Mathematics Curriculum Guide

High School Algebra 1

2017-18

Topic 8: Polynomials & Factoring (Part 1)

Transfer Goals

1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.
2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience.
3) Construct viable arguments and critique the reasoning of others using precise mathematical language.

Timeframe: 2 weeks/12 days
Start Date: March 9, 2018
Assessment Dates: March 23-24, 2018

Standards

A-SSE Seeing Structure in Expressions
Interpret the structure of expressions ~~[Linear, exponential, and~~ quadratic]

1. Interpret expressions that represent a quantity in terms of its context. ★
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* ★
2. Use the structure of an expression to identify ways to rewrite it.

A-APR Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials [Linear and quadratic]

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Meaning-Making

Understandings

Students will understand that…

Equivalence*:* A single quantity may be represented by many different expressions.

- *Monomials may be used to form larger expressions called polynomials.*
- *Monomials can be written as a product of two or more monomials.*
- *Polynomials can be added and subtracted.*
- *There are several ways to find the product of two binomials, including models, algebra, and tables.*
- *Similar to integers, polynomials are closed under the operations of addition, subtraction, and multiplication.*

Properties: *All the facts of arithmetic and algebra follow from certain properties.*

- *The properties of real numbers can be used to multiply a monomial by a polynomial, a binomial by a trinomial, or to simplify the product of binomials.*

Essential Questions

Students will keep considering…

- Can two algebraic expressions that appear to be different be equivalent?
- How are the properties of real numbers related to polynomials?
- How do polynomials form a system that is analogous to integers?

Acquisition

Knowledge

Students will know…

Vocabulary: monomial, binomial, trinomial, polynomial, standard form of a polynomial, degree of a polynomial, coefficient, Distributive Property, square of a binomial, product of a sum and difference

Procedures for:

- Adding polynomials by combining like terms.
- Subtracting polynomials by adding the additive inverse of the polynomial being subtracted.
- Multiplying a monomial with a polynomial using the distributive property.
- Multiplying two binomials using the distributive property (FOIL).
  - Using special rules to simplify the square of a binomial or the product of a sum and difference.

Skills

Students will be skilled at and able to do the following…

- Applying the properties of real numbers to add, subtract, multiply, and factor polynomials.
- Use the Commutative and Associative Properties to manipulate polynomial expressions.
- Use the Distributive Property to multiply polynomials and factor polynomials.

Topic 8: Polynomials & Factoring (Part 1)

Transfer is a student's ability to independently apply understanding in a novel or unfamiliar situation. In mathematics, this requires that students use reasoning and strategy, not merely plug in numbers in a familiar-looking exercise, via a memorized algorithm.

Transfer goals highlight the effective uses of understanding, knowledge, and skills we seek in the long run – that is, what we want students to be able to do when they confront new challenges, both in and outside school, beyond the current lessons and unit. These goals were developed so all students can apply their learning to mathematical or real-world problems while simultaneously engaging in the Standards for Mathematical Practices. In the mathematics classroom, assessment opportunities should reflect student progress towards meeting the transfer goals.

With this in mind, the revised **PUSD transfer goals** are:

1) **Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.**
2) **Effectively communicate orally, in writing, and by using models (e.g., concrete, representational, abstract) for a given purpose and audience.**
3) **Construct viable arguments and critique the reasoning of others using precise mathematical language.**

Multiple measures will be used to evaluate student acquisition, meaning-making and transfer. Formative and summative assessments play an important role in determining the extent to which students achieve the desired results in stage one.

Formative Assessment	Summative Assessment
Aligning Assessment to Stage One	
• What constitutes evidence of understanding for this lesson? • Through what other evidence during the lesson (e.g. response to questions, observations, journals, etc.) will students demonstrate achievement of the desired results? • How will students reflect upon, self-assess, and set goals for their future learning?	• What evidence must be collected and assessed, given the desired results defined in stage one? • What is evidence of understanding (as opposed to recall)? • Through what task(s) will students demonstrate the desired understandings?
Opportunities	
• Discussions and student presentations • Checking for understanding (using response boards) • Ticket out the door, Cornell note summary, and error analysis • *Performance Tasks* within a Unit • Teacher-created assessments/quizzes	• Unit assessments • Teacher-created quizzes and/or mid-unit assessments • *Illustrative Mathematics* tasks (https://www.illustrativemathematics.org/) • Performance tasks

Topic 8: Polynomials & Factoring (Part 1)

Transfer Goals
1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution. 2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience. 3) Construct viable arguments and critique the reasoning of others using precise mathematical language.

Essential Questions:	**Standards:** A-SSE 1a, A-SSE 1B, A-SSE 2, A-APR 1 **Timeframe:** 2 weeks/12 days **Start Date:** March 9, 2018 **Assessment Dates:** March 23-24, 2018
• Can two algebraic expressions that appear to be different be equivalent? • How are the properties of real numbers related to polynomials? • How do polynomials form a system that is analogous to integers?	

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Resources
1 Day	***Topic Opener: Planning a Garden Plot*** ***p. 485 Common Core Performance Task***					
2 Days	**Lesson 8-1: Adding and Subtracting Polynomials** (pp. 486-491) SMP 1,2,3,4,6 **A-APR 1**	**Focus Question(s):** • How do polynomials form a system that is analogous to integers with respect to addition and subtraction? • What is the rule for simplifying like terms when you are adding and subtracting polynomials? **Inquiry Question:** p. 485 Planning a Garden Plot	• Monomials can be used to form larger expressions called polynomials. • Polynomials can be added and subtracted. • Similar to integers, polynomials are closed under the operations of addition and subtraction.	**Vocabulary**: monomial, degree of a monomial, polynomial, standard form of a polynomial, degree of a polynomial, binomial, trinomial **Students will know…** • That polynomials can be classified by degrees and by the number of terms. • The standard form of a polynomial. • How to add and subtract polynomials.	• Classify, add, and subtract polynomials. • Simplify polynomial expressions.	**Notes:** • Emphasize to students that their final answer must be written in standard form. • Include an example where students have to find the perimeter of a figure. **Problems to Emphasize:** • 8-1 Think About a Plan. • **CC Problems:** p. 489-490 #7,41,43 **Thinking Maps:** Create a Bridge Map to see the relationship b/w polynomials and their degrees.

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
1 Day	**Lesson 8-2: Multiplying and Factoring** (pp. 492-496) SMP 1,2,3,4 **A-APR 1**	**Focus Question(s):** • How do polynomials form a system that is analogous to integers with respect to multiplication? • How can the properties of real numbers be applied to multiplying a monomial to a polynomial? **Inquiry Question:** p. 492 Solve It!	• A monomial can be multiplied by a polynomial using the Distributive Property. • Similar to integers, polynomials are closed under the operation of multiplication.	**Vocabulary**: monomial, polynomial, Distributive Property **Students will know…** • The Distributive Property. • The appropriate rules of exponents that can be applied when multiplying a power to a polynomial.	• Multiply a monomial by a polynomial using the Distributive Property.	**Note:** • Include concepts from sections 8.1-8.2 on quiz. **Problems to emphasize:** Page: 495 #8, 35, 36, 41 **Thinking Maps:** Use Flow Maps to give the steps for multiplying a monomial and a trinomial. **CC Problems:** #8, 35, 36, 40, 41 **STEM:** #43
2 Days	**Lesson 8-3: Multiplying Binomials (FOIL & with Trinomials)** (pp. 498-503) SMP 1,2,3,4,7,8 **A-APR 1** Day 1: Multiplying binomials; Finding areas Day 2: Multiplying binomials and trinomials; Finding volumes	**Focus Question(s):** • How do polynomials form a system that is analogous to integers with respect to multiplication? • What are the methods for multiplying two binomials? What do all of the methods have in common? **Inquiry Question:** p. 498 Solve It!	• There are several ways to find the product of two binomials, including models, algebra, and tables. • The properties of real numbers can be used to multiply two binomials. • Similar to integers, polynomials are closed under the operation of multiplication.	**Vocabulary**: binomial, trinomial **Students will know…** • The procedures for using the Distributive Property to multiply polynomials. • That multiplying polynomials using algebra tiles or with a table will result in the same expression as using the Distributive Property.	• Multiply two binomials. • Multiply a binomial by a trinomial.	**Notes:** Include examples where students have to… • Multiply binomials before adding or subtracting it to a polynomial. • Find the area of a rectangle. • Find the area of a shaded region. • Find the volume of a prism. **Problems to emphasize:** • Page: 501 and 502 # 5, 6, 42 • Think About a Plan 8-3 • P. 503 Apply What You've Learned **Thinking Maps:** Use Flow Maps to give the steps for using the distributive property with binomials. **CC Problems:** #5, 6, 7, 42, 43, 44, 45

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
2 Days	**Lesson 8-4: Multiplying Special Cases** (pp. 504-509) SMP 1,2,3,4,7,8 **A-APR 1** Day 1: Lesson Day 2: Review Day 3: Teacher Generated Quiz #2	**Focus Question(s):** • How do polynomials form a system that is analogous to integers with respect to multiplication? • How do you model the square of a binomial? • How do you model the product of a sum and difference? **Inquiry Question:** p. 504 Solve It!	• There are special rules you can use to simplify the square of a binomial or the product of a sum and difference. • Similar to integers, polynomials are closed under the operation of multiplication.	**Vocabulary**: square of a binomial, product of a sum and difference **Students will know…** • That the square of a binomial can be modeled using $a^2+2ab+b^2$ or $a^2-2ab+b^2$. • The product of a sum and difference can be modeled using a^2-b^2.	• Simplify square of a binomial using special rules. • Simplify product of a sum and difference using special rules.	**Note:** • Include concepts from sections 8.3-8.4 on quiz. **Problems to emphasize:** • Page: 508 #50, 51, 54, 57 • 8-3 Think About a Plan **Thinking Maps:** Use Tree Map to give details for "special cases." **CC Problems:** #8, 50, 51, 53, 54, 55, 57 **STEM:** #52
1 Day	***Performance Task*** ***(See attached documents for details)***					
1 Day	**Review Topic 8 Concepts & Skills** Use Textbook Resources and/or Teacher Created Items					
2 Days	**Topic 8 Assessment** (Created and provided by PUSD)					

Powers of 11

A-APR.C.5 and A-APR.A.1

Situation:

Felicia notices what appears to be an interesting pattern between 11 and powers of $x+1$:

$$11^0 = 1 \qquad (x+1)^0 = 1$$

$$11^1 = 11 \qquad (x+1)^1 = x+1$$

$$11^2 = 121 \qquad (x+1)^2 = x^2+2x+1$$

The digits of the number of 11^n are the same as the coefficients of the polynomial $(x+1)^n$. Is this always true?

Task 1:

Does this pattern continue for $n=3$ and $n=4$?

Task 2:

What is the answer to Felicia's question?

Solution

The pattern continues! We have $11^3 = 1331$ and $(x+1)^3 = x^3 + 3x^2 + 3x + 1$, and $11^4 = 14641$ and $(x+1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$. To see why this is happening, let's start with the simple case of $n = 2$. Here we have $11^2 = 121$ and $(x+1)^2 = x^2 + 2x + 1$. The hundreds digit of 11^2 is the same as the coefficient of x^2 in $(x+1)^2$ and similarly for the tens digit and ones digit. In this case, however, we can start to see what is happening by substituting $x = 10$ in the expression $(x+1)^2$. When $x = 10$ we have $x + 1 = 11$. Moreover, $x^2 = 100$ and $x = 10$ so the coefficient of x^2 in $(x+1)^2$ tells us how many hundreds we have (1), the coefficient of x tells us how many tens (2), and the constant term is the number of ones (1).

Now we see why the pattern continues: the binomial theorem tells us how to expand $(x+1)^n$, and the digits of 11^n are what we get when we substitute $x = 10$. When $n = 3$, we have

$$(x+1)^3 = x^3 + 3x^2 + 3x + 1.$$

Substituting $x = 10$ we find

$$\begin{aligned} 11^3 &= 1000 + 3 \times 100 + 3 \times 10 + 1 \\ &= 1331 \end{aligned}$$

When $n = 4$ we have (using the binomial theorem or successively multiplying polynomials)

$$(x+1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1.$$

Substituting $x = 10$ we find

$$\begin{aligned} 11^4 &= 10,000 + 4 \times 1,000 + 6 \times 100 + 4 \times 10 + 1 \\ &= 14,641 \end{aligned}$$

So Felicia is right that there is a very close relationship between the coefficients of the polynomial $(x+1)^n$ and the digits of the number 11^n. The two are not always the same, however, because the coefficients of $(x+1)^n$ depend on n and become larger as n grows. The digits of 11^n, however, can only range from 0 to 9 because of our decimal system for writing numbers. So when $n = 5$, for example,

$$(x+1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1.$$

Substituting $x = 10$ we find

$$\begin{aligned} 11^5 &= 100,000 + 5 \times 10,000 + 10 \times 1,000 + 10 \times 100 + 5 \times 10 + 1 \\ &= 161,051 \end{aligned}$$

In this case, both the hundreds and the thousands get regrouped when we write this number in base 10. If we were allowed to put any whole number we like in each place value, then the coefficients of $(x+1)^n$ would give us directly an expression for 11^n for all values of n.