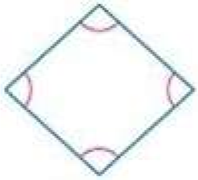
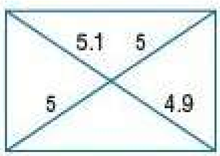
8-3 Tests for Parallelograms

Determine whether each quadrilateral is a parallelogram. Justify your answer.

1. 

SOLUTION:

From the figure, all 4 angles are congruent. Since each pair of opposite angles are congruent, the quadrilateral is a parallelogram by Theorem 8.10.

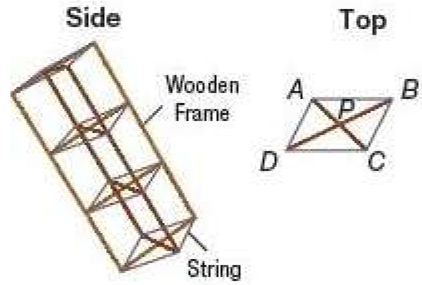
2. 

SOLUTION:

No; none of the tests for ▱ are fulfilled.

We cannot get any information on the angles, so we cannot meet the conditions of Theorem 8.10. We cannot get any information on the sides, so we cannot meet the conditions of Theorems 8.9 or 8.12. One of the diagonals is bisected, but the other diagonal is not because it is split into unequal sides. So the conditions of Theorem 8.11 are not met. Therefore, the figure is not a parallelogram.

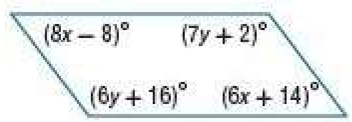
3. **KITES** Charmaine is building the kite shown below. She wants to be sure that the string around her frame forms a parallelogram before she secures the material to it. How can she use the measures of the wooden portion of the frame to prove that the string forms a parallelogram? Explain your reasoning.



SOLUTION:

Sample answer: Charmaine can use Theorem 6.11 to determine if the string forms a parallelogram. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram, so if $AP = CP$ and $BP = DP$, then the string forms a parallelogram.

ALGEBRA Find *x* and *y* so that the quadrilateral is a parallelogram.

4. 

SOLUTION:

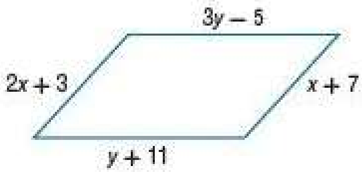
Opposite angles of a parallelogram are congruent. So, $8x - 8 = 6x + 14$ and $7y + 2 = 6y + 16$.

Solve for *x*.

$$\begin{aligned} 8x - 8 &= 6x + 14 \\ 2x - 8 &= 14 \\ 2x &= 22 \\ x &= 11 \end{aligned}$$

Solve for *y*.

$$\begin{aligned} 7y + 2 &= 6y + 16 \\ y + 2 &= 16 \\ y &= 14 \end{aligned}$$

5. 

SOLUTION:

Opposite sides of a parallelogram are congruent.
So, $2x+3=x+7$ and $3y-5=y+11$.

Solve for *x*.

$$\begin{aligned}2x+3&=x+7\\x+3&=7\\x&=4\end{aligned}$$

Solve for y.

$$\begin{aligned}3y-5&=y+11\\2y-5&=11\\2y&=16\\y&=8\end{aligned}$$

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

6. *A*(–2, 4), *B*(5, 4), *C*(8, –1), *D*(–1, –1); Slope Formula

SOLUTION:

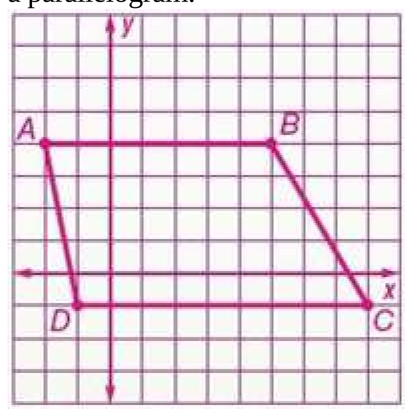
$$\text{Slope of } \overline{AB} = \frac{4-4}{5+2} = 0$$

$$\text{Slope of } \overline{BC} = \frac{-1-4}{8-5} = -\frac{5}{3}$$

$$\text{Slope of } \overline{CD} = \frac{-1-(-1)}{-1-8} = 0$$

$$\text{Slope of } \overline{AD} = \frac{-1-4}{-1-(-2)} = -5$$

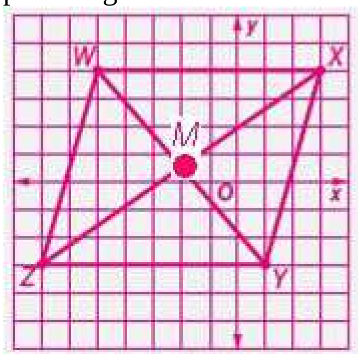
Since the slope of $\overline{BC} \neq$ slope of $\overline{AD}$, *ABCD* is not a parallelogram.



7. $W(-5, 4)$, $X(3, 4)$, $Y(1, -3)$, $Z(-7, -3)$; Midpoint Formula

SOLUTION:

Yes; the midpoint of $\overline{WY}$ is $\left(\frac{-5+1}{2}, \frac{4-3}{2}\right)$ or $\left(-2, \frac{1}{2}\right)$. The midpoint of $\overline{XZ}$ is $\left(\frac{3-7}{2}, \frac{4-3}{2}\right)$ or $\left(-2, \frac{1}{2}\right)$. So the midpoint of $\overline{WY}$ and $\overline{XZ}$ is $M\left(-2, \frac{1}{2}\right)$. By the definition of midpoint, $\overline{WM} \cong \overline{MY}$ and $\overline{ZM} \cong \overline{MX}$. Since the diagonals bisect each other, *WXYZ* is a parallelogram.



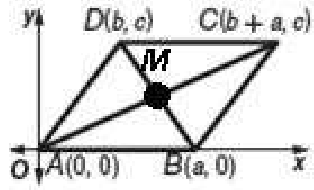
8. Write a coordinate proof for the statement: *If a quadrilateral is a parallelogram, then its diagonals bisect each other.*

SOLUTION:

Begin by positioning parallelogram *ABCD* on the coordinate plane so *A* is at the origin and the figure is in the first quadrant. Let the length of each base be *a* units so vertex *B* will have the coordinates $(a, 0)$. Let the height of the parallelogram be *c*. Since *D* is further to the right than *A*, let its coordinates be (b, c) and *C* will be at $(b + a, c)$. Once the parallelogram is positioned and labeled, use the midpoint formula to determine whether the diagonals bisect each other.

Given: *ABCD* is a parallelogram.

Prove: $\overline{AC}$ and $\overline{DB}$ bisect each other.



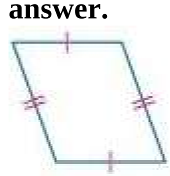
Proof:

midpoint of $\overline{AC} = \left(\frac{0+(a+b)}{2}, \frac{0+c}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$

midpoint of $\overline{DB} = \left(\frac{a+b}{2}, \frac{0+c}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$

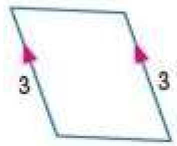
$\overline{AM} \cong \overline{MC}$, $\overline{DM} \cong \overline{MB}$ by definition of midpoint so $\overline{AC}$ and $\overline{DB}$ bisect each other.

CCSS ARGUMENTS Determine whether each quadrilateral is a parallelogram. Justify your answer.

9. 

SOLUTION:

Yes; both pairs of opposite sides are congruent, which meets the conditions stated in Theorem 6.9. No other information is needed.

10. 

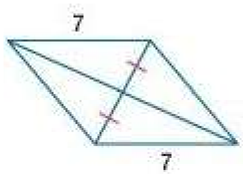
SOLUTION:

Yes; one pair of opposite sides are parallel and congruent. From the figure, one pair of opposite sides has the same measure and are parallel. By the definition of congruence, these segments are congruent. By Theorem 6.12 this quadrilateral is a parallelogram.

11. 

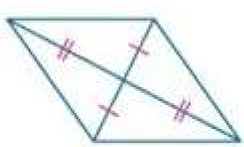
SOLUTION:

No; none of the tests for ▱ are fulfilled. Only one pair of opposite sides have the same measure. We don't know if they are parallel.

12. 

SOLUTION:

No; none of the tests for ▱ are fulfilled. We know that one pair of opposite sides are congruent and one diagonal bisected the second diagonal of the quadrilateral. These do not meet the qualifications to be a parallelogram.

13. 

SOLUTION:

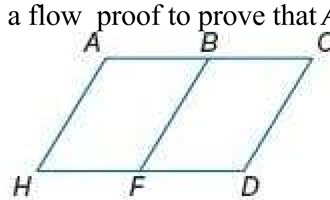
Yes; the diagonals bisect each other. By Theorem 6.11 this quadrilateral is a parallelogram.

14. 

SOLUTION:

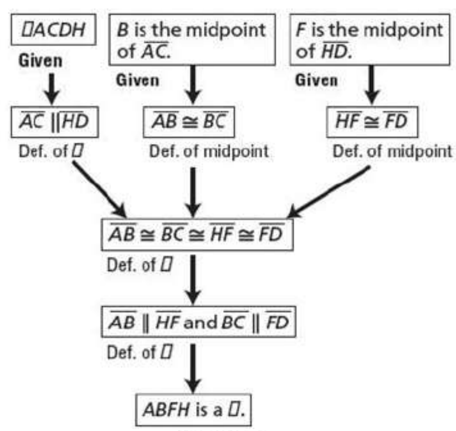
No; none of the tests for ▱ are fulfilled. Consecutive angles are supplementary but no other information is given. Based on the information given, this is not a parallelogram.

15. **PROOF** If *ACDH* is a parallelogram, *B* is the midpoint of $\overline{AC}$, and *F* is the midpoint of $\overline{HD}$, write a flow proof to prove that *ABFH* is a parallelogram.



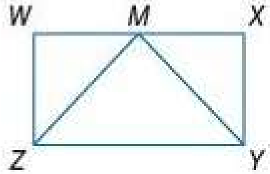
SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ACDH* is a parallelogram, *B* is the midpoint of $\overline{AC}$ and *F* is the midpoint of $\overline{HD}$. You need to prove that *ABFH* is a parallelogram. Use the properties that you have learned about parallelograms and midpoints to walk through the proof.



Opp. sides are ∥ and ≅.

16. **PROOF** If *WXYZ* is a parallelogram, $\angle W \cong \angle X$, and *M* is the midpoint of $\overline{WX}$, write a paragraph proof to prove that *ZMY* is an isosceles triangle.



SOLUTION:

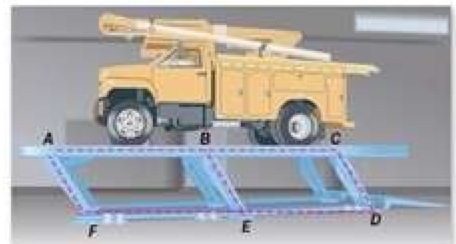
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given WXYZ is a parallelogram, $\angle W \cong \angle X$, and *M* is the midpoint of $\overline{WX}$. You need to prove that *ZMY* is an isosceles triangle. Use the properties that you have learned about parallelograms, triangle congruence, and midpoints to walk through the proof.

Given: *WXYZ* is a parallelogram, $\angle W \cong \angle X$, and *M* is the midpoint of $\overline{WX}$.

Prove: *ZMY* is an isosceles triangle.

Proof: Since *WXYZ* is a parallelogram, $\overline{WZ} \cong \overline{XY}$. *M* is the midpoint of $\overline{WX}$, so $\overline{WM} \cong \overline{MX}$. It is given that $\angle W \cong \angle X$, so by SAS $\Delta ZWM \cong \Delta YXM$. By CPCTC, $\overline{ZM} \cong \overline{YM}$. So, *ZMY* is an isosceles triangle, by the definition of an isosceles triangle.

17. **REPAIR** Parallelogram lifts are used to elevate large vehicles for maintenance. In the diagram, *ABEF* and *BCDE* are parallelograms. Write a two-column proof to show that *ACDF* is also a parallelogram.

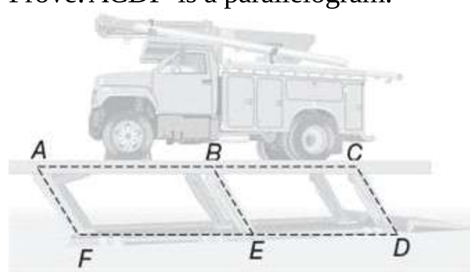


SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABEF* and *BCDE* are parallelograms. You need to prove that *ACDH* is a parallelogram. Use the properties that you have learned about parallelograms to walk through the proof.

Given: *ABEF* is a parallelogram; *BCDE* is a parallelogram.

Prove: *ACDF* is a parallelogram.

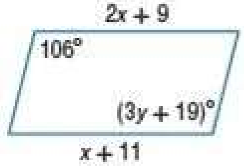


Proof:

Statements (Reasons)

1. *ABEF* is a parallelogram; *BCDE* is a parallelogram. (Given)
2. $\overline{AF} \cong \overline{BE}, \overline{BE} \cong \overline{CD}, \overline{AF} \parallel \overline{BE}, \overline{BE} \parallel \overline{CD}$ (Def. of ▱)
3. $\overline{AF} \cong \overline{CD}, \overline{AF} \parallel \overline{CD}$ (Trans. Prop.)
4. *ACDF* is a parallelogram. (If one pair of opp. sides is $\cong$ and $\parallel$, then the quad. is a ▱.)

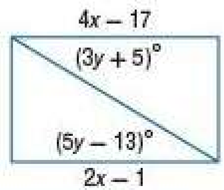
ALGEBRA Find x and y so that the quadrilateral is a parallelogram.

18. 

SOLUTION:
Opposite sides of a parallelogram are congruent.

Solve for x.
$$2x+9=x+11$$
$$x+9=11$$
$$x=2$$

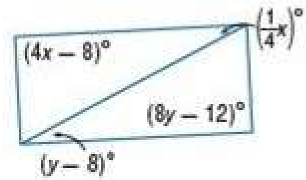
Solve for y.
$$3y+19=106$$
$$3y=87$$
$$y=29$$

19. 

SOLUTION:
Opposite sides of a parallelogram are congruent.

Solve for x.
$$4x-17=2x-1$$
$$2x-17=-1$$
$$2x=16$$
$$x=8$$

Solve for y.
$$5y-13=3y+5$$
$$2y-13=5$$
$$2y=18$$
$$y=9$$

20. 

SOLUTION:
Opposite angles of a parallelogram are congruent.

Alternate interior angles are congruent.

$$\frac{1}{4}x=y-8$$
$$x=4y-32$$

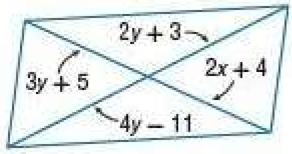
Use substitution.

$$4x-8=8y-12$$
$$4(4y-32)-8=8y-12$$
$$16y-128-8=8y-12$$
$$16y-136=8y-12$$
$$8y=124$$
$$y=15.5$$

Substitute $y=15.5$ in $x=4y-32$.

$$x=4(15.5)-32$$
$$x=62-32$$
$$x=30$$

ALGEBRA Find *x* and *y* so that the quadrilateral is a parallelogram.

21. 

SOLUTION:

Diagonals of a parallelogram bisect each other.

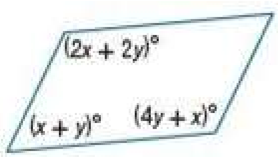
So, $3y+5=2x+4$ and $4y-11=2y+3$.

