

Exemplar Early High School Mathematics Test Questions



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Introduction

This booklet explains ACT Aspire® Early High School (EHS) Mathematics test questions by presenting, with their answer keys, sample questions aligned to each reporting category on the test. A key includes the question's depth-of-knowledge (DOK) level,¹ an explanation of the task posed by each question, a thorough explanation of correct responses, ideas for improvement, and more. The exemplar test questions included here are representative of the range of content and types of questions found on the ACT Aspire Early High School Mathematics test. Educators can use this resource in several ways:

- Become familiar with ACT Aspire question types.
- See what typical questions in each ACT Aspire reporting category look like.
- Help reinforce or adjust teaching and learning objectives.
- Learn how ACT Aspire improvement idea statements can help students identify key skills they have not yet mastered.

ACT Aspire Mathematics tests provide a picture of the whole of a student's mathematical development, including a look at the concepts and skills new to the grade level as well as whether the student has continued to strengthen, integrate, and apply mathematics from earlier grades. These components are important in judging how a student is progressing and what next steps are appropriate.

Reporting Categories

The following ACT Aspire reporting categories help to provide this picture.

Grade Level Progress

The Grade Level Progress reporting category represents a student's achievement related to the mathematical topics new to the grade. To allow for an analysis of student strengths, the category also includes a reporting category for each of the grade-level domains that constitute Grade Level Progress for that grade.

¹ Norman L. Webb, "Depth-of-Knowledge Levels for Four Content Areas," last modified March 28, 2002, http://facstaff. wcer.wisc.edu/normw/All%20content%20areas%20%20DOK%20levels%2032802.doc.

INTRODUCTION

Foundation

The Foundation reporting category looks at the mathematical growth of the student with topics learned in previous grades. This mathematics should not be static, but should be strengthened as the student progresses through the grades. Students should integrate and become more fluent in these topics, using them flexibly as needed to solve problems, give explanations, and accomplish tasks of greater complexity that reflect grade-level expectations for mathematical practice.

Together, the Grade Level Progress and Foundation categories make up the entirety of the ACT Aspire Mathematics test. Two other reporting categories, Modeling and Justification and Explanation, pull out information that crosses the other reporting categories.

Modeling

The Modeling reporting category highlights questions that assess understanding of mathematical models and their creation, interpretation, evaluation, and improvement. Modeling is closely tied to problem solving, and because models are frequently used to teach mathematics—especially in the early grades—modeling is also closely tied to learning mathematics. Modeling expectations increase from one grade to the next. To ensure that the Modeling reporting category provides a better indication of being on track, some modeling skills are a part of the reporting category in lower grades but not in upper grades.

Justification and Explanation

The Justification and Explanation (JE) category focuses on giving reasons for why things work as they do, where students create a mathematical argument to justify. The evidence is collected through constructed-response tasks designed around a progression of justification skills connecting grades 3 and up.

Structure of the Mathematics Test

The structure of the ACT Aspire Mathematics test is the same from grade 3 through early high school (grades 9 and 10), assessing new topics for the grade and whether students continue to strengthen their mathematical core. (For the Early High School test, grade 8 topics are included in the Grade Level Progress component to keep together formal algebra, functions, and geometry topics. This makes Grade Level Progress and its subcategories more coherent.) Within this structure of content comes a level of rigor represented in part by a distribution of depth of knowledge through Webb's level 3. The Foundation component includes only DOK level 2 and level 3 because that component is about assessing how well students have continued to strengthen their mathematical core. Across all parts of the test, students can apply Mathematical Practices to help them demonstrate their mathematical achievement.

Mathematical justification is a way of knowing. In theory, students will be able to learn new mathematics more reliably if they have a strong framework to build upon. Mathematical justification is glue for that framework. The Common Core State Standards for Mathematics (CCSSM) recognizes this in its Mathematical Practice 3 (MP3): "Create viable arguments and critique the reasoning of others." The ACT Aspire Mathematics test focuses attention on student justification.

INTRODUCTION

Students respond to JE tasks with a grade-level-appropriate mathematical argument. These tasks utilize a constructed-response format, allowing students flexibility in the way they shape their arguments. Each response is evaluated on the basis of demonstrated evidence of particular skills associated with mathematical justification. These JE skills include stating relevant properties and definitions that support the justification, constructing an argument that includes reasons for claims, and demonstrating indirect proof or command of counterexample.

