NEW JERSEY CENTER
FOR TEACHING & LEARNING

8th Grade

Pythagorean Theorem, Distance & Midpoint

2016-01-15

www.njctl.org

Table of Contents

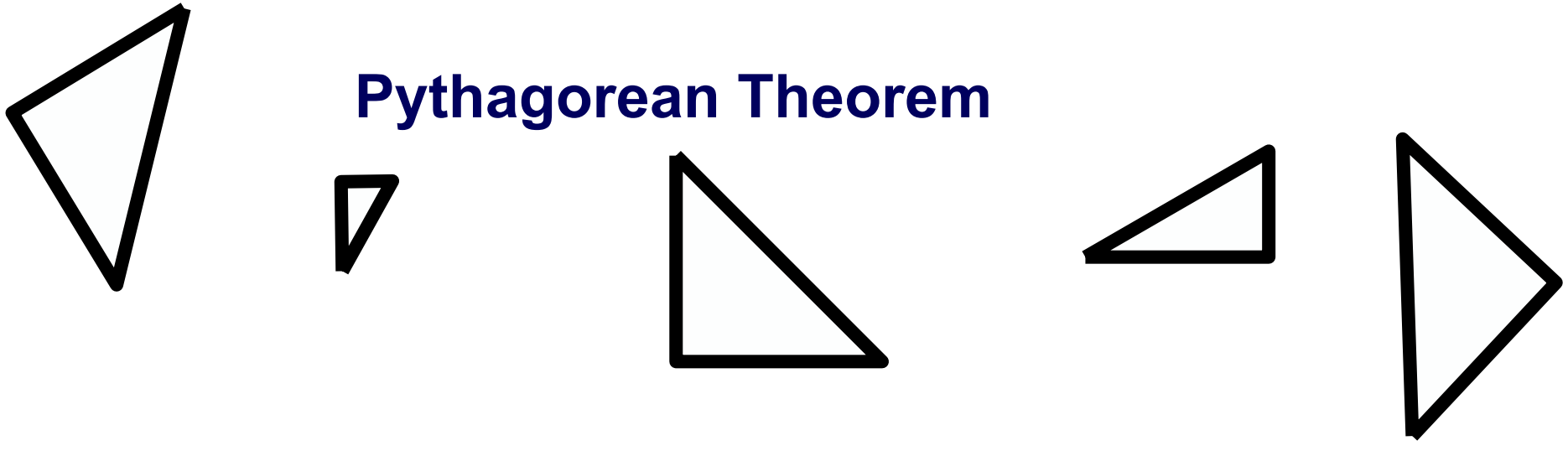
Click on a topic to go to that section

Teacher Notes

Pythagorean Theorem

Click to return to
the table of contents

Pythagorean Theorem

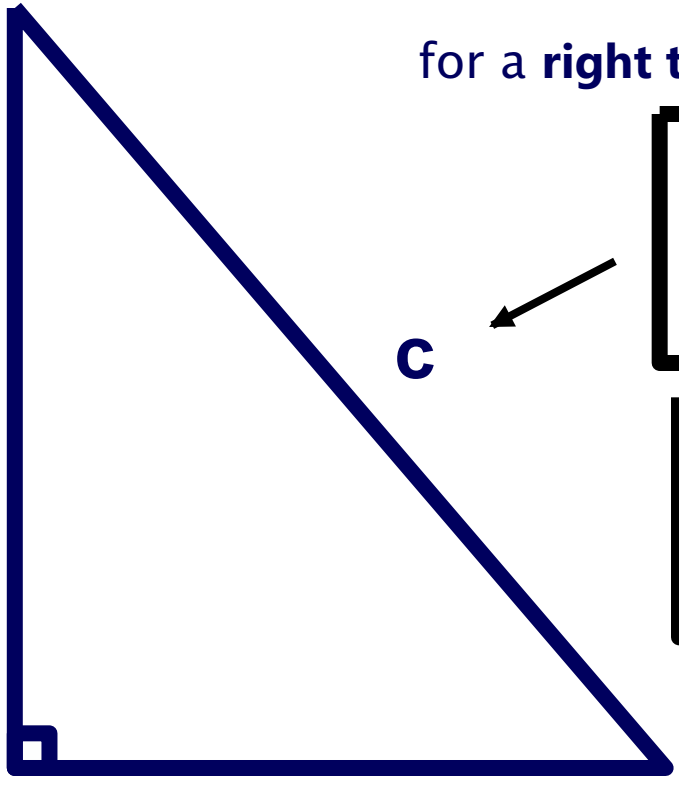


Pythagorean theorem is used for right triangles. It was first known in ancient Babylon and Egypt beginning about 1900 B.C. However, it was not widely known until Pythagoras stated it.

Pythagoras lived during the 6th century B.C. on the island of Samos in the Aegean Sea. He also lived in Egypt, Babylon, and southern Italy. He was a philosopher and a teacher.

Labels

for a **right triangle**



click to reveal

click to reveal

click to reveal

click to reveal

Pythagorean Theorem Proofs

In a right triangle, the sum of the squares of the lengths of the legs (*a* and *b*) is equal to the square of the length of the hypotenuse (*c*).

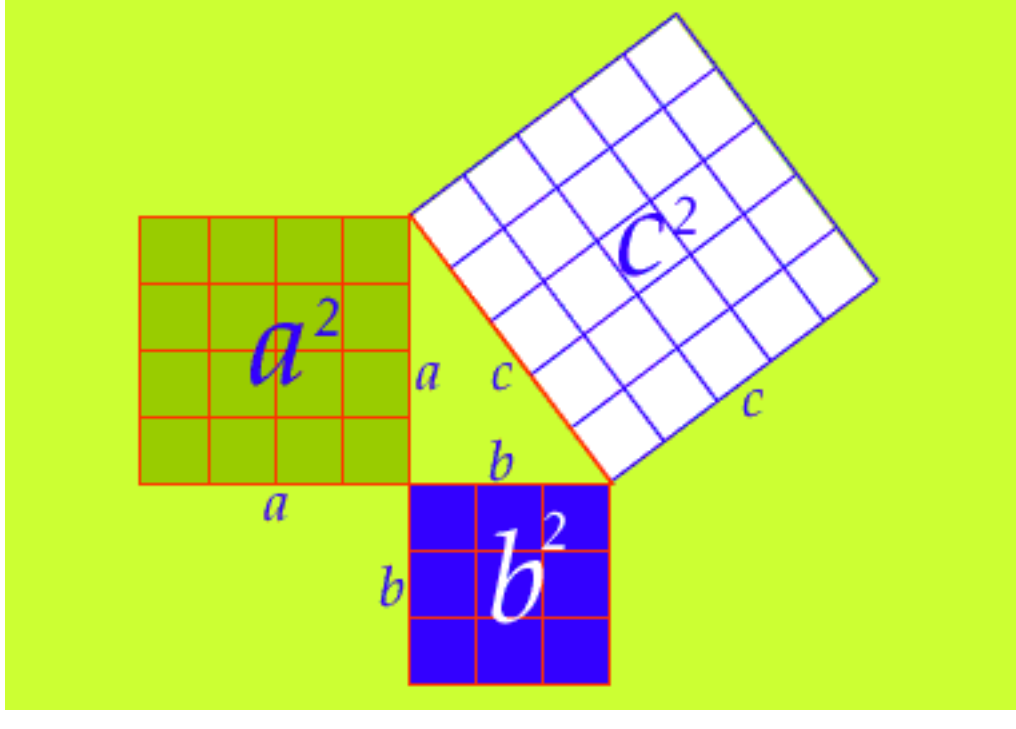
a2 + b2 = c2

Click on the links below to see several animations of the proof

Water demo

Move slider to show c2

Moving of squares



Pythagorean Theorem

How to use the formula to find missing sides.