Solve for y.

$$4y-11=2y+3$$
$$2y=14$$
$$y=7$$

Substitute $y=7$ in $3y+5=2x+4$.

$$3(7)+5=2x+4$$
$$21+5=2x+4$$
$$26=2x+4$$
$$-2x=-22$$
$$x=11$$

22. 

SOLUTION:

Opposite angles of a parallelogram are congruent.

So, $2x+2y=4y+x$. We know that consecutive angles in a parallelogram are supplementary.

So, $x+y+4y+x=180$.

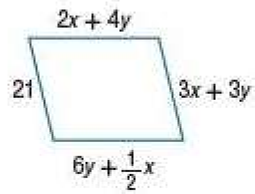
Solve for x.

$$2x+2y=4y+x$$
$$x+2y=4y$$
$$x=2y$$

Substitute $x=2y$ in $x+y+4y+x=180$.

$$x+y+4y+x=180$$
$$2y+y+4y+2y=180$$
$$9y=180$$
$$y=20$$

So, $x=2(20)$ or 40.

23. 

SOLUTION:

Opposite sides of a parallelogram are congruent.

So, $3x+3y=21$ and $2x+4y=6y+\frac{1}{2}x$.

Solve for x in terms y.

$$3x+3y=21$$
$$x+y=7$$
$$x=7-y$$

Substitute $x=7-y$ in $2x+4y=6y+\frac{1}{2}x$.

$2x+4y=6y+\frac{1}{2}x$	Original equation
$2(7-y)+4y=6y+\frac{1}{2}(7-y)$	Substitute.
$14-2y+4y=6y+\frac{7}{2}-\frac{1}{2}y$	Distributive Property
$-2y+4y-6y+\frac{1}{2}y=\frac{7}{2}-14$	Combine like terms.
$-2y+4y-\frac{11}{2}y=\frac{7}{2}-14$	Simplify.
$-\frac{7}{2}y=-\frac{21}{2}$	Simplify.
$y=3$	Multiply each side

Substitute $y=3$ in $x=7-y$ to solve for x.

$$x=7-3$$
$$x=4$$

So, $x=4$ and $y=3$.

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

24. $A(-3, 4)$, $B(4, 5)$, $C(5, -1)$, $D(-2, -2)$; Slope Formula

SOLUTION:

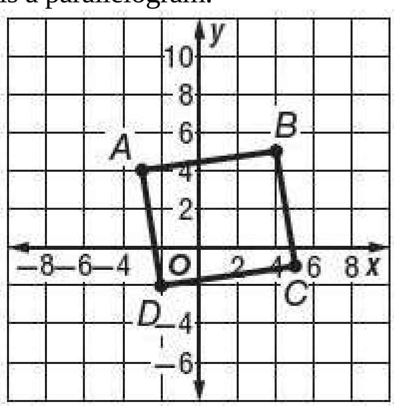
$$\text{Slope of } \overline{AB} = \frac{5-4}{4+3} = \frac{1}{7}$$

$$\text{Slope of } \overline{BC} = \frac{-1-5}{5-4} = -6$$

$$\text{Slope of } \overline{CD} = \frac{-2-(-1)}{-2-5} = \frac{1}{7}$$

$$\text{Slope of } \overline{AD} = \frac{-2-4}{-2-(-3)} = -6$$

Since both pairs of opposite sides are parallel, *ABCD* is a parallelogram.



25. $J(-4, -4)$, $K(-3, 1)$, $L(4, 3)$, $M(3, -3)$; Distance Formula

SOLUTION:

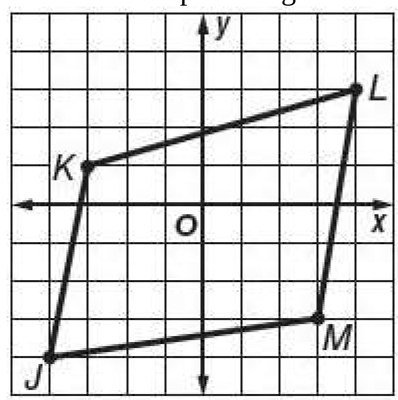
$$JK = \sqrt{(-3-(-4))^2 + (1-(-4))^2} = \sqrt{26}$$

$$KL = \sqrt{(4-(-3))^2 + (3-1)^2} = \sqrt{53}$$

$$LM = \sqrt{(3-4)^2 + (-3-3)^2} = \sqrt{37}$$

$$JM = \sqrt{(3-(-4))^2 + (-3-(-4))^2} = \sqrt{50}$$

Since the pairs of opposite sides are not congruent, *JKLM* is not a parallelogram.



26. $V(3, 5)$, $W(1, -2)$, $X(-6, 2)$, $Y(-4, 7)$; Slope Formula

SOLUTION:

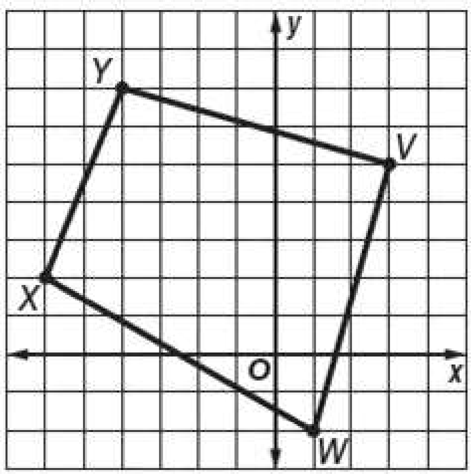
$$\text{Slope of } \overline{YV} = \frac{5-7}{3-(-4)} = -\frac{2}{7}$$

$$\text{Slope of } \overline{WX} = \frac{2-(-2)}{-6-1} = -\frac{4}{7}$$

$$\text{Slope of } \overline{YX} = \frac{2-7}{-6-(-4)} = \frac{5}{2}$$

$$\text{Slope of } \overline{VW} = \frac{-2-5}{1-3} = \frac{7}{2}$$

Since the slope of $\overline{YV} \neq$ slope of $\overline{XW}$ and the slope of $\overline{YX} \neq$ slope of $\overline{VW}$, *VWXY* is not a parallelogram.



27. $Q(2, -4)$, $R(4, 3)$, $S(-3, 6)$, $T(-5, -1)$; Distance and Slope Formulas

SOLUTION:

$$\text{Slope of } \overline{QR} = \frac{3-(-4)}{4-2} = \frac{7}{2}$$

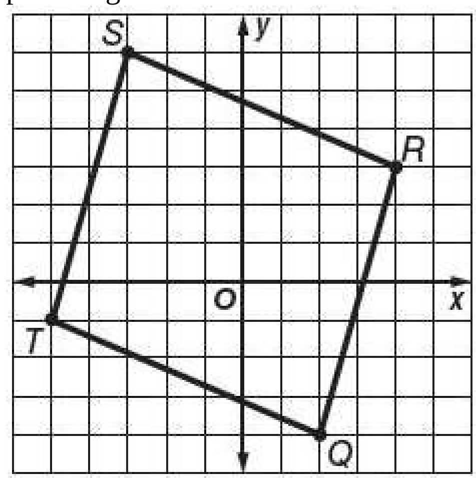
$$\text{Slope of } \overline{ST} = \frac{-1-6}{-5-(-3)} = \frac{7}{2}$$

Slope of $\overline{QR} = \frac{7}{2} =$ slope of $\overline{ST}$, so $\overline{QR} \parallel \overline{ST}$.

$$QR = \sqrt{(4-2)^2 + (3-(-4))^2} = \sqrt{53}$$

$$ST = \sqrt{(-5-(-3))^2 + (-1-6)^2} = \sqrt{53}$$

Since $QR = ST$, $\overline{QR} \cong \overline{ST}$. So, *QRST* is a parallelogram.

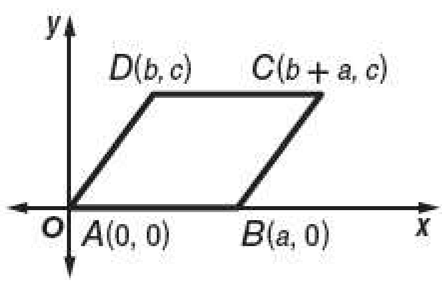


28. Write a coordinate proof for the statement: *If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.*

SOLUTION:

Begin by positioning quadrilateral *ABCD* on a coordinate plane. Place vertex *A* at the origin. Let the length of the bases be *a* units and the height be *c* units. Then the rest of the vertices are *B*(*a*, 0), *C*(*b* + *a*, *c*), and *D*(*b*, *c*). You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given $\overline{AB} \cong \overline{CD}, \overline{AD} \cong \overline{BC}$ and you need to prove that *ABCD* is a parallelogram. Use the properties that you have learned about parallelograms to walk through the proof.

Given: $\overline{AB} \cong \overline{CD}, \overline{AD} \cong \overline{BC}$
Prove: *ABCD* is a parallelogram



Proof:

slope of $\overline{AD} = \frac{c-0}{b-0} = \frac{c}{b}$. The slope of $\overline{AB}$ is 0.

slope of $\overline{BC} = \frac{c-0}{b+a-a} = \frac{c}{b}$. The slope of $\overline{CD}$ is 0.

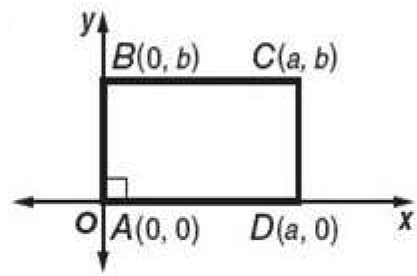
Therefore, $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{CD}$. So by definition of a parallelogram, *ABCD* is a parallelogram.

29. Write a coordinate proof for the statement: *If a parallelogram has one right angle, it has four right angles.*

SOLUTION:

Begin by positioning parallelogram *ABCD* on a coordinate plane. Place vertex *A* at the origin. Let the length of the bases be *a* units and the height be *b* units. Then the rest of the vertices are *B*(0, *b*), *C*(*a*, *b*), and *D*(*a*, 0). You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given that *ABCD* is a parallelogram and $\angle A$ is a right angle. You need to prove that rest of the angles in *ABCD* are right angles. Use the properties that you have learned about parallelograms to walk through the proof.

Given: *ABCD* is a parallelogram.
$\angle A$ is a right angle.
Prove: $\angle B, \angle C$, and $\angle D$ are right angles.



Proof:

slope of $\overline{BC} = \left(\frac{b-b}{a-0}\right)$ or 0. The slope of $\overline{CD}$ is undefined.

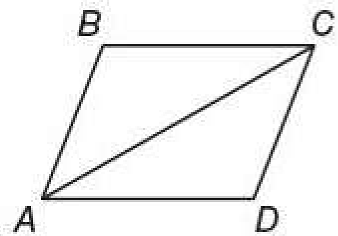
slope of $\overline{AD} = \left(\frac{0-0}{a-0}\right)$ or 0. The slope of $\overline{AB}$ is undefined.

Therefore, $\overline{BC} \perp \overline{CD}, \overline{CD} \perp \overline{AD}$, and $\overline{AB} \perp \overline{BC}$.
So, $\angle B, \angle C$, and $\angle D$ are right angles.

30. **PROOF** Write a paragraph proof of Theorem 6.10.

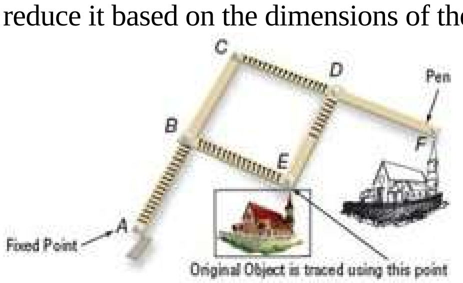
SOLUTION:
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given $\angle A \cong \angle C, \angle B \cong \angle D$. You need to prove that *ABCD* is a parallelogram. Use the properties that you have learned about parallelograms and angles and parallel lines to walk through the proof.

Given: $\angle A \cong \angle C, \angle B \cong \angle D$
Prove: *ABCD* is a parallelogram.



Proof: Draw $\overline{AC}$ to form two triangles. The sum of the angles of one triangle is 180, so the sum of the angles for two triangles is 360.
So, $m\angle A + m\angle B + m\angle C + m\angle D = 360$.
Since $\angle A \cong \angle C$ and $\angle B \cong \angle D, m\angle A = m\angle C$ and $m\angle B = m\angle D$. By substitution, $m\angle A + m\angle A + m\angle B + m\angle B = 360$. So, $2(m\angle A) + 2(m\angle B) = 360$. Dividing each side by 2 yields $m\angle A + m\angle B = 180$. So, the consecutive angles are supplementary and $\overline{AD} \parallel \overline{BC}$. Likewise, $2(m\angle A) + 2(m\angle D) = 360$ *or* $m\angle A + m\angle D = 180$. So, these consecutive angles are supplementary and $\overline{AB} \parallel \overline{DC}$. Opposite sides are parallel, so *ABCD* is a parallelogram.

31. **PANTOGRAPH** A pantograph is a device that can be used to copy an object and either enlarge or reduce it based on the dimensions of the pantograph.



a. If $\overline{AC} \cong \overline{CF}, \overline{AB} \cong \overline{CD} \cong \overline{BE}$, and $\overline{DF} \cong \overline{DE}$, write a paragraph proof to show that $\overline{BE} \parallel \overline{CD}$.
b. The scale of the copied object is the ratio of *CF* to *BE*. If *AB* is 12 inches, *DF* is 8 inches, and the width of the original object is 5.5 inches, what is the width of the copy?

SOLUTION:
a. You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given $\overline{AC} \cong \overline{CF}, \overline{AB} \cong \overline{CD} \cong \overline{BE}$, and $\overline{DF} \cong \overline{DE}$. You need to prove that *BCDE* is a parallelogram. Use the properties that you have learned about parallelograms to walk through the proof.

Given: $\overline{AC} \cong \overline{CF}, \overline{AB} \cong \overline{CD} \cong \overline{BE}$, and $\overline{DF} \cong \overline{DE}$
Prove: *BCDE* is a parallelogram.
Proof: We are given that $\overline{AC} \cong \overline{CF}, \overline{AB} \cong \overline{CD} \cong \overline{BE}$, and $\overline{DF} \cong \overline{DE}$. $AC = CF$ by the definition of congruence. $AC = AB + BC$ and $CF = CD + DF$ by the Segment Addition Postulate and $AB + BC = CD + DF$ by substitution. Using substitution again, $AB + BC = AB + DF$, and $BC = DF$ by the Subtraction Property. $\overline{BC} \cong \overline{DF}$ by the definition of congruence, and $\overline{BC} \cong \overline{DE}$ by the Transitive Property. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram, so *BCDE* is a parallelogram. By the definition of a parallelogram, $\overline{BE} \parallel \overline{CD}$.

b. The scale of the copied object is $\frac{CF}{BE}$.

$BE = CD$. So, $BE = 12$.

$CF = CD + DF$.
$= 12 + 8$
$= 20$

Therefore, $\frac{CF}{BE} = \frac{20}{12}$.

Write a proportion. Let x be the width of the copy.

$\frac{20}{12} = \frac{x}{5.5}$.
Solve for x.
$12x = 110$
$x \approx 9.2$
The width of the copy is about 9.2 in.

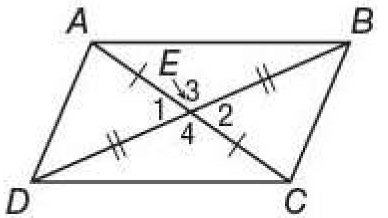
PROOF Write a two-column proof.

32. Theorem 8.11

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given $\overline{AE} \cong \overline{EC}, \overline{DE} \cong \overline{EB}$. You need to prove that *ABCD* is a parallelogram. Use the properties that you have learned about parallelograms and triangle congruence to walk through the proof

Given: $\overline{AE} \cong \overline{EC}, \overline{DE} \cong \overline{EB}$
Prove: *ABCD* is a parallelogram.



Statements (Reasons)

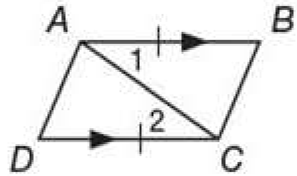
1. $\overline{AE} \cong \overline{EC}, \overline{DE} \cong \overline{EB}$ (Given)
2. $\angle 1 \cong \angle 2$, and $\angle 3 \cong \angle 4$ (Vertical $\angle s$ are $\cong$.)
3. $\Delta ABE \cong \Delta CDE, \Delta ADE \cong \Delta CBE$ (SAS)
4. $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$ (CPCTC)
5. *ABCD* is a parallelogram. (If both pairs of opp. sides are $\cong$, then quad is a ▱.)

33. Theorem 8.12

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given $\overline{AB} \cong \overline{DC}, \overline{AB} \parallel \overline{DC}$. You need to prove that *ABCD* is a parallelogram. Use the properties that you have learned about parallelograms and triangle congruence to walk through the proof

Given: $\overline{AB} \cong \overline{DC}, \overline{AB} \parallel \overline{DC}$
Prove: *ABCD* is a parallelogram.



Statements (Reasons)

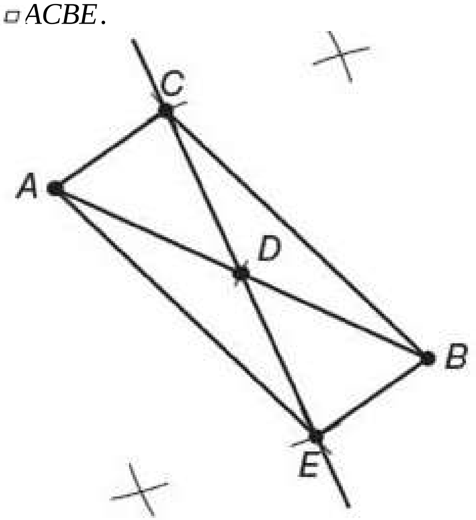
1. $\overline{AB} \cong \overline{DC}, \overline{AB} \parallel \overline{DC}$ (Given)
2. Draw $\overline{AC}$. (Two points determine a line.)
3. $\angle 1 \cong \angle 2$ (If two lines are $\parallel$, then alt. int. $\angle s$ are $\cong$.)
4. $\overline{AC} \cong \overline{AC}$ (Refl. Prop.)
5. $\Delta ABC \cong \Delta CDA$ (SAS)
6. $\overline{AD} \cong \overline{BC}$ (CPCTC)
7. *ABCD* is a parallelogram. (If both pairs of opp. sides are $\cong$, then the quad. is ▱.)

34. **CONSTRUCTION** Explain how you can use Theorem 8.11 to construct a parallelogram. Then construct a parallelogram using your method.

SOLUTION:

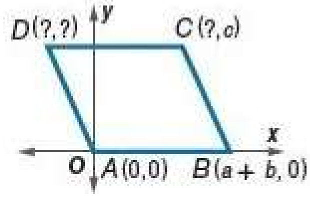
Analyze the properties of parallelograms, the aspects of Theorem 8.11, and the process of constructing geometric figures. What do you need to know to begin your construction? What differentiates a parallelogram from other quadrilaterals? How does Theorem 8.11 help in determining that the constructed figure is a parallelogram?

By Theorem 8.11, if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. Begin by drawing and bisecting a segment $\overline{AB}$. Then draw a line that intersects the first segment through its midpoint *D*. Mark a point *C* on one side of this line and then construct a segment $\overline{DE}$ congruent to $\overline{CD}$ on the other side of *D*. You now have intersecting segments which bisect each other. Connect point *A* to point *C*, point *C* to point *B*, point *B* to point *E*, and point *E* to point *A* to form ▱*ACBE*.



CCSS REASONING Name the missing coordinates for each parallelogram.

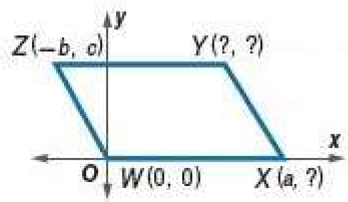
35.



SOLUTION:

Since *AB* is on the *x*-axis and horizontal segments are parallel, position the endpoints of $\overline{DC}$ so that they have the same *y*-coordinate, *c*. The distance from *D* to *C* is the same as *AB*, also *a* + *b* units, let the *x*-coordinate of *D* be –*b* and of *C* be *a*. Thus, the missing coordinates are *C*(*a*, *c*) and *D*(–*b*, *c*).

36.

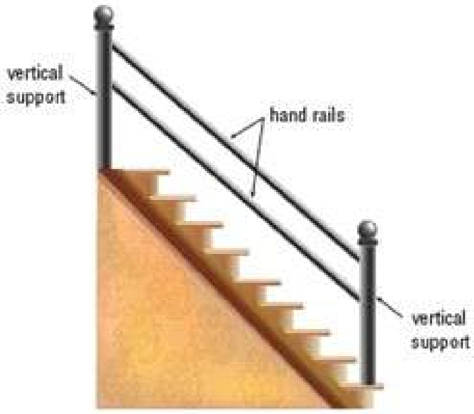


SOLUTION:

From the *x*-coordinates of *W* and *X*, $\overline{WX}$ has a length of *a* units. Since *X* is on the x-axis it has coordinates (*a*, 0). Since horizontal segments are parallel, the endpoints of $\overline{ZY}$ have the same *y*-coordinate, *c*. The distance from *Z* to *Y* is the same as *WX*, *a* units. Since *Z* is at (-*b*, *c*), what should be added to -*b* to get *a* units? The *x*-coordinate of *Y* is *a* – *b*.

Thus the missing coordinates are *Y(a* – *b*, *c)* and *X* (*a*, *0)*.

37. **SERVICE** While replacing a hand rail, a contractor uses a carpenter's square to confirm that the vertical supports are perpendicular to the top step and the ground, respectively. How can the contractor prove that the two hand rails are parallel using the fewest measurements? Assume that the top step and the ground are both level.

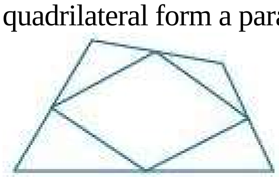


SOLUTION:

What are we asking to prove? What different methods can we use to prove it? How does the diagram help us choose the method of proof that allows for the fewest measurements? What can we deduce from diagram without measuring anything?

Sample answer: Since the two vertical rails are both perpendicular to the ground, he knows that they are parallel to each other. If he measures the distance between the two rails at the top of the steps and at the bottom of the steps, and they are equal, then one pair of sides of the quadrilateral formed by the handrails is both parallel and congruent, so the quadrilateral is a parallelogram. Since the quadrilateral is a parallelogram, the two hand rails are parallel by definition.

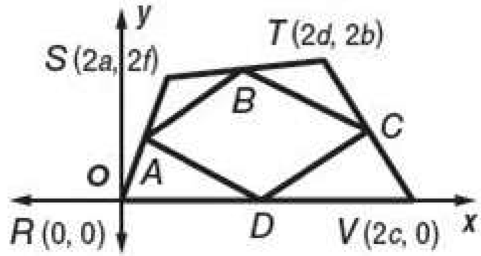
38. **PROOF** Write a coordinate proof to prove that the segments joining the midpoints of the sides of any quadrilateral form a parallelogram.



SOLUTION:

Begin by positioning quadrilateral *RSTV* and *ABCD* on a coordinate plane. Place vertex *R* at the origin. Since *ABCD* is formed from the midpoints of each side of *RSTV*, let each length and height of *RSTV* be in multiples of 2. Since *RSTV* does not have any congruent sides or any vertical sides, let *R* be (0, 0), *V*(2*c*, 0), *T*(2*d*, 2*b*), and *S*(2*a*, 2*f*).
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given
RSTV is a quadrilateral and *A*, *B*, *C*, and *D* are midpoints of sides $\overline{RS}, \overline{ST}, \overline{TV}$, and $\overline{VR}$, respectively.
You need to prove that *ABCD* is a parallelogram.
Use the properties that you have learned about parallelograms to walk through the proof

Given: *RSTV* is a quadrilateral.
A, *B*, *C*, and *D* are midpoints of sides $\overline{RS}, \overline{ST}, \overline{TV}$, and $\overline{VR}$, respectively.
Prove: *ABCD* is a parallelogram.



Proof:
Place quadrilateral *RSTV* on the coordinate plane and label coordinates as shown. (Using coordinates that are multiples of 2 will make the computation easier.) By the Midpoint Formula, the coordinates of *A*, *B*, *C*, and *D* are $A\left(\frac{2a}{2}, \frac{2f}{2}\right) = (a, f)$;

$B\left(\frac{2d+2a}{2}, \frac{2f+2b}{2}\right) = (d+a, f+b)$;

$C\left(\frac{2d+2c}{2}, \frac{2b}{2}\right) = (d+c, b)$; and $D\left(\frac{2c}{2}, \frac{0}{2}\right) = (c, 0)$

Find the slopes of $\overline{AB}$ and $\overline{DC}$.

slope of $\overline{AB}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(f+b)-f}{(d+a)-a} = \frac{b}{d}$$

slope of $\overline{DC}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-b}{c-(d+c)} = \frac{-b}{-d} \text{ or } \frac{b}{d}$$

The slopes of $\overline{AB}$ and $\overline{DC}$ are the same so the segments are parallel. Use the Distance Formula to find *AB* and *DC*.

$$AB = \sqrt{(d+a-a)^2 + (f+b-f)^2}$$
$$= \sqrt{d^2 + b^2}$$
$$DC = \sqrt{(d+c-c)^2 + (b-0)^2}$$
$$= \sqrt{d^2 + b^2}$$

Thus, $AB = DC$ and $\overline{AB} \cong \overline{DC}$. Therefore, *ABCD* is a parallelogram because if one pair of opposite sides of a quadrilateral are both parallel and congruent, then the quadrilateral is a parallelogram.

39. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the properties of rectangles. A rectangle is a quadrilateral with four right angles.

a. GEOMETRIC Draw three rectangles with varying lengths and widths. Label one rectangle *ABCD*, one *MNOP*, and one *WXYZ*. Draw the two diagonals for each rectangle.

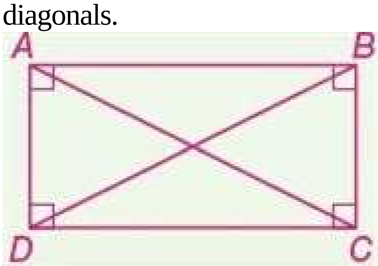
b. TABULAR Measure the diagonals of each rectangle and complete the table.

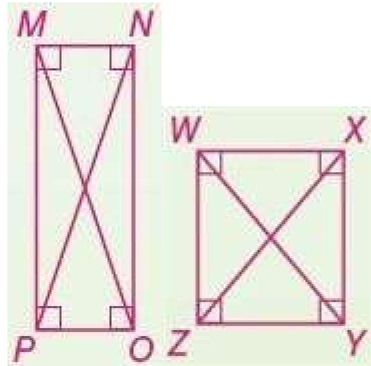
Rectangle	Side	Length
ABCD	$\overline{AC}$	
	$\overline{BD}$	
MNOP	$\overline{MO}$	
	$\overline{NP}$	
WXYZ	$\overline{WY}$	
	$\overline{XZ}$	

c. VERBAL Write a conjecture about the diagonals of a rectangle.

SOLUTION:

a. Rectangles have 4 right angles and have opposite sides congruent. Draw 4 different rectangles with diagonals.





b. Use a ruler to measure the length of each diagonal.

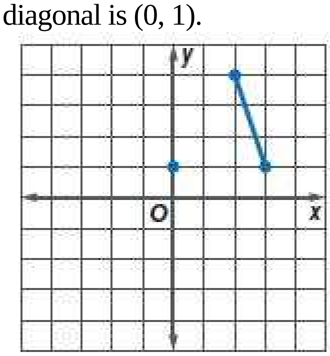
Rectangle	Side	Length
ABCD	$\overline{AC}$	3.3 cm
	$\overline{BD}$	3.3 cm
MNOP	$\overline{MO}$	2.8 cm
	$\overline{NP}$	2.8 cm
WXYZ	$\overline{WY}$	2.0 cm
	$\overline{XZ}$	2.0 cm

c. Sample answer: The measures of the diagonals for each rectangle are the same. The diagonals of a rectangle are congruent.

40. **CHALLENGE** The diagonals of a parallelogram meet at the point (0, 1). One vertex of the parallelogram is located at (2, 4), and a second vertex is located at (3, 1). Find the locations of the remaining vertices.

SOLUTION:

First graph the given points. The midpoint of each diagonal is (0, 1).



Let (x_1, y_1) and (x_2, y_2) be the coordinates of the remaining vertices. Here, diagonals of a parallelogram meet at the point (0, 1).

So, $\left(\frac{x_1+2}{2}, \frac{y_1+4}{2}\right) = (0,1)$ and

$\left(\frac{x_2+3}{2}, \frac{y_2+1}{2}\right) = (0,1)$.

Consider $\left(\frac{x_1+2}{2}, \frac{y_1+4}{2}\right) = (0,1)$.

$\Rightarrow \frac{x_1+2}{2} = 0$ and $\frac{y_1+4}{2} = 1$

$\Rightarrow x_1 + 2 = 0$ and $y_1 + 4 = 2$

$\Rightarrow x_1 = -2$ and $y_1 = -2$

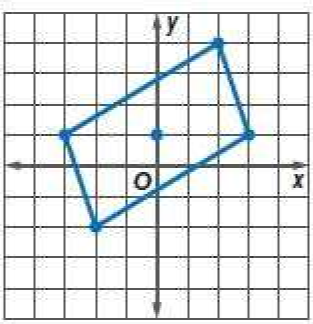
Consider $\left(\frac{x_2+3}{2}, \frac{y_2+1}{2}\right) = (0,1)$.

$\Rightarrow \frac{x_2+3}{2} = 0$ and $\frac{y_2+1}{2} = 1$

$\Rightarrow x_2 + 3 = 0$ and $y_2 + 1 = 2$

$\Rightarrow x_1 = -3$ and $y_1 = 1$

Therefore, the coordinates of the remaining vertices are (–3, 1) and (–2, –2).



41. **WRITING IN MATH** Compare and contrast Theorem 8.9 and Theorem 8.3.

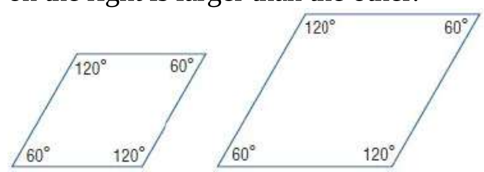
SOLUTION:

Sample answer: Theorem 8.9 states "If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram." Theorem 8.3 states "If a quadrilateral is a parallelogram, then its opposite sides are congruent." The theorems are converses of each other since the hypothesis of one is the conclusion of the other. The hypothesis of Theorem 8.3 is "a figure is a ▱", and the hypothesis of 8.9 is "both pairs of opp. sides of a quadrilateral are ≅". The conclusion of Theorem 8.3 is "opp. sides are ≅", and the conclusion of 8.9 is "the quadrilateral is a ▱".

42. **CCSS ARGUMENTS** If two parallelograms have four congruent corresponding angles, are the parallelograms *sometimes*, *always*, or *never* congruent?

SOLUTION:

Sometimes; sample answer: The two parallelograms could be congruent, but you can also make the parallelogram bigger or smaller without changing the angle measures by changing the side lengths.
For example, these parallelograms have corresponding congruent angles but the parallelogram on the right is larger than the other.

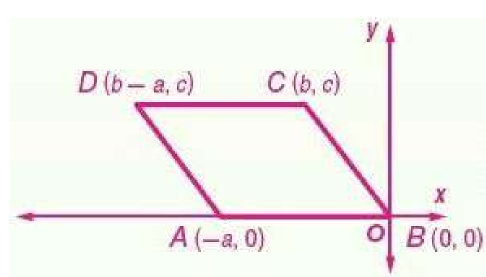


43. **OPEN ENDED** Position and label a parallelogram on the coordinate plane differently than shown in either Example 5, Exercise 35, or Exercise 36.

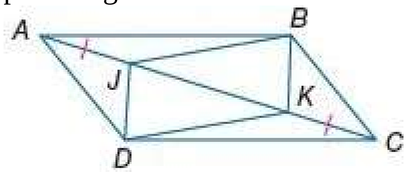
SOLUTION:

Sample answer:

- Position the parallelogram in Quadrant IV with vertex *B* at the origin.
- Let side *AB* be the base of the parallelogram with length *a* units. Place *A* on the *x*-axis at (-*a*, 0).
- The *y*-coordinates of *DC* are the same. Let them be *c*.
- *DC* is the same as *AB*, *a* units long. Since *D* is to the left of *A*, let the *x*-coordinate be *b* – *a*.
- To find the *x*-coordinate of *C* add a units to the *x*-coordinate of *D* to get *b*.
- The coordinates of the vertices are *A*(-*a*, 0), *B* (0, 0), *C*(*b*, *c*), and *D*(*b* – *a*, *c*).



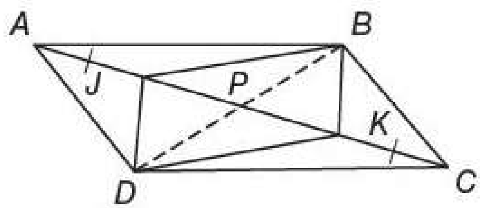
44. **CHALLENGE** If *ABCD* is a parallelogram and $\overline{AJ} \cong \overline{KC}$, show that quadrilateral *JBKD* is a parallelogram.



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is a parallelogram and $\overline{AJ} \cong \overline{KC}$. You need to prove that *JBKD* is a parallelogram. Use the properties that you have learned about parallelograms to walk through the proof.

Given: *ABCD* is a parallelogram and $\overline{AJ} \cong \overline{KC}$.
Prove: Quadrilateral *JBKD* is a parallelogram.



Proof:

Draw in segment $\overline{DB}$. Since *ABCD* is a parallelogram, then by Theorem 6.3, diagonals $\overline{DB}$ and $\overline{AC}$ bisect each other. Label their point of intersection *P*. By the definition of bisect, $\overline{AP} \cong \overline{PC}$, so $AP = PC$. By Segment Addition, $AP = AJ + JP$ and $PC = PK + KC$. So $AJ + JP = PK + KC$ by Substitution. Since $\overline{AJ} \cong \overline{KC}$, $AJ = KC$ by the definition of congruence. Substituting yields $KC + JP = PK + KC$. By the Subtraction Property, $JP = KC$. So by the definition of congruence, $\overline{JP} \cong \overline{PK}$. Thus, *P* is the midpoint of $\overline{JK}$. Since $\overline{JK}$ and $\overline{DB}$ bisect each other and are diagonals of quadrilateral *JBKD*, by Theorem 6.11, quadrilateral *JBKD* is a parallelogram.

45. **WRITING IN MATH** How can you prove that a quadrilateral is a parallelogram?

SOLUTION:

You will need to satisfy only one of Theorems 6.9, 6.10, 6.11, and 6.12.

Sample answer: You can show that: both pairs of opposite sides are congruent or parallel, both pairs of opposite angles are congruent, diagonals bisect each other, or one pair of opposite sides is both congruent and parallel.

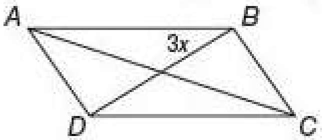
46. If sides *AB* and *DC* of quadrilateral *ABCD* are parallel, which additional information would be sufficient to prove that quadrilateral *ABCD* is a parallelogram?

A $\overline{AB} \cong \overline{AC}$
B $\overline{AB} \cong \overline{DC}$
C $\overline{AC} \cong \overline{BD}$
D $\overline{AD} \cong \overline{BC}$

SOLUTION:

If sides *AB* and *DC* are parallel, then the quadrilateral must be either a trapezoid or a parallelogram.
If the parallel sides are also congruent, then it must be a parallelogram. (Note that if the other pair of opposite sides were congruent, as in option C, or if the diagonals were congruent, as in option D, then the figure could be an isosceles trapezoid, not a parallelogram.)
The correct answer is B.

47. **SHORT RESPONSE** Quadrilateral *ABCD* is shown. *AC* is 40 and *BD* is $\frac{3}{5}AC$. $\overline{BD}$ bisects $\overline{AC}$. For what value of *x* is *ABCD* a parallelogram?



SOLUTION:

$$BD = \frac{3}{5}AC$$
$$= \frac{3}{5}(40)$$
$$= 24$$

The diagonals of a parallelogram bisect each other.
So, $3x = 12$.
Therefore, $x = 4$.
At $x = 4$, *ABCD* a parallelogram.

48. **ALGEBRA** Jarod's average driving speed for a 5-hour trip was 58 miles per hour. During the first 3 hours, he drove 50 miles per hour. What was his average speed in miles per hour for the last 2 hours of his trip?

F 70 **H** 60
G 66 **J** 54

SOLUTION:

Distance = Speed × Time taken
Form an equation for the given situation. Let *x* be the average speed in miles per hour for the last 2 hours of Jarod's trip.

$$58(5) = 50(3) + x(2)$$
$$290 = 150 + 2x$$
$$140 = 2x$$
$$x = 70$$

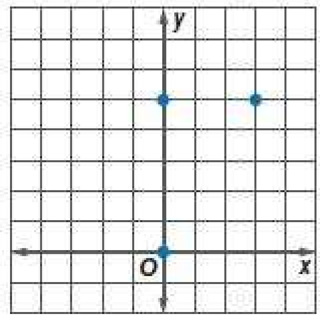
So, the correct option is F.

49. **SAT/ACT** A parallelogram has vertices at (0, 0), (3, 5), and (0, 5). What are the coordinates of the fourth vertex?

A (0, 3)
B (5, 3)
C (5, 0)
D (0, –3)
E (3, 0)

SOLUTION:

First graph the given points.



The vertices (3, 5) and (0, 5) lie on the same horizontal line. The distance between them is 3. So, the fourth vertex must also lie in the same horizontal line as (0, 0) and should be 3 units away.
The only point which lie on the same horizontal line as (0, 0) is (3, 0) and it is also 3 units away from (0, 0). So, the correct choice is E.

COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of ▱*ABCD* with the given vertices.

50. *A*(–3, 5), *B*(6, 5), *C*(5, –4), *D*(–4, –4)

SOLUTION:

Since the diagonals of a parallelogram bisect each other, their intersection point is the midpoint of $\overline{AC}$ and $\overline{BD}$. Find the midpoint of $\overline{AC}$ with endpoints (–3, 5) and (5, –4).
Use the Midpoint Formula.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute.

$$\left(\frac{-3+5}{2}, \frac{5-4}{2}\right) = (1, 0.5)$$

The coordinates of the intersection of the diagonals of parallelogram *ABCD* are (1, 0.5).

51. A(2, 5), B(10, 7), C(7, –2), D(–1, –4)

SOLUTION:

Since the diagonals of a parallelogram bisect each other, their intersection point is the midpoint of $\overline{AC}$ and $\overline{BD}$. Find the midpoint of $\overline{AC}$ with endpoints (2, 5) and (7, –2).

Use the Midpoint Formula.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

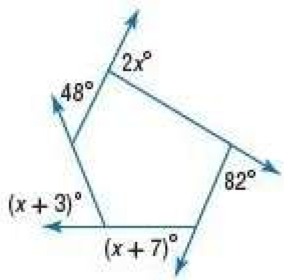
Substitute.

$$\left(\frac{2+7}{2}, \frac{5-2}{2}\right) = (4.5, 1.5)$$

The coordinates of the intersection of the diagonals of parallelogram *ABCD* are (4.5, 1.5).

Find the value of *x*.

52.

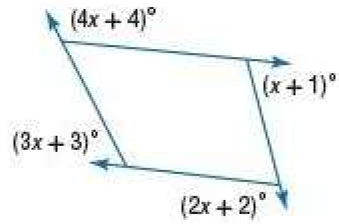


SOLUTION:

Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for *x*.

$$\begin{aligned} 2x+82+(x+7)+(x+3)+48 &= 360 \\ 2x+82+x+7+x+3+48 &= 360 \\ 4x+140 &= 360 \\ 4x &= 220 \\ x &= 55 \end{aligned}$$

53.

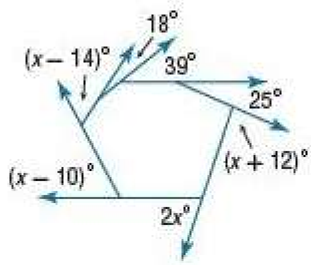


SOLUTION:

Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for *x*.

$$\begin{aligned} (x+1)+(2x+2)+(3x+3)+(4x+4) &= 360 \\ x+1+2x+2+3x+3+4x+4 &= 360 \\ 10x+10 &= 360 \\ 10x &= 350 \\ x &= 35 \end{aligned}$$

54.



SOLUTION:

Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for *x*.

$$\begin{aligned} 18+39+25+(x+12)+2x+(x-10)+(x-14) &= 360 \\ 18+39+25+x+12+2x+x-10+x-14 &= 360 \\ 5x+70 &= 360 \\ 5x &= 290 \\ x &= 58 \end{aligned}$$

55. **FITNESS** Toshiro was at the gym for just over two hours. He swam laps in the pool and lifted weights. Prove that he did one of these activities for more than an hour.

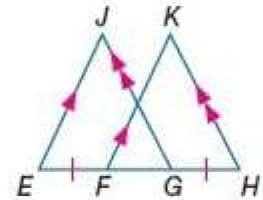
SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given that Toshiro swam laps in the pool and lifted weights for a total of just over 2 hours. You need to prove that he did one of these activities for more than 1 hour. Use a proof by contradiction method. Assume the opposite of what you need to prove and then find a contradiction.

Given: $P + W > 2$ (P is time spent in the pool; W is time spent
lifting weights.)
Prove: $P > 1$ or $W > 1$
Proof:
Step 1: Assume $P \leq 1$ and $W \leq 1$.
Step 2: $P + W \leq 2$
Step 3: This contradicts the given statement. Therefore he did at least one of these activities for more than an hour.

PROOF Write a flow proof.

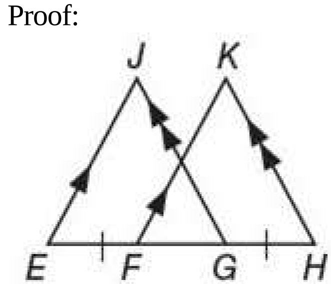
56. **Given:** $\overline{EJ} \parallel \overline{FK}, \overline{JG} \parallel \overline{KH}, \overline{EF} \cong \overline{GH}$
Prove: $\triangle EJG \cong \triangle FKH$

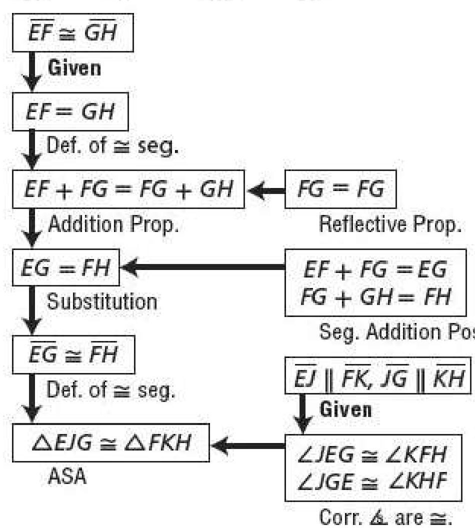


SOLUTION:

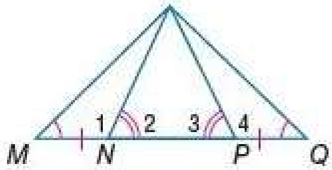
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given $\overline{EJ} \parallel \overline{FK}, \overline{JG} \parallel \overline{KH}, \overline{EF} \cong \overline{GH}$. You need to prove $\triangle EJG \cong \triangle FKH$. Use the properties that you have learned about triangles to walk through the proof.

Proof:



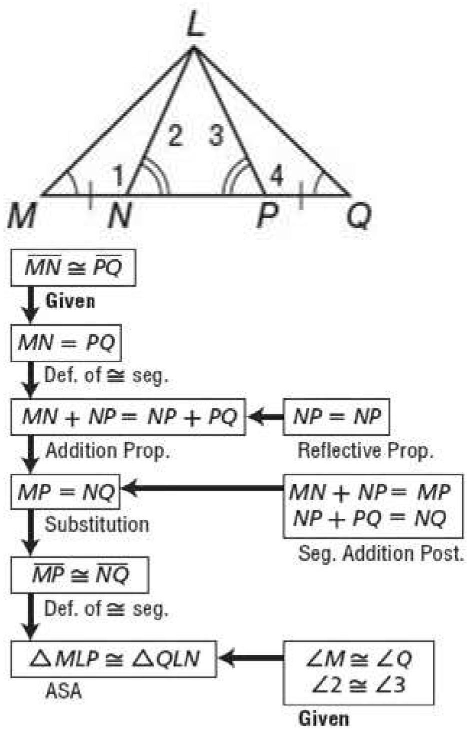


57. **Given:** $\overline{MN} \cong \overline{PQ}, \angle M \cong \angle Q, \angle 2 \cong \angle 3$
Prove: $\Delta MLP \cong \Delta QLN$



SOLUTION:
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given $\overline{MN} \cong \overline{PQ}, \angle M \cong \angle Q, \angle 2 \cong \angle 3$. You need to prove $\Delta MLP \cong \Delta QLN$. Use the properties that you have learned about triangles to walk through the proof.

Proof:



Use slope to determine whether *XY* and *YZ* are *perpendicular or not perpendicular*.

58. $X(-2, 2)$, $Y(0, 1)$, $Z(4, 1)$

SOLUTION:
Substitute the coordinates of the points in slope formula to find the slopes of the lines.

$$\text{Slope of } \overline{XY} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-2}{0-(-2)} = -\frac{1}{2} \qquad \text{Slope of } \overline{YZ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-1}{4-0} = 0$$

The product of the slopes of the lines is not –1. Therefore, the lines are not perpendicular.

59. $X(4, 1)$, $Y(5, 3)$, $Z(6, 2)$

SOLUTION:
Substitute the coordinates of the points in slope formula to find the slopes of the lines.

$$\text{Slope of } \overline{XY} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{5-4} = 2 \qquad \text{Slope of } \overline{YZ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{6-5} = -1$$

The product of the slopes of the lines is not –1. Therefore, the lines are not perpendicular.