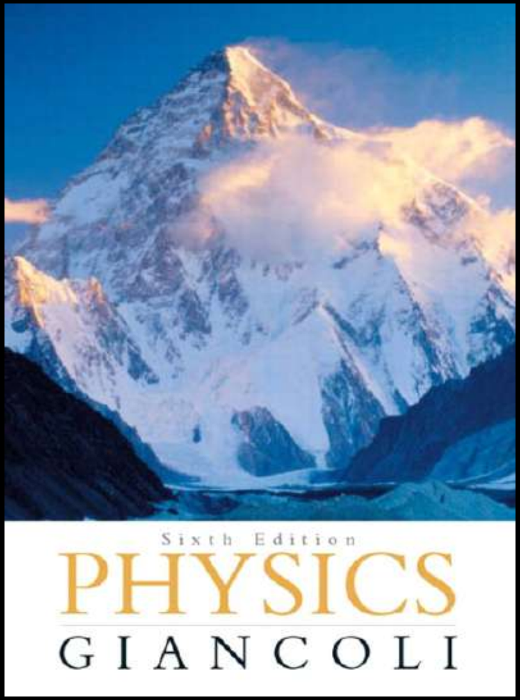



# Lecture PowerPoints

## Chapter 8

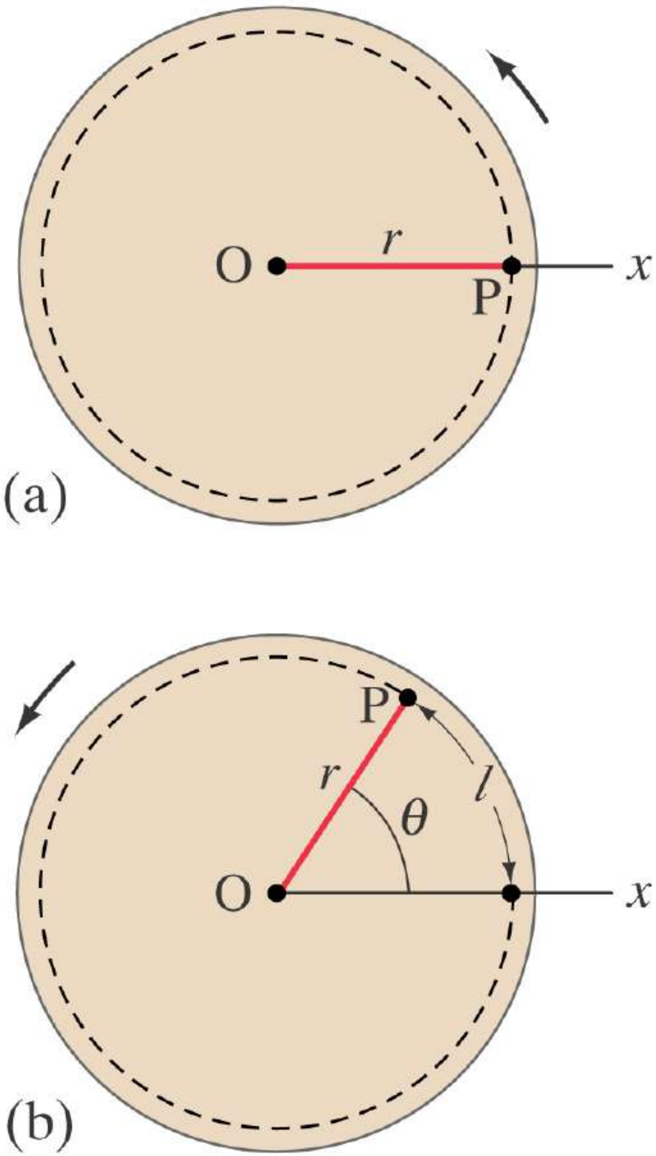
***Physics: Principles with Applications, 6th edition***

**Giancoli**

# Chapter 8

# Rotational Motion

# 8-1 Angular Quantities



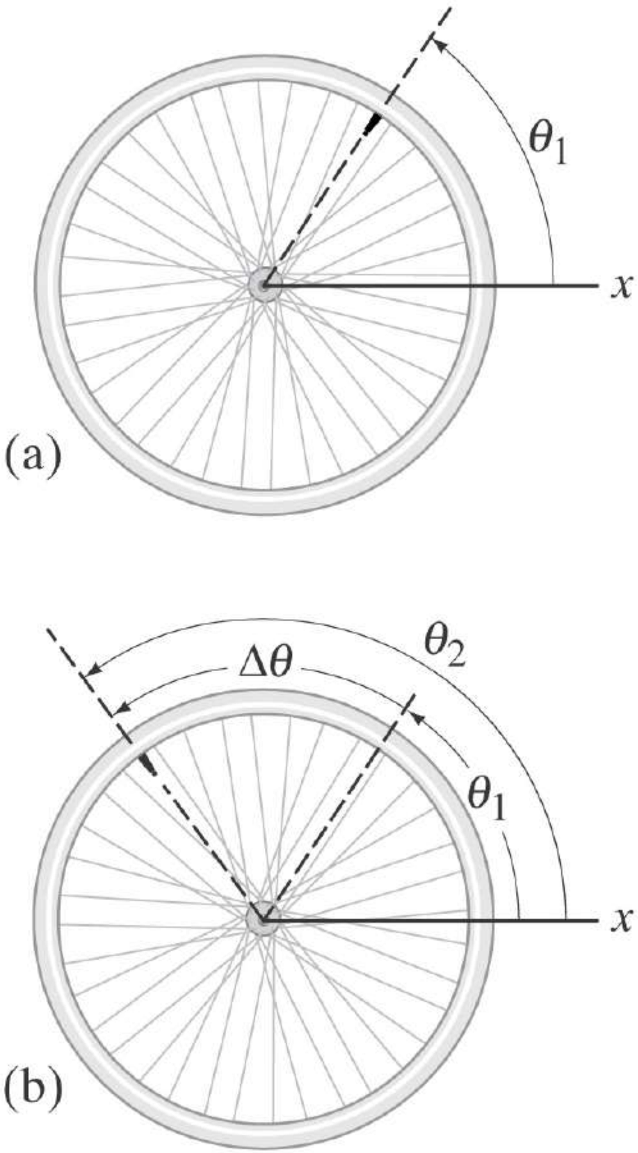


In purely **rotational** motion, all points on the object move in **circles** around the **axis** of rotation ("*O*"). The **radius** of the circle is *r*. All points on a straight line drawn through the axis move through the same **angle** in the same **time**. The angle $\theta$ in radians is defined:

$$\theta = \frac{l}{r} \qquad \textbf{(8-1a)}$$

where $l$ is the **arc length**.