Missing Leg

Write Equation

Substitute in numbers

Square numbers

Subtract

Find the Square Root

Label Answer

Missing Hypotenuse

Write Equation

Substitute in numbers

Square numbers

Add

Find the Square Root

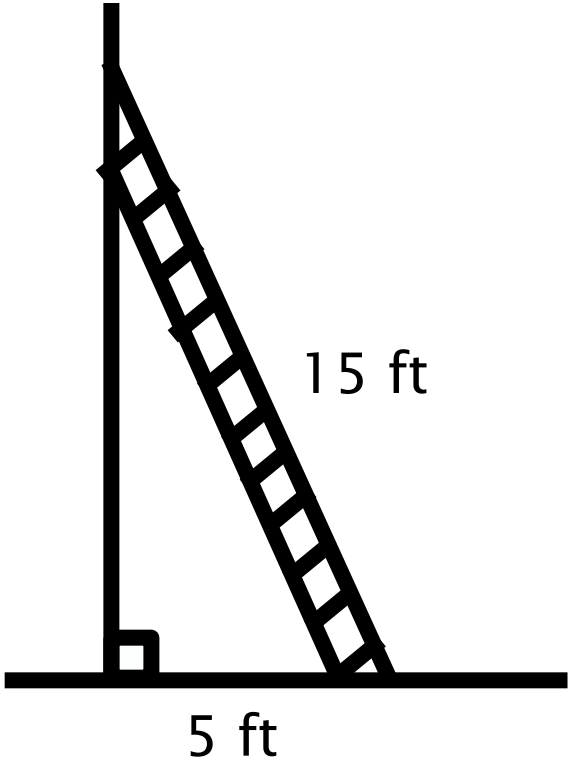
Label Answer

Pythagorean Theorem

Missing Leg

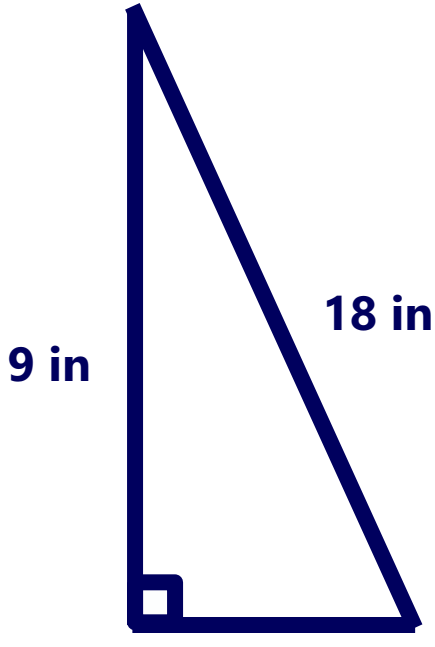
Write Equation

$a^2 + b^2 = c^2$



Math Practice

Pythagorean Theorem



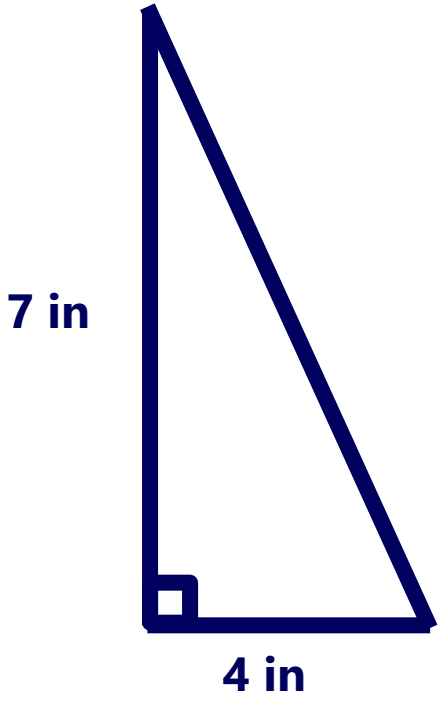
Missing Leg

Write Equation

$a^2 + b^2 = c^2$

Substitute in numbers

Pythagorean Theorem



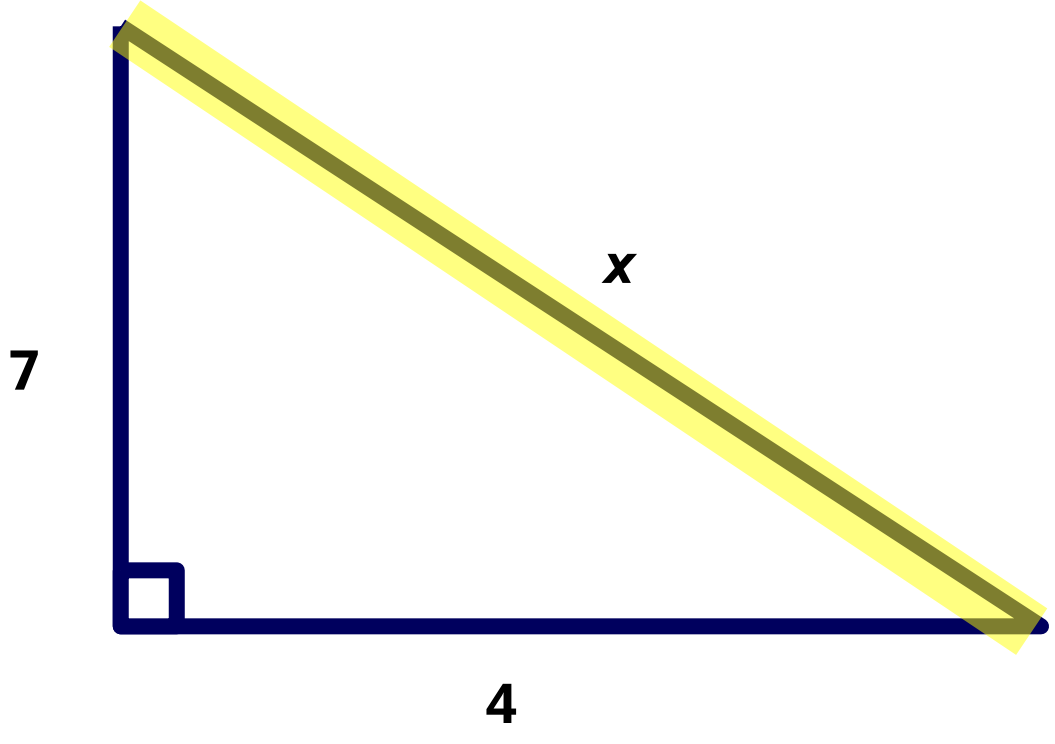
Missing Hypotenuse

Write Equation

$a2 + b2 = c2$

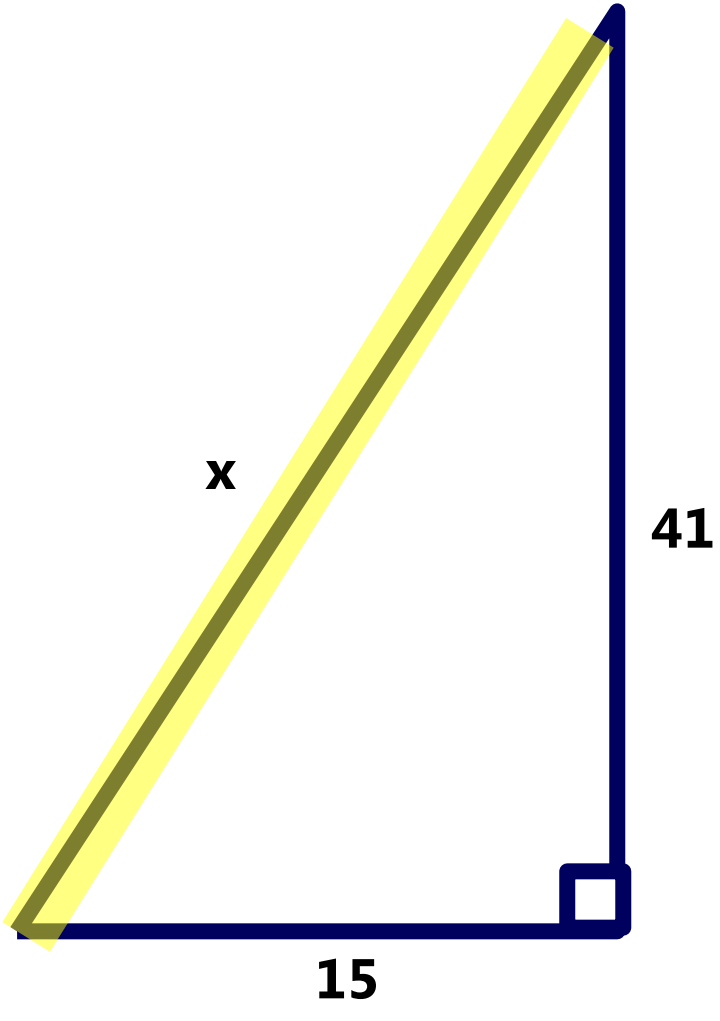
Substitute in numbers

1 What is the length of the third side?



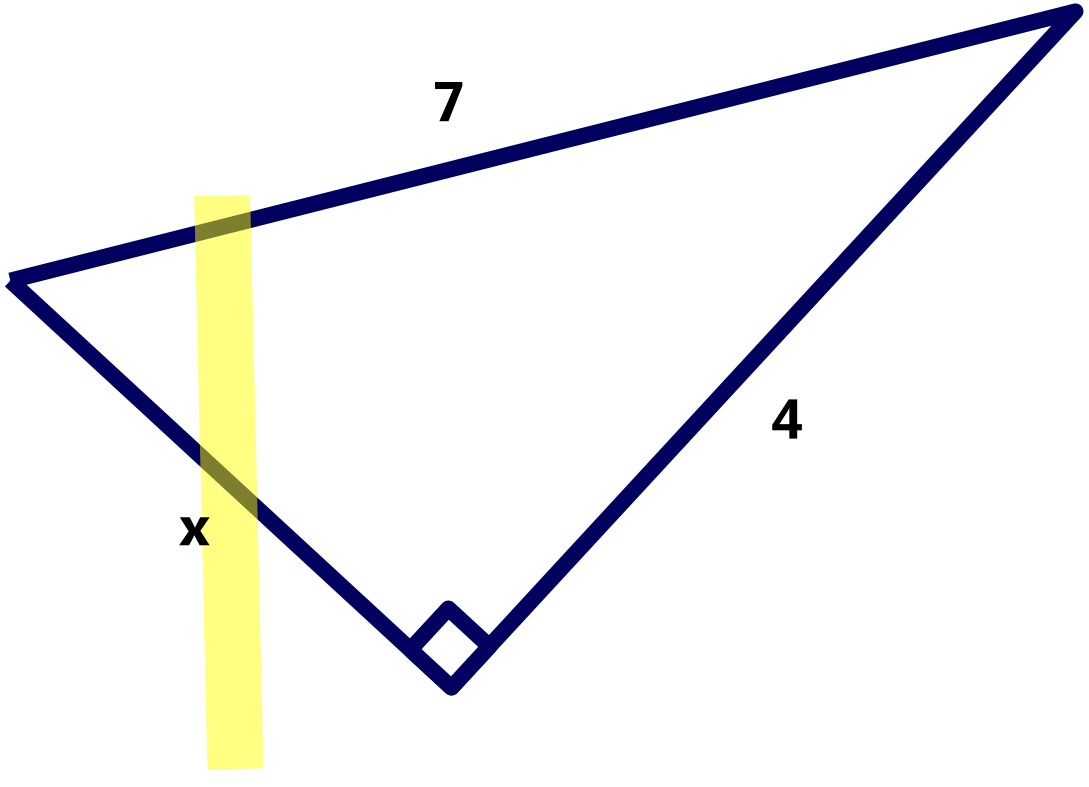
Answer

2 What is the length of the third side?



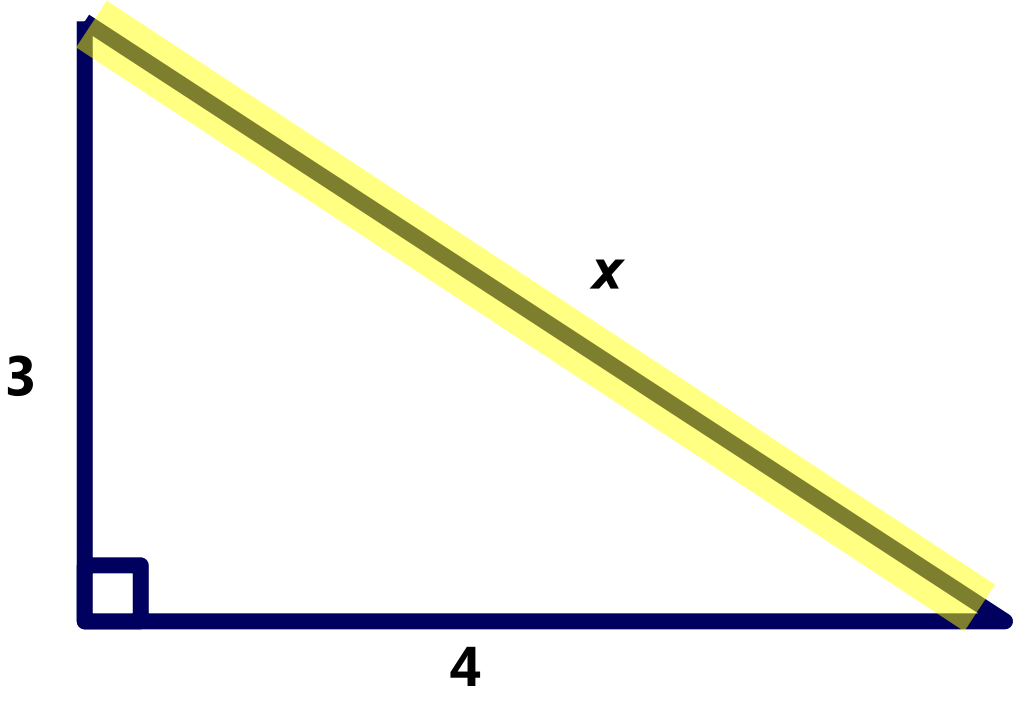
Answer

3 What is the length of the third side?



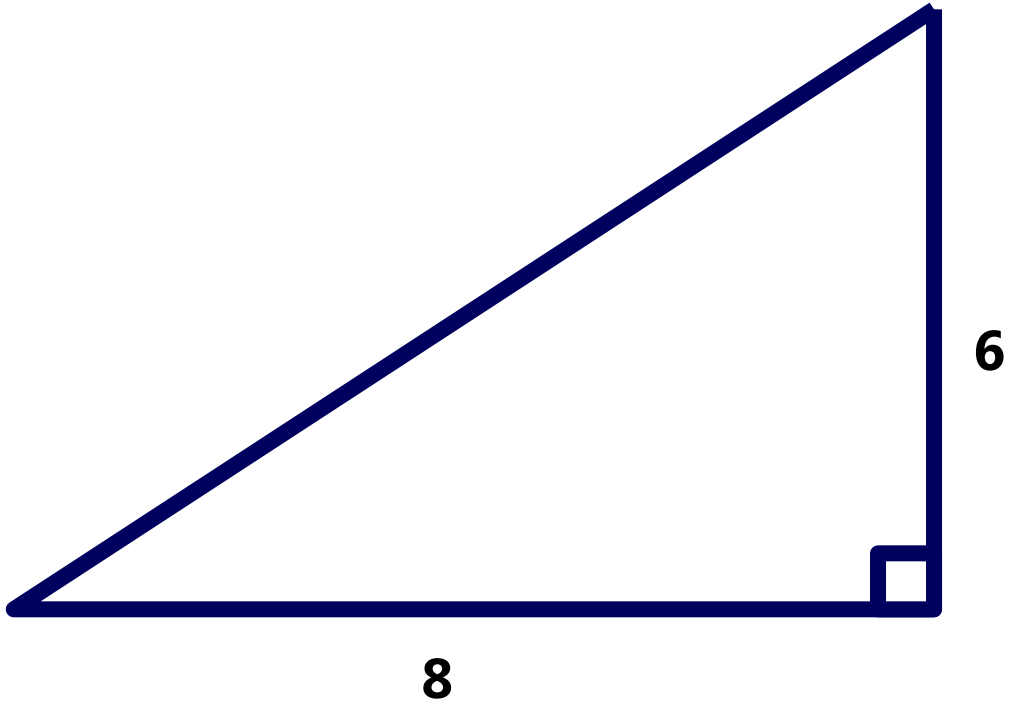
Answer

4 What is the length of the third side?



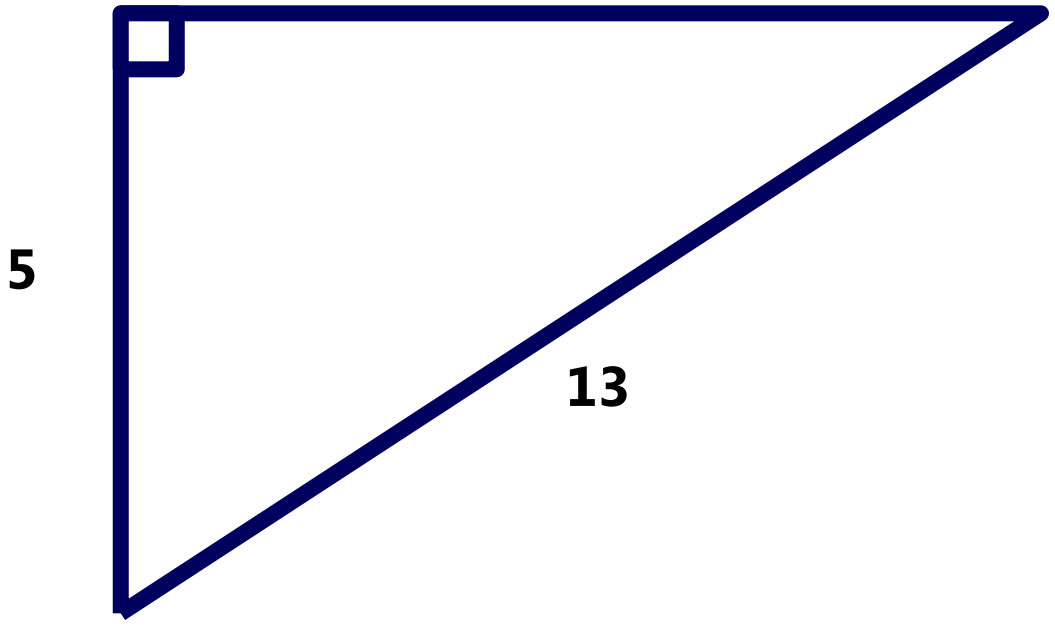
Answer

5 What is the length of the third side?



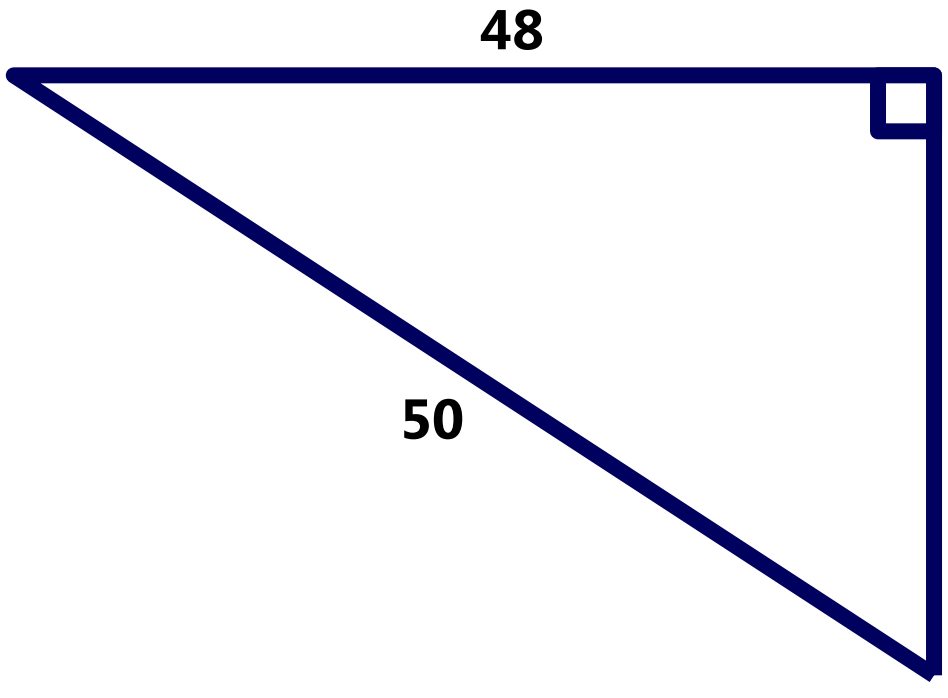
Answer

6 What is the length of the third side?



Answer

7 What is the length of the third side?



Answer

8 The legs of a right triangle are 7.0 and 3.0, what is the length of the hypotenuse?

Answer

9 The legs of a right triangle are 2.0 and 12, what is the length of the hypotenuse?

Answer

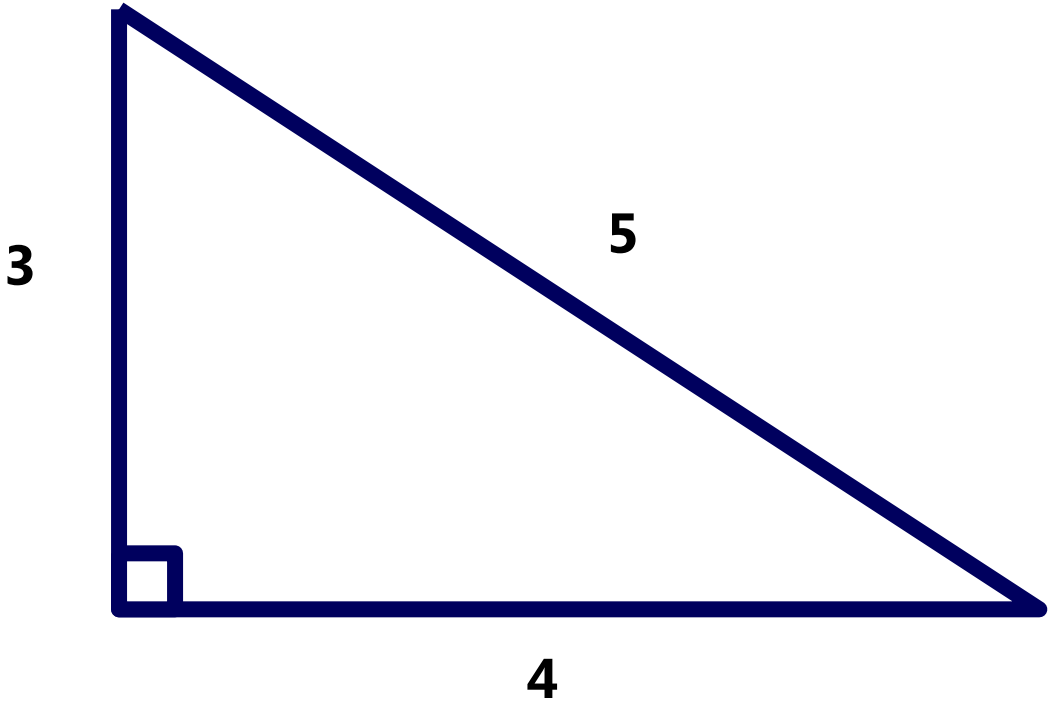
10 The hypotenuse of a right triangle has a length of 4.0 and one of its legs has a length of 2.5. What is the length of the other leg?

Answer

11 The hypotenuse of a right triangle has a length of 9.0 and one of its legs has a length of 4.5. What is the length of the other leg?

Answer

Pythagorean Triples



There are combinations of whole numbers that work in the Pythagorean Theorem. These sets of numbers are known as **Pythagorean Triples**.

3-4-5 is the most famous of the triples. If you recognize the sides of the triangle as being a triple (or multiple of one), you won't need a calculator!

Pythagorean Triples

Can you find any other Pythagorean Triples?

Use the list of squares to see if any other triples work.

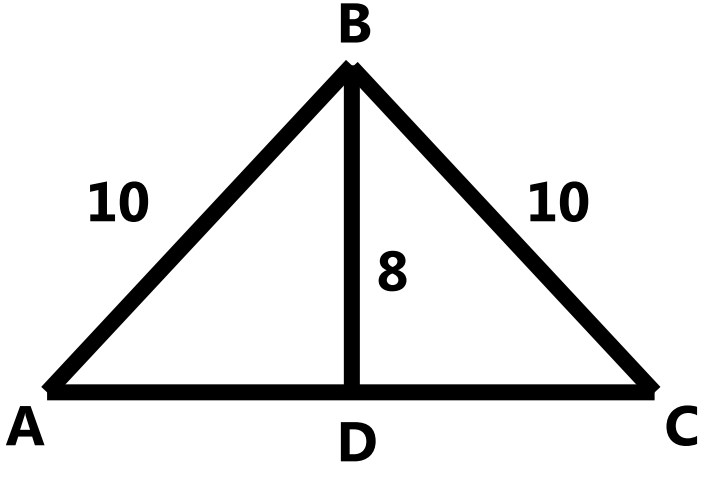
$1^2 = 1$ $11^2 = 121$ $21^2 = 441$
$2^2 = 4$ $12^2 = 144$ $22^2 = 484$
$3^2 = 9$ $13^2 = 169$ $23^2 = 529$
$4^2 = 16$ $14^2 = 196$ $24^2 = 576$
$5^2 = 25$ $15^2 = 225$ $25^2 = 625$
$6^2 = 36$ $16^2 = 256$ $26^2 = 676$
$7^2 = 49$ $17^2 = 289$ $27^2 = 729$
$8^2 = 64$ $18^2 = 324$ $28^2 = 784$
$9^2 = 81$ $19^2 = 361$ $29^2 = 841$
$10^2 = 100$ $20^2 = 400$ $30^2 = 900$

Answer & Math Practice

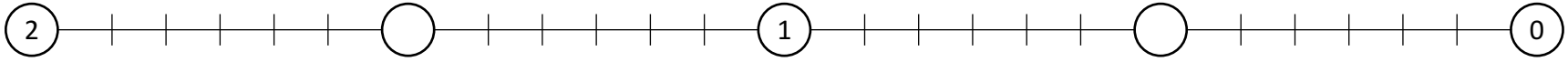
This is a great problem and draws on a lot of what we've learned.

Try it in your groups. Then we'll work on it step by step together by asking questions that break the problem into pieces. You have TWO minutes – GO!

In ΔABC, $\overline{BD}$ is perpendicular to $\overline{AC}$. The dimensions are shown in centimeters.



What is the length of $\overline{AC}$?



From PARCC EOY sample test calculator #1

12 What have we learned that will help solve this problem?

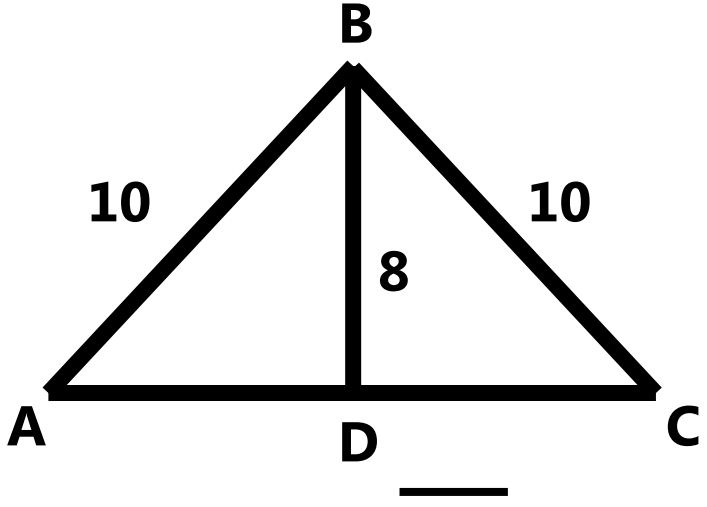
A Pythagorean Theorem

B Pythagorean Triples

C Distance Formula

D A and B only

In ΔABC, $\overline{BD}$ is perpendicular to $\overline{AC}$. The dimensions are shown in centimeters.

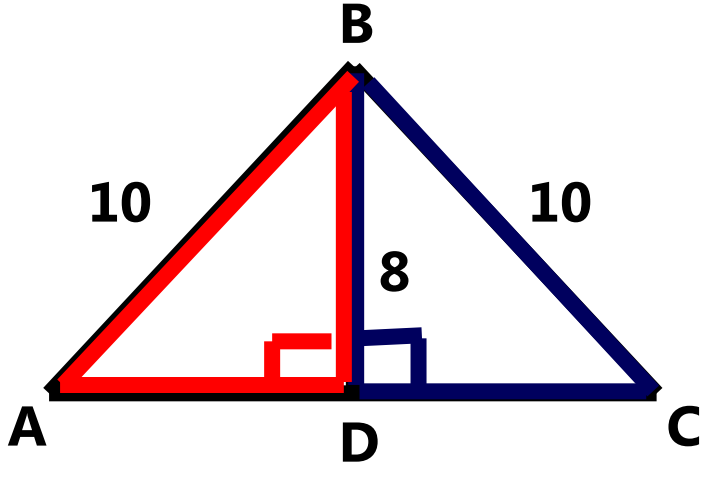


What is the length of $\overline{AC}$?

Answer

First, notice that we have two right triangles (perpendicular lines make right angles). The triangles are outlined red & blue in the diagram below.

In ΔABC, $\overline{BD}$ is perpendicular to $\overline{AC}$. The dimensions are shown in centimeters.



What is the length of $\overline{AC}$?

13 What is the length of the 3rd side in the red triangle?

A 3 cm

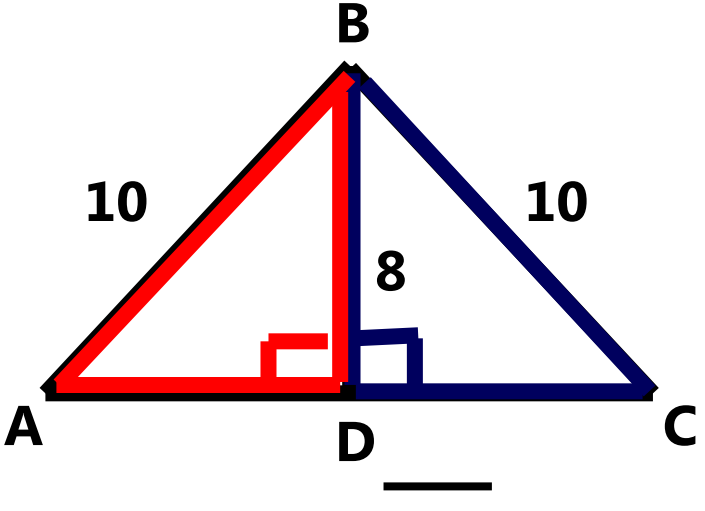
B 6 cm

C 9 cm

D 13.45 cm

Answer

In ΔABC, $\overline{BD}$ is perpendicular to $\overline{AC}$. The dimensions are shown in centimeters.



What is the length of $\overline{AC}$?

