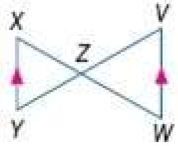
9-3 Similar Triangles

Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.

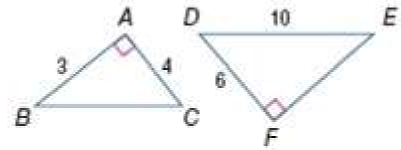
1. 

SOLUTION:

We can prove $\Delta YXZ \sim \Delta VWZ$ by AA Similarity.

1) We can prove that $\angle XYZ \cong \angle WVZ$ because they are alternate interior angles and $\overline{YX} \parallel \overline{VW}$. (Alternate Interior Angles Theorem)

2) We can prove that $\angle XZY \cong \angle WZV$ because they are vertical angles. (Vertical angles Theorem)

2. 

SOLUTION:

We can prove $\Delta BAC \sim \Delta DFE$ by SAS Similarity.

1) We can prove that $\angle A \cong \angle F$ because they are both right angles.(All right angles are congruent.)

2) Since these are right triangles, we can use the Pythagorean Theorem to find the missing sides

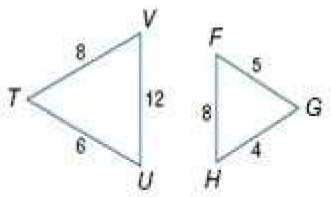
$$FD^2 + FE^2 = DE^2$$
$$6^2 + FE^2 = 10^2$$
$$36 + FE^2 = 100$$
$$64 = FE^2$$
$$\sqrt{64} = \sqrt{FE^2}$$
$$8 = FE$$

$$AB^2 + AC^2 = BC^2$$
$$3^2 + 4^2 = BC^2$$
$$9 + 16 = BC^2$$
$$25 = BC^2$$
$$\sqrt{25} = \sqrt{BC^2}$$
$$5 = BC$$

Now, since we are using SAS Similarity to prove this relationship, we can set up ratios of corresponding sides to see if they are equal. We will match short side to short side and middle side to middle side.

$$\frac{AB}{DF} = \frac{3}{6} = \frac{1}{2}$$
$$\frac{AC}{FE} = \frac{4}{8} = \frac{1}{2}$$
$$\text{Therefore, } \frac{AB}{DF} = \frac{AC}{FE} = \frac{1}{2}$$

So, $\Delta BAC \sim \Delta DFE$ by SAS Similarity.

3. 

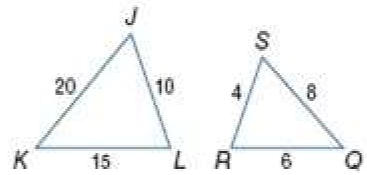
SOLUTION:

Since no angles measures are provided in these triangles, we can determine if these triangles can be proven similar by using the SSS Similarity Theorem.This requires that we determine if each pair of corresponding sides have an equal ratio.

We know the following correspondences exist because we are matching longest side to longest side, middle to middle, and shortest to shortest:

$$\frac{TU}{GH}=\frac{6}{4}=\frac{3}{2}$$
$$\frac{TV}{GF}=\frac{8}{5}$$
$$\frac{VU}{FH}=\frac{12}{8}=\frac{3}{2}$$

Since the ratios of the corresponding sides are not all the same, these triangles are not similar.

4. 

SOLUTION:

Since no angles measures are provided in these triangles, we can determine if these triangles are similar by using the SSS Similarity Theorem.This requires that we determine if each pair of corresponding sides have an equal ratio.

We know the following correspondences exist because we are matching longest side to longest side, middle to middle, and shortest to shortest:

$$\frac{SR}{JL}=\frac{4}{10}=\frac{2}{5}$$
$$\frac{RQ}{LK}=\frac{6}{15}=\frac{2}{5}$$
$$\frac{SQ}{JK}=\frac{8}{20}=\frac{2}{5}$$

Since the ratios of the corresponding sides are equal, $\Delta JLK \sim \Delta SRQ$ by SSS Similarity.

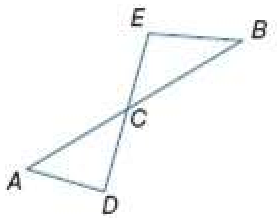
5. **MULTIPLE CHOICE** In the figure, $\overline{AB}$ intersects $\overline{DE}$ at point *C*. Which additional information would be enough to prove that $\Delta ADC \sim \Delta BEC$?

A $\angle DAC$ and $\angle ECB$ are congruent.

B $\overline{AC}$ and $\overline{BC}$ are congruent.

C $\overline{AD}$ and $\overline{EB}$ are parallel.

D $\angle CBE$ is a right angle.

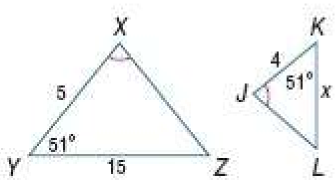


SOLUTION:

Since $\angle BCE \cong \angle ACD$, by the Vertical Angle Theorem, option C is the best choice. If we know that $\overline{AD} \parallel \overline{BE}$, then we know that the alternate interior angles formed by these segments and sides $\overline{AB}$ and $\overline{ED}$ are congruent. This would allow us to use AA Similarity to prove the triangles are similar.

CCSS STRUCTURE Identify the similar triangles. Find each measure.

6. *KL*



SOLUTION:

By AA Similarity, $\Delta XYZ \sim \Delta JKL$.

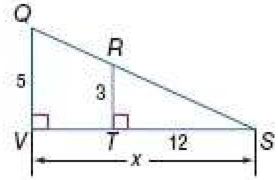
Use the corresponding side lengths to write a proportion.

$$\frac{KL}{KJ} = \frac{YZ}{XY}$$
$$\frac{x}{4} = \frac{15}{5}$$

Solve for *x*.

$$5x = 60$$
$$x = 12$$

7. *VS*



SOLUTION:

We can see that $\angle QVS \cong \angle RTS$ because all right triangles are congruent.
Additionally, $\angle S \cong \angle S$, by Reflexive Property.

Therefore, by AA Similarity, $\Delta QVS \sim \Delta RTS$.

Use the corresponding side lengths to write a proportion.

$$\frac{VS}{TS} = \frac{QV}{RT}$$
$$\frac{x}{12} = \frac{5}{3}$$

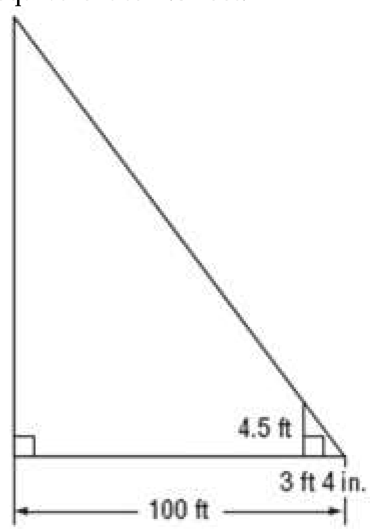
Solve for *x*.

$$3x = 60$$
$$x = 20$$

8. **COMMUNICATION** A cell phone tower casts a 100-foot shadow. At the same time, a 4-foot 6-inch post near the tower casts a shadow of 3 feet 4 inches. Find the height of the tower.