The JE skills identified in table 1 are arranged in a progression from grade 3 through EHS. At each grade, the JE skills are divided into three levels. Trained scorers weigh evidence and then make an overall determination about the evidence for or against each skill level. Demonstrating JE skills at one level is evidence of having learned the skills in previous levels. In addition to looking at the JE skills, each response is rated according to how successful the student was in completing the task assigned; this is the Progress rating. A full-credit response shows evidence of the required level of JE skills needed to solve the problem and applies these skills to complete the task.

For each of the JE tasks, evidence for and against each of the JE levels is combined with the Progress rating and mapped to a 0–4 scale. These task scores contribute to the JE reporting category and to the total Mathematics score. Some of the tasks contribute to the Grade Level Progress reporting category, and the others contribute to the Foundation reporting category.

Level 2 JE skills are those most closely aligned with grade-level focus. Level 3 JE skills are more advanced, and level 1 JE skills are those where students should have a fluent command. As the research base increases for this progression, the list will grow and become more refined. Note that there are two JE statements for evidence of misconceptions. These are marked with asterisks in table 1.

As students progress from grade to grade, expectations increase according to which JE skill belongs to which level. Some level 3 JE skills will become level 2, and some level 2 will become level 1.

	JE level at grade:				
Justification statement	3–4	8	EHS		
Provide an example.	1	1	1	1	1
State a definition, theorem, formula, or axiom.	1	1	1	1	1
State a property or classification of an object.	1	1	1	1	1
State a relationship between two or more objects.	1	1	1	1	1
State one or more steps in a procedure.	1	1	1	1	1
Provide a visual representation.	1	1	1	1	1
Provide a computation.	1	1	1	1	1
Use a Specific Statement to draw a Conclusion or Provide Specific Support for a Statement.	1	1	1	1	1
Explain a pattern using words, algebraic expressions, or numeric operations – OR – generate a sequence from a rule.	2	1	1	1	1
Use two or more Specific Statements to draw a Conclusion.	2	1	1	1	1
Indicate an error occurred.	2	1	1	1	1
Explain why a step in a procedure is necessary.	2	2	2	1	1
Make a conditional statement (e.g., If-Then, When-Then).	2	2	2	1	1
Draw and label a visual representation that illustrates a mathematical concept, property, or relationship.	2	2	2	1	1
Use a pattern or sequence to support a Statement or Conclusion.	2	2	2	1	1
Provide a counterexample of a conditional statement.	2	2	2	2	2
Use a General Statement to draw a Conclusion or Provide General Support for a Statement.	2	2	2	2	2
Use a Claim to draw a Conclusion and provide Specific Support for the Claim.	2	2	2	2	2
Use a Claim to draw a Conclusion and provide General Support for the Claim.	3	3	2	2	2
Use a Specific Statement and a General Statement to draw a Conclusion.	3	3	2	2	2
Draw and label a visual representation that illustrates a mathematical concept, property, or relationship, and use the labeling in one's prose to clarify an argument.	3	3	3	2	2
Provide a computation and reference the computation in one's prose to clarify an argument.	3	3	3	2	2
Use proof by example.*	3	3	3	2	2
Conclude from a conditional statement.	3	3	3	2	2
Indicate an error and use a mathematical concept (definition, theorem, or axiom) to explain why an error occurred.	3	3	3	3	2
Provide a counterexample and verify that the conditional conclusion does not hold for the example.	3	3	3	3	2

Table 1. Justification and Explanation Skills Progression

* This statement represents evidence of misconceptions.

Table 1 (continued)

	JE level at grade:				
Justification statement	3–4	5	6–7	8	EHS
Understand that a statement can be true and its converse or inverse can be false.	3	3	3	3	2
State that the converse or inverse of a conditional statement is true because the original statement is true.*	3	3	3	3	2
State that an object belongs (or does not belong) to a class, state at least one of the common characteristics of the class, and state that the object has (or does not have) those characteristics.	3	3	3	3	2
Use two or more Specific Statements to draw a Conclusion and provide Specific Support for at least one of the Statements.	3	3	3	3	2
Use two General Statements to draw a Conclusion.	3	3	3	3	2
Introduce a pattern or sequence and use it to support a Statement or Conclusion.	3	3	3	3	3
Provide a counterexample and verify that the conditional hypotheses do hold for the example, while the conditional conclusion does not.	3	3	3	3	3
Conclude from a conditional statement and verify that the statement's hypotheses hold.	3	3	3	3	3
Use cases in a proof.	3	3	3	3	3
Use indirect proof (i.e., proof by contradiction).	3	3	3	3	3
Use two or more Claims to draw a Conclusion and provide Support for at least one Claim—at least one Claim or Support must be General.	3	3	3	3	3
State what is required to be a member of a class, verify that an object meets all of those requirements, and then state that the object belongs to that class.	3	3	3	3	3

