Advanced Algebra II with Trigonometry

Algebra II with Trigonometry is a course designed to extend students' knowledge of Algebra I with additional algebraic and trigonometric content. Mastery of the content standards for this course is necessary for student success in higher-level mathematics. The use of appropriate technology is encouraged for numerical and graphical investigations that enhance analytical comprehension.

Algebra II with Trigonometry is required for all students pursuing the Alabama High School Diploma with Advanced Academic Endorsement. Prerequisites for this course are Algebra I and Geometry. If a student chooses to take the Algebraic Connections course, it must be taken prior to the Algebra II with Trigonometry course.

Students will:

NUMBER AND QUANTITY

The Complex Number System

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. (N-CN1)

2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. (N-CN2)

3. (+)Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. (N-CN3)

Use complex numbers in polynomial identities and equations. ***(Polynomials with real coefficients.)***

4. Solve quadratic equations with real coefficients that have complex solutions. (N-CN7)

5. (+) Extend polynomial identities to the complex numbers. (N-CN8)

   Example: Rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.

6. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. (N-CN9)

Vector and Matrix Quantities

Perform operations on matrices and use matrices in applications.

7. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. (Use technology to approximate roots.) (N-VM6)

8. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. (N-VM7)

9. (+) Add, subtract, and multiply matrices of appropriate dimensions. (N-VM8)

10. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. (N-VM9)

11. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. (N-VM10)

ALGEBRA

Seeing Structure in Expression

Interpret the structure of expressions. ***(Polynomials and rational.)***

12. Interpret expressions that represent a quantity in terms of its context. * (A-SSE1)

    a. Interpret parts of an expression such as terms, factors, and coefficients. (A-SSE1a)
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. (A-SSE1b)

    Example: Interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.

13. Use the structure of an expression to identify ways to rewrite it. (A-SSE2)

    Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

14. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.* (A-SSE4)

    Example: Calculate mortgage payments.

Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials. ***(Beyond quadratic.)***

15. Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A-APR1)

Understand the relationship between zeros and factors of polynomials.

16. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. (A-APR2)

17. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. (A-APR3)

Use polynomial identities to solve problems.

18. Prove polynomial identities and use them to describe numerical relationships. (A-APR4)

    Example: The polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

Rewrite rational expressions. ***(Linear and quadratic denominators.)***

19. Rewrite simple rational expression in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system. (A-APR6)

Creating Equations*

Create equations that describe numbers or relationships. ***(Equations using all available types of expressions, including simple root functions.)***

20. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* (A-CED1)

21. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A-CED2)

22. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. (A-CED3)

    Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods.

23. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (A-CED4)

Example: Rearrange Ohm's law $V = IR$ to highlight resistance R.

Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning, and explain the reasoning. ***(Simple rational and radical.)***

24. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (A-REI2)

Solve equations and inequalities in one variable.

25. Recognize when the quadratic formula gives complex solutions, and write them as $a \pm bi$ for real numbers a and b. (A-REI4b)

Solve systems of equations.

26. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater). (A-REI9)

Represent and solve equations and inequalities graphically. ***(Combine polynomial, rational, radical, absolute value, and exponential functions.)***

27. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations, Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* (A-REI11)

Conic Sections

Understand the graphs and equations of conic sections. (Emphasize understanding graphs and equations of circles and parabolas.)

28. Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.

Example: Graph $x^2 - 6x + y^2 - 12y + 41 = 0$ or $y^2 - 4x + 2y + 5 = 0$

a. Formulate equations of conic sections from their determining characteristics.

Example: Write the equation of an ellipse with center (5, -3), a horizontal major axis of length 10, and a minor axis of length 4.

Answer: $\frac{(x-5)^2}{} + \frac{(y+3)^2}{} = 1$.

FUNCTIONS

Interpreting Functions

Interpret functions that arise in applications in terms of the context. ***(Emphasize selection of appropriate models.)***

29. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.* (F-IF5)

    Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Analyze functions using different representations. ***(Focus on using key features to guide selection of appropriate type of model function.)***

30. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* (F-IF7)

    a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (F-IF7b)
    b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. (F-IF7c)
    c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (F-IF7e)

31. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (F-IF8)

32. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (F-IF9)

    Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

Build a function that models a relationship between two quantities. ***(Include all types of functions studied.)***

33. Write a function that describes a relationship between two quantities.* (F-BF1)

    a. Combine standard function types using arithmetic operations. (F-BF1b)

Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Build new functions from existing functions. ***(Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types.)***

34. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (F-BF3)

35. Find inverse function. (F-BF4)

    a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse, and write an expression for the inverse. (F-BF4a)

       Example: $f(x) = 2x^3$ or $f(x) = (x+1)/(x–1)$ for $x \neq 1$.

Linear, Quadratic, and Exponential Models*

Construct and compare linear, quadratic, and exponential models and solve problems. ***(Logarithms as solutions for exponentials.)***

36. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers, and the base b is 2, 10, or e; evaluate the logarithm using technology. (F-LE4)

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

37. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (F-TF1)

38. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (F-TF2)

39. Define the six trigonometric functions using ratios of the sides of a right triangle, coordinates on the unit circle, and the reciprocal of other functions.

Model periodic phenomena with trigonometric functions.

40. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* (F-TF5)

STATISTICS AND PROBABILITY

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions. ***(Include more complex situations.)***

41. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). (S-MD6)

42. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of game). (S-MD7)

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data. ***(Link to data from simulations or experiments.)***

43. Describe events as subsets of a sample space (the set of outcomes), using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). (S-CP1)

44. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. (S-CP3)

45. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. (S-CP4)

    Example: Collect data from a random sample of students in your school on their favorite subject among mathematics, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

46. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. (S-CP5)

    Example: Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

47. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. (S-CP6)

48. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. (S-CP7)

49. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. (S-CP8)

50. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. (S-CP9)