SOLUTION:

Make a sketch of the situation. 4 feet 6 inches is equivalent to 4.5 feet.



In shadow problems, you can assume that the angles formed by the sun's rays with any two objects are congruent and that the two objects form the sides of two right triangles.

Since two pairs of angles are congruent, the right triangles are similar by the AA Similarity Postulate. So, the following proportion can be written:

$$\frac{\text{Tower's height}}{\text{Post's height}} = \frac{\text{Tower's shadow length}}{\text{Post's shadow length}}$$

Let *x* be the tower's height. (Note that 1 foot = 12 inches and covert all the dimensions to inches)

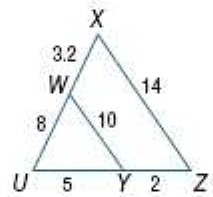
100 ft = 1200 inches
4 feet 6 inches = 54 inches
3 feet 4 inches = 40 inches.

Substitute these corresponding values in the proportion.

$$\frac{x}{54} = \frac{1200}{40}$$
$$40x = 64800$$
$$x = 1620$$

So, the cell phone tower is 1620 inches or 135 feet tall.

Determine whether the triangles are similar. If so, write a similarity statement. If not, what would be sufficient to prove the triangles similar? Explain your reasoning.

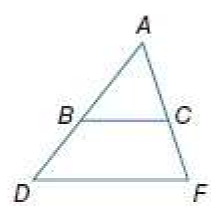
9. 

SOLUTION:

Matching up short to short, middle to middle, and long to long sides, we get the following ratios:

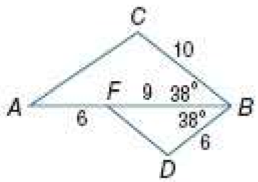
$$\frac{UZ}{UY} = \frac{7}{5} = 1.4$$
$$\frac{UX}{UW} = \frac{11.2}{8} = 1.4$$
$$\frac{XZ}{WY} = \frac{14}{10} = 1.4$$

Since, $\frac{UZ}{UY} = \frac{UX}{UW} = \frac{XZ}{WY} = 1.4$ then $\Delta XUZ \sim \Delta WUY$ by SSS Similarity

10. 

SOLUTION:

No; $\overline{BC}$ needs to be parallel to $\overline{DF}$ for $\Delta DAF \sim \Delta BAC$ by AA Similarity. Additionally, there are no given side lengths to compare to use SAS or SSS Similarity theorems.

11. 

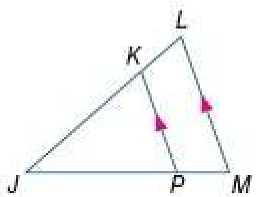
SOLUTION:

We know that $\angle ABC \cong \angle FBD$, because their measures are equal. We also can match up the adjacent sides that include this angle and determine if they have the same ratio. We will match short to short and middle to middle lengths.

$$\frac{BD}{BC} = \frac{6}{10} = \frac{3}{5}$$
$$\frac{BF}{BA} = \frac{9}{9+6} = \frac{9}{15} = \frac{3}{5}$$

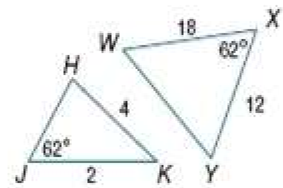
Yes; since $\frac{BD}{BC} = \frac{BF}{BA}$ and $\angle ABC \cong \angle FBD$, we know that $\Delta CBA \sim \Delta DBF$ by SAS Similarity.

Determine whether the triangles are similar. If so, write a similarity statement. If not, what would be sufficient to prove the triangles similar? Explain your reasoning.

12. 

SOLUTION:

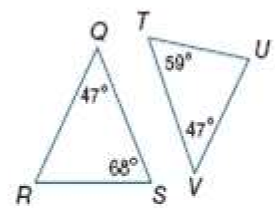
We know that $\angle J \cong \angle J$ due to the Reflexive property. Additionally, we can prove that $\angle JPK \cong \angle JML$, because they are corresponding angles formed by parallel lines . Therefore, $\Delta MLJ \sim \Delta PKJ$ by AA Similarity.

13. 

SOLUTION:

The known information for ΔWXY relates to a SAS relationship, whereas the known information for ΔHJK is a SSA relationship. Since they are no the same relationship, there is not enough information to determine if the triangles are similar.

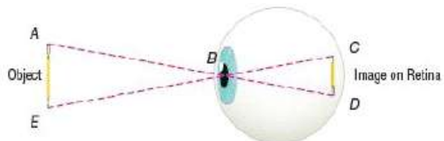
If *JH* = 3 or *WY* = 24, then all the sides would have the same ratio and we could prove $\Delta JHK \sim \Delta WXY$ by SSS Similarity.

14. 

SOLUTION:

No; the angles of ΔTUV are 59, 47, and 74 degrees and the angles of ΔQRS are 47, 68, and 65 degrees. Since the angles of these triangles won't ever be congruent, so the triangles can never be similar.

15. **CCSS MODELING** When we look at an object, it is projected on the retina through the pupil. The distances from the pupil to the top and bottom of the object are congruent and the distances from the pupil to the top and bottom of the image on the retina are congruent. Are the triangles formed between the object and the pupil and the object and the image similar? Explain your reasoning.

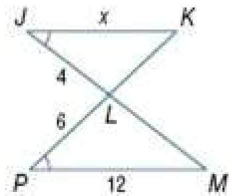


SOLUTION:

Yes; sample answer: $\overline{AB} \cong \overline{EB}$ and $\overline{CB} \cong \overline{DB}$, therefore, we can state that their ratios are proportional, or $\frac{AB}{CB} = \frac{EB}{DB}$. We also know that $\angle ABE \cong \angle CBD$ because vertical angles are congruent. Therefore, $\Delta ABE \sim \Delta CBD$ by SAS Similarity.

ALGEBRA Identify the similar triangles. Then find each measure.

16. *JK*



SOLUTION:

We know that vertical angles are congruent. So, $\angle JLK \cong \angle PLM$.

Additionally, we are given that $\angle J \cong \angle P$.

.

Therefore, by AA Similarity, $\Delta JLK \sim \Delta PLM$.

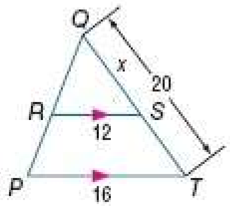
Use the corresponding side lengths to write a proportion.

$$\frac{JK}{PM} = \frac{JL}{PL}$$
$$\frac{x}{12} = \frac{4}{6}$$

Solve for *x*.

$$6x = 48$$
$$x = 8$$

17. *ST*



SOLUTION:

By the Reflexive Property, we know that $\angle Q \cong \angle Q$.

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Also, since $\overline{RS} \parallel \overline{PT}$, we know that $\angle QRS \cong \angle QPT$ (Corresponding Angle Postulate).

Therefore, by AA Similarity, $\Delta QRS \sim \Delta QPT$.

Use the corresponding side lengths to write a proportion.

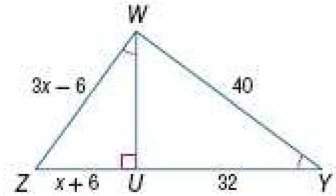
$$\frac{QS}{QT} = \frac{RS}{PT}$$
$$\frac{x}{20} = \frac{12}{16}$$

Solve for x.

$$16x = 240$$
$$x = 15$$

$$ST = 20 - x$$
$$= 20 - 15$$
$$= 5$$

18. *WZ*, *UZ*



SOLUTION:

We are given that $\angle ZWU \cong \angle WYU$ and we also know that $\angle WUZ \cong \angle YUW$ (All right angles are congruent.)

Therefore, by AA Similarity, $\Delta WUZ \sim \Delta YUW$.

Use the Pythagorean Theorem to find *WU*.

$$WU^2 + 32^2 = 40^2$$
$$WU^2 + 1024 = 1600$$
$$WU^2 = 576$$
$$WU = \sqrt{576} = \pm 24$$

Since the length must be positive, *WU* = 24.

Use the corresponding side lengths to write a proportion.

$$\frac{WZ}{WY} = \frac{WU}{UY}$$
$$\frac{3x-6}{40} = \frac{24}{32}$$

Solve for x.

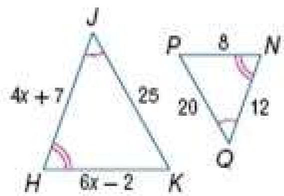
$$32(3x-6) = 40 \cdot 24$$
$$96x - 192 = 960$$
$$96x = 1152$$
$$x = 12$$

Substitute $x = 12$ in *WZ* and *UZ*.

$WZ = 3x - 6$
$= 3(12) - 6$
$= 30$

$UZ = x + 6$
$= 12 + 6$
$= 18$

19. *HJ*, *HK*



SOLUTION:

Since we are given two pairs of congruent angles, we know that $\triangle HJK \sim \triangle NQP$, by AA Similarity.

Use the corresponding side lengths to write a proportion.

$$\frac{HJ}{NQ} = \frac{JK}{QP}$$
$$\frac{4x+7}{12} = \frac{25}{20}$$

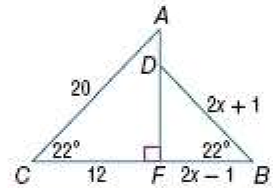
Solve for *x*.

$$20(4x+7) = 12 \cdot 25$$
$$80x + 140 = 300$$
$$80x = 160$$
$$x = 2$$

Substitute *x* = 2 in *HJ* and *HK*.

HJ = 4*(2)* + 7
=15
HK = 6*(2)* – 2
= 10

20. *DB*, *CB*



SOLUTION:

We know that $\angle CFA \cong \angle DFB$ (All right angles are congruent.) and we are given that $m\angle C = m\angle B$.

Therefore, $\triangle DFB \sim \triangle AFC$, by AA Similarity.

Use the corresponding side lengths to write a proportion.

$$\frac{DB}{AC} = \frac{FB}{FC}$$
$$\frac{2x+1}{20} = \frac{2x-1}{12}$$

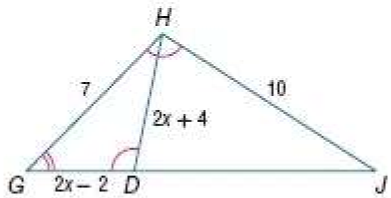
Solve for *x*.

$$12(2x+1) = 20(2x-1)$$
$$24x + 12 = 40x - 20$$
$$-16x = -32$$
$$x = 2$$

Substitute *x* = 2 in *DB* and *CB*.

DB = 2(2) + 1
= 5
CB = 2 (2) – 1 + 12
= 15

21. *GD, DH*



SOLUTION:

We know that $\angle G \cong \angle G$ (Reflexive Property) and are given $\angle GDH \cong \angle GHJ$.
Therefore, $\Delta GHJ \sim \Delta GDH$ by AA Similarity.

Use the corresponding side lengths to write a proportion:

$$\frac{DH}{HJ} = \frac{GD}{GH}$$
$$\frac{2x+4}{10} = \frac{2x-2}{7}$$

Solve for *x*.

$$7(2x+4) = 10(2x-2)$$
$$14x + 28 = 20x - 20$$
$$-6x = -48$$
$$x = 8$$

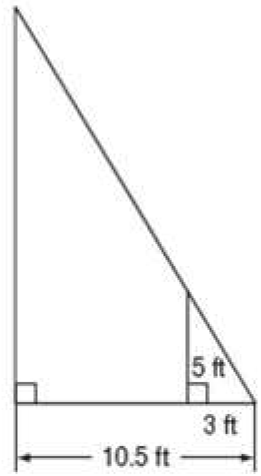
Substitute *x* = 8 in *GD* and *DH*.

GD = 2 *(8)* – 2
= 14
DH = 2 (8) + 4
= 20

22. **STATUES** Mei is standing next to a statue in the park. If Mei is 5 feet tall, her shadow is 3 feet long, and the statue's shadow is $10\frac{1}{2}$ feet long, how tall is the statue?

SOLUTION:

Make a sketch of the situation. 4 feet 6 inches is equivalent to 4.5 feet.



In shadow problems, you can assume that the angles formed by the Sun's rays with any two objects are congruent and that the two objects form the sides of two right triangles.

Since two pairs of angles are congruent, the right triangles are similar by the AA Similarity Postulate.

So, the following proportion can be written:

$$\frac{\text{Statue's height}}{\text{Mei's height}} = \frac{\text{Statue's shadow length}}{\text{Mei's shadow length}}$$

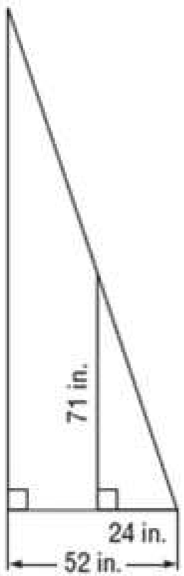
Let *x* be the statue's height and substitute given values into the proportion:

$$\frac{x}{5} = \frac{10.5}{3}$$
$$3x = 52.5$$
$$x = 17.5$$

So, the statue's height is 17.5 feet tall.

23. **SPORTS** When Alonzo, who is $5'11''$ tall, stands next to a basketball goal, his shadow is $2'$ long, and the basketball goal's shadow is $4'4''$ long. About how tall is the basketball goal?

SOLUTION:
Make a sketch of the situation. 4 feet 6 inches is equivalent to 4.5 feet.



In shadow problems, you can assume that the angles formed by the Sun's rays with any two objects are congruent and that the two objects form the sides of two right triangles.

Since two pairs of angles are congruent, the right triangles are similar by the AA Similarity Postulate. So, the following proportion can be written:

$$\frac{\text{basketball goal's height}}{\text{Alonzo's height}} = \frac{\text{baseketball goal's shadow length}}{\text{Alonzo's shadow length}}$$

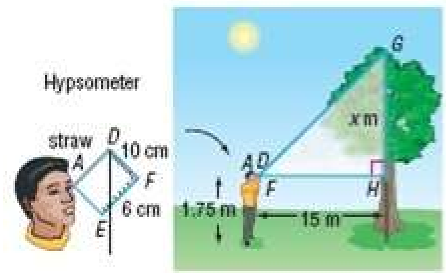
Let *x* be the basketball goal's height. We know that 1 ft = 12 in.. Convert the given values to inches.

$5'11'' = 71$ inches
$2' = 24$ inches
$4'4'' = 52$ inches

Substitute.

$$\frac{x}{71} = \frac{52}{24}$$
$$x = \frac{3692}{24}$$
$$x \approx 154 \text{ inches}$$
$$x \approx 12.8 \text{ ft}$$

24. **FORESTRY** A hypsometer, as shown, can be used to estimate the height of a tree. Bartolo looks through the straw to the top of the tree and obtains the readings given. Find the height of the tree.



SOLUTION:
Triangle *EFD* in the hypsometer is similar to triangle *GHF*.

$$\frac{GH}{FH} = \frac{EF}{DF}$$
$$\frac{x}{15} = \frac{6}{10}$$
$$x = \frac{15(6)}{10}$$
$$x = \frac{90}{10}$$
$$x = 9$$

Therefore, the height of the tree is (9 + 1.75) or 10.75 meters.

PROOF Write a two-column proof.

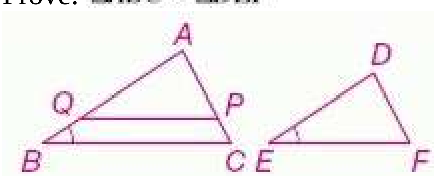
25. Theorem 9.3

SOLUTION:

A good way to approach this proof is to consider how you can get $\Delta ABC \sim \Delta DEF$ by AA Similarity. You already have one pair of congruent angles ($\angle B \cong \angle E$) , so you just need one more pair. This can be accomplished by proving that $\Delta AQP \cong \Delta DEF$ and choosing a pair of corresponding angles as your CPCTC. To get those triangles congruent, you will need to have proven that $\Delta ABC \sim \Delta AQP$ but you have enough information in the given statements to do this. Pay close attention to how the parallel line statement can help. Once these triangles are similar, you can create a proportion statement and combine it with the given statements $\overline{QP} \cong \overline{EF}$ *and* $\frac{AB}{DE} = \frac{BC}{EF}$ to create the relationship that $\overline{AQ} \cong \overline{DE}$.

Given: $\angle B \cong \angle E$, $\overline{QP} \parallel \overline{BC}$, $\overline{QP} \cong \overline{EF}$, $\frac{AB}{DE} = \frac{BC}{EF}$

Prove: $\Delta ABC \sim \Delta DEF$



Proof:

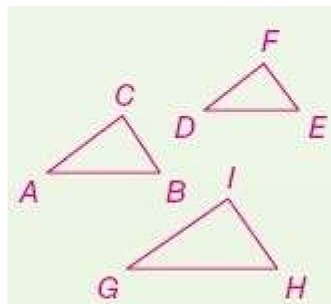
Statements (Reasons)

1. $\angle B \cong \angle E$, $\overline{QP} \parallel \overline{BC}$, $\overline{QP} \cong \overline{EF}$, $\frac{AB}{DE} = \frac{BC}{EF}$ (Given)
2. $\angle APQ \cong \angle C$, $\angle AQP \cong \angle B$ (Corr. $\angle$'s Post.)
3. $\angle AQP \cong \angle E$ (Trans. Prop.)
4. $\Delta ABC \sim \Delta AQP$ (AA Similarity)
5. $\frac{AB}{AQ} = \frac{BC}{QP}$ (Def. of $\sim \Delta$s)
6. $AB \cdot QP = AQ \cdot BC$; $AB \cdot EF = DE \cdot BC$ (Cross products)
7. $QP = EF$ (Def. of $\cong$ segs.)
8. $AB \cdot EF = AQ \cdot BC$ (Subst.)
9. $AQ \cdot BC = DE \cdot BC$ (Subst.)
10. $AQ = DE$ (Div. Prop.)
11. $\overline{AQ} \cong \overline{DE}$ (Def. of $\cong$ segs.)
12. $\Delta AQP \cong \Delta DEF$ (SAS)
13. $\angle APQ \cong \angle F$ (CPCTC)
14. $\angle C \cong \angle F$ (Trans. Prop.)
15. $\Delta ABC \sim \Delta DEF$ (AA Similarity)

26. Theorem 9.4

SOLUTION:

This is a three-part proof, as you need to prove three different relationships - that Reflexive, Symmetric, and Transitive properties are true for similar triangles. For each part of this proof, the key is to find a way to get two pairs of congruent angles which will allow you to use AA Similarity Postulate.As you try these, remember that you already know that these three properties already hold for congruent triangles and can use these relationships in your proofs.



Reflexive Property of Similarity

Given: ΔABC

Prove: $\Delta ABC \sim \Delta ABC$

Proof:

Statements (Reasons)

1. ΔABC (Given)
2. $\angle A \cong \angle A$, $\angle B \cong \angle B$ (Refl. Prop of $\cong$.)
3. $\Delta ABC \sim \Delta ABC$ (AA Similarity)

Transitive Property of Similarity

Given: $\Delta ABC \sim \Delta DEF$ and $\Delta DEF \sim \Delta GHI$

Prove: $\Delta ABC \sim \Delta GHI$

Statements (Reasons)

1. $\Delta ABC \sim \Delta DEF$, $\Delta DEF \sim \Delta GHI$ (Given)
2. $\angle A \cong \angle D, \angle B \cong \angle E, \angle D \cong \angle G, \angle E \cong \angle H$ (Def. of $\sim$ polygons)
3. $\angle A \cong \angle G, \angle B \cong \angle H$ (Trans. Prop.)
4. $\Delta ABC \sim \Delta GHI$ (AA Similarity)

Symmetric Property of Similarity

Given: $\Delta ABC \sim \Delta DEF$

Prove: $\Delta DEF \sim \Delta ABC$

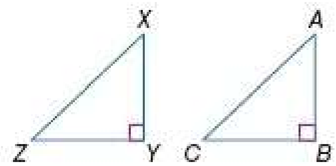
Statements (Reasons)

1. $\Delta ABC \sim \Delta DEF$ (Given)

2. $\angle A \cong \angle D, \angle B \cong \angle E$ (Def. of $\sim$ polygons)
3. $\angle D \cong \angle A, \angle E \cong \angle B$ (Symm. Prop of.)
4. $\Delta DEF \sim \Delta ABC$ (AA Similarity)

PROOF Write a two-column proof.

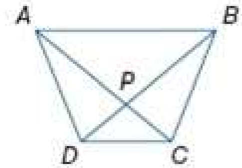
27. **Given:** ΔXYZ and ΔABC are right triangles; $\frac{XY}{AB} = \frac{YZ}{BC}$.
Prove: $\Delta YXZ \sim \Delta BAC$



SOLUTION:
The given information in this proof is almost all you need to prove $\Delta YXZ \sim \Delta BAC$ by SAS Similarity theorem. You already have two pairs of proportional corresponding sides. You just need to think about how to get the included angles congruent to each other.

Proof:
Statements (Reasons)
1. ΔXYZ and ΔABC are right triangles. (Given)
2. $\angle XYZ$ and $\angle ABC$ are right angles. (Def. of rt. Δ)
3. $\angle XYZ \cong \angle ABC$ (All rt. angles are $\cong$.)
4. $\frac{XY}{AB} = \frac{YZ}{BC}$ (Given)
5. $\Delta YXZ \sim \Delta BAC$ (SAS Similarity)

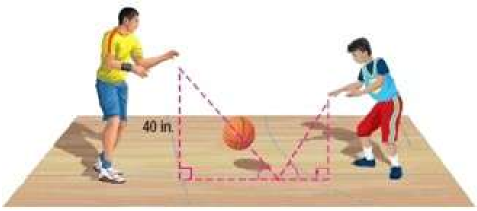
28. **Given:** *ABCD* is a trapezoid.
Prove: $\frac{DP}{PB} = \frac{CP}{PA}$



SOLUTION:
Think backwards when attempting this proof. In order to prove that $\frac{DP}{PB} = \frac{CP}{PA}$, we need to show that $\Delta DCP \sim \Delta BAP$. To prove triangles are similar, you need to prove two pairs of corresponding angles are congruent. Think about what you know about trapezoids and how that can help you get $\angle BDC \cong \angle ABD, \angle BAC \cong \angle DCA$.

Proof:
Statements (Reasons)
1. *ABCD* is a trapezoid. (Given)
2. $\overline{AB} \parallel \overline{DC}$ (Def. of trap.)
3. $\angle BDC \cong \angle ABD, \angle BAC \cong \angle DCA$ (Alt. Int. angle Thm.)
4. $\Delta DCP \sim \Delta BAP$ (AA Similarity)
5. $\frac{DP}{PB} = \frac{CP}{PA}$ (Corr. sides of $\sim \Delta$ s are proportional.)

29. **CCSS MODELING** When Luis's dad threw a bounce pass to him, the angles formed by the basketball's path were congruent. The ball landed $\frac{2}{3}$ of the way between them before it bounced back up. If Luis's dad released the ball 40 inches above the floor, at what height did Luis catch the ball?



SOLUTION:

Since the ball landed $\frac{2}{3}$ of the way between them, the horizontal line is in the ratio of 2:1.
By AA Similarity, the given two triangles are similar.

Form a proportion and solve for x. Assume that Luis will catch the ball at a height of x inches.

$$\frac{\text{ball release height (father)}}{\text{ball catch height(Luis)}} = \frac{\text{distance of bounce to father}}{\text{distance of bounce to Luis}}$$

$$\frac{40}{x} = \frac{2}{1}$$
$$2x = 40$$
$$x = 20$$

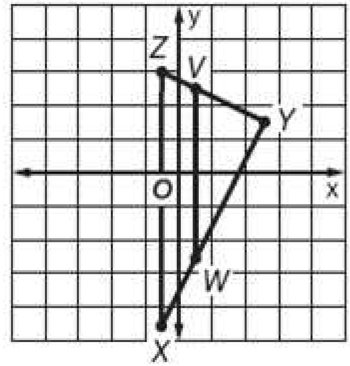
So, Luis will catch the ball 20 inches above the floor.

COORDINATE GEOMETRY ΔXYZ and ΔWYV have vertices $X(-1, -9)$, $Y(5, 3)$, $Z(-1, 6)$, $W(1, -5)$, and $V(1, 5)$.

30. Graph the triangles, and prove that $\Delta XYZ \sim \Delta WYV$.

SOLUTION:

We can prove that $\Delta XYZ \sim \Delta WYV$ by using the distance formula to determine the lengths of each side of the triangles. Then, we can set up ratios to determine if the ratios of corresponding sides are equal and use SSS Similarity theorem to prove the triangles are similar.



The lengths of the sides of ΔXYZ are:

$XY = \sqrt{12^2 + 6^2} = \sqrt{180}$ or $6\sqrt{5}$;

$YZ = \sqrt{3^2 + (-6)^2} = \sqrt{45}$ or $3\sqrt{5}$;

$ZX = 6 - (-9) = 15$;

The lengths of the sides of ΔWYV are:

$VW = 5 - (-5) = 10$;

$WY = \sqrt{8^2 + 4^2} = \sqrt{80}$ or $4\sqrt{5}$;

$YV = \sqrt{2^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}$.

Now, find the ratios of the corresponding sides:

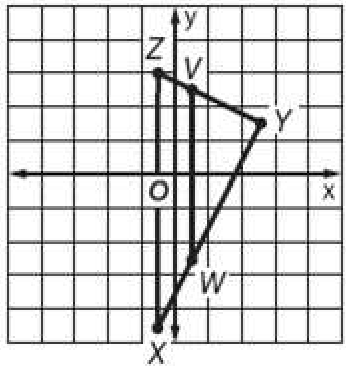
$$\frac{XY}{WY} = \frac{6\sqrt{5}}{4\sqrt{5}} = \frac{6}{4} = \frac{3}{2}$$
$$\frac{YZ}{YV} = \frac{3\sqrt{5}}{2\sqrt{5}} = \frac{3}{2}$$
$$\frac{ZW}{VW} = \frac{15}{10} = \frac{3}{2}$$

Since $\frac{XY}{WY} = \frac{YZ}{YV} = \frac{ZX}{VW} = \frac{3}{2}$, *then* $\Delta XYZ \sim \Delta WYV$ by SSS Similarity.

31. Find the ratio of the perimeters of the two triangles.

SOLUTION:

We can prove that $\Delta XYZ \sim \Delta WYV$ by using the distance formula to determine the lengths of each side of the triangles. Then, we can set up ratios to determine if the ratios of corresponding sides are equal and use SSS Similarity theorem to prove the triangles are similar.



The lengths of the sides of ΔXYZ are:

$XY = \sqrt{12^2 + 6^2} = \sqrt{180}$ or $6\sqrt{5}$;

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$ZX = 6 - (-9) = 15$;

The lengths of the sides of ΔWYV are:

$VW = 5 - (-5) = 10$;

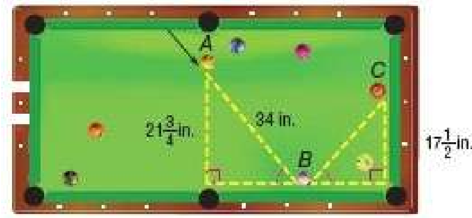
$WY = \sqrt{8^2 + 4^2} = \sqrt{80}$ or $4\sqrt{5}$;

$YV = \sqrt{2^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}$.

Now, find the perimeter of each triangle:

$$\begin{aligned}\frac{\text{Perimeter of triangle } XYZ}{\text{Perimeter of triangle } WYV} &= \frac{6\sqrt{5} + 3\sqrt{5} + 15}{4\sqrt{5} + 2\sqrt{5} + 10} \\ &= \frac{9\sqrt{5} + 15}{6\sqrt{5} + 10} \\ &= \frac{3\left(3\sqrt{5} + 5\right)}{2\left(3\sqrt{5} + 5\right)} \\ &= \frac{3}{2}\end{aligned}$$

32. **BILLIARDS** When a ball is deflected off a smooth surface, the angles formed by the path are congruent. Booker hit the orange ball and it followed the path from A to B to C as shown below. What was the total distance traveled by the ball from the time Booker hit it until it came to rest at the end of the table?



SOLUTION:

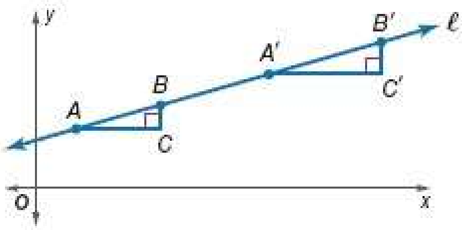
By AA Similarity, the given triangles are similar. Form a proportion and solve for BC. Convert the fractions to decimals.

$$\frac{BC}{17.5} = \frac{34}{21.75}$$
$$BC \approx 27$$

$$\begin{aligned}\text{Total distance traveled by the ball} &= AB + BC \\ &\approx 34 + 27 \\ &= 61\end{aligned}$$

So, the total distance traveled by the ball is about 61 in..

33. **PROOF** Use similar triangles to show that the slope of the line through any two points on that line is constant. That is, if points *A*, *B*, A' and B' are on line ℓ, use similar triangles to show that the slope of the line from *A* to *B* is equal to the slope of the line from A' to B'.



SOLUTION:

In this proof, it is important to recognize that $\overline{BC}$ and $\overline{B'C'}$ are both vertical lines and are, therefore, parallel to each other. Using this relationship, along with the fact that line ℓ is a transversal of these segments, we can prove that $\Delta ABC \sim \Delta A'B'C'$. Once this is proven, you can use a proportion statement to complete the proof.

$\angle C \cong \angle C'$, since all rt. angles are $\cong$. Line ℓ is a transversal of $\|$ segments $\overline{BC}$ and $\overline{B'C'}$, so $\angle ABC \cong \angle A'B'C'$ since corresponding angles of $\|$ lines are $\cong$. Therefore, by AA Similarity, $\Delta ABC \sim \Delta A'B'C'$. So $\frac{BC}{AC}$, the slope of line ℓ through points *A* and *B*, is equal to $\frac{B'C'}{A'C'}$, the slope of line ℓ through points A' and B'.

34. **CHANGING DIMENSIONS** Assume that $\Delta ABC \sim \Delta JKL$.
a. If the lengths of the sides of ΔJKL are half the length of the sides of ΔABC, and the area of ΔABC is 40 square inches, what is the area of ΔJKL? How is the area related to the scale factor of ΔABC to ΔJKL?
b. If the lengths of the sides of ΔABC are three times the length of the sides of ΔJKL, and the area of ΔABC is 63 square inches, what is the area of ΔJKL? How is the area related to the scale factor of ΔABC to ΔJKL?

SOLUTION:

a. Let *b* and *h* be the base and height of the triangle *ABC* respectively.

$$\begin{aligned}\text{Area of } \Delta JKL &= \left(\frac{1}{2}\cdot\frac{b}{2}\cdot\frac{h}{2}\right)\\ &= \frac{1}{4}\left(\frac{1}{2}bh\right)\\ &= \frac{1}{4}(\text{Area of } \Delta ABC)\\ &= \frac{1}{4}(40)\\ &= 10\end{aligned}$$

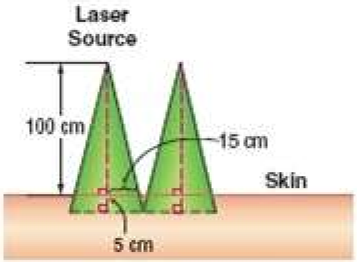
Thus, the area of the triangle *JKL* is 10 square inches.
The ratio of the areas is the square of the scale factor.

b. Let *b* and *h* be the base and height of the triangle *ABC* respectively.

$$\begin{aligned}\text{Area of } \Delta JKL &= \left(\frac{1}{2}\cdot\frac{b}{3}\cdot\frac{h}{3}\right)\\ &= \frac{1}{9}\left(\frac{1}{2}bh\right)\\ &= \frac{1}{9}(\text{Area of } \Delta ABC)\\ &= \frac{1}{9}(63)\\ &= 7\end{aligned}$$

Thus, the area of the triangle *JKL* is 7 square inches.
The ratio of the areas is the cube of the scale factor.

35. **MEDICINE** Certain medical treatments involve laser beams that contact and penetrate the skin, forming similar triangles. Refer to the diagram. How far apart should the laser sources be placed to ensure that the areas treated by each source do not overlap?



SOLUTION:

For 100 cm, it covers an area that has a radius of 15 cm. It penetrates and go inside the skin for 5 cm. so, the total height is 105 cm. Assume that for 105 cm, laser source covers an area that has a radius of x cm.

Form a proportion.

$$\frac{105}{x} = \frac{100}{15}$$

$x = 15.75$

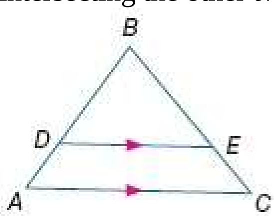
So, the laser beam covers 15.75 + 15.75 or 31.5 cm.

36. **MULTIPLE REPRESENTATIONS** In this problem, you will explore proportional parts of triangles.

a. GEOMETRIC Draw a ΔABC with $\overline{DE}$ parallel to $\overline{AC}$ as shown.

b. TABULAR Measure and record the lengths AD, DB, CD, and EB and the ratios $\frac{AD}{DB}$ and $\frac{CE}{EB}$ in a table.

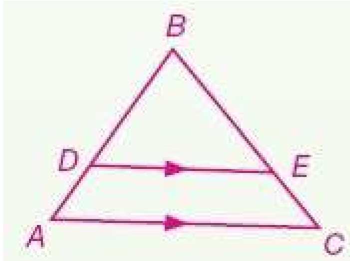
c. VERBAL Make a conjecture about the segments created by a line parallel to one side of a triangle and intersecting the other two sides.



SOLUTION:

a. The triangle you draw doesn't have to be congruent to the one in the text. However, measure carefully so that $\overline{DE}$ is parallel to side $\overline{AC}$.

Sample answer:



b. When measuring the side lengths, it may be easiest to use centimeters. Fill in the table with the corresponding measures.

Sample answer:

Lengths		Ratios	
AD	0.9 cm	$\frac{AD}{DB}$	$\frac{1}{2}$
DB	1.8 cm		
CE	1.1 cm	$\frac{CE}{EB}$	$\frac{1}{2}$
EB	2.2 cm		

c. Observe patterns you notice in the table that are formed by the ratios of sides of a triangle cut by a parallel line.

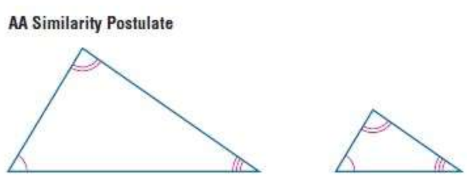
Sample answer: The segments created by a line || to one side of a triangle and intersecting the other two sides are proportional.

37. **WRITING IN MATH** Compare and contrast the AA Similarity Postulate, the SSS Similarity Theorem, and the SAS similarity theorem.

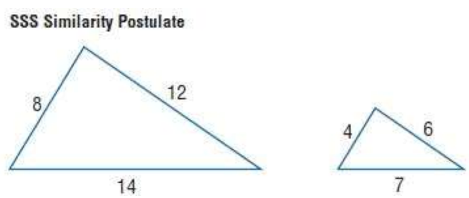
SOLUTION:

Sample answer: The AA Similarity Postulate, SSS Similarity Theorem, and SAS Similarity Theorem are all tests that can be used to determine whether two triangles are similar.

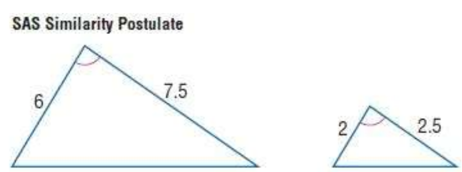
The AA Similarity Postulate is used when two pairs of congruent angles of two triangles are given.



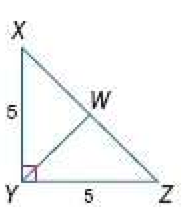
The SSS Similarity Theorem is used when the corresponding side lengths of two triangles are given.



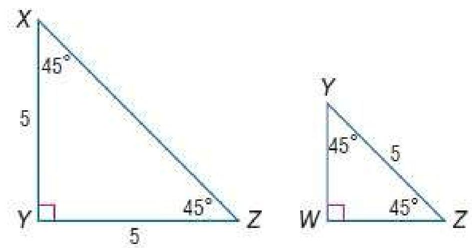
The SAS Similarity Theorem is used when two proportional side lengths and the included angle of two triangles are given.



38. **CHALLENGE** $\overline{YW}$ is an altitude of $\triangle XYZ$. Find *YW*.



SOLUTION:



Both $\triangle XYZ$ *and* $\triangle YWZ$ are isosceles right triangles, so by AA Similarity postulate, we know that they are similar. This allows us to set up a proportion of corresponding side lengths to find YW:

$$\frac{YW}{XY} = \frac{YZ}{XZ}$$

First, we need to find the length of XZ.

$$XY^2 + YZ^2 = XZ^2$$
$$5^2 + 5^2 = XZ^2$$
$$25 + 25 = XZ^2$$
$$50 = XZ^2$$
$$\sqrt{50} = XZ$$
$$5\sqrt{2} = XZ$$

Now, substitute the side lengths you know into the proportion $\frac{YW}{XY} = \frac{YZ}{XZ}$.

$$\frac{YW}{5} = \frac{5}{5\sqrt{2}}$$

$$5\sqrt{2} \cdot YW = 25$$

$$YW = \frac{25}{5\sqrt{2}}$$

$$YW = \frac{5}{\sqrt{2}} = \frac{5\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

Therefore, YW= $\frac{5\sqrt{2}}{2}$.

39. **REASONING** A pair of similar triangles has angle measures of 45°, 50°, and 85°. The sides of one triangle measure 3, 3.25, and 4.23 units, and the sides of the second triangle measure $x - 0.46$, x, and $x + 1.81$ units. Find the value of x.

SOLUTION:

Using the given information, sketch two triangles and label the corresponding sides and angles. Make sure you use the Angle- Sides relationships of triangles to place the shortest sides across from the smallest angles, etc.

Form a proportion and solve for x.

$$\frac{x-0.46}{x}=\frac{3}{3.25}$$

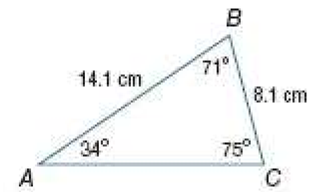
$$3.25(x-0.46)=3x$$

$$3.25x-1.5=3x$$

$$0.25x=6$$

$$x=6$$

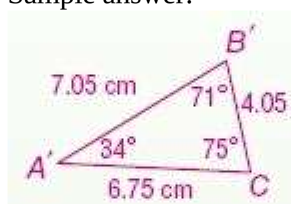
40. **OPEN ENDED** Draw a triangle that is similar to ΔABC shown. Explain how you know that it is similar.



SOLUTION:

When making a triangle similar to ΔABC, keep in mind the relationships that exist between the angles of similar triangles, as well as the sides. We know that the corresponding sides of similar triangles are proportional and the corresponding angles are congruent.

Sample answer:



$\Delta ABC \sim \Delta A'B'C'$ because the measures of each side are half the measure of the corresponding side and the measures of corresponding angles are equal.

41. **WRITING IN MATH** How can you choose an appropriate scale?

SOLUTION:

Sample answer: You could consider the amount of space that the actual object occupies and compare it to the amount of space that is available for the scale model or drawing. Then, you could determine the amount of detail that you want the scale model or drawing to have, and you could use these factors to choose an appropriate scale.

42. **PROBABILITY** $\frac{x!}{(x-3)!} =$

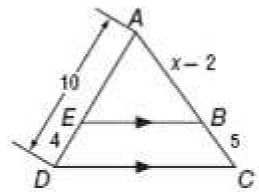
A 3.0
B 0.33
C $x^2 - 3x + 2$
D $x^3 - 3x^2 + 2x$

SOLUTION:

$$\frac{x!}{(x-3)!} = \frac{x(x-1)(x-2)\cancel{(x-3)!}}{\cancel{(x-3)!}}$$
$$= x(x-1)(x-2)$$
$$= x^3 - 3x^2 + 2x$$

So, the correct option is D.

43. **EXTENDED RESPONSE** In the figure below, $\overline{EB} \parallel \overline{DC}$.



a. Write a proportion that could be used to find x.

b. Find the value of x and the measure of $\overline{AB}$.

SOLUTION:

Since we know $\overline{EB} \parallel \overline{DC}$, $\angle AEB \cong \angle ADC$ and $\angle ABE \cong \angle ACD$ because they are corresponding angles formed by parallel lines. Therefore, $\triangle AEB \sim \triangle ADC$ and corresponding sides are proportional.

a. $\frac{6}{x-2} = \frac{4}{5}$

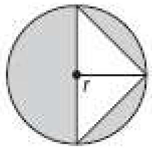
b. $\frac{6}{x-2} = \frac{4}{5}$

Solve for x.

$$4(x-2) = 30$$
$$4x - 8 = 30$$
$$4x = 38$$
$$x = 9.5$$

Substitute $x = 9.5$ in AB.
AB = $x - 2$
= 9.5 – 2
= 7.5

44. **ALGEBRA** Which polynomial represents the area of the shaded region?



F πr^2
G $\pi r^2 + r^2$
H $\pi r^2 + r$
J $\pi r^2 - r^2$

SOLUTION:

The area of the circle is πr^2.

The area of one white triangle is $\frac{1}{2}r^2$, because the radius of the circle is both the height and the base of the triangle.

The area of two white triangles would be

$\frac{1}{2}r^2 + \frac{1}{2}r^2$.

To find the area of the shaded region, you can subtract the area of the two white triangles from the circle's area.

$$\text{Area of shaded region} = \pi r^2 - \left(\frac{1}{2}r^2 + \frac{1}{2}r^2\right)$$
$$= \pi r^2 - r^2$$

So, the correct option is J.

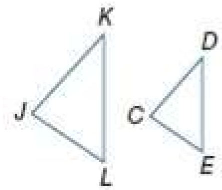
45. **SAT/ACT** The volume of a certain rectangular solid is 16x cubic units. If the dimensions of the solid are integers x, y, and z units, what is the greatest possible value of z?
A 32
B 16
C 8
D 4
E 2

SOLUTION:

The volume of a rectangular solid with dimensions x, y, and z is given by xyz. So $xyz = 16$. Since all dimensions are integers, and since lengths must be positive, the least possible value of x and y is 1. In that case, $z = 16$. So the correct answer is B.

List all pairs of congruent angles, and write a proportion that relates the corresponding sides for each pair of similar polygons.

46. $\triangle JKL \sim \triangle CDE$

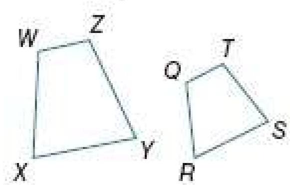


SOLUTION:

The order of vertices in a similarity statement identifies the corresponding angles and sides. Since we know that $\triangle LKJ \sim \triangle EDC$, we can take the corresponding angles of this statement and set them congruent to each other. Then, since the corresponding sides of similar triangles are proportional to each other, we can write a proportion that relates the corresponding sides to each other.

$$\angle L \cong \angle E, \angle K \cong \angle D, \angle J \cong \angle C; \frac{KL}{DE} = \frac{JK}{CD} = \frac{JL}{CE}$$

47. $WXYZ \sim QRST$

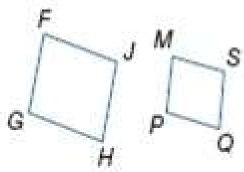


SOLUTION:

The order of vertices in a similarity statement identifi corresponding angles and sides. Since we know that $\triangle XWY \sim \triangle RQT$, we can take the corresponding a this statement and set them congruent to each other. the corresponding sides of similar triangles are propor each other, we can write a proportion that relates the corresponding sides to each other.

$$\angle X \cong \angle R, \angle W \cong \angle Q, \angle Y \cong \angle S, \angle Z \cong \angle T; \frac{WX}{QR} = \frac{ZY}{TS} =$$

48. $FGHJ \sim MPQS$



SOLUTION:

The order of vertices in a similarity statement identifi corresponding angles and sides. Since we know that $GFJH \sim PMSQ$, we can take the corresponding statement and set them congruent to each other. The corresponding sides of similar polygons are proportio other, we can write a proportion that relates the corr sides to each other.

$\angle G \cong \angle P, \angle F \cong \angle M, \angle J \cong \angle S, \angle H \cong \angle Q;\ \dfrac{JH}{SQ} = \dfrac{GH}{PQ} =$

Solve each proportion.

49. $\dfrac{3}{4} = \dfrac{x}{16}$

SOLUTION:

$\dfrac{3}{4} = \dfrac{x}{16}$

Cross multiply.

$x(4) = 3(16)$

Solve for *x*.

$$\begin{aligned} 4x &= 48 \\ x &= 12 \end{aligned}$$

50. $\dfrac{x}{10} = \dfrac{22}{50}$

SOLUTION:

$\dfrac{x}{10} = \dfrac{22}{50}$

Cross multiply.

$50x = 220$

Solve for *x*.

$x = 4.4$

51. $\dfrac{20.2}{88} = \dfrac{12}{x}$

SOLUTION:

$\dfrac{20.2}{88} = \dfrac{12}{x}$

Cross multiply.

$x(20.2) = 88(12)$

Solve for *x*.

$$\begin{aligned} 20.2x &= 1056 \\ x &\approx 52.3 \end{aligned}$$

52. $\dfrac{x-2}{2} = \dfrac{3}{8}$

SOLUTION:

$\dfrac{x-2}{2} = \dfrac{3}{8}$

Cross multiply.

$8(x-2) = 6$

Solve for *x*.

$$\begin{aligned} 8x - 16 &= 6 \\ 8x &= 22 \\ x &\approx 2.8 \end{aligned}$$

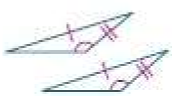
53. **TANGRAMS** A tangram set consists of seven pieces: a small square, two small congruent right triangles, two large congruent right triangles, a medium-sized right triangle, and a quadrilateral. How can you determine the shape of the quadrilateral? Explain.

SOLUTION:

Consider the properties of different quadrilaterals when answering this question. The shape appears to be a parallelogram, therefore you can test the conditions of a parallelogram to see if they are true for this shape.

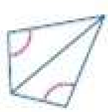
Sample answer: If one pair of opposite sides are congruent and parallel, the quadrilateral is a parallelogram.

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write *not possible*.

54. 

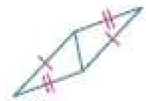
SOLUTION:

We are given two pairs of congruent sides and a pair of congruent angles. However, the congruent angles are not the include angle between the two sides. Therefore, it is a SSA relationship, not a SAS relationship and it is not possible to prove these triangles congruent.

55. 

SOLUTION:

We are given one pair of congruent angles and one pair of congruent sides (by Reflexive property). However, it not possible to prove these triangles congruent because we can't prove any other pair of sides or angles congruent.

56. 

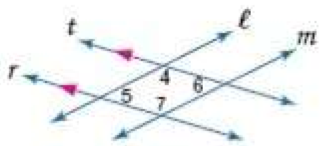
SOLUTION:

We are given that two pairs of sides are congruent and can prove the third pair of sides is, as well, by using Reflexive property. Therefore, these triangles can be proven congruent with SSS.

Write a two-column proof.

57. **Given:** $r \parallel t$; $\angle 5 \cong \angle 6$

Prove: $\ell \parallel m$

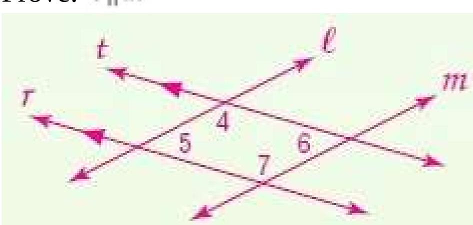


SOLUTION:

There are many angles in this diagram, so it is easy to get confused by which ones to use. Notice how the given statement guides you to using $\angle 5$ and $\angle 6$. Because they have different transversals, they are not related to the same set of parallel lines. However, they are both related to $\angle 4$. If you can think about how to get $\angle 4$ supplementary to $\angle 6$, then you can prove that $\ell \parallel m$.

Given: $r \parallel t$; $\angle 5 \cong \angle 6$

Prove: $\ell \parallel m$



Proof:

Statements (Reasons)

1. $r \parallel t$; $\angle 5 \cong \angle 6$ (Given)
2. $\angle 4$ supp $\angle 5$ (Consecutive Interior Angle Theorem)
3. $m\angle 4 + m\angle 5 = 180$ (Definition of supplementary angles)
4. $m\angle 5 = m\angle 6$ (Definition of congruent angles)
5. $m\angle 4 + m\angle 6 = 180$ (Substitution)
6. $\angle 4$ supp $\angle 6$ (Definition of supplementary)
7. $\ell \parallel m$ (If cons. int. $\angle$s are suppl., then lines are ||.)