

Advanced Geometry

Advanced Geometry builds on Algebra I concepts and increases students' knowledge of shapes and their properties through geometry-based applications, many of which are observable in aspects of everyday life. This knowledge helps develop visual and spatial sense and strong reasoning skills. The Advanced Geometry course requires students to make conjectures and to use reasoning to validate or negate these conjectures. The use of proofs and constructions is a valuable tool that enhances reasoning skills and enables students to better understand more complex mathematical concepts. Technology should be used to enhance the students' mathematical experience, not replace their reasoning abilities.

Students will:

GEOMETRY

Congruence

Experiment with transformations in the plane.

1. Know the precise definitions of angle, circle, perpendicular line, parallel line, and line segment based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (G-CO1)
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch.) (G-CO2)
3. Given a rectangle, parallelogram, trapezoid, or rectangular polygon, describe the rotations and reflections that carry it onto itself. (G-CO3)
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (G-CO4)
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. (G-CO5)

Understand congruence in terms of rigid motions. (*Build on rigid motions as a familiar starting point for development of concept of geometric proof.*)

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (G-CO6)
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (G-CO7)

8. Explain why the criteria for triangle congruence, angle-side-angle (ASA), side-angle-side (SAS), and side-side-side (SSS), follow from the definition of congruence in terms of rigid motions. (G-CO8)

Prove geometric theorems. (Focus on validity of underlying reasoning while using variety of ways of writing proofs.)

9. Prove theorems about lines and angles. *Theorems include vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; and points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* (G-CO9)
10. Prove theorems about triangles. *Theorems include measures of interior angles of a triangle sum to 180° , base angles of isosceles triangles are congruent, the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length, and the medians of a triangle meet at a point.* (G-CO10)
11. Prove theorems about parallelograms. *Theorems include opposite sides are congruent, opposite angles are congruent; the diagonals of a parallelogram bisect each other; and conversely, rectangles are parallelograms with congruent diagonals.* (G-CO11)

Make geometric constructions. (Formalize and explain processes.)

12. Make formal geometric constructions with a variety of tools and methods such as compass and straightedge, string, reflective devices, paper folding, and dynamic geometric software. *Constructions include copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* (G-CO12)
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (G-CO13)

Similarity, Right Triangles, and Trigonometry

Understand similarity in terms of similarity transformations.

14. Verify experimentally the properties of dilations given by a center and a scale factor. (G-SRT1)
- A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. (G-SRT1a)
 - The dilation of a line segment is longer or shorter in the ratio given by the scale factor. (G-SRT1b)
15. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (G-SRT2)
16. Use the properties of similarity transformations to establish the angle-angle (AA) criterion for two triangles to be similar. (G-SRT3)

Prove theorems involving similarity.

17. Prove theorems about triangles. *Theorems include a line parallel to one side of a triangle divides the other two proportionally, and conversely; and the Pythagorean Theorem proved using triangle similarity.* (G-SRT4)
18. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (G-SRT5)

Define trigonometric ratios and solve problems involving right triangles.

19. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle leading to definitions of trigonometric ratios for acute angles. (G-SRT6)
20. Explain and use the relationship between the sine and cosine of complementary angles. (G-SRT7)
21. Use trigonometric ratios and Pythagorean Theorem to solve right triangles in applied problems.* (G-SRT8)

Apply trigonometry to general triangles.

22. (+)Derive the formula $A = (\frac{1}{2})ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. (G-SRT9)
23. (+)Prove the Law of Sines and the Law of Cosines and use them to solve problems. (G-SRT10)
24. (+)Understand and apply the Law of Sines and the Law of Cosines to find the unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). (G-SRT11)

Circles

Understand and apply theorems about circles.

25. Prove that all circles are similar. (G-C1)
26. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* (G-C2)
27. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. (G-C3)
28. Construct a tangent line from a point outside a given circle to the circle. (G-C4)

Find area lengths and areas of sectors of circles. (*Radian introduced only as unit of measure.*)

29. Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. (G-C5)

Expressing Geometric Properties with Equations

Translate between the geometric description and equation for a conic section.

30. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. (G-GPE1)

Use coordinates to prove simple geometric theorems algebraically. (Include distance formula; relate to Pythagorean Theorem.)

31. Use coordinates to prove simple geometric theorems algebraically. (G-GPE4)

Example: Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0,2)$.

32. Prove the slope criterion for parallel and perpendicular lines, and use them to solve geometric problems, (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). (G-GPE5)

33. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (G-GPE6)

34. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.* (G-GPE7)

Use coordinates to prove simple geometric theorems algebraically. ▮

35. Determine areas and perimeters of regular polygons, including inscribed or circumscribed polygons, given the coordinates of vertices and other characteristics. ▮

Geometric Measurement and Dimension

Explain volume formulas and use them to solve problems.

36. Give an informal argument for the formulas for the circumference of a circle; area of a circle; and volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* (G-GMD1)

37. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.* (G-GMD3)

38. Determine the relationship between surface areas of similar figures and volumes of similar figures. ▮

Visualize relationships between two-dimensional and three-dimensional objects.

39. Identify the shapes of two-dimensional cross-sections of three-dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects. (G-GMD4)

Modeling with Geometry

Apply geometric concepts in modeling situations.

40. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* (G-MG1)
41. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, British Thermal Units (BTUs) per cubic foot).* (G-MG2)
42. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost, working with typographic grid systems based on ratios).* (G-MG3)

STATISTICS AND PROBABILITY

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data. (Link to data from simulations or experiments.)

43. Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A , and the conditional probability of B given A is the same as the probability of B . (S-CP3)
44. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample to decide if events are independent and to approximate conditional probabilities. (S-CP4)

Example: Collect data from a random sample of students in your school on their favorite subject among mathematics, science, and English. Estimate the probability that a randomly selected student from your school will favor science given the student is in tenth grade. Do the same for other subjects and compare the results.

45. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. (S-CP5)

Example: Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

46. Find the conditional probability of A given B as the fraction of B 's outcomes that also belong to A , and interpret the answer in terms of the model. (S-CP6)

47. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. (S-CP7)
48. (+)Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. (S-CP8)
49. (+)Use permutations and combinations to compute probabilities of compound events and solve problems. (S-CP9)

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions. (Introductory; apply counting rules.)

50. (+)Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
(S-MD6)
51. (+)Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). (S-MD7)