# 8-1 Angular Quantities






**Angular displacement:**

$$\Delta\theta = \theta_2 - \theta_1$$

The average angular **velocity** is defined as the total angular **displacement** divided by **time**:

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t} \quad \text{(8-2a)}$$

The **instantaneous** angular velocity:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \quad \text{(8-2b)}$$

# 8-1 Angular Quantities

The angular **acceleration** is the rate at which the angular velocity changes with time:

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{\Delta \omega}{\Delta t} \qquad \text{(8-3a)}$$

The **instantaneous** acceleration:

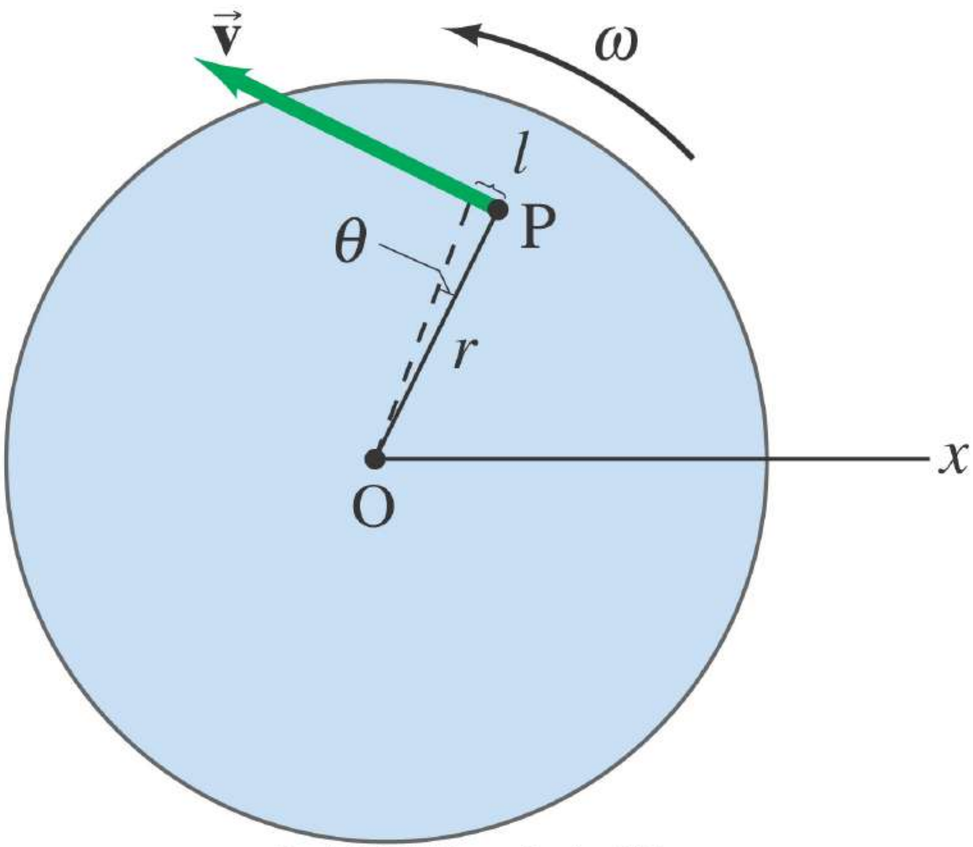
$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} \qquad \text{(8-3b)}$$