* This statement represents evidence of misconceptions.

Improvement Ideas

ACT Aspire includes simple improvement ideas at the reporting category (skill) level on student and parent reports. These improvement ideas are provided for the lowest performing skill for each subject tested. The skills are always ordered from highest performing to lowest performing based on the percentage of points correct. If the percentages for two or more skills are tied, the skill with the lower number of total points is displayed first.

Keep in mind that the order of skills listed on reports may not always be exemplary of where to focus learning. For example, the skills in which a student performed within the ACT Readiness Range may not always be listed first, and the skills in which a student did not perform within the ACT Readiness Range may not always be listed last. Also, keep in mind the total number of points possible in each skill when interpreting the percentage correct.

There are two levels of improvement idea statements (low and high) for ACT Aspire summative reporting. Low statements are given on the report if the student's lowest skill score is below the ACT Readiness Range for that particular skill. High statements are given on the report if the student's lowest skill score is at or above the ACT Readiness Range for that particular skill.

Answer Key

This section presents the grade, question type, DOK level, alignment to the ACT Aspire reporting categories, and correct response for each of several test questions. Each question is also accompanied by an explanation of the question and by the correct response as well as improvement idea statements for ACT Aspire Mathematics.

Some test questions are appropriate at several grades: as a part of Grade Level Progress when the topic is new to the grade and then in later grades as a part of Foundation (as long as the question is at least DOK level 2 for that grade).

Question 1



Question type	CCSSM topic C	Correct response	
Selected Respon	se 3.NF.A, MP4, Recognize equivalent fractions E and fractions in lowest terms (N 13-15)		
Appropriate	Foundation and Orada Louel Duraneas remarking action	vice Medaline	DOK

grade level(s)	Foundation and Grade Level Progress reporting categories	Modeling	DOK level
3	Grade Level Progress > Number & Operations—Fractions	Yes	3
4–5	Foundation	Yes	3
6-EHS	Foundation	Yes	2

In this selected-response (multiple-choice) question, students must analyze the number line given and determine what fraction is being represented (CCSSM.3.NF.A.3). Because this question requires students to analyze the situation and connect different representations, it is a DOK level 3 question for the Grades 3, 4, and 5 tests. For all other ACT Aspire tests, it is a DOK level 2 question. Because students are interpreting models, this question is a part of the Modeling reporting category (MP4).

Correct Response

After determining that the fraction at point M is 3/4, students must then determine which of the circles provided has 3/4 of its area shaded. The circle in answer option E has 9 out of 12 equally sized sectors shaded, and 9/12 is equivalent to 3/4.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.
Modeling	9	Work on interpreting models. Identify the numbers and variables in the model and describe what they represent. Explain places where the model does not exactly reflect reality.	Find some real-world situations and create models to describe and predict information.
Modeling	Modeling 10 Work on interpreting models. Identify the numbers and variables in the model and describe what they represent. Compare different models and explain trade-offs between accuracy and simplicity.		Find some real-world situations and create models to describe and predict information.

Explain what a line of symmetry is.
Explain why the dashed line drawn in the figure below is NOT a line of symmetry for the figure.

Question type	CCSSM topic	Correct response
Justification & Explanation (Constructed Response)	4.G.A, MP4	See explanation.

Appropriate grade level(s)	Foundation and Grade Level Progress reporting categories	JE level	Modeling	DOK level
4	Grade Level Progress	3	Yes	3
5–7	Foundation	3	Yes	3
8, EHS	Foundation	2	Yes	3

This Justification and Explanation task elicits an explanation of why something is not true. The task is crafted carefully so that successful students must give a definition and tie it to their explanation—an important way of reasoning in mathematics and in many areas of life. The context here is symmetry, a topic from grade 4 (CCSSM.4.G.A.3, "Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry"). A response that successfully justifies the result will contain a general definition of a line of symmetry and show why that definition does not fit the specific situation shown by the drawing. This is JE level 3 reasoning for grade 4 students and would be a part of the Grade Level Progress reporting category. The task is also a part of the JE reporting category. This task would also be appropriate for the Grades 5, 6, 7, 8, and Early High School tests where it would be a part of the Foundation and the JE reporting categories. The reasoning skills assessed by this task are at JE level 3 for grades 4–7 and JE level 2 for grade 8 and EHS. At all grades, this is a task at DOK level 3.

The figure is a possible model for the definition of a line of symmetry, and the student must judge whether the model fits, so this question contributes to the Modeling reporting category (MP4).

Correct Response

The reasoning in the following sample response is within reach of grade 4 students and would receive full credit.

A line of symmetry is a line that divides a figure into two equal parts where you can fold along the line and make the edges match up. Folding along the given line will not make the edges match up because it does not divide the picture into two equal parts, so the dashed line is not a line of symmetry. The primary justification skills in this response are captured by the JE statements "State that an object belongs (or does not belong) to a class, state at least one of the common characteristics of the class, and state that the object has (or does not have) those characteristics" and "Use a Specific Statement and a General Statement to draw a Conclusion." The response also demonstrates direct evidence of "State a property or classification of an object" and "State a definition, theorem, formula, or axiom."

This response successfully completes the assigned task by stating the definition of a line of symmetry and using that definition to conclude that the line in question was not, in fact, a line of symmetry. In addition to successfully completing the task, the response also shows understanding of the given information and the required goal, and it expresses the argument in a clear and organized manner. Note that the definition provided in this response may not be adequate for higher grade levels.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Justification & Explanation	9	Understand mathematical derivations and justification in your textbooks. Know the concepts behind math terms and why procedures work.	Read a proof you found in a college mathematics textbook or on the internet.
Justification & Explanation	10	Understand not just what to do but why that works. Create reasoning based on different cases, being sure to cover all of the cases and then summarizing the result.	Read a proof you found in a college mathematics textbook or on the internet.
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.
Modeling	9	Work on interpreting models. Identify the numbers and variables in the model and describe what they represent. Explain places where the model does not exactly reflect reality.	Find some real-world situations and create models to describe and predict information.
Modeling	10	Work on interpreting models. Identify the numbers and variables in the model and describe what they represent. Compare different models and explain trade-offs between accuracy and simplicity.	Find some real-world situations and create models to describe and predict information.

Liam is making chocolate chip cookies. The recipe calls for 1 cup of sugar for every 3 cups of flour. Liam has only 2 cups of flour.

- · How much sugar should Liam use?
- · Explain why your answer is correct.

Question type		CCSSM topic	Correct response			
Justification & Explanation (Constructed Response)		5.NF.B, MP3, MP4	See explanation.			
Appropriate grade level(s)	Foundation	and Grade Level Prog	ress reporting categories	JE level	Modeling	DOK level
5	5 Grade Level Progress			3	Yes	3
6–8	Foundation			3	No	3
EHS	Foundation		2	No	3	

This questions prompts students to explain their reasoning and tie it to a real-world problem. Logical flow, number sense, and computation are key justification elements in this question. A successful student will make sense of the real-world problem involving fractions (CCSSM.5.NF.B) and provide appropriate justification and explanation (MP3).

For grade 5 students, this task would be a part of the Grade Level Progress reporting category. This task would also be appropriate for the Grades 6, 7, 8, and Early High School tests; it would be a part of the Foundation reporting category for those grades. The task is part of the JE reporting category. The reasoning required is at JE level 3 for grades 5–8. That same reasoning is JE level 2 for the Early High School test. It is a DOK level 3 task at all grade levels.

For grade 5, this task contributes to the Modeling reporting category. Students produce a numerical model and provide an interpretation. In higher grades this should be automatic, so this question does not contribute to the Modeling reporting category for the Grade 6 through Early High School tests.

Correct Response

A student could receive full credit for the following sample response:

Liam has 2 cups of flour, which is 2/3 of the 3 cups of flour that the recipe talks about. So he should use 2/3 of the 1 cup of sugar that the recipe talks about. 2/3 of 1 cup is 2/3 cup. Liam should use 2/3 cup of sugar.

The JE statement "Use two or more Specific Statements to draw a Conclusion and provide Specific Support for at least one of those Statements" captures the complexity of the argument. This response also uses "State one or more steps in a procedure," "Explain why a step in a procedure is necessary," and "State a relationship between two or more objects." The response successfully completes the task assigned by giving the correct amount of sugar that Liam should use and thoroughly explaining why that amount is correct. The response demonstrates one successful pathway and presents a cohesive and well-organized argument.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Justification & Explanation	9	Understand mathematical derivations and justification in your textbooks. Know the concepts behind math terms and why procedures work.	Read a proof you found in a college mathematics textbook or on the internet.
Justification & Explanation	10	Understand not just what to do but why that works. Create reasoning based on different cases, being sure to cover all of the cases and then summarizing the result.	Read a proof you found in a college mathematics textbook or on the internet.
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.

Nigel's class placed 10 empty rain gauges on the playground Monday morning. afternoon.	The line plot below shows the number of inches of rainwater in each gauge after it rained Monday
Numbe	r of Inches of Rainwater
	X X X X X X X X X 3 1 5 3 7
What is the mean amount of rainwater per rain gauge, in inches, in the 10 rain	8 2 8 4 8 gauges?
$A = \frac{25}{30}$	
В. ⁵ / ₈	
\odot C. $\frac{51}{80}$	
D. 37 56	
\odot E $\frac{51}{3}$	

Question type	CCSSM topic	CCSSM topic Correc		
Selected Respor	se 6.SP.B, Extract relevant data from a basic table or chart and use the data in a computation (S 16-19)	С		
Appropriate grade level(s)	Foundation and Grade Level Progress reporting ca	tegories	Modeling	DOK level
6	arade Level Progress > Statistics & Probability		No	3
7–EHS	Foundation		No	2

This question requires students to provide evidence that they can summarize and describe distributions (CCSSM.6.SP.B). This question is part of the Statistics and Probability reporting category within the Grade Level Progress component of the Grade 6 test as a DOK level 3 question. This question requires that students be able to interpret a line plot and use that information to summarize the data by calculating and differentiating quantitative measures of center. This question builds on previous skills regarding line plots and computation with fractions of unlike denominators (CCSSM.5.MD.B.2 and CCSSM.5.NF.A.1). This question could be part of the Foundation component for the Grades 7, 8, and Early High School tests as a DOK level 2 question.

Correct Response

The key for this question is 51/80, which is answer option C. The correct response demonstrates evidence that the student can identify equivalent fractions with like denominators, add those fractions, and find the mean by dividing the total by 10 (the number of rain gauges).

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.

Ryan and Tomas walked to school and then to the park, as described below.

Ryan walked 2.3 miles from his home to meet Tomas at school.

Tomas walked 2.7 miles from his home to meet Ryan at school.

Once they were at school, the boys walked x miles to the park and then x miles back to the school



The sum of the distance Ryan walked and the distance Tomas walked was at least 15 miles but not more than 21 miles. One of the following is the graph of the possible values of x. Which one?

0	A		0	1• 2	4	6	8	 10	1 12	14	16	18	20	11 22	×
0	В.	-	0	2	4	6	8	+ + 10	12	14	16	18	20	22	×
0	С	-	0	2	4	6	+ 8	 10	1 12	14	16	18	20	22	x
0	D.	-	0	2	4	+ + 6	8	10	12	14	16	18	20	22	x
0	E		+ 0	2	4	1 6	8	 10	1 12	14	16	18	20	• 22	x

Question type	CCSSM topic	Correc	t response	
Selected Respon	se 7.EE.B, MP4, Match compound inequalities with their graphs on the number line (e.g., -10.5 < x < 20.3) (A 24–27)	A		
Appropriate grade level(s)	Foundation and Grade Level Progress reporting cate	gories	Modeling	DOK level
7	Grade Level Progress > Expressions & Equations	Yes	3	
8, EHS	Foundation		Yes	2

In this selected-response question, the student needs to find the total distance that Ryan and Tomas walked by setting up an inequality with the variable *x* and solving. The solution then needs to be matched to the correct number line representation (CCSSM.7.EE.B.4).

Correct Response

The correct response is the graph of the solution to the inequality $15 \le (2.3 + 2x + 2.7 + 2x) \le 21$, which is answer option A. This question assesses DOK level 3 skills on the Grade 7 test and is part of the Expressions and Equations reporting category within the Grade Level Progress reporting category. On the Grade 8 or Early High School tests, the question would assess DOK level 2 skills as part of the Foundation reporting category. Students produce an inequality to model the situation, and this question is part of the Modeling reporting category.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.
Modeling	9	Work on interpreting models. Identify the numbers and variables in the model and describe what they represent. Explain places where the model does not exactly reflect reality.	Find some real-world situations and create models to describe and predict information.
Modeling	10	Work on interpreting models. Identify the numbers and variables in the model and describe what they represent. Compare different models and explain trade-offs between accuracy and simplicity.	Find some real-world situations and create models to describe and predict information.

Click and drag the signs below to place them in order from least amount of discount to greatest amount of discount .							
Least Greatest							
Question type CCSSM topic Correct response							
Technology Enhanced 7.RPA, MP4, Solve routine two-step or three-step arithmetic problems involving concepts such as rate and proportion, tax added, percentage off, and estimating by using a given average value in place of actual values (A 20–23) Original Price: \$30.00 Original Price: \$30.00 Take \$1.50 off, and then take \$1.00 off that amount. Original Price: \$30.00 Take \$1.50 off, and then take \$1.00 off that amount. Original Price: \$25.00 Take \$1.50 off, and then take \$1.00 off that amount. Original Price: \$20.00 Take \$1.50 off, and then take \$1.00 off that amount. Take \$1.50 off, and then take \$1.00 off that amount. Take \$1.50 off, and then take \$1.00 off that amount. Take \$1.50 off, and then take \$1.00 off that amount. Take \$1.50 off, and then take \$1.00 off that amount. Take \$1.50 off, and then take \$1.00 off that amount. Take \$1.50 off, and then take \$1.00 off that amount. Take \$1.50 off, and then take \$1.00 off that amount. Take \$1.50 off, and then take \$1.00 off that amount. Take \$1.50 off, and then take \$1.00 off that amount. Take \$1.50 off, and then take \$1.00 off that amount. Take \$1.50 off, and then take \$1.00 off that amount. Take \$1.50 off, and then take \$1.00 off that amount. Take \$1.50 off, and then take \$1.00 off, and then take \$1.00 off that amount. Take \$1.50 off, and \$1.50 off, and \$1.50 off, and \$1.50 off, amount. Take \$1.50 off, and \$1.50 off, and \$1.50 off, and \$1.50 off, and \$1.50 off, amount. Take \$1.50 off, and \$1.50 off, amount. Take \$1.50 off, and \$1.50 off, amount. Take \$1.50 off, amount. Take \$1.50 off, amount. <td< td=""></td<>							
Appropriate grade level(s) Foundation and Grade Level Progress reporting categories Modeling DOK level							
7 Grade Level Progress > Ratios & Proportional Relationships Yes 3							
8, EHS Foundation No 2							

This technology-enhanced problem provides evidence of the student's ability to use proportional relationships to solve percent problems (CCSSM.7.RP.A.3), a skill that is frequently used in everyday encounters outside the classroom. When placed on the Grade 7 test, this question is part of the Ratios and Proportional Relationships reporting category within the Grade Level Progress reporting category. It is part of the Foundation reporting category on the Grade 8 and Early High School tests.

Correct Response

The problem requires that students calculate multiple percentages and perform subtraction in the correct order according to the given information. They must pay careful attention to the fact that the question is asking for the order of the discount and not the final sale price. The correct answer puts the original prices in the order (from left to right) \$25.00, \$30.00, and \$24.00.

The process followed to solve this question requires some interpretation for students in grade 7 and is therefore a DOK level 3 skill on the Grade 7 test. The process becomes more routine in grade 8, so it would be a DOK level 2 problem on the Grade 8 and Early High School tests. The problem contributes to the Modeling reporting category at grade 7 because the student is modeling the descriptions on the signs with numerical calculations. For higher grades, this skill should be more automatic and drops off the list of what contributes to the Modeling reporting category.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.

The function machine below takes an input value, x, processes that value according to the processing equation inside the function machine and creates an output value, y. From the numbers provided below the function machine, drag an input value to box x and the correct output value to box y.





Students use the previously developed skill of evaluating algebraic expressions containing whole number exponents (CCSSM.6.EE.A.1, 2c) to evaluate the given function—that is, to find the output of a function for a given input (8.F.1). This problem is part of the Functions reporting category within the Grade Level Progress reporting category for the Grade 8 and Early High School tests.

Correct Response

Evaluating the expression inside the function machine at each of the 6 listed values will show that only 2 of the 6 values in the list yield integer outputs that are also in the list. For example, 8 = 22 + 3(2) - 2, so x = 2 and y = 8 is one solution. This problem assesses DOK level 2 skills on the both the Grade 8 and Early High School tests.

Technology-enhanced problems like this provide students with the opportunity to supply one of several possible solutions. Students are not limited by the notion that they must give the one correct answer. Students could instead select x = 1 and y = 2 and also receive credit for the question.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Grade Level Progress and Functions	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.

Elaine's Grocery has a lunch counter where people can order 3 types of sandwiches: egg salad, chicken salad, or falafel. Sandwiches can be made with 2 types of bread: pita or baguette. Create a tree diagram by dragging and dropping the sandwich choices into the empty boxes to show all the possible combinations for a sandwich from this lunch counter.

pita	pita	pita	
baguette	baguette	baguette	
egg salad	egg salad	egg salad	
chicken salad	chicken salad	chicken salad	
falafel	falafel	falafel	

Question type		CCSSM topic		Correct response		
Technology Enhanced		7.SP.C, MP4, Apply counting techniques (S 28–32)	See explanati			
Appropriate grade level(s)	Foun	dation and Grade Level Progress reporting cate	egories	Modeling	DOK level	
7	Grad	e Level Progress > Statistics & Probability		Yes	2	
8, EHS	Foun	dation		Yes	2	

In this technology-enhanced problem, the student must demonstrate the ability to create a tree diagram to represent all the possible combinations of three sandwich types and two bread types (CCSSM.7.SP.C.8). Because the student is creating a mathematical model representing a real-world situation, this problem assesses a student's modeling skills (MP4). The student is given the framework of the model and must use the connections to the real-world situation

to create a specific model for the situation. This problem is considered DOK level 2 and contributes to the Statistics and Probability reporting category within the Grade Level Progress reporting category for the Grade 7 test. It is still DOK level 2 for the Grade 8 and Early High School tests, but on those tests the question would be part of the Foundation reporting category and the Modeling reporting category.

Correct Response

To achieve a correct solution, "type of sandwich" boxes (i.e., falafel, egg salad, and chicken salad) should be moved to the three empty boxes closest to the point of origin of the tree diagram but can be placed in any order; then branching off the "type of sandwich" boxes would be the "type of bread" boxes (i.e., pita and baguette) in any order. There are 48 different correct response variations because the order within each type does not matter.

Sample Correct Response



Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.
Modeling	9	Work on interpreting models. Identify the numbers and variables in the model and describe what they represent. Explain places where the model does not exactly reflect reality.	Find some real-world situations and create models to describe and predict information.
Modeling	10	Work on interpreting models. Identify the numbers and variables in the model and describe what they represent. Compare different models and explain trade-offs between accuracy and simplicity.	Find some real-world situations and create models to describe and predict information.

In square ACEG shown below, B, D, F, and H are the midpoints of \overline{AC} , \overline{CE} , \overline{EG} , and \overline{AG} , respectively. A student thinks that the area of $\triangle DEF$, shown shaded, is $\frac{1}{4}$ the area of square ACEG. Explain why the student is NOT correct.



Question type		CCSSM topic	Correct respo	onse	
Justification & Explanation (Constructed Response)		6.G.A, MP3, Compute the area and perimeter See explanation. of triangles and rectangles in simple problems (G 20–23)		on.	
Appropriate grade level(s)	Foundatio	on and Grade Level Progress reporting categori	es JE level	Modeling	DOK level
6	Grade Level Progress		3	No	3
7–EHS	Foundatio	on	3	No	3

This Justification and Explanation problem assesses students' ability to identify an error and use mathematical concepts to explain why it is an error. A successful student must give general mathematical truths (formulas and definitions) to support claims—an important way of justifying an argument in mathematics and everyday life. At an early age, many students incorrectly assume that if a region is divided into 4 parts and 1 part is shaded, then the area of the shaded part is one-fourth the area of the total region. In their responses, students will critique the reasoning of others (MP3).

This problem is a good example of a JE problem that could appear on any of the Grades 6, 7, 8, or Early High School tests. If appearing on the Grade 6 test, this task would be a part of the Grade Level Progress reporting category. If placed on the Grade 7, 8, or Early High School tests, it would be a part of the Foundation reporting category. Due to the justification and explanation required to answer this question, it would be classified as a DOK level 3 at all grade levels. At all grade levels, this task is part of the Justification and Explanation reporting category and requires JE level 3 reasoning.

Correct Response

A student could receive full credit for the following sample response:

Let's say the length of a side of the square is *x*. Since a midpoint will divide a segment in half, $DE = EF = \frac{1}{2}x$. The formula for the area of a triangle is $A = \frac{1}{2}bh$, so the area of $\Delta DEF = (\frac{1}{2})(\frac{1}{2}x)(\frac{1}{2}x) = \frac{1}{8}x^2$. The area of *ACEG* is x^2 because it is a square. The student said the area of ΔDEF would be $\frac{1}{4}$ of the area of square *ACEG*. The correct answer is $\frac{1}{8}$ of the area of the square, so the student is wrong. The primary justification skills appearing in this response are captured by the JE statements "Indicate an error occurred" and "Indicate an error and use a mathematical concept (definition, theorem, or axiom) to explain why an error occurred." Additionally, the complex structure of the argument in the response is direct evidence of the statement "Use two or more Claims to draw a Conclusion and provide Support for at least one Claim—at least one Claim or Support must be General." Finally, the response also demonstrates direct evidence of "State a property or classification of an object," "State a definition, theorem, formula, or axiom," "State a relationship between two objects," and "Provide a computation."

The response successfully completes the task assigned by constructing an argument that uses multiple claims, definitions, and formulas to explain why the student in the problem is incorrect. The response successfully synthesizes the given information with outside knowledge, as evidenced in the use of formulas and definitions. Furthermore, the response demonstrates understanding of the given information, uses logically consistent reasons to support mathematical claims, and expresses the argument in a clear, organized manner.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Justification & Explanation	9	Understand mathematical derivations and justification in your textbooks. Know the concepts behind math terms and why procedures work.	Read a proof you found in a college mathematics textbook or on the internet.
Justification & Explanation	10	Understand not just what to do but why that works. Create reasoning based on different cases, being sure to cover all of the cases and then summarizing the result.	Read a proof you found in a college mathematics textbook or on the internet.
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.

A pattern exists among the units digits of the powers of 7, as shown below. What is the units digit of 7^{50} ?
$7^0 =$ 1 $7^3 =$ 343 $7^6 =$ 117,649 $7^1 =$ 7 $7^4 =$ 2,401 $7^7 =$ 823,543 $7^2 =$ 49 $7^5 =$ 16,807 $7^8 =$ 5,764,801
(Note: The units digit of 2,401 is 1.)
O A. 1
● B. 3
© C. 4
O D. 7
O E. 9

Question type	CCSSM topic	Correct I	response	
Selected Respor	5.OA.B, MP1, MP7, Exhibit knowledge of elementary number concepts such as rounding, the ordering of decimals, pattern identification, primes, and greatest common factor (N 20–23)	E		
Appropriate grade level(s)	Foundation and Grade Level Progress reporting cate	egories	Modeling	DOK leve
5	Grade Level Progress > Operations & Algebraic Thin	king	No	3
6-EHS	Foundation		No	3

Mathematics is sometimes described as a study of patterns. The word "pattern" is found throughout CCSSM. This exemplar assesses a student's ability to recognize a pattern and use the pattern to solve a problem, a part of CCSSM Mathematical Practice 7 (MP7): "Look for and make use of structure." The question is based on content from CCSSM cluster 5.OA.B. The question involves a relatively high level of competence with Mathematical Practice 1 (MP1): "Make sense of problems and persevere in solving them." Understanding the place-value structure of whole numbers and operations on whole numbers, as well as more advanced relationships involving factors, multiples, and remainders are useful for finding the solution. The question is at a DOK level of 3—students must make decisions on to how to approach finding a solution.

Correct Response

A student solