5.

Check It Out! Example 4

The fastest recorded Hawaiian lava flow moved at an average speed of 6 miles per hour. The function $y = 6x$ describes the distance y the lava moved on average in x hours. Graph the function. Use the graph to estimate how many miles the lava moved after 5.5 hours.

Check It Out! Example 4 Continued

Understand the Problem

The **answer** is a graph that can be used to find the value of y when x is 5.5.

List the **important information:**

- The function $y = 6x$ describes how many miles the lava can flow.

Check It Out! Example 4 Continued

Make a Plan

Think: What values should I use to graph this function? Both the speed of the lava and the number of hours it flows cannot be negative. Use only nonnegative values for both the domain and the range. The function will be graphed in Quadrant I.

Check It Out! Example 4 Continued

3 **Solve**

Choose several nonnegative values of x to find values of y.

x	$y = 6x$	(x, y)
1	$y = 6(1) = 6$	(1, 6)
3	$y = 6(3) = 18$	(3, 18)
5	$y = 6(5) = 30$	(5, 30)

Check It Out! Example 4 Continued

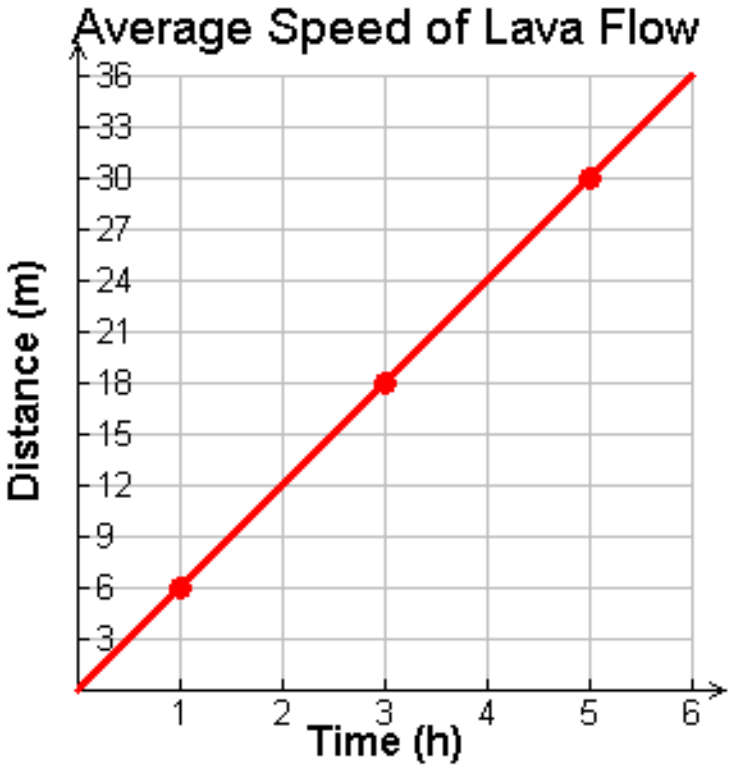
3 **Solve**

Graph the ordered pairs.

Draw a line through the points to show all the ordered pairs that satisfy this function.

Use the graph to estimate the *y*-value when *x* is 5.5.

The lava will travel about 32.5 meters in 5.5 seconds.



Check It Out! Example 4 Continued

Look Back

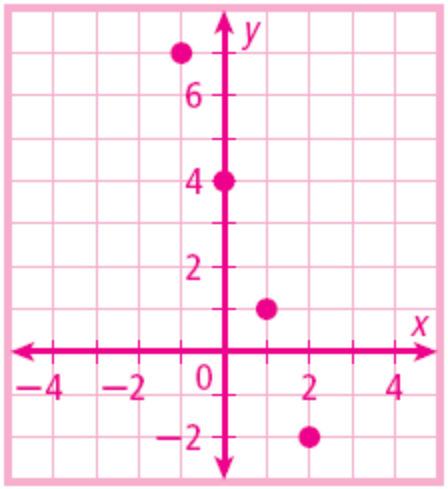
As the amount of time increases, the distance traveled by the lava also increases, so the graph is reasonable. When x is between 5 and 6, y is between 30 and 36. Since 5.5 is between 5 and 6, it is reasonable to estimate y to be 32.5 when x is 5.5.

Lesson Quiz: Part I

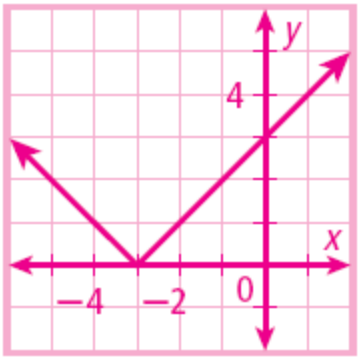
1. Graph the function for the given domain.

$3x + y = 4$

D: $\{-1, 0, 1, 2\}$

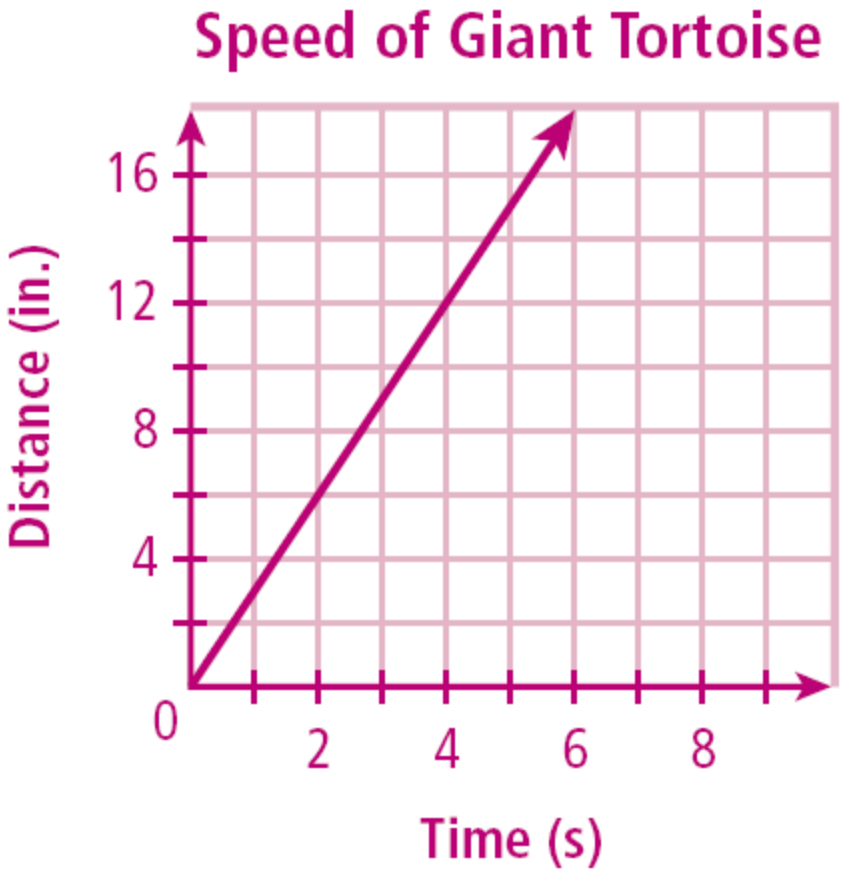


2. Graph the function $y = |x + 3|$.



Lesson Quiz: Part II

3. The function $y = 3x$ describes the distance (in inches) a giant tortoise walks in x seconds. Graph the function. Use the graph to estimate how many inches the tortoise will walk in 5.5 seconds.



About 16.5 in.