# 8-1 Angular Quantities

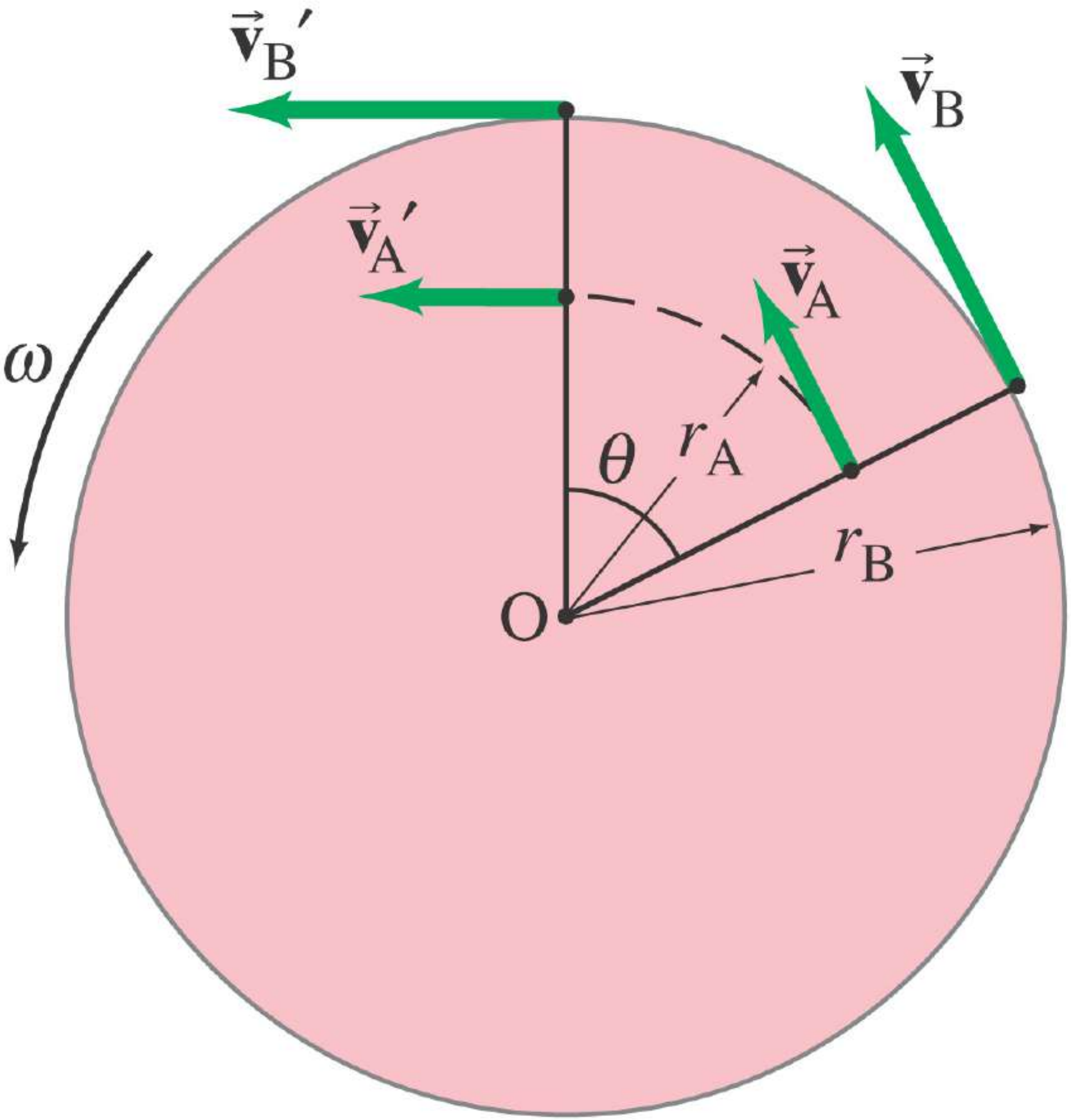
Every point on a rotating body has an **angular velocity** $\omega$ and a **linear velocity** v.

They are related:

$$v = r\omega \qquad \text{(8-4)}$$



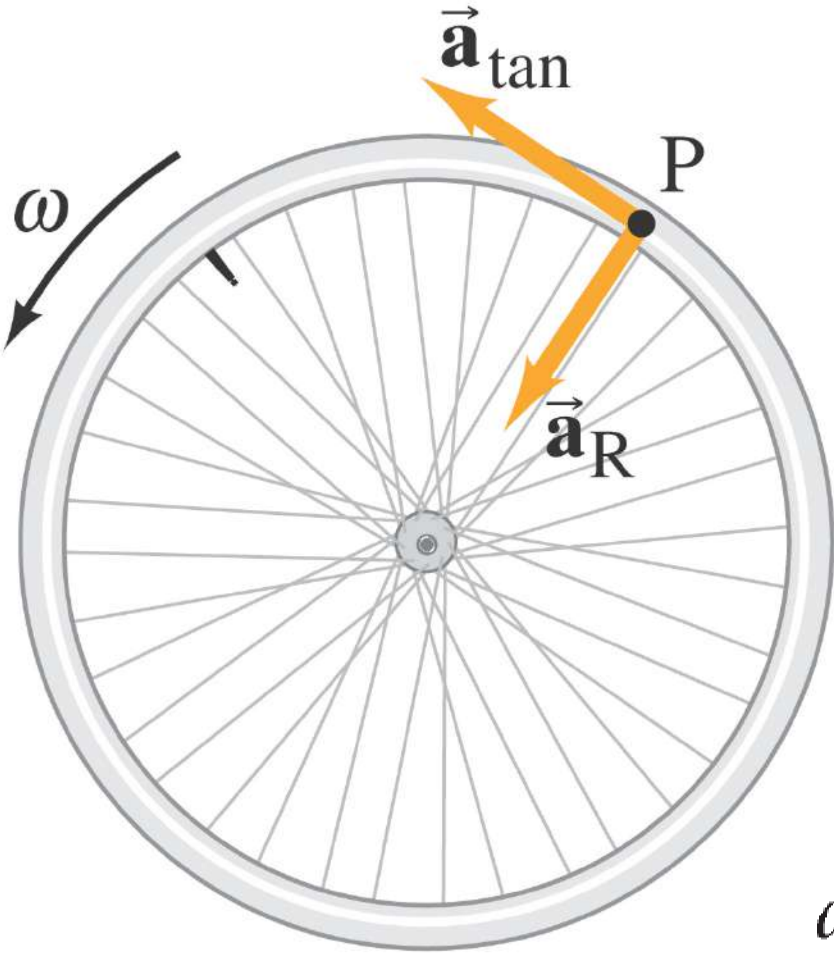
# 8-1 Angular Quantities






Therefore, objects **farther** from the axis of rotation will move **faster**.

# 8-1 Angular Quantities





If the angular velocity of a rotating object **changes**, it has a **tangential** acceleration:

$$a_{tan} = r\alpha \qquad \text{(8-5)}$$

Even if the angular velocity is constant, each point on the object has a **centripetal** acceleration:

$$a_R = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r \qquad \text{(8-6)}$$

# 8-1 Angular Quantities

Here is the correspondence between linear and rotational quantities:

**TABLE 8–1 Linear and Rotational Quantities**

| Linear | Type | Rotational | Relation |
|---|---|---|---|
| $x$ | displacement | $\theta$ | $x = r\theta$ |
| $v$ | velocity | $\omega$ | $v = r\omega$ |
| $a_{\tan}$ | acceleration | $\alpha$ | $a_{\tan} = r\alpha$ |

# 8-1 Angular Quantities

The **frequency** is the number of complete **revolutions** per second: $f = \frac{\omega}{2\pi}$