14 How is AD related to CD?

A $AD > CD$

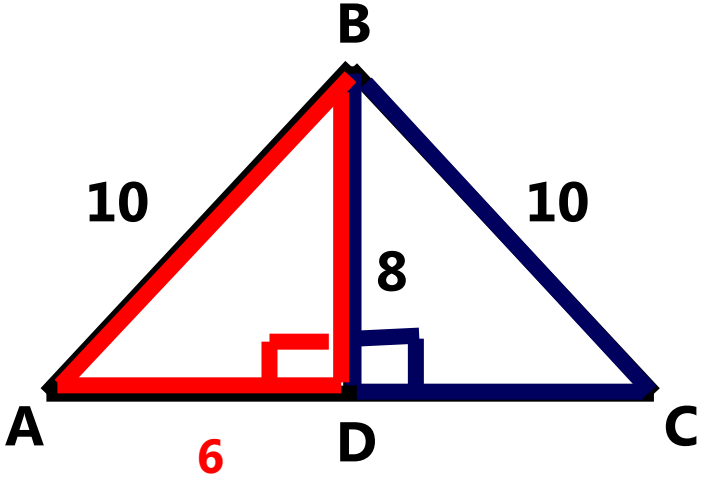
B $AD < CD$

C $AD = CD$

D not enough information to relate these segments

Answer

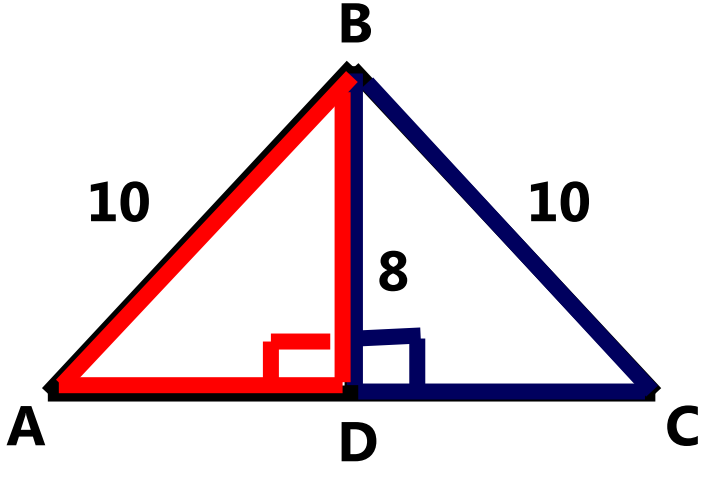
In ΔABC, $\overline{BD}$ is perpendicular to $\overline{AC}$. The dimensions are shown in centimeters.



What is the length of $\overline{AC}$?

15 What is the length of $\overline{AC}$?

In ΔABC, $\overline{BD}$ is perpendicular to $\overline{AC}$. The dimensions are shown in centimeters.



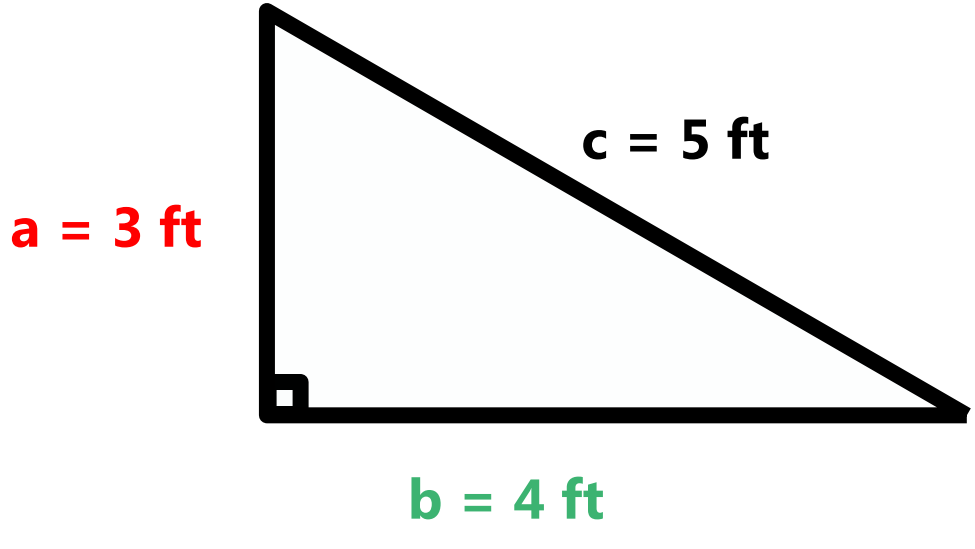
What is the length of $\overline{AC}$?

Answer

Converse of the Pythagorean Theorem

If a and b are measures of the shorter sides of a triangle,
c is the measure of the longest side, and
$c^2 = a^2 + b^2$, then the triangle is a right triangle.

If $c^2 \neq a^2 + b^2$, then the triangle is not a right triangle. This is the Converse of the Pythagorean Theorem.



Converse of the Pythagorean Theorem

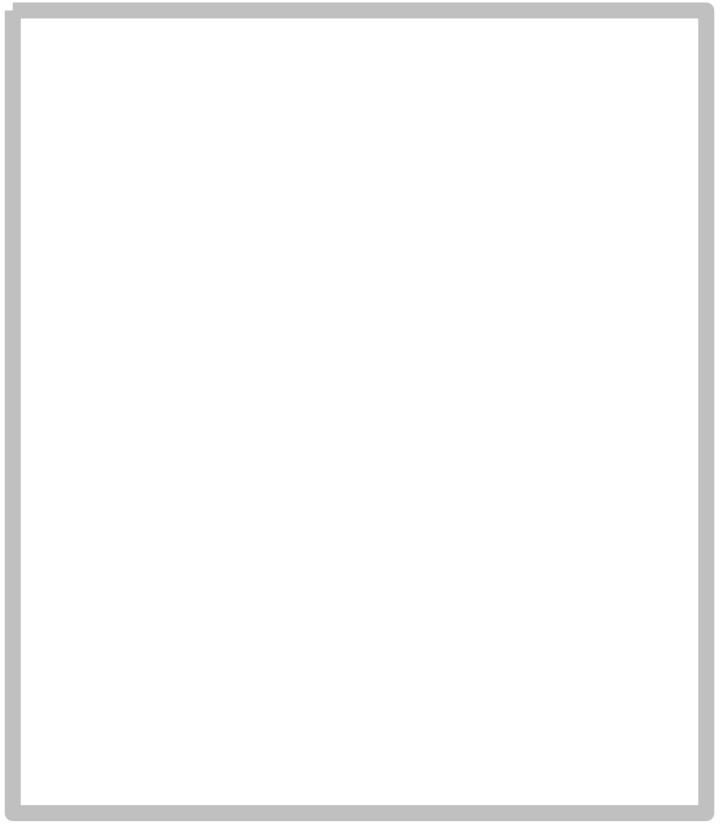
In other words, you can check to see if a triangle is a right triangle by seeing if the Pythagorean Theorem is true.

Test the Pythagorean Theorem. If the final equation is true, then the triangle is right. If the final equation is false, then the triangle is not right.

Converse of the Pythagorean Theorem

8 in, 17 in, 15 in

Is it a Right Triangle?

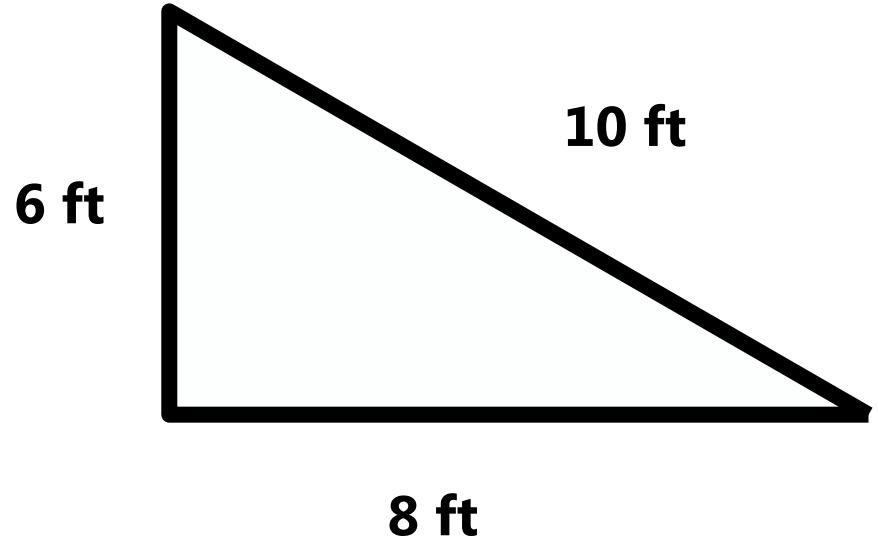


Math Practice

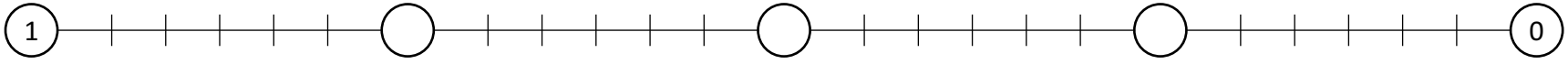
16 Is the triangle a right triangle?

Yes

No



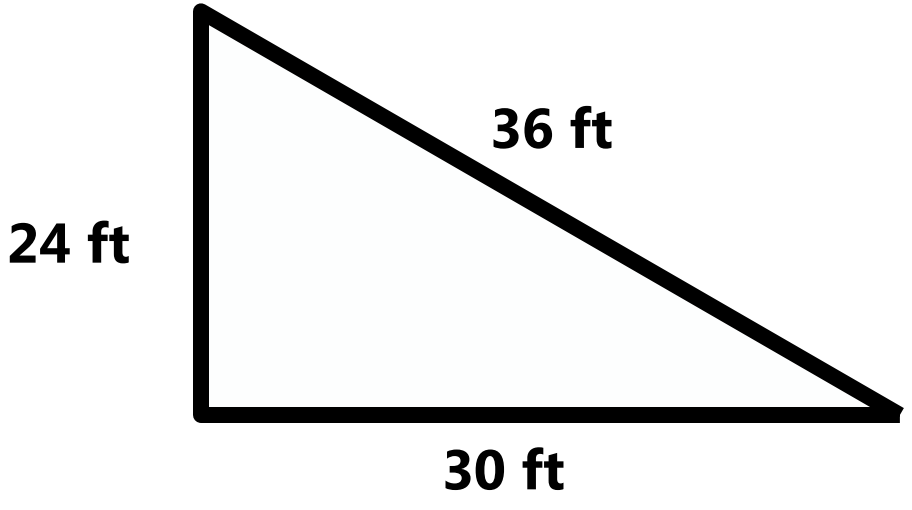
Answer



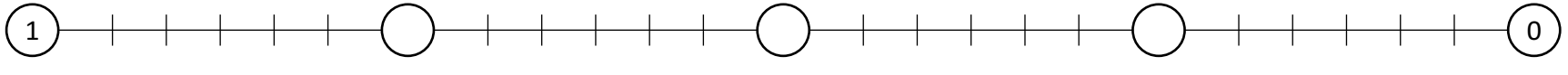
17 Is the triangle a right triangle?

Yes

No



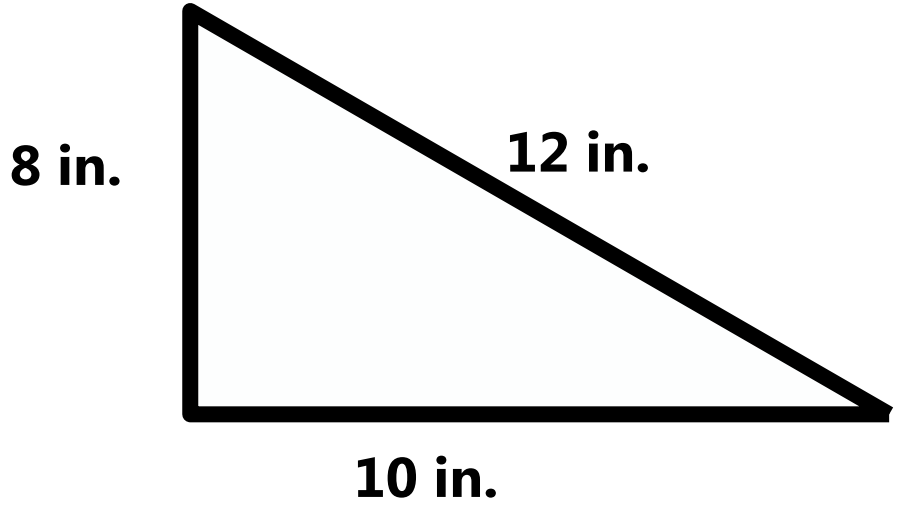
Answer



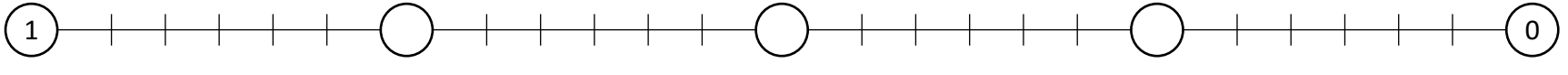
18 Is the triangle a right triangle?

Yes

No



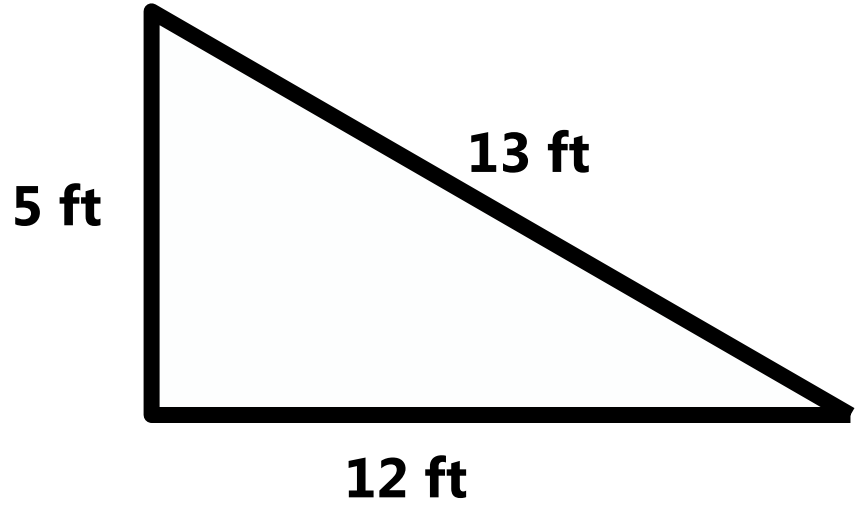
Answer



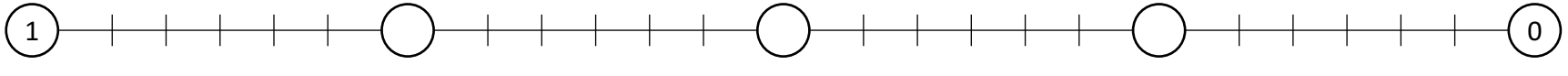
19 Is the triangle a right triangle?

Yes

No



Answer

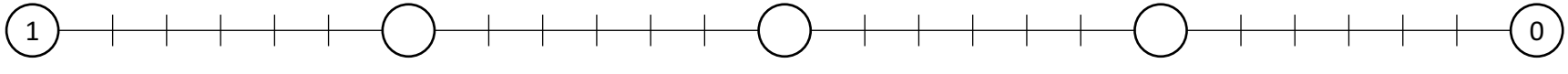


20 Can you construct a right triangle with three lengths of wood that measure 7.5 in, 18 in and 19.5 in?

Yes

No

Answer



Applications of Pythagorean Theorem

Steps to Pythagorean Theorem Application Problems.

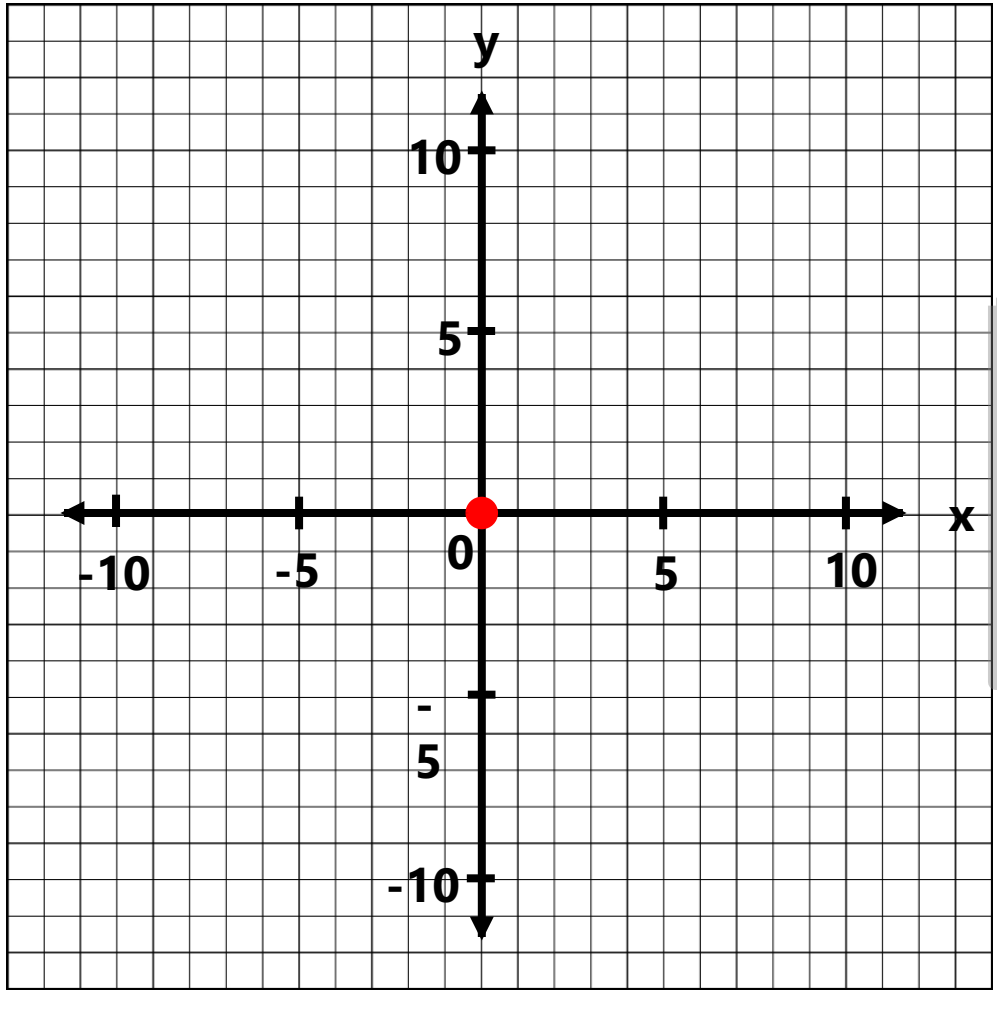
1. Draw a right triangle to represent the situation.
2. Solve for unknown side length.
3. Round to the nearest tenth.

Math Practice

Applications of Pythagorean Theorem

Work with your partners to complete:

To get from his high school to his home, Jamal travels 5.0 miles east and then 4.0 miles north. When Sheila goes to her home from the same high school, she travels 8.0 miles east and 2.0 miles south. What is the measure of the shortest distance, to the *nearest tenth of a mile,* between Jamal's home and Sheila's home?



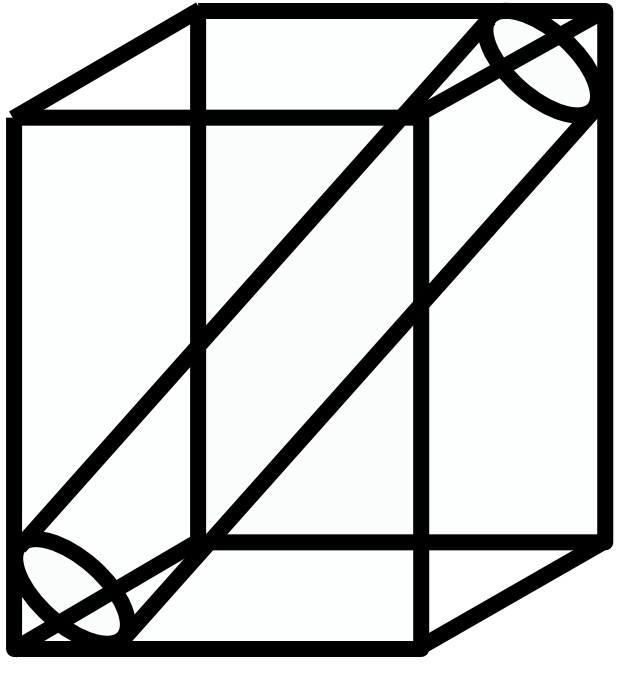
Answer

From the New York State Education Department. Office of Assessment Policy, Development and Administration. Internet. Available from www.nysedregents.org/IntegratedAlgebra; accessed 17, June, 2011.

Applications of Pythagorean Theorem

Work with your partners to complete:

A straw is placed into a rectangular box that is 3 inches by 4 inches by 8 inches, as shown in the accompanying diagram. If the straw fits exactly into the box diagonally from the bottom left front corner to the top right back corner, how long is the straw, to the *nearest tenth of an inch*?



Answer

From the New York State Education Department. Office of Assessment Policy, Development and Administration. Internet. Available from www.nysedregents.org/IntegratedAlgebra; accessed 17, June, 2011.

Applications of Pythagorean Theorem

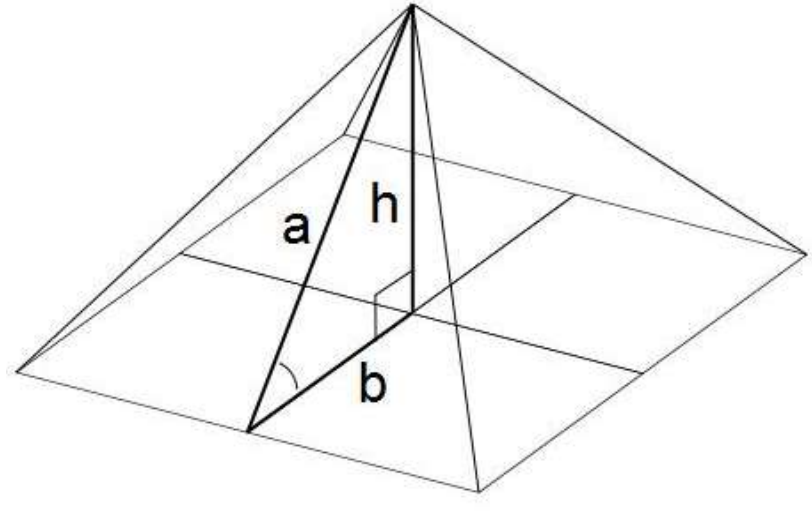
The Pythagorean Theorem can be applied to 3 Dimensional Figures

In this figure:

a = *slant height (height of triangular face)*

b = 1/2 base length (from midpoint of side of base to center of the base of the pyramid)

h = height of pyramid

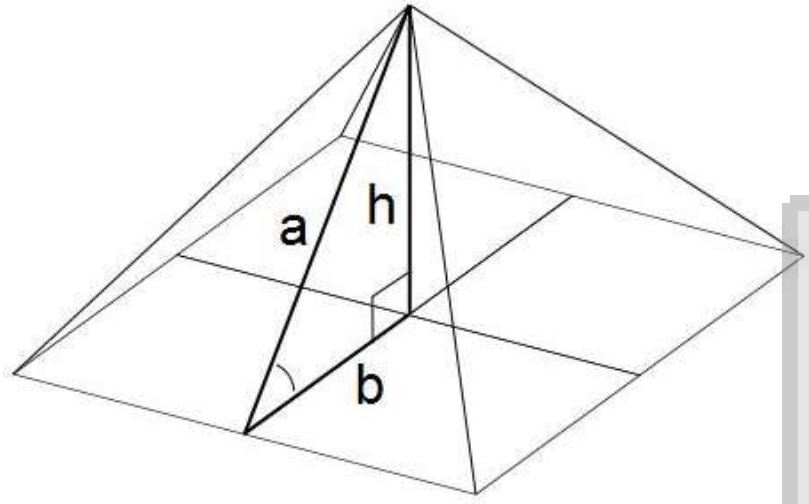


Applications of Pythagorean Theorem

A right triangle is formed between the three lengths.

If you know two of the measurements, you can calculate the third.

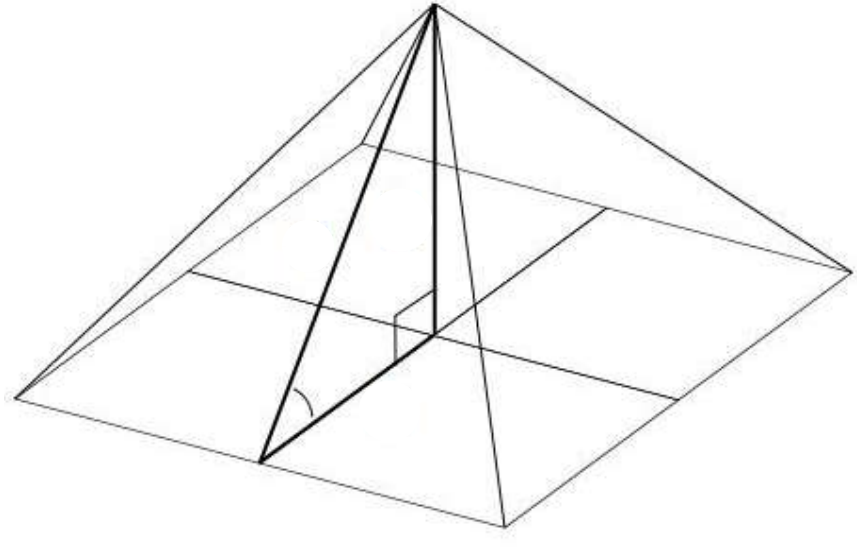
EXAMPLE:
Find the slant height of a pyramid whose height is 5 cm and whose base has a length of 8cm.



Answer

Applications of Pythagorean Theorem

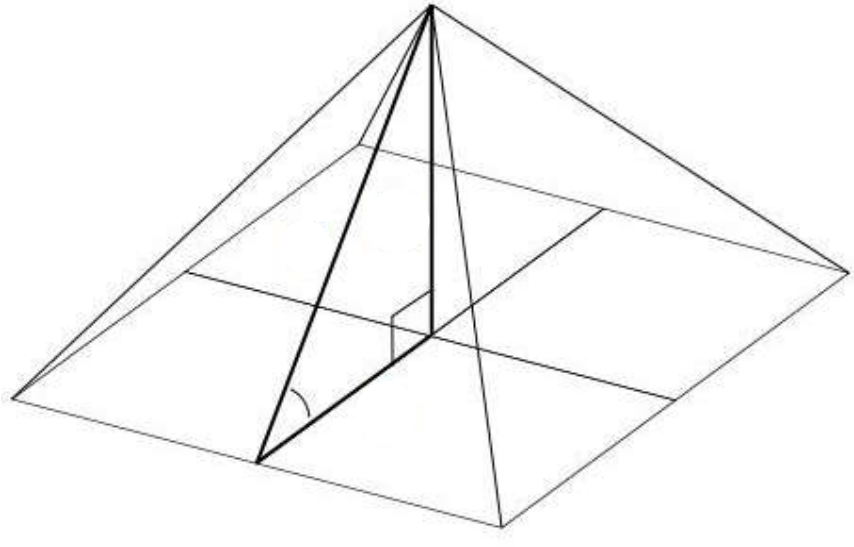
Find the slant height of the pyramid whose base length is 10 cm and height is 12 cm. Label the diagram with the measurements.



Answer

Applications of Pythagorean Theorem

Find the base length of the pyramid whose height is 21 m and slant height is 29 m. Label the diagram with the measurements.

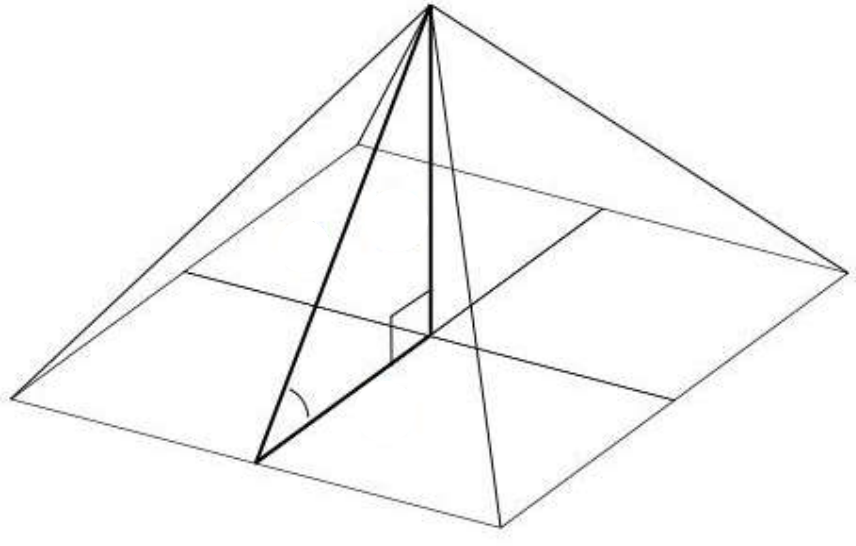


Answer

21 The sizes of television and computer monitors are given in inches. However, these dimensions are actually the diagonal measure of the rectangular screens. Suppose a 14-inch computer monitor has an actual screen length of 11-inches. What is the height of the screen?

Answer

22 Find the height of the pyramid whose base length is 16 in and slant height is 17 in. Label the diagram with the measurements.



Answer

23 A tree was hit by lightning during a storm. The part of the tree still standing is 3 meters tall. The top of the tree is now resting 8 meters from the base of the tree, and is still partially attached to its trunk. Assume the ground is level. How tall was the tree originally?

Answer

24 Suppose you have a ladder of length 13 feet. To make it sturdy enough to climb you myct place the ladder exactly 5 feet from the wall of a building. You need to post a banner on the building 10 feet above ground. Is the ladder long enough for you to reach the location you need to post the banner?

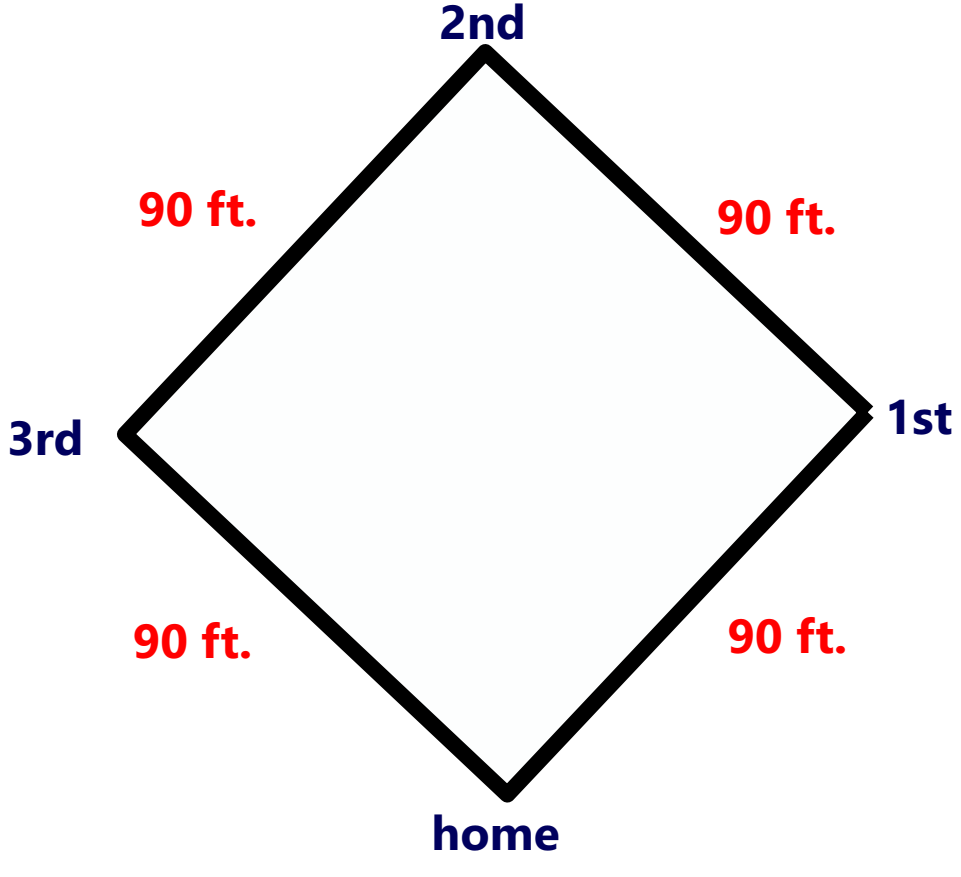
Yes

No

Answer

(Derived from engage^ny)

25 You've just picked up a ground ball at 3rd base, and you see the other team's player running towards 1st base. How far do you have to throw the ball to get it from third base to first base, and throw the runner out? (A baseball diamond is a square)



Answer

26 You're locked out of your house and the only open window is on the second floor, 25 feet above ground. There are bushes along the edge of your house, so you'll have to place a ladder 10 feet from the house. What length of ladder do you need to reach the window?

Answer

27 Scott wants to swim across a river that is 400 meters wide. He begins swimming perpendicular to the shore, but ends up 100 meters down the river because of the current. How far did he actually swim from his starting point?

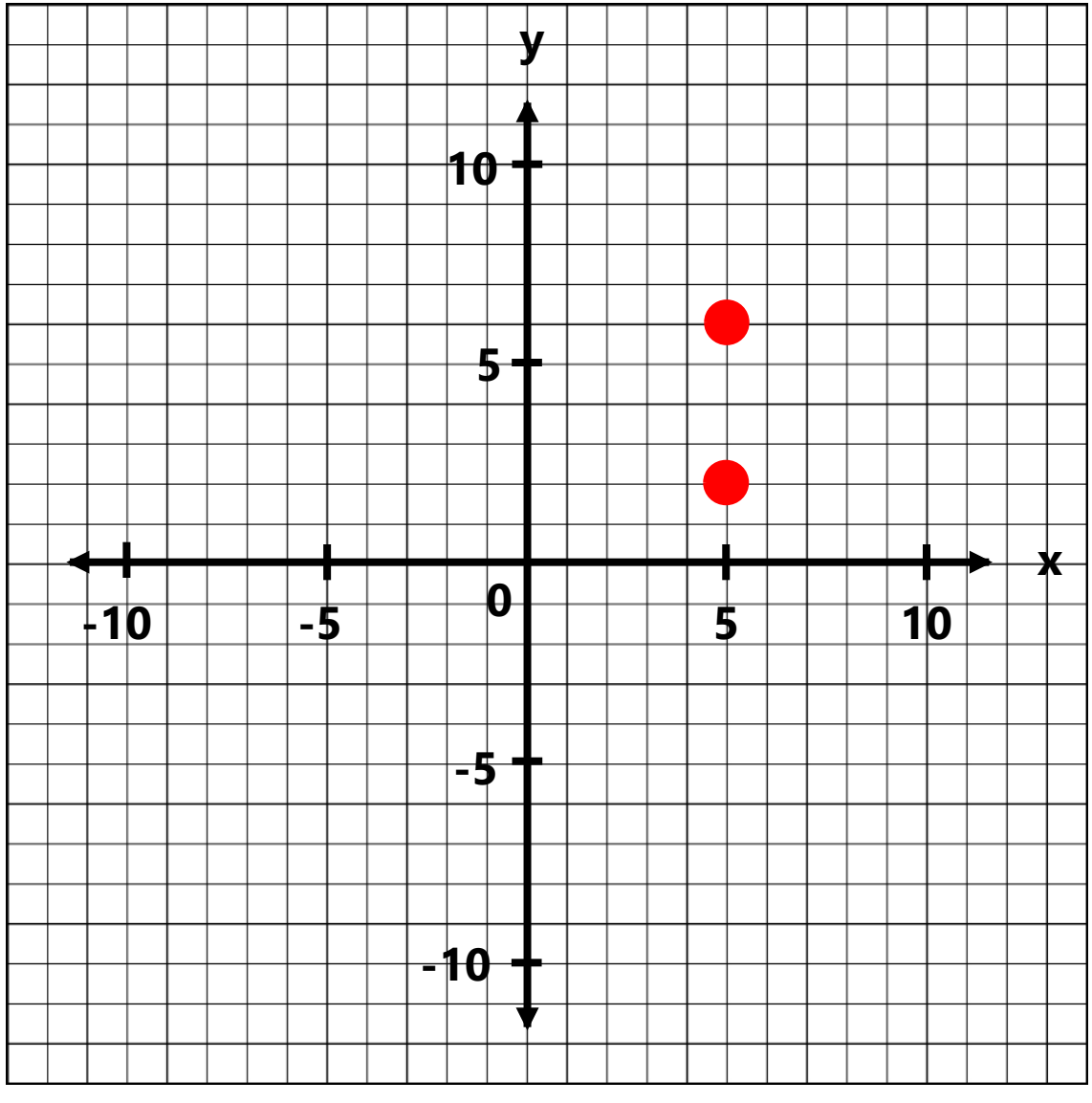
Answer

Distance Formula

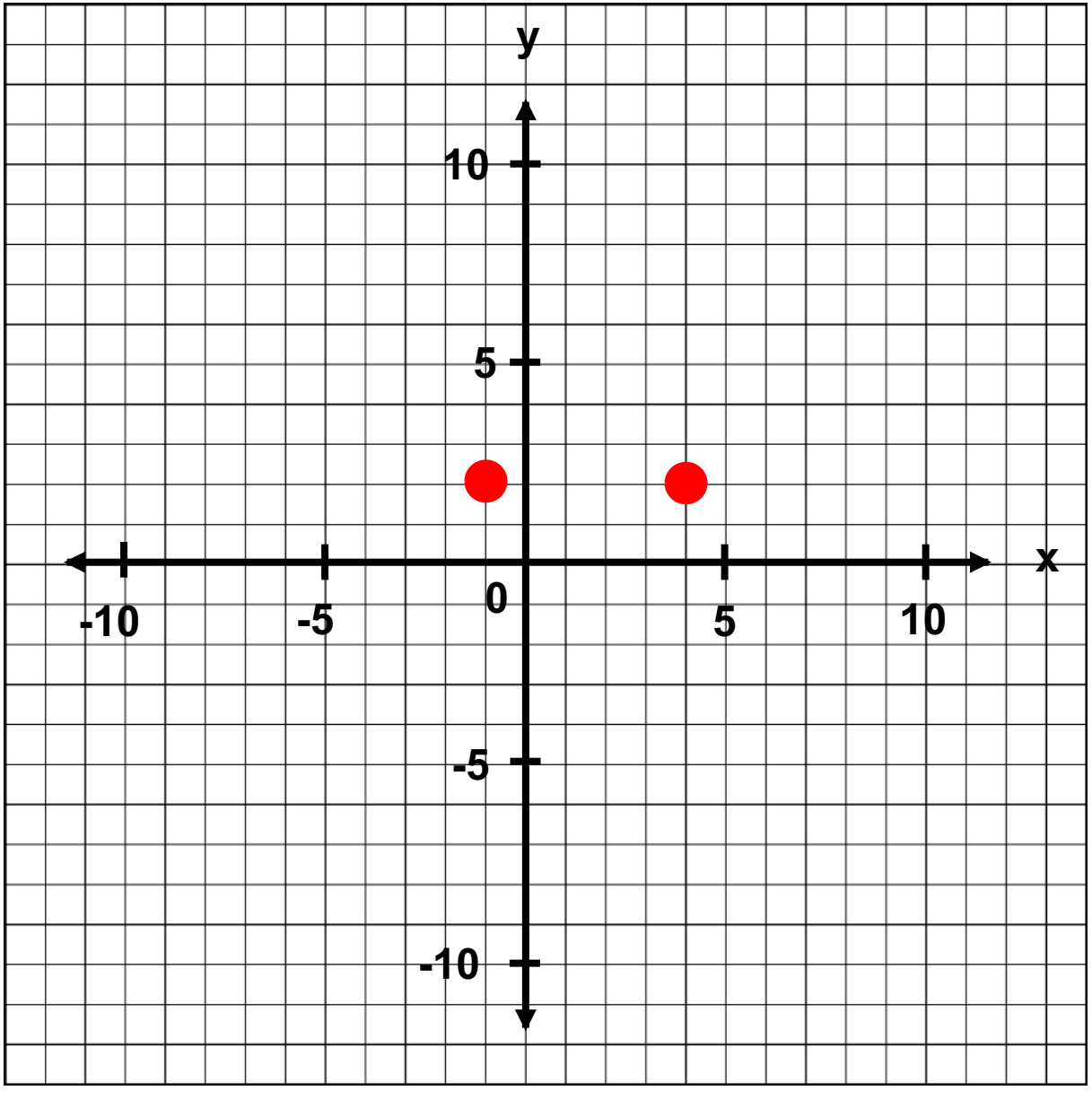
Click to return to
the table of contents

Distance Between Two Points

If you have two points on a graph, such as (5,2) and (5,6), you can find the **distance** between them by simply counting units on the graph, since they lie in a vertical line.

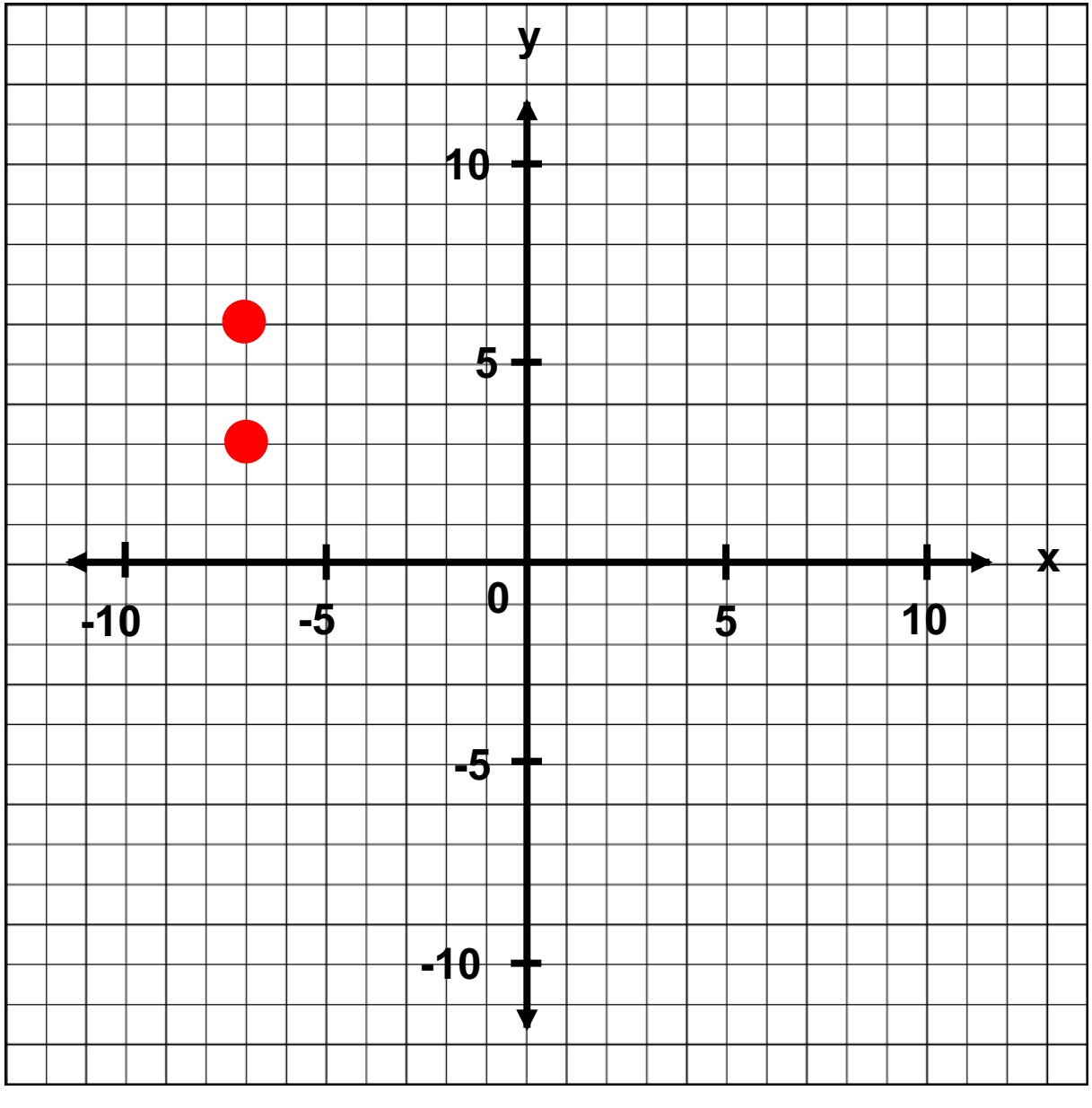


28 What is the distance between these two points?



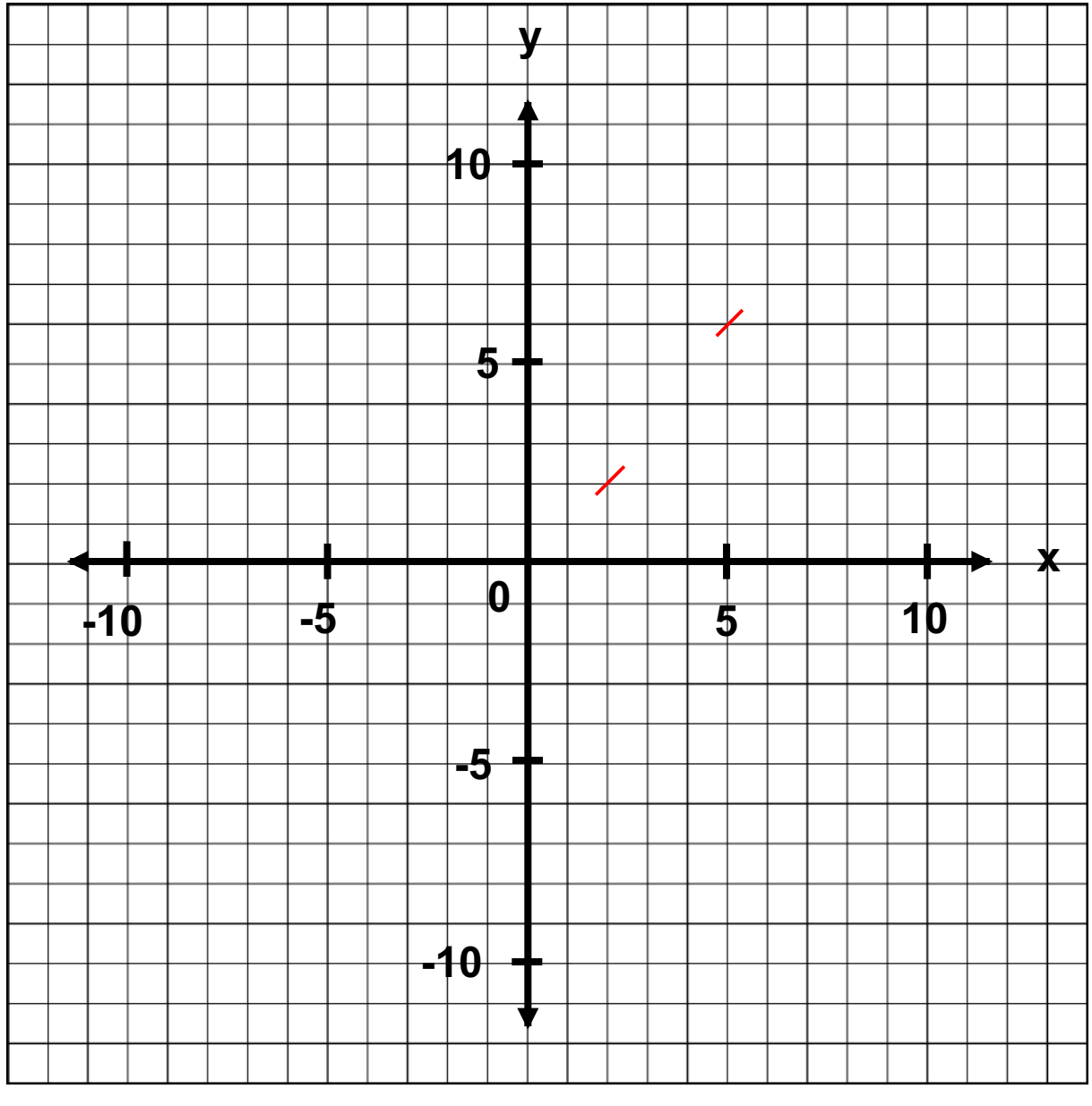
Answer

29 What is the distance between these two points?



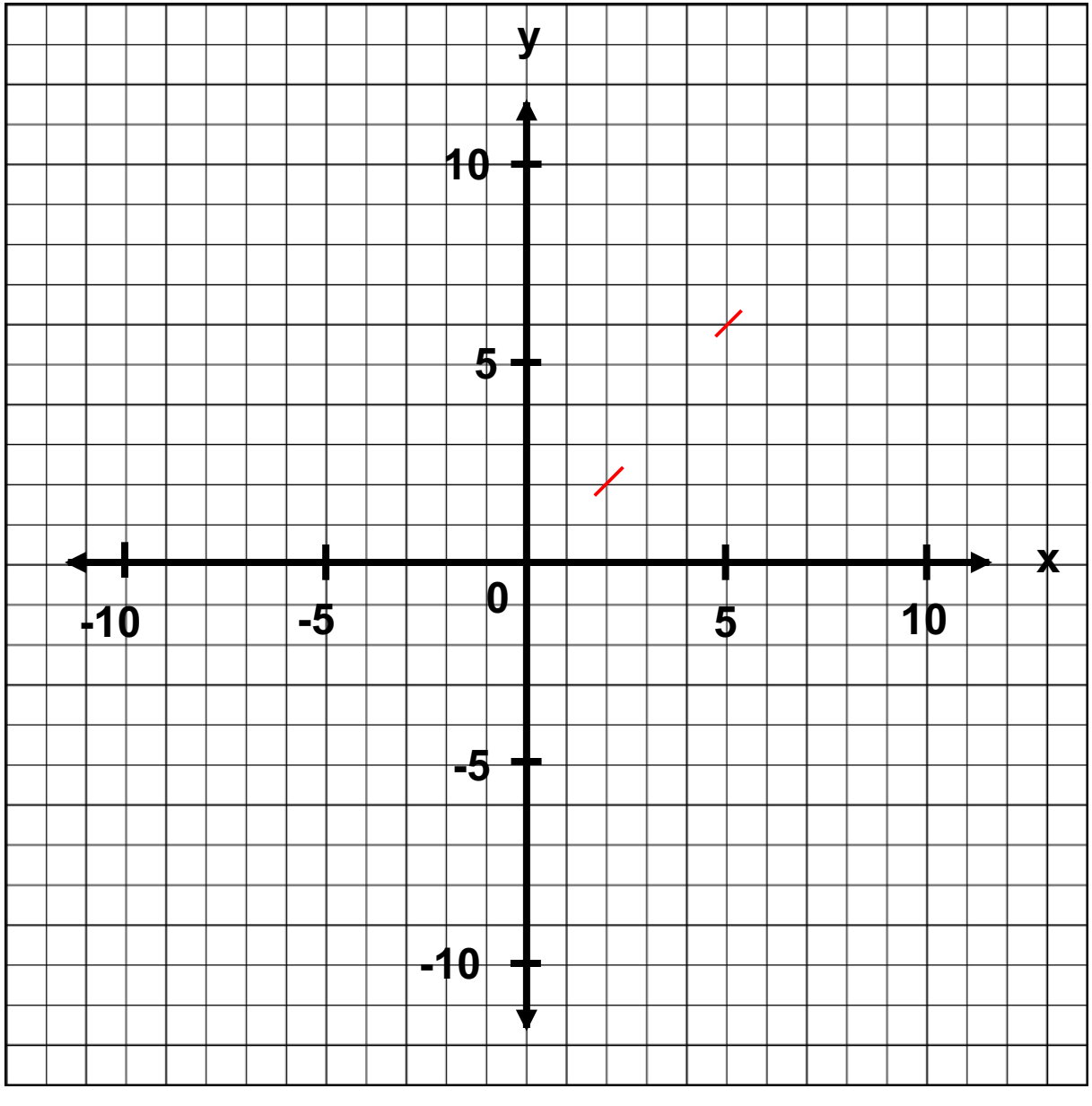
Answer

30 What is the distance between these two points?



Distance Between Two Points

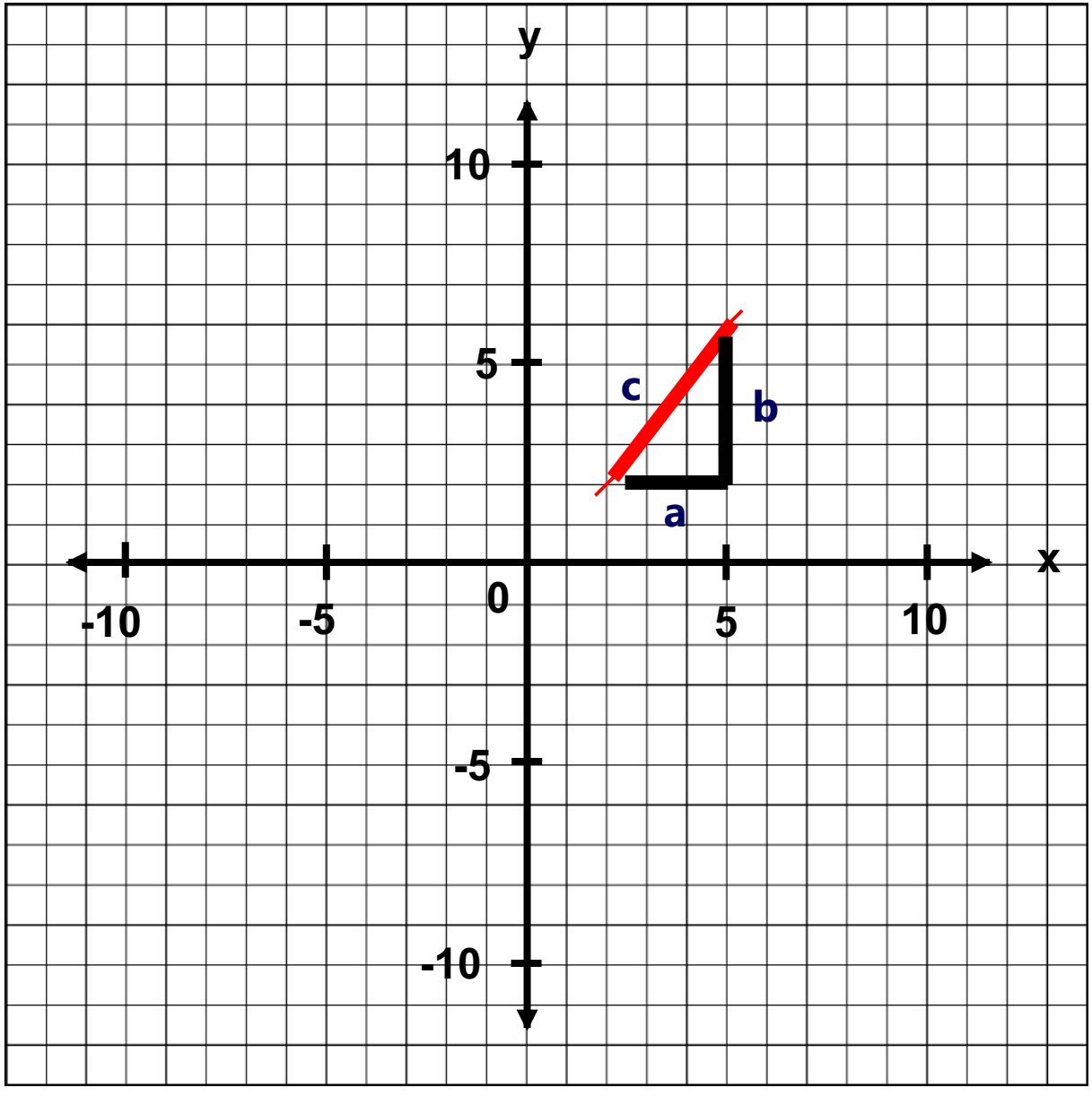
Most sets of points do not lie in a vertical or horizontal line.
For example:



Counting the units between these two points is impossible. So mathematicians have developed a formula using the Pythagorean theorem to find the distance between two points.

Distance Between Two Points

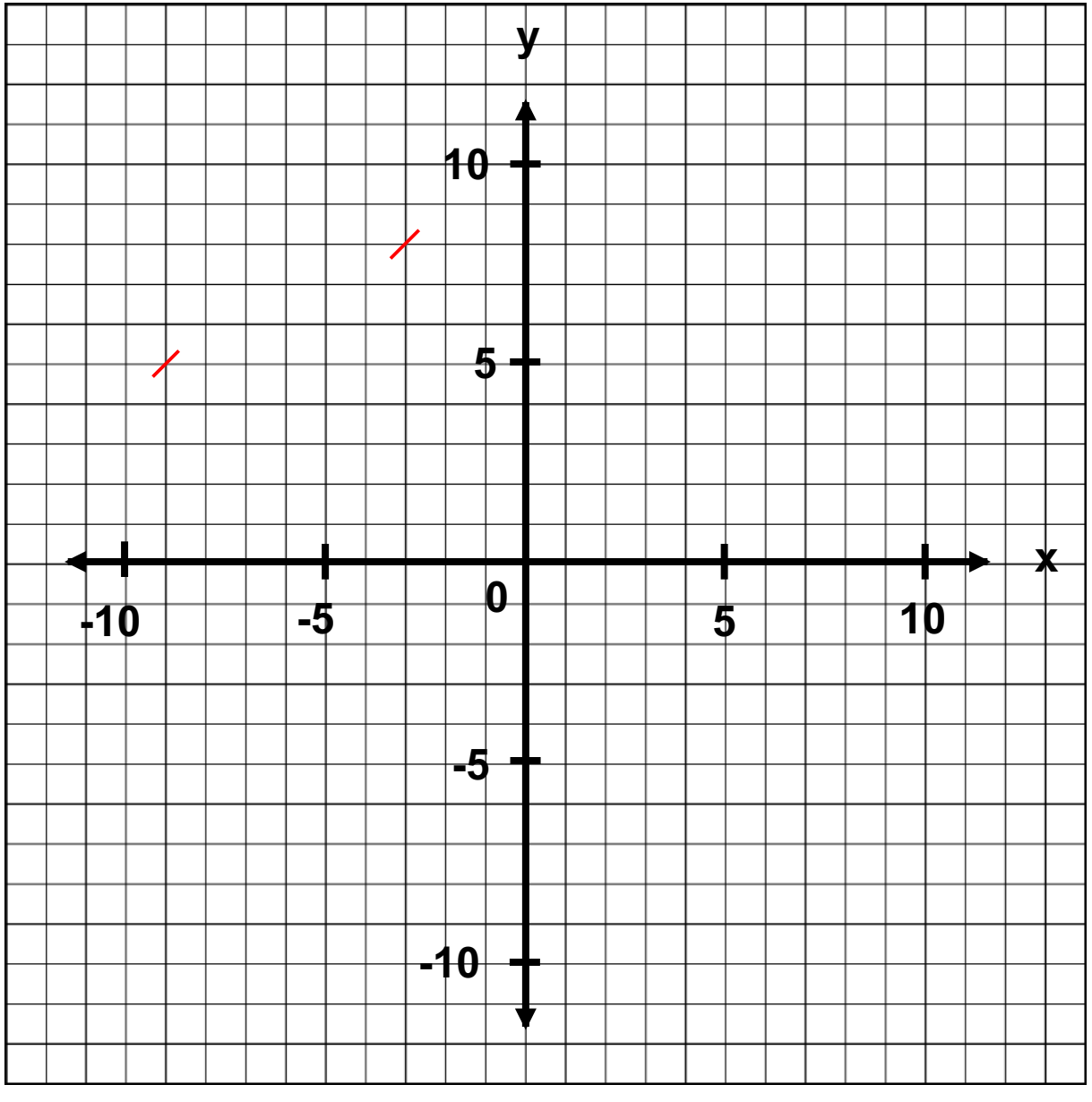
Draw the right triangle around these two points. Then use the Pythagorean theorem to find the distance in red.



Answer

Distance Between Two Points

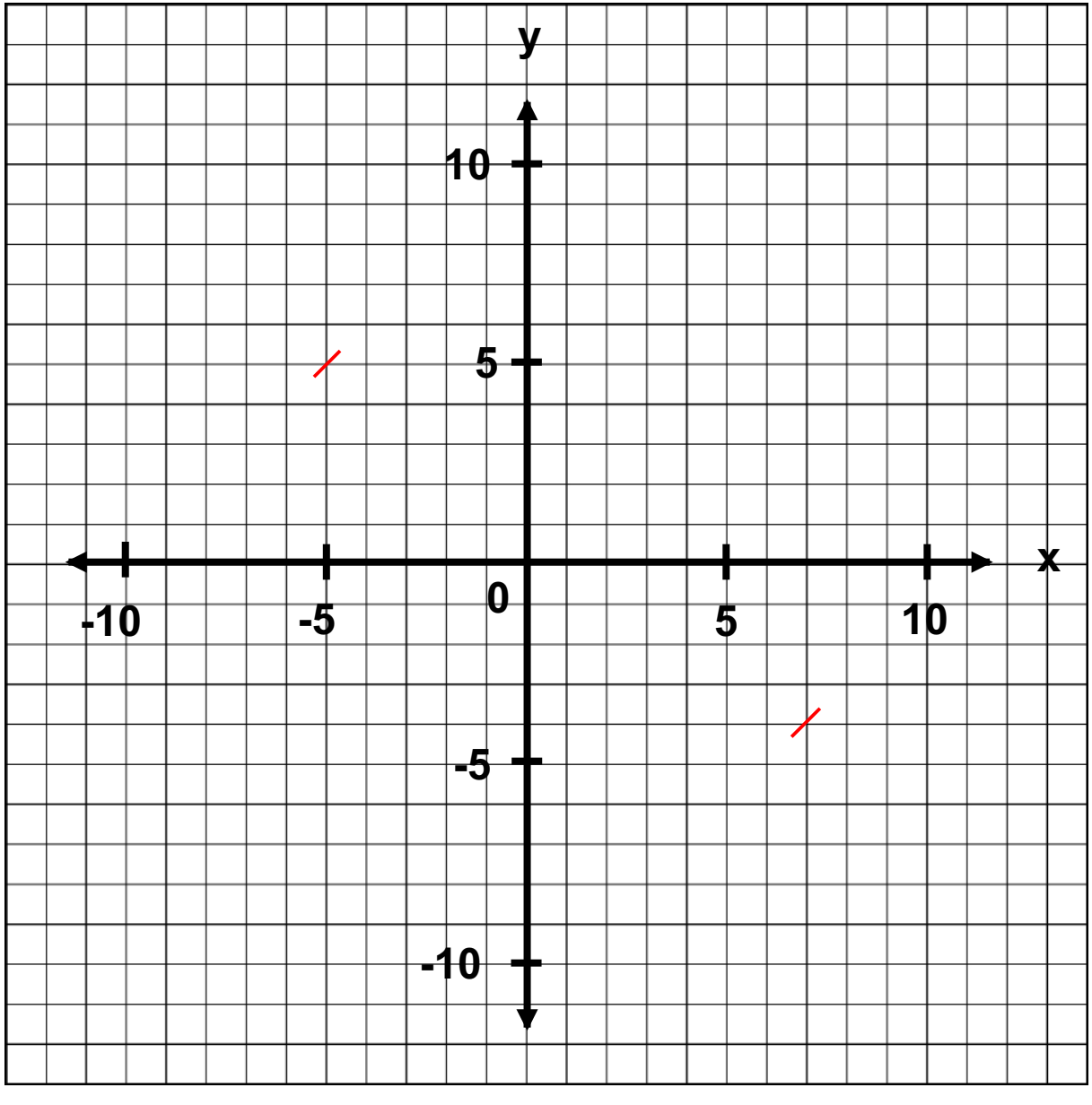
Example:



Answer

Distance Between Two Points

Try This:



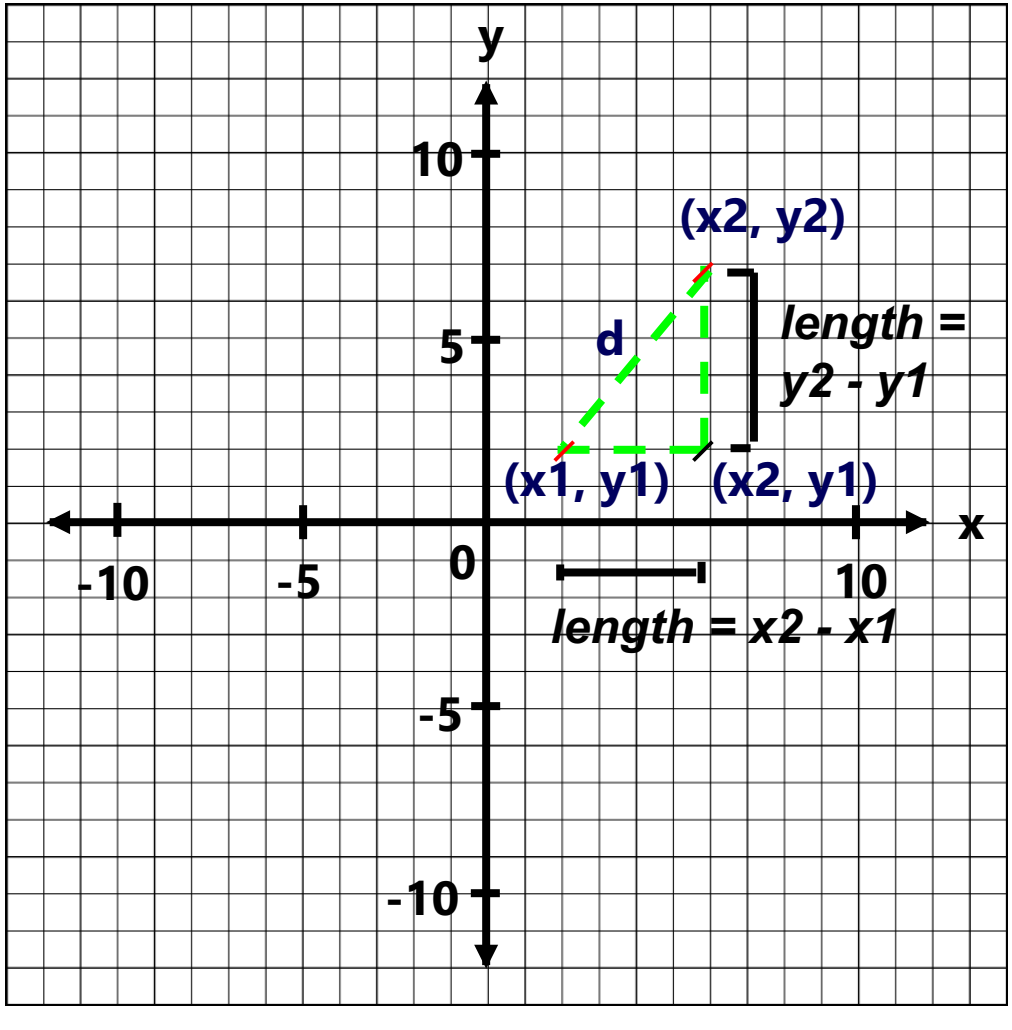
Answer

Distance Formula

Deriving a formula for calculating distance...

Distance Formula

Create a right triangle around the two points. Label the points as shown. Then substitute into the Pythagorean Theorem.



$c2 = a2 + b2$

$d2 = (x2 - x1)2 + (y2 - y1)2$

$$d = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$$

This is the distance formula now substitute in values.

Answer

Distance Formula

You can find the distance d between any two points (x1, y1) and (x2, y2) using the formula below.

$$d = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$$

how far between the x-coordinate

how far between the y-coordinate

Distance Formula

When only given the two points, use the formula.

Find the distance between:
Point 1 (−4, −7)
Point 2 (−5, −2)

Answer

31 Find the distance between (2, 3) and (6, 8).
Round answer to the nearest tenth.

Answer

hint

32 Find the distance between (−7, −2) and (11, 3).
Round answer to the nearest tenth.

Answer

hint

33 Find the distance between (4, 6) and (1, 5).
Round answer to the nearest tenth.

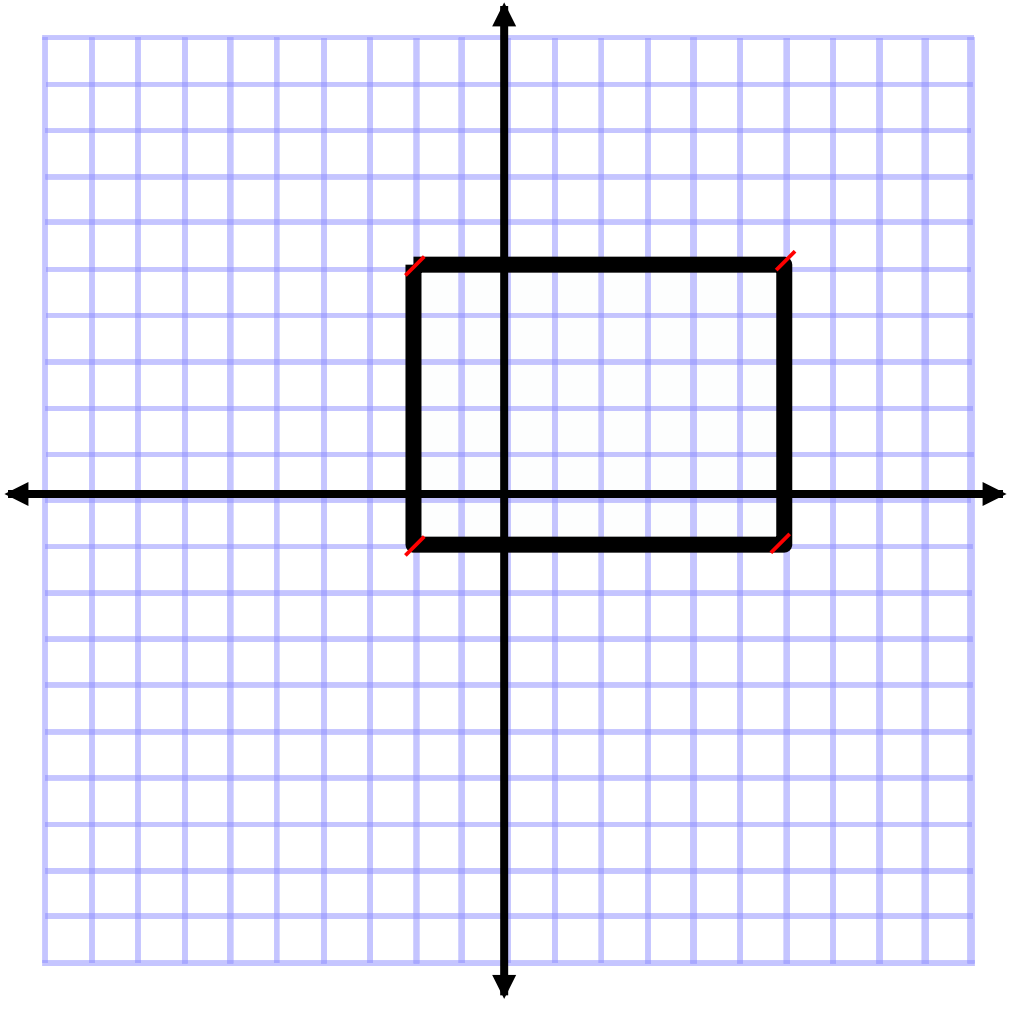
Answer

34 Find the distance between (7, −5) and (9, −1).
Round answer to the nearest tenth.

Answer

Applications of the Distance Formula

How would you find the perimeter of this rectangle?

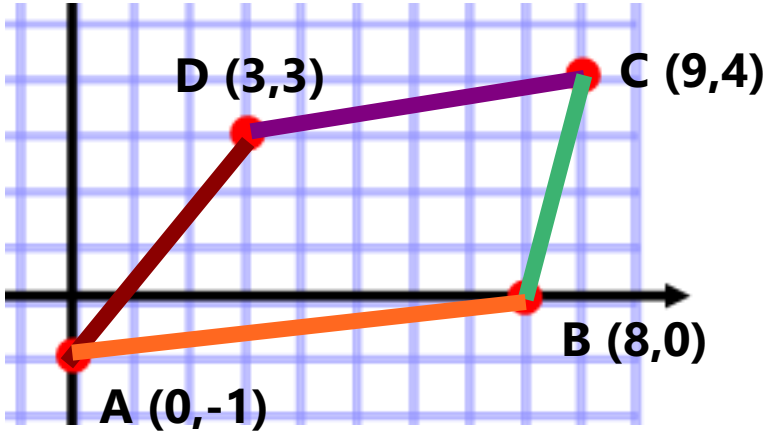


Either just count the units or find the distance between the points from the ordered pairs.

Answer

Applications of the Distance Formula

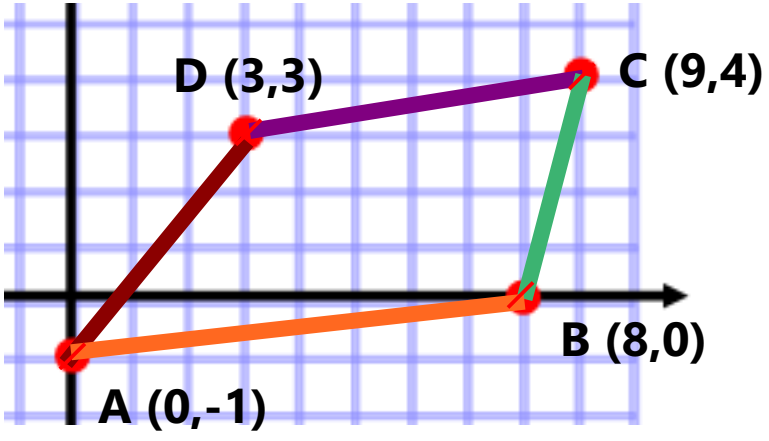
Can we just count how many units long each line segment is in this quadrilateral to find the perimeter?



Math Practice

Applications of the Distance Formula

You can use the Distance Formula
to solve geometry problems.



Find the perimeter of *ABCD*.
Use the distance formula to
find all four of the side lengths.
Then add then together.

AB =

AB =

BC =

BC =

CD =

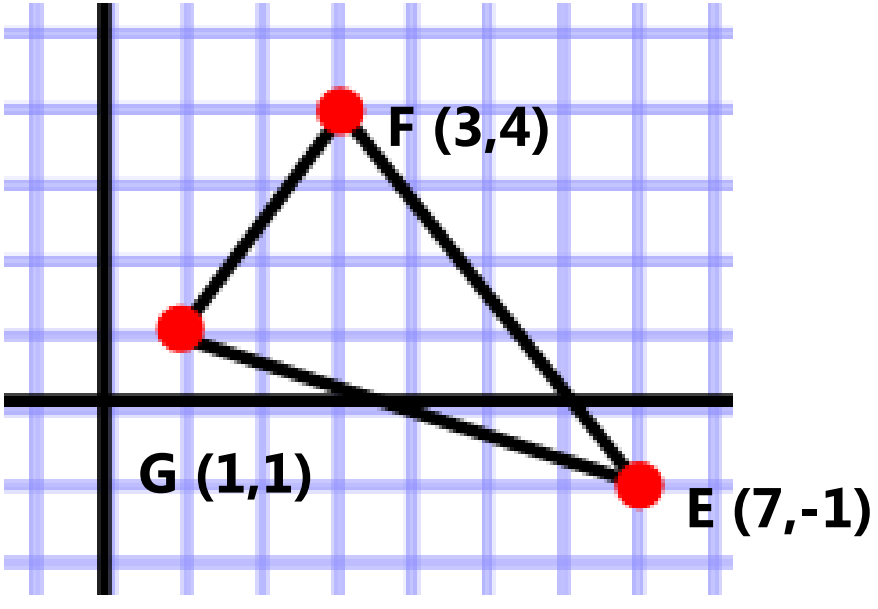
CD =

DA =

DA =

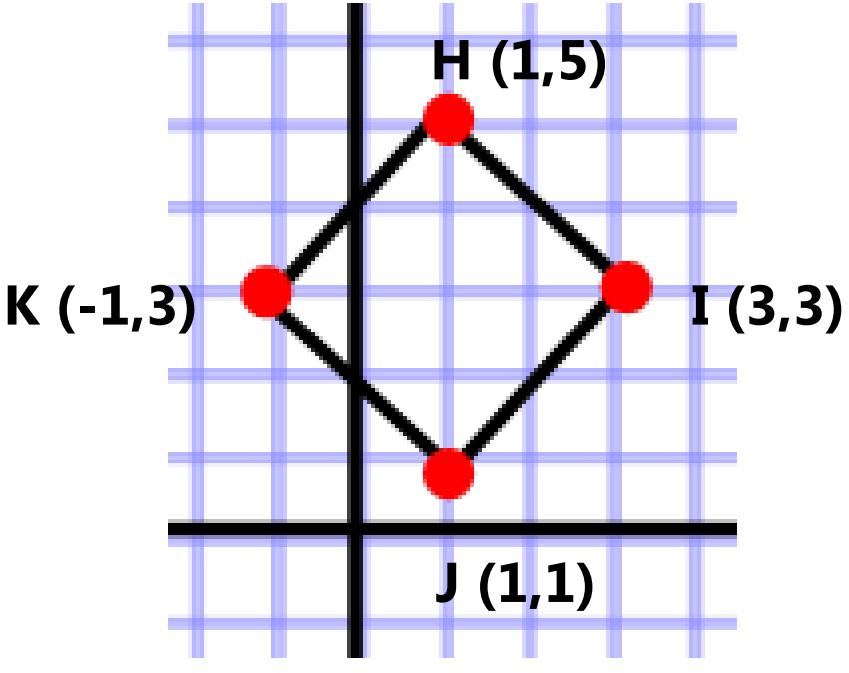
Answer

35 Find the perimeter of ΔEFG. Round the answer to the nearest tenth.



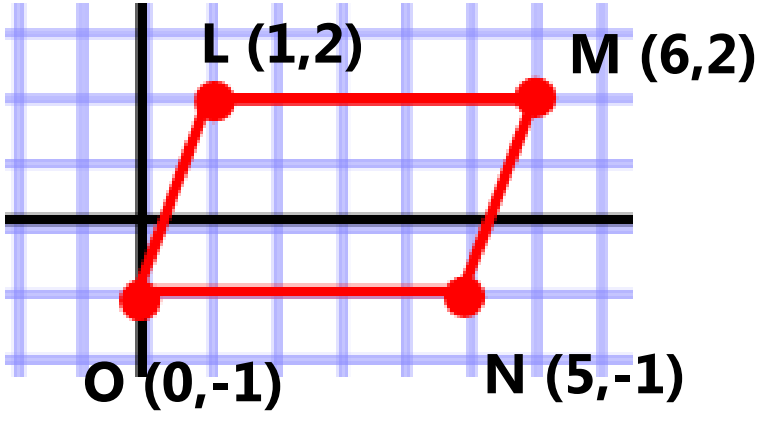
Answer

36 Find the perimeter of the square.
Round answer to the nearest tenth.



Answer

37 Find the perimeter of the parallelogram.
Round answer to the nearest tenth.

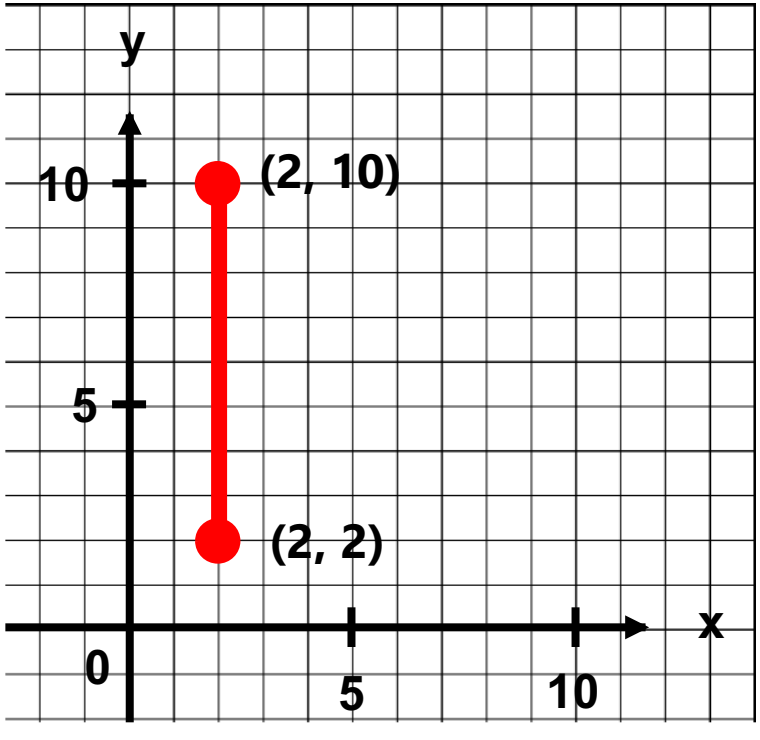


Answer

Midpoints

Click to return to
the table of contents

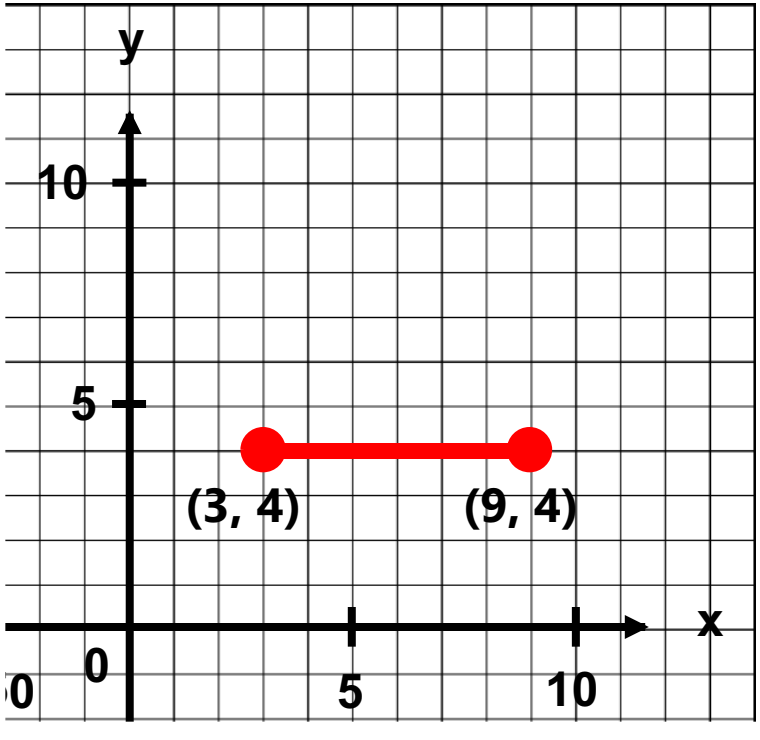
Midpoint



Find the **midpoint** of the line segment.

What is a midpoint?
How did you find the midpoint?
What are the coordinates of the midpoint?

Midpoint



Find the midpoint of the line segment.

What are the coordinates of the midpoint?

How is it related to the coordinates of the endpoints?

Answer & Math Practice

The Midpoint Formula

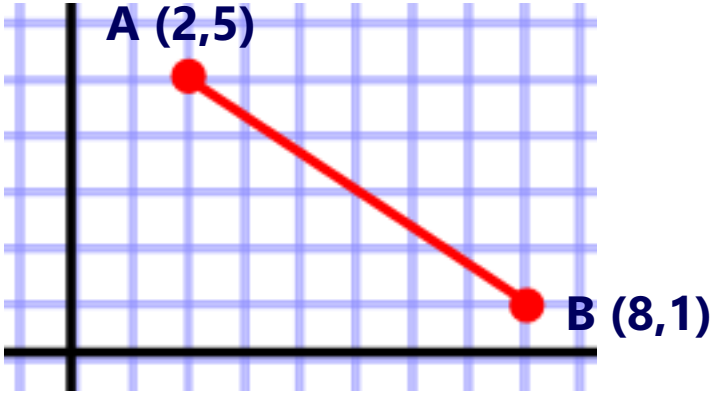
To calculate the midpoint of a line segment with endpoints (x1,y1) and (x2,y2) use the formula:

$$\left(\frac{x1 + x2}{2} , \frac{y1 + y2}{2} \right)$$

The x and y coordinates of the midpoint are the averages of the x and y coordinates of the endpoints, respectively.

The Midpoint Formula

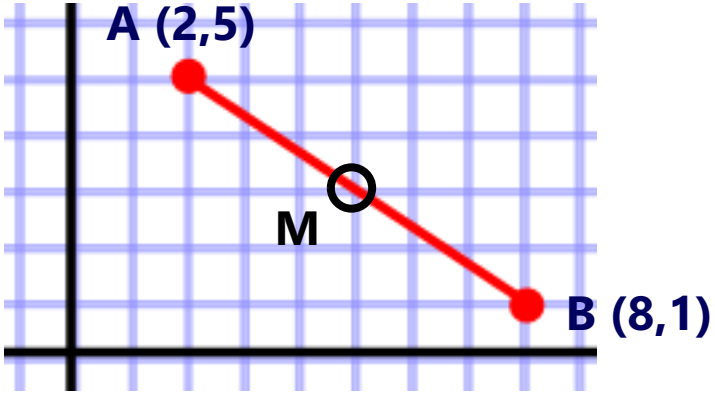
The midpoint of a segment $\overline{AB}$ is the point M on $\overline{AB}$ halfway between the endpoints A and B.



See next page for answer

The Midpoint Formula

The midpoint of a segment $\overline{AB}$ is the point M on $\overline{AB}$ halfway between the endpoints A and B.



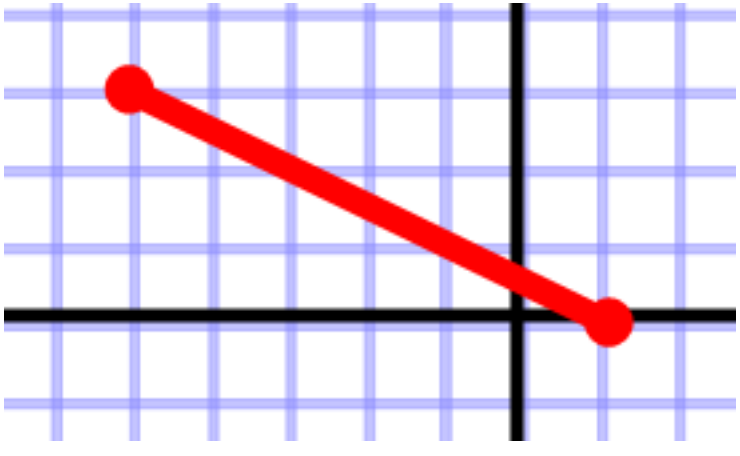
Use the midpoint formula:

$$\left(\frac{x1 + x2}{2} , \frac{y1 + y2}{2} \right)$$

Answer

The Midpoint Formula

Find the midpoint of (1, 0) and (−5, 3).



Use the midpoint formula:

$$\left(\frac{x1 + x2}{2} , \frac{y1 + y2}{2} \right)$$

Answer

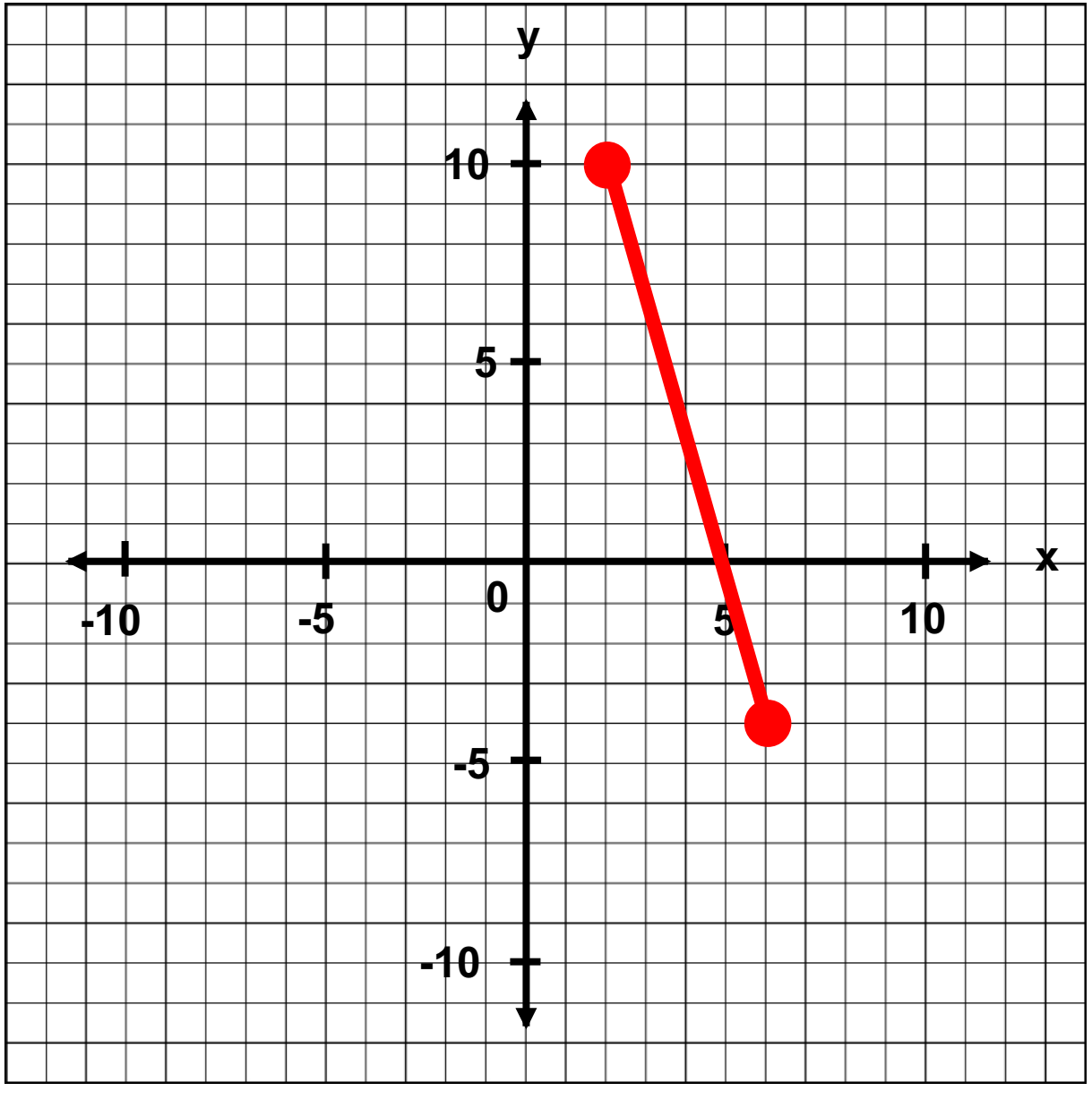
38 What is the midpoint of the line segment that has the endpoints (2, 10) and (6 ,−4)?

A

B

C

D



Answer

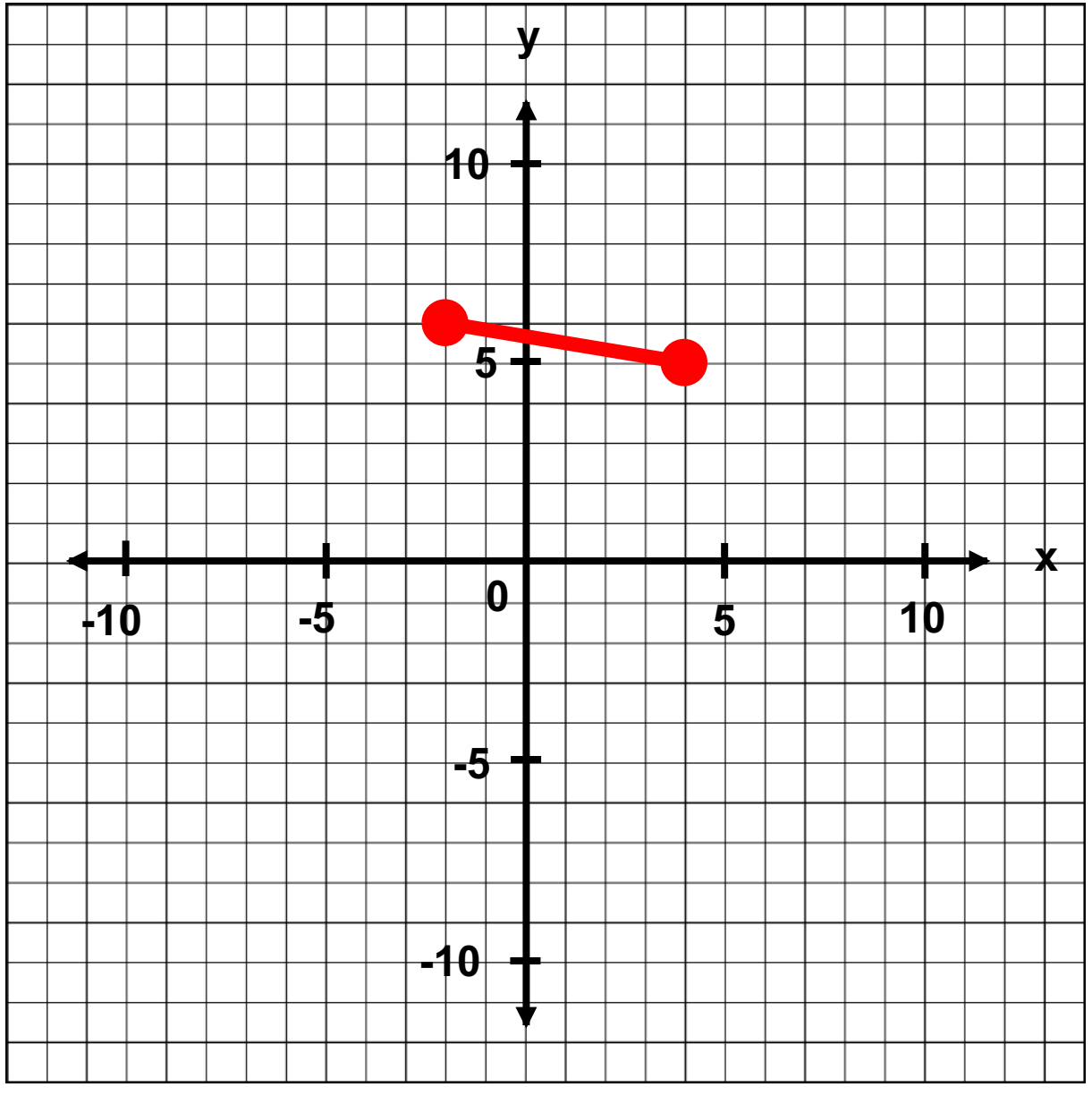
39 What is the midpoint of the line segment that has the endpoints (4, 5) and (−2, 6)?

A

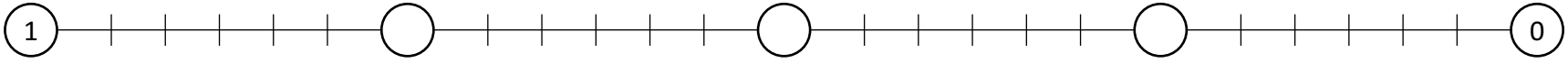
B

C

D



Answer



40 What is the midpoint of the line segment that has the endpoints (−4, −7) and (−12, 2)?

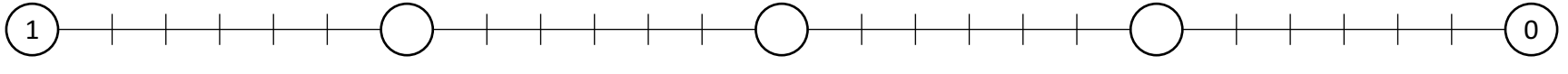
A (−8, −2.5)

B (−4, −4.5)

C (−1, −6.5)

D (−8, −4)

Answer



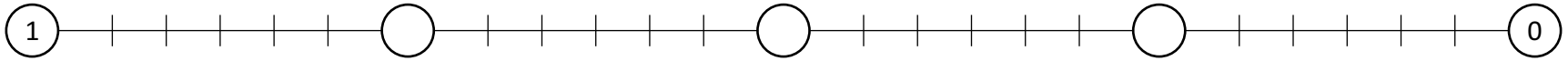
41 What is the midpoint of the line segment that has the endpoints (10, 9) and (5, 3)?

A (6.5, 2)

B (6, 7.5)

C (7.5, 6)

D (15, 12)



Answer

42 Find the center of the circle with a diameter having endpoints at (−4, 3) and (0, 2).

Which formula should be used to solve this problem?

A Pythagorean Formula

B Distance Formula

C Midpoint Formula

D Formula for Area of a Circle

Answer

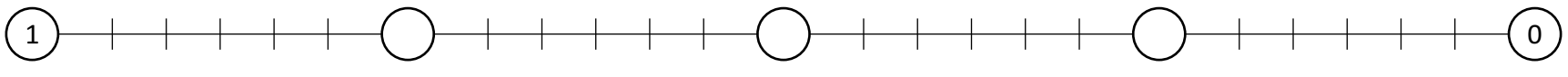
43 Find the center of the circle with a diameter having endpoints at (–4, 3) and (0, 2).

A

B

C

D



Answer

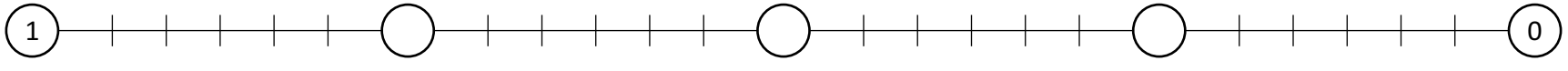
44 Find the center of the circle with a diameter having endpoints at (–12, 10) and (2, 6).

A

B

C

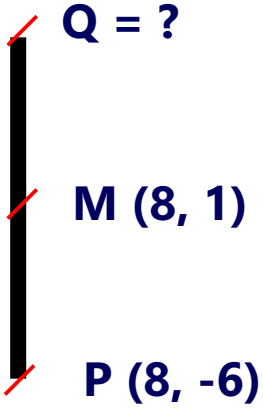
D



Answer

Using Midpoint to Find the Missing Endpoint

If point M is the midpoint between the points P and Q. Find the coordinates of the missing point.



Use the midpoint formula and solve for the unknown.

$$\left(\frac{x1 + x2}{2}, \frac{y1 + y2}{2} \right)$$

Substitute	$8 = \frac{8 + x_1}{2}$	$1 = \frac{-6 + y_1}{2}$
Multiply both sides by 2	$16 = 8 + x_1$	$2 = -6 + y_1$
Add or subtract	$8 = x_1$	$8 = y_1$

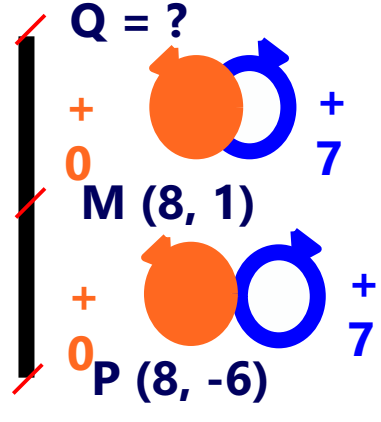
(8, 8)

Teacher Notes

Using Midpoint to Find the Missing Endpoint

If point M is the midpoint between the points P and Q. Find the coordinates of the missing point.

Another method that can be used to find the missing endpoint is to look at the relationship between both the x- and y-coordinates and use the relationship again to calculate the missing endpoint.



Following the pattern, we see that the coordinates for point Q are (8, 8), which is exactly the same answer that we found using the midpoint formula.

45 If Point M is the midpoint between the points P and Q. What are the coordinates of the missing point?

A (−13, −22)

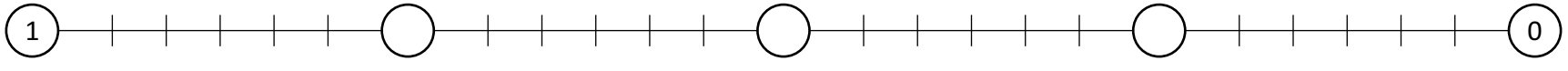
B (−8.5, −9.5)

C (−4.5, −7.5)

D (−12.5, −6.5)

P = (−4,3)
M = (−8.5,−9.5)
Q = ?

Answer



46 If Point M is the midpoint between the points P and Q. What are the coordinates of the missing point?

A (1, −1)

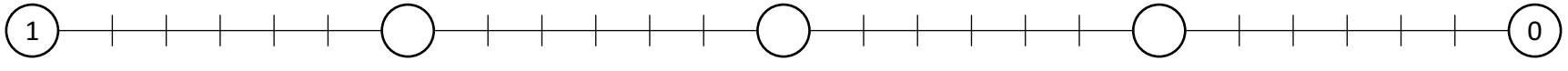
B (−13, 19)

C (−8, 11)

D (−19, 8)

Q = (−6, 9)
M = (−7, 10)
P = ?

Answer



Glossary & Standards

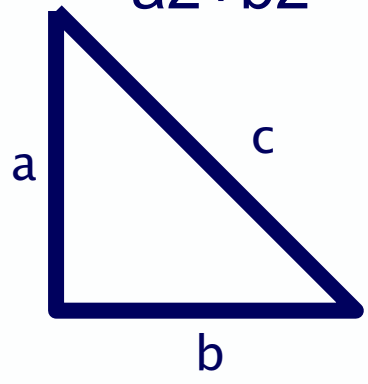
Teacher Notes

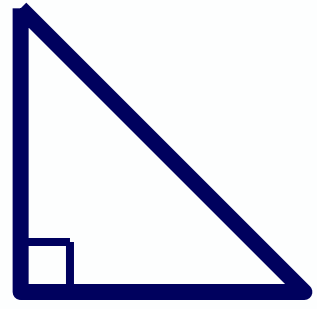
Click to return to the table of contents

Converse of Pythagorean Theorem

If a and b are measures of the shorter sides of a triangle, c is the measure of the longest side, and c squared equals a squared plus b squared, then the triangle is a right triangle.

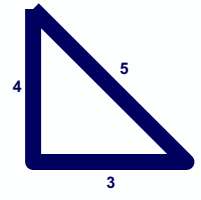
a2+b2 = c2





right triangle

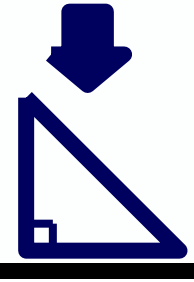
Example:



42+32 = 52

16+9 = 25

25 = 25



Back to Instruction

Distance

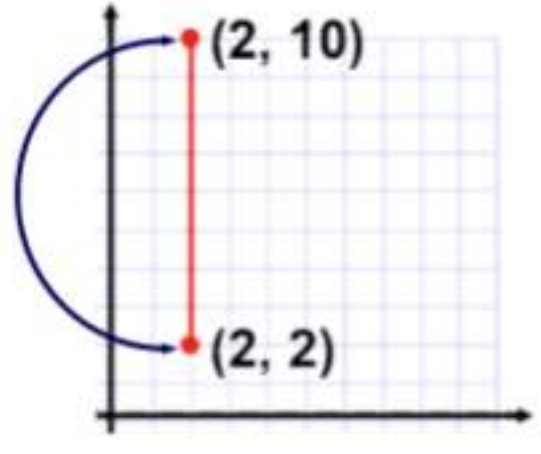
Length

Measurement of how far two points are through space.

Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

how far between the x-coord. | how far between the y-coord



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(2 - 2)^2 + (10 - 2)^2}$$

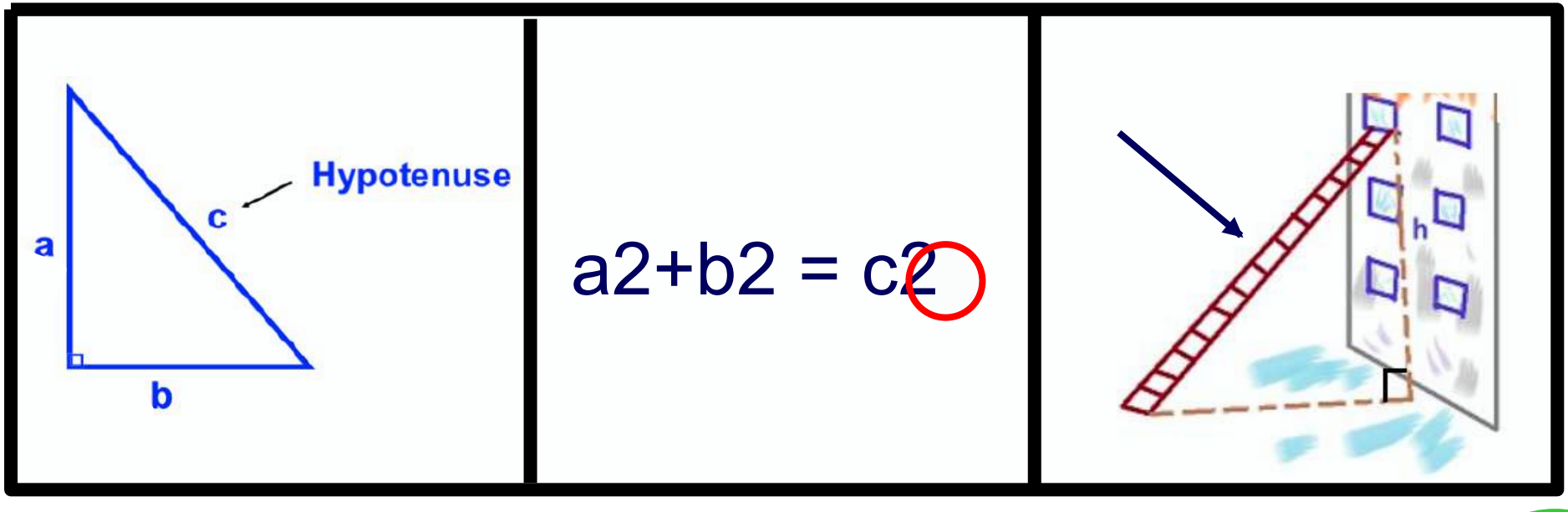
$$d = \sqrt{(0)^2 + (8)^2}$$

$$d = \sqrt{64} = 8$$

Back to Instruction

Hypotenuse

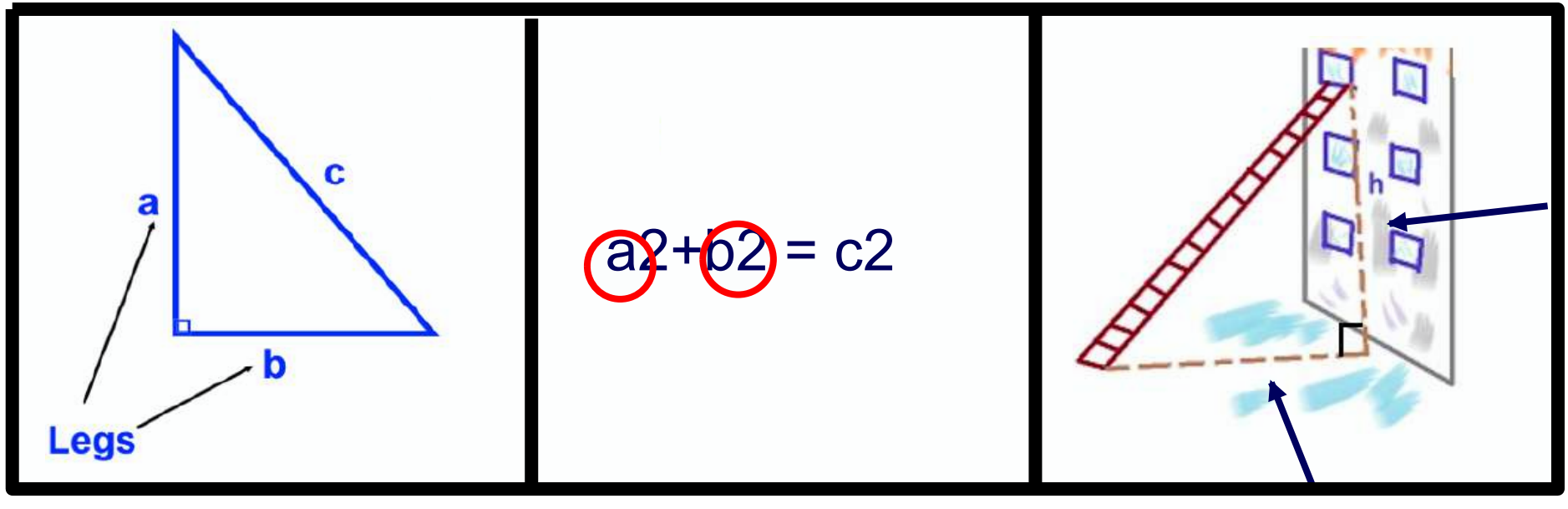
The longest side of a right triangle that is opposite the right angle.



Back to Instruction

Leg

2 sides that form the right angle of a right triangle.



Back to Instruction

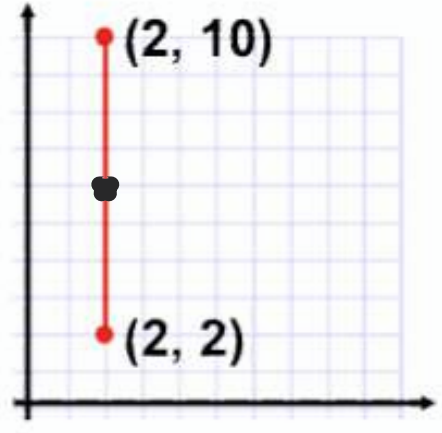
Midpoint

The middle of something.
The point halfway along a line.

Midpoint

Formula:

$$\left(\frac{x1 + x2}{2} , \frac{y1 + y2}{2} \right)$$



$$\left(\frac{x1 + x2}{2} , \frac{y1 + y2}{2} \right)$$

$$\left(\frac{2 + 2}{2} , \frac{10 + 2}{2} \right)$$

$$\left(\frac{4}{2} , \frac{12}{2} \right)$$

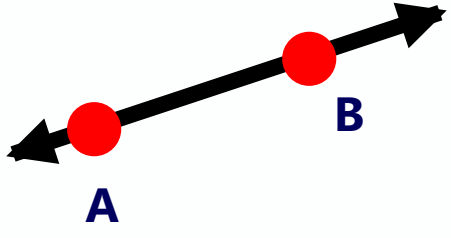
$$(2 , 6)$$

Back to Instruction

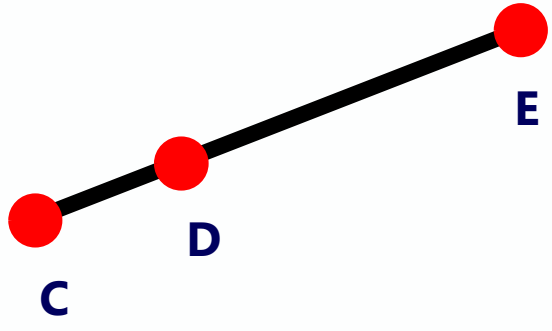
Postulate

A property that is accepted without proof

Through any two points, there is exactly one line.

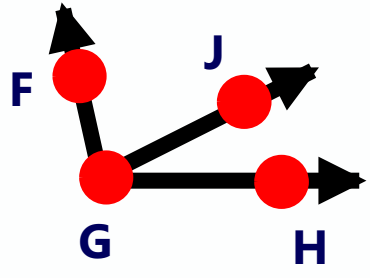


Segment Addition Postulate



CD + DE = CE

Angle Addition Postulate



m∠FGJ + m∠JGH = m∠FGH

Back to Instruction

Proof

Reasoned, logical explanations that use definitions, algebraic properties, postulates, and previously proven theorems to arrive at a conclusion

Given: $3x - 24 = 0$
Prove: $x = 8$

Statements	Reasons
1. $3x - 24 = 0$	1. Given
2. $3x = 24$	2. Addition
3. $x = 8$	3. Division

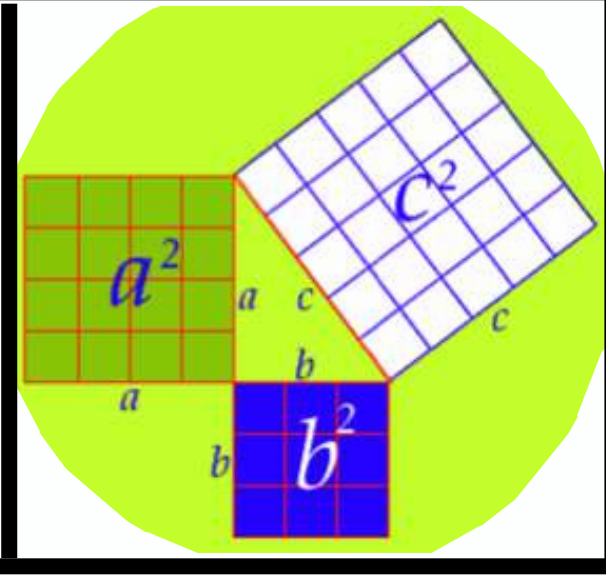
Back to Instruction

Pythagorean Theorem

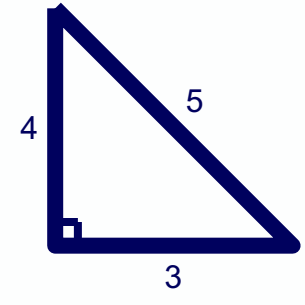
In a right triangle, the sum of the squares of the lengths of the legs (a and b) is equal to the square of the length of hypotenuse (c).

Formula:

$$a^2 + b^2 = c^2$$



Example:



$4^2+3^2 = 5^2$

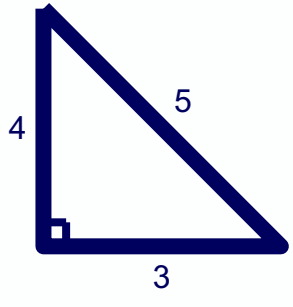
$16+9 = 25$

$25 = 25$ ✔

Back to Instruction

Pythagorean Triples

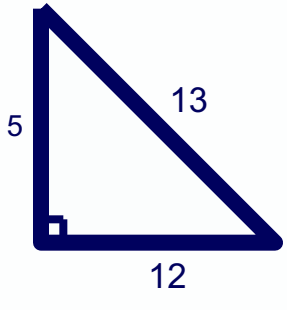
Combinations of whole numbers that work in the Pythagorean Theorem.



$4^2+3^2=5^2$

$16+9=25$

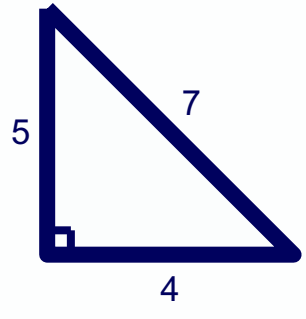
$25=25$ ✔



$5^2+12^2=13^2$

$25+144=169$

$169=169$ ✔



$5^2+4^2=7^2$

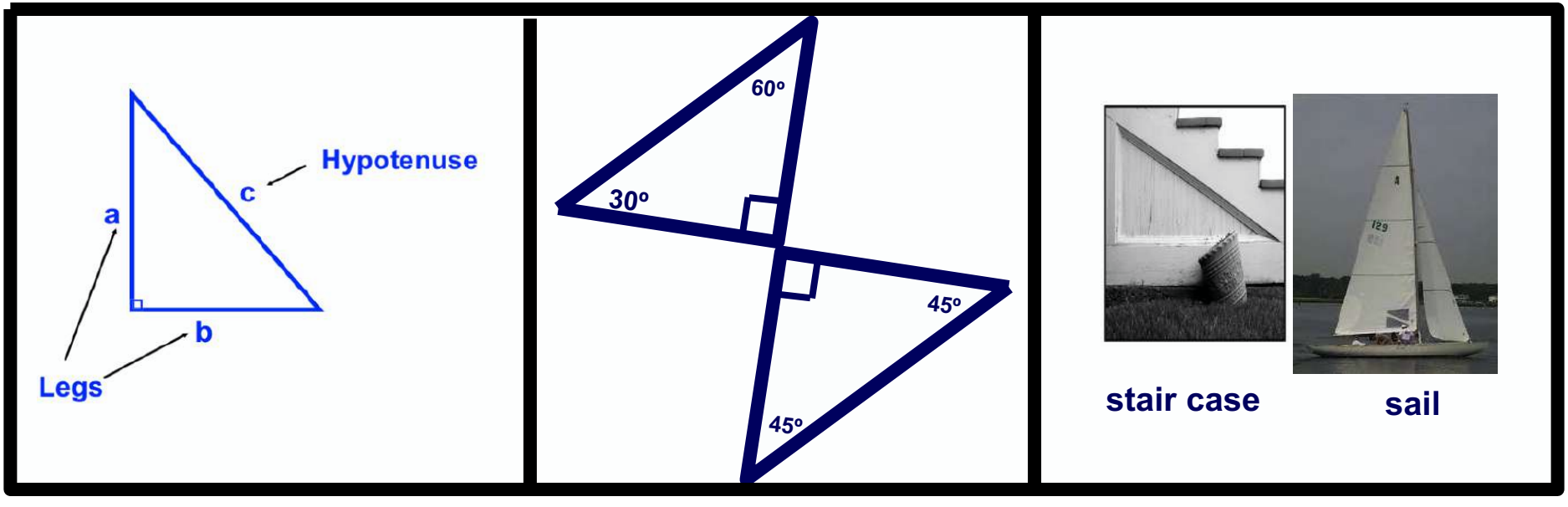
$25+16=49$

$41=49$ ✘

Back to Instruction

Right Triangle

A triangle that has a right angle (90°).



Back to Instruction

Two-Column Proof

A tool to organize your reasoning into two columns. Statements are written in the left column. Reasons are written in the right column.

Given: $3x - 24 = 0$
Prove: $x = 8$

Statements	Reasons
1. $3x - 24 = 0$	1. Given
2. $3x = 24$	2. Addition
3. $x = 8$	3. Division

Back to Instruction

Standards for Mathematical Practices

MP1 Make sense of problems and persevere in solving them.

MP2 Reason abstractly and quantitatively.

MP3 Construct viable arguments and critique the reasoning of others.

MP4 Model with mathematics.

MP5 Use appropriate tools strategically.

MP6 Attend to precision.

MP7 Look for and make use of structure.

MP8 Look for and express regularity in repeated reasoning.

Click on each standard to bring you to an example of how to meet this standard within the unit.

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER