2-6 Continuous Compounding

Advanced Financial Algebra

How can interest be compounded continuously?

- Compound interest means that you are paid interest on your balance AND on previous interest you have earned.

- Compound interest allows your money to grow FASTER and we have compounded annually (once a year), quarterly (4 times per year), monthly (12 times per year), daily (365 times per year), etc.

- With computers, we can compound interest every second or microsecond or faster which becomes continuously.

Developing formula

- Start with % increase and decrease formula: A = P$(1 + \;)$ = Principal $(1 + \quad)$

- **Periodic compounding formula from Section 2-5** (not just yearly/annual): **A = P**$(\; + -)^{(\;)}$

A = amount \$ after time   P = Principal (original \$)   r = rate as a decimal

n = number of compounds per year   t = time in years

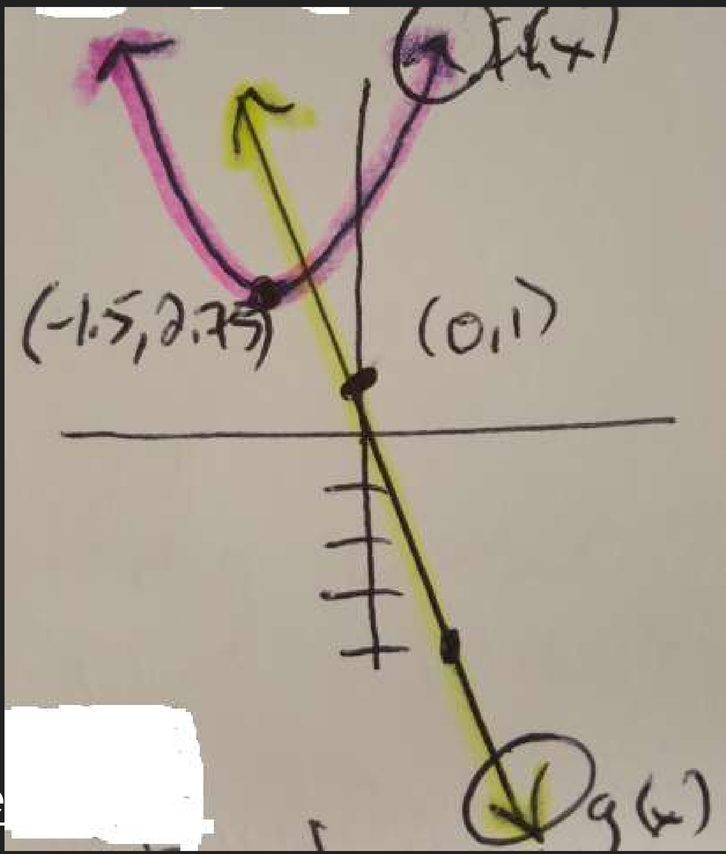
With limits that I will explain some during next class, that formula becomes **A = P**$(\;)^{(\;)}$

Example 1 – as x increases, what happens to y?

- Given the quadratic function $f(x) = x^2 + 3x + 5$ and the linear function $g(x) = -5x + 1$, as the values of x increase to infinity, what happens to the values of y?

- SOLUTION:

- Look at the graphs to the right. F(x) is pink and g(x) is yellow.
- For f(x), as the x values increase to the right, the **y values also increase**
- For g(x), as the x values increase to the right, the **y values decrease**.



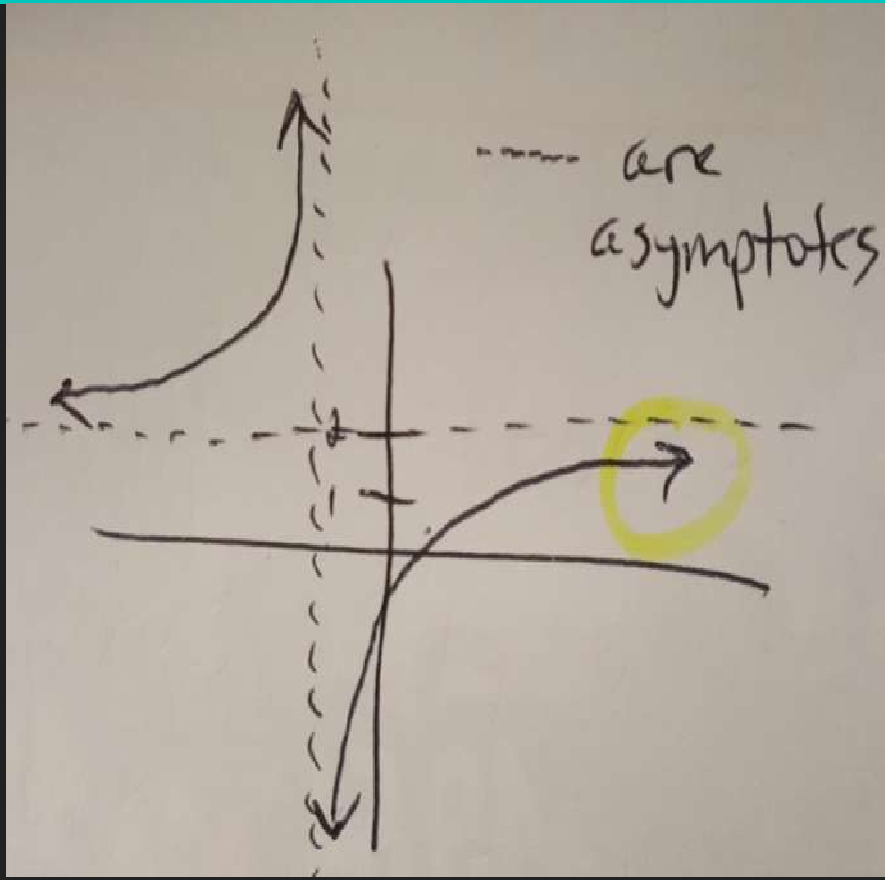
Example 2 – hyperbola limit

- Given the function f(x) = $\frac{6\quad -1}{3\quad +2}$, as the values of x increase to infinity, what happens to the values of *f(x)*?

- SOLUTION:
- Look at the yellow circle on the graph to the right. It is a hyperbola

- As the x values increase to the right, the **y values approach 2.**

- Mathematical representation: $\lim_{\to +\infty} (\quad) = 2$



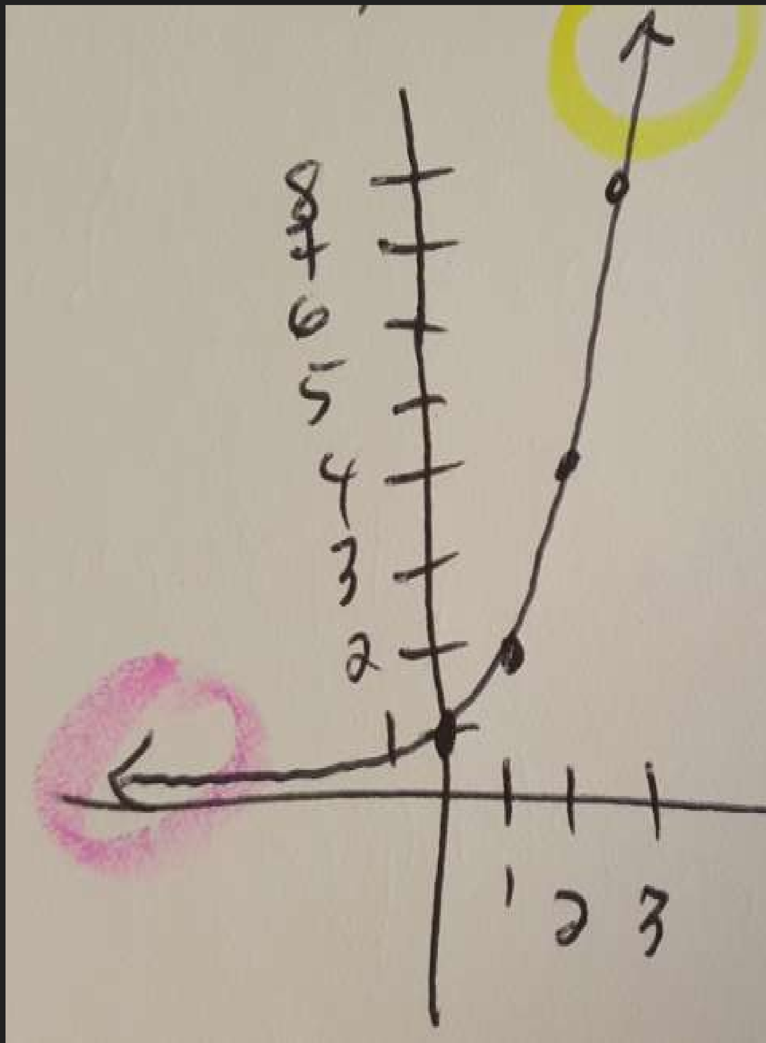
Example 3 – evaluate the limit using graph

- Given the exponential curve function f(x) = 2 , find : $\lim_{\to +\infty} (\ ) = ?$ and $\lim_{\to -\infty} (\ ) = ?$

- SOLUTION:

- Use the table to graph the function f(x). See graph at right and analyze function behavior at each end.

- $\lim_{\to +\infty} (\ ) =$ **+ ∞** (see yellow) $\lim_{\to -\infty} (\ ) =$ (see pink)



Example 4 – where does the e in A = P( )( ) come from?

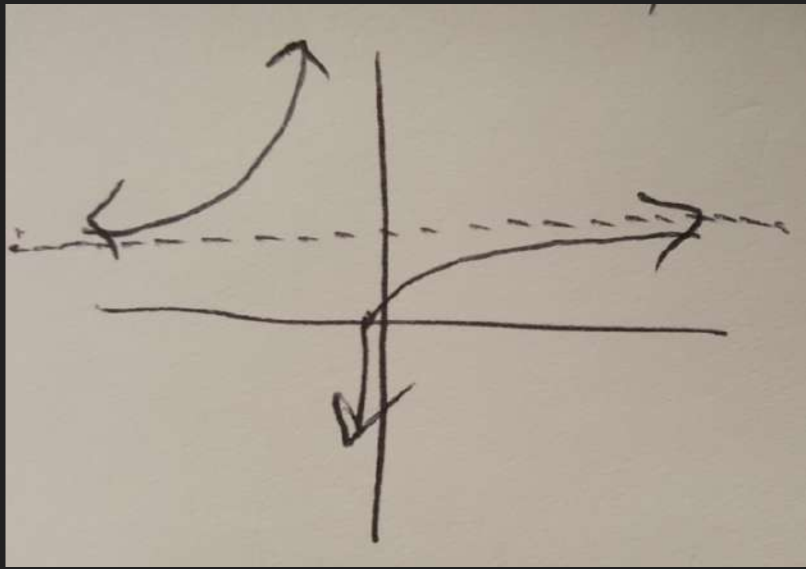
- If $f(x) = \left(1 + \frac{1}{\ }\right)$, find $\lim_{\to + \infty} (\ ) = ?$

- SOLUTION:

- See graph at right. It has an asymptote at y = e $\approx$ 2.718…

- Therefore, $\lim_{\to + \infty} (\ ) =$

which we use when compounding continuously since that is like compounding an infinite number of times (n = ∞) .



Example 5 – how do we use $A = P(\ )^{(\ )}$ to solve a problem?

- If you deposited $1,000 at 100% interest, compounded continuously, what would your ending balance be after 1 year?

- SOLUTION:

- Use the old formula $A = P(1 + -)^{(\ )} = 1000(1 + \frac{1}{\infty})^{(\infty * 1)}$ = 1000 * e $\approx$ $2,718.28
- Or use the easier formula $A = P(\ )^{(\ )}$ which is only for compounding continuously.
- $A = P(\ )^{(\ )} = 1000(\ )^{(1 * 1)} \approx$ **$2,718.28**

Continuous Compounding Formula

- $A = P(\quad)^{(\quad)}$

A = ending amount P = principal r = rate as a decimal t = time

Example 6 – another scenario

- If you deposit $1,000 at 2.3% interest, compounded continuously, what would your ending balance be to the nearest cent after 5 years?

- SOLUTION:

- Use the formula $A = P(\ )^{(\ \ )}$ which is only for compounding continuously.
- $A = P(\ )^{(\ \ )} = 1000(\ )^{(.023 * 5)} \approx$ **<u>$1,121.87</u>**

Assignment: pg 107 #2 b-i only, #4, 5, 7

- #2

A bank representative studies compound interest, so she can better serve customers. She analyses what happens when $2,000 earns interest several different ways at a rate of 2% for 3 years.

b. Find the interest if it is compounded annually.

c. Find the interest if it is compounded semi-annually.

d. Find the interest if it is compounded quarterly.

e. Find the interest if it is compounded monthly.

f. Find the interest if it is compounded daily.

g. Find the interest if it is compounded hourly.

h. Find the interest if it is compounded every minute.

i. Find the interest if it is compounded continuously.

Assignment: pg 107 #2 b-i only, #4, 5, 7 cont.

- #4

Find the interest earned on a $50,000 deposited for six years at $1\frac{1}{8}$ % interest, compounded continuously. ____

- #5

Whitney deposits $9,000 for two years. She compares two different banks. State Bank will pay her 2.1% interest, compounded monthly. Kings Savings will pay her 2.01% interest, compounded continuously.

a. How much interest does State Bank pay? ____

b. How much interest does Kings Savings pay? ____

c. Which bank pays higher interest? How much higher? ____ ▼ ____

- #7

Find the interest earned on a $30,000 deposit for six months at $1\frac{1}{2}$ % interest, compounded continuously. ____