Frequencies are measured in **hertz**.

$$1\ \mathrm{Hz} = 1\ \mathrm{s}^{-1}$$

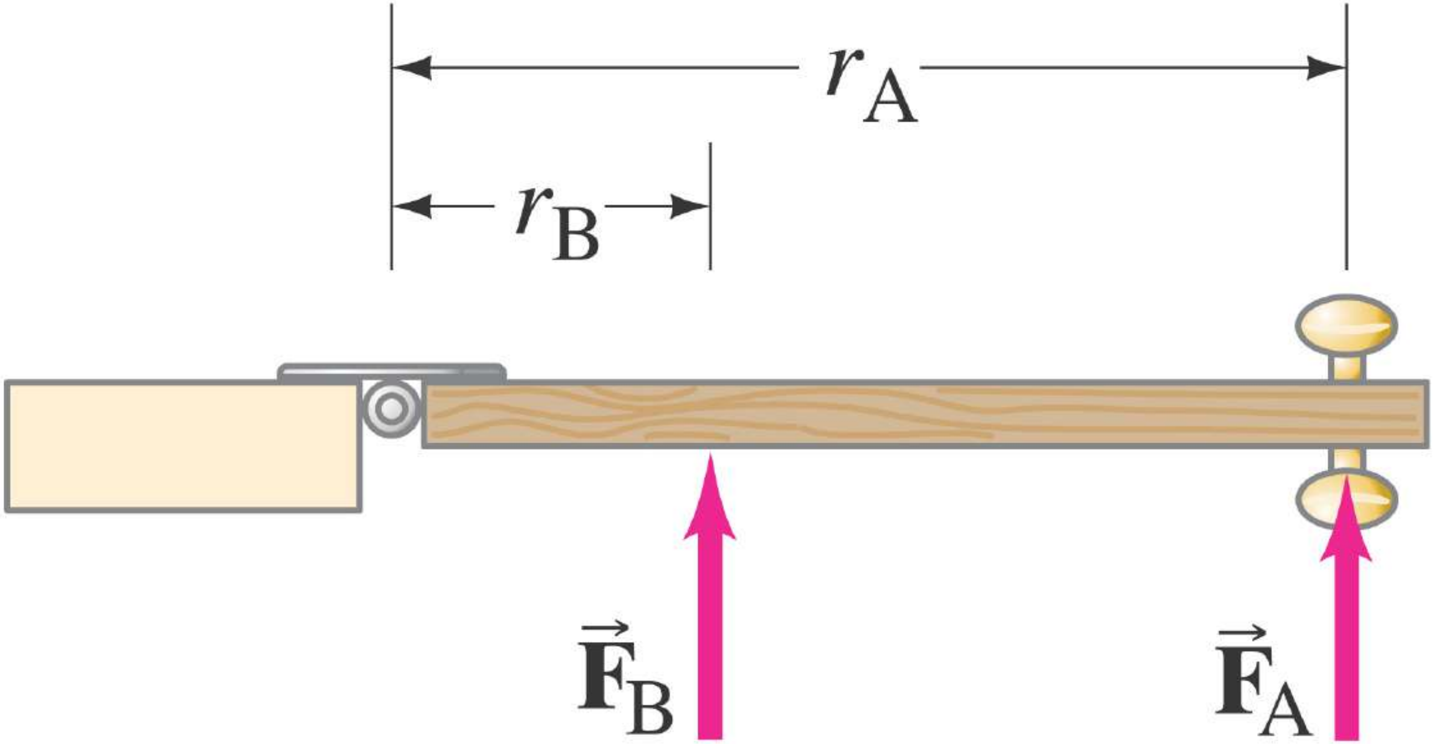
The **period** is the time one revolution takes:

$$T = \frac{1}{f} \qquad (8\text{-}8)$$

# 8-4 Torque

To make an object **start** rotating, a **force** is needed; the **position** and **direction** of the force matter as well.

The **perpendicular** distance from the axis of rotation to the line along which the force acts is called the **lever arm**.



# 8-4 Torque

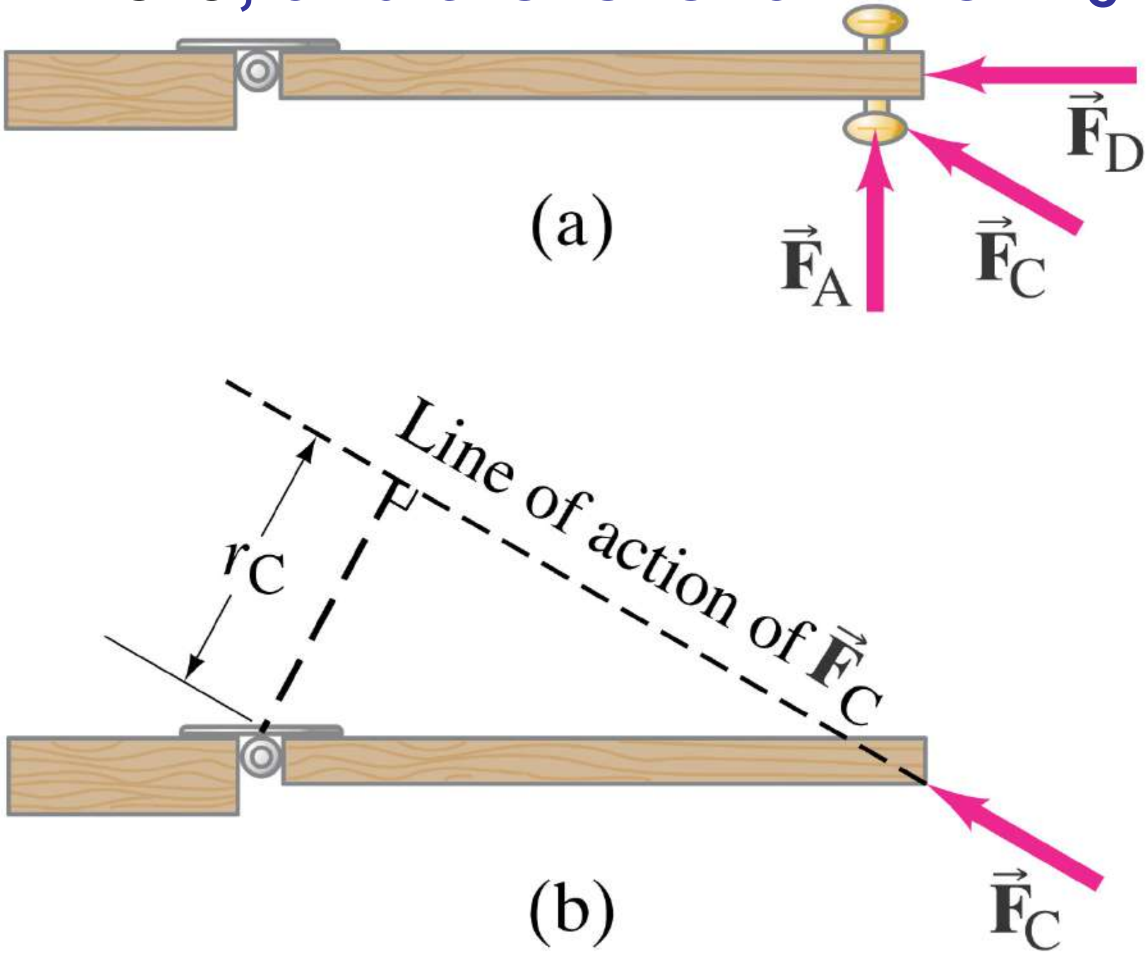
(a) (b)



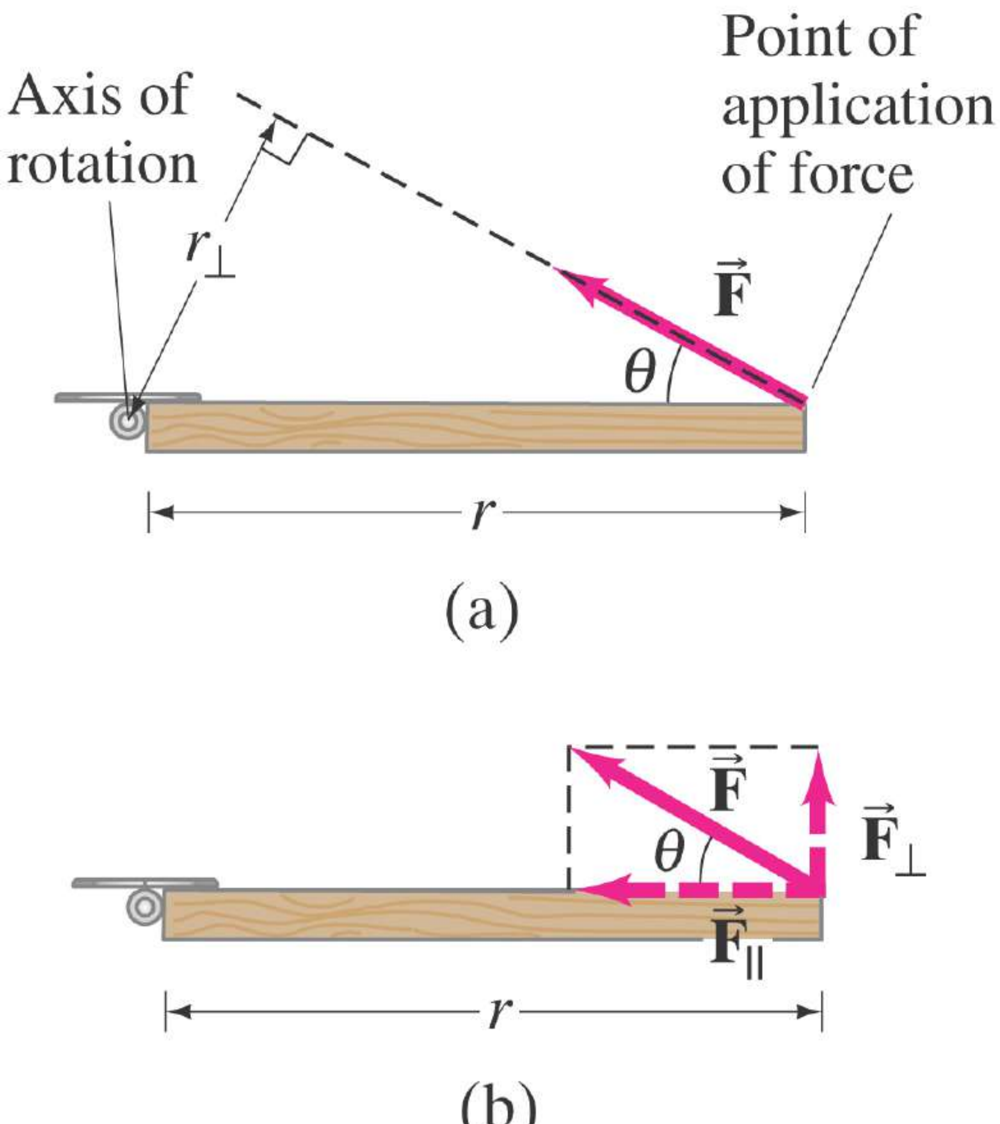
**A longer lever arm is very helpful in rotating objects.**

# 8-4 Torque

**Here, the lever arm for $F_A$ is the distance from the knob to the hinge; the lever arm for $F_D$ is zero; and the lever arm for $F_C$ is as shown.**



# 8-4 Torque





The **torque** is defined as:

$$\tau = r_\perp F$$

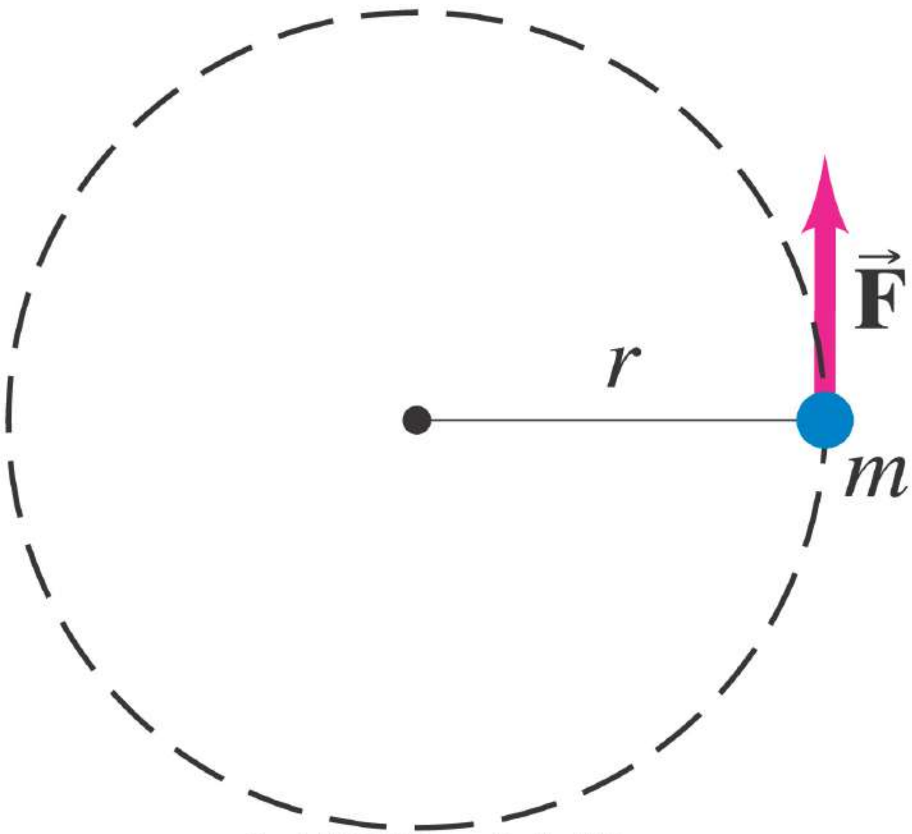
$$= r F \sin\theta$$

**(on formula sheet)**

# 8-5 Rotational Dynamics; Torque and Rotational Inertia

Knowing that $F = ma$ we see that $\tau = mr^2\alpha$ **(8-11)**

and $a_{\tan} = r\alpha$






This is for a single point mass; what about an **extended** object?

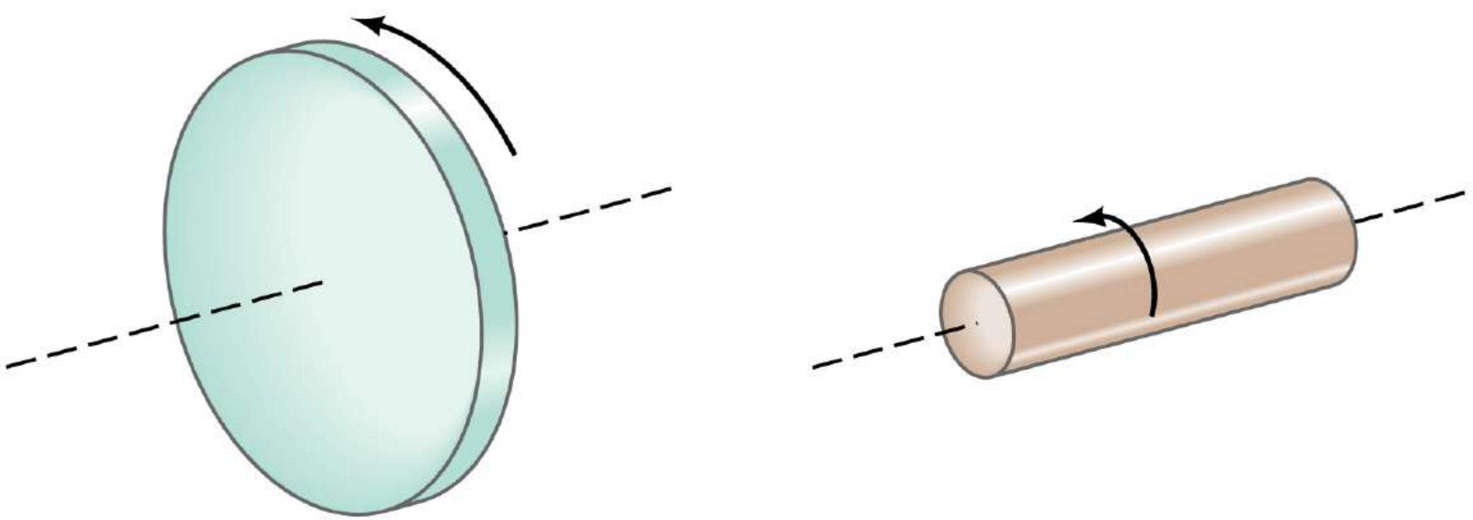
As the angular acceleration is the same for the whole object, we can write:

$$\Sigma\tau = \left(\Sigma mr^2\right)\alpha$$ **(8-12)**

# 8-5 Rotational Dynamics; Torque and Rotational Inertia

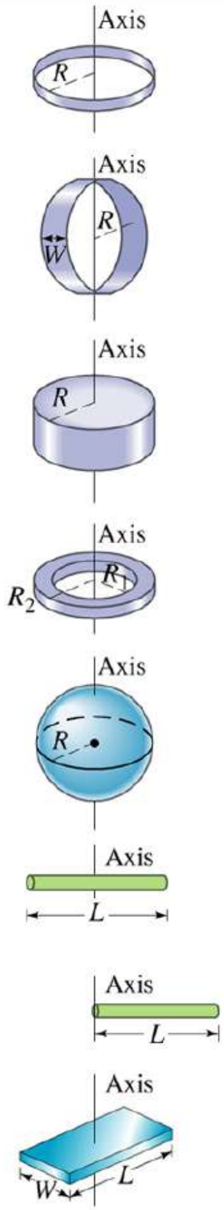
The quantity $I = \Sigma mr^2$ is called the **rotational inertia** (or moment of inertia) of an object. **Moment of Inertia** is the resistance to angular acceleration.

The **distribution** of mass matters here – these two objects have the same mass, but the one on the left has a greater **rotational inertia**, as so much of its mass is far from the axis of rotation.



# 8-5 Rotational Dynamics; Torque and Rotational Inertia

| | Object | Location of axis | Moment of inertia |
|---|---|---|---|
| (a) | **Thin hoop,** radius $R$ | Through center | $MR^2$ |
| (b) | **Thin hoop,** radius $R$ width $W$ | Through central diameter | $\frac{1}{2}MR^2 + \frac{1}{12}MW^2$ |
| (c) | **Solid cylinder,** radius $R$ | Through center | $\frac{1}{2}MR^2$ |
| (d) | **Hollow cylinder,** inner radius $R_1$ outer radius $R_2$ | Through center | $\frac{1}{2}M(R_1^2 + R_2^2)$ |
| (e) | **Uniform sphere,** radius $R$ | Through center | $\frac{2}{5}MR^2$ |
| (f) | **Long uniform rod,** length $L$ | Through center | $\frac{1}{12}ML^2$ |
| (g) | **Long uniform rod,** length $L$ | Through end | $\frac{1}{3}ML^2$ |
| (h) | **Rectangular thin plate,** length $L$, width $W$ | Through center | $\frac{1}{12}M(L^2 + W^2)$ |



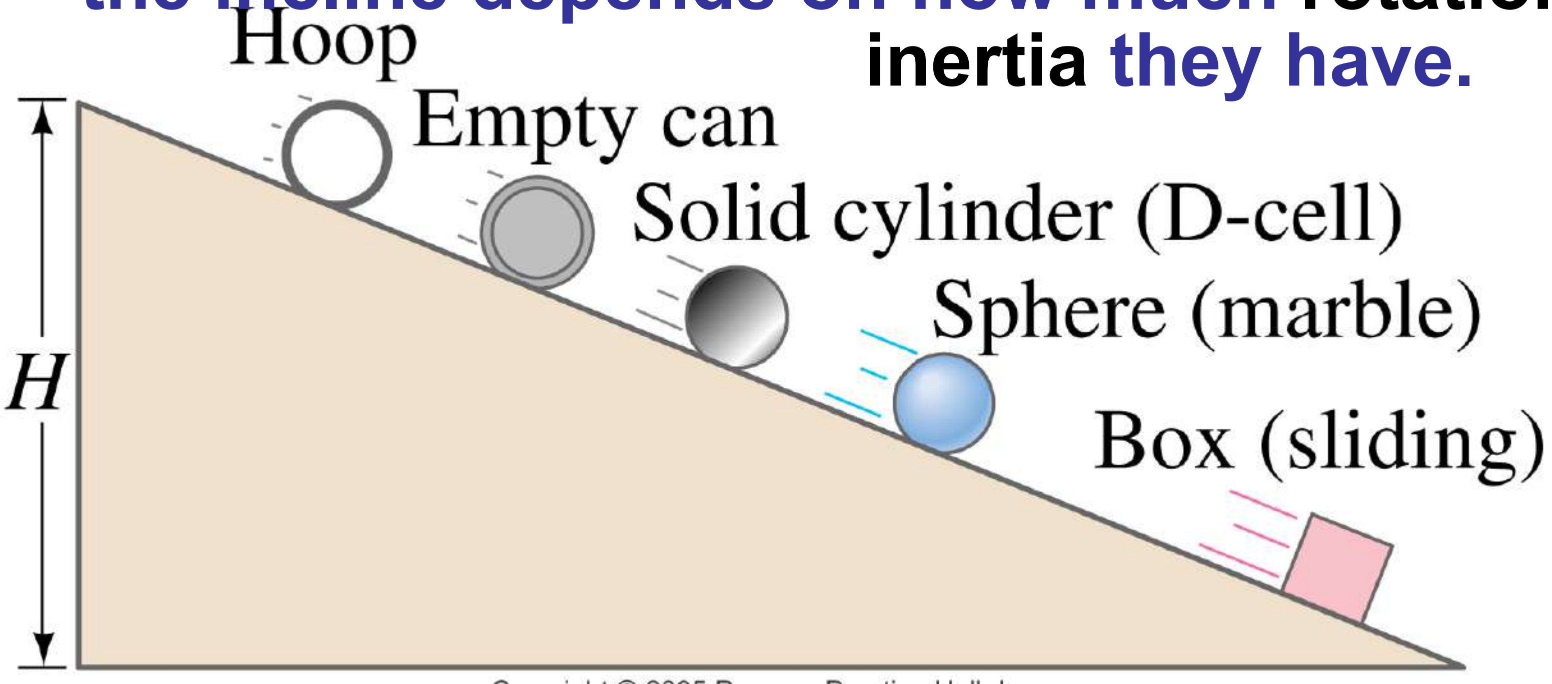



The **rotational inertia** of an object depends not only on its **mass distribution** but also the location of the **axis** of rotation – compare (f) and (g), for example.

# 8-7 Rotational Kinetic Energy

When using **conservation** of energy, both **rotational** and **translational** kinetic energy must be taken into account.

All these objects have the same **potential** energy at the top, but the time it takes them to get down the incline depends on how much **rotational inertia** they have.



# 8-8 Angular Momentum and Its Conservation

In analogy with linear momentum, we can define **angular momentum** *L*:

$$L = I\omega \quad (8\text{-}18)$$

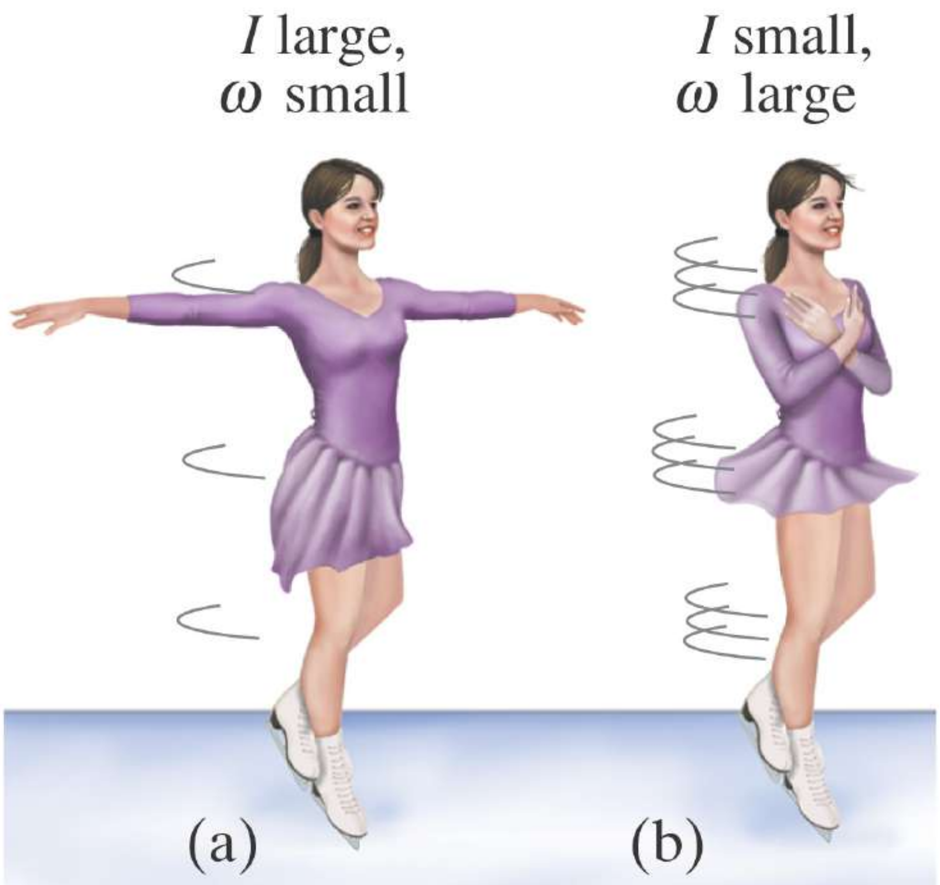
We can then write the total **torque** as being the rate of **change** of angular **momentum**.

If the net torque on an object is zero, the total angular momentum is **constant**.

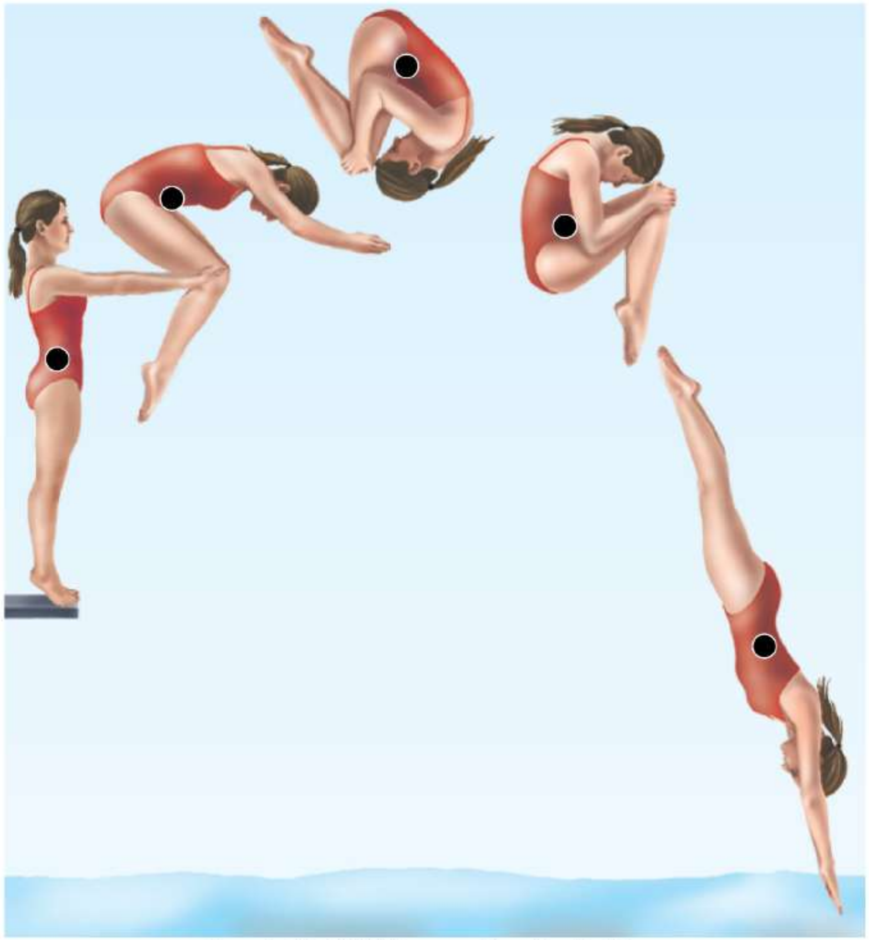
$$I\omega = I_0\omega_0 = \text{constant}$$

# 8-8 Angular Momentum and Its Conservation

Therefore, systems that can **change** their rotational inertia through internal forces will also change their **rate** of rotation:





# 8-9 Vector Nature of Angular Quantities

The **angular velocity** vector points along the **axis** of rotation; its direction is found using a **right hand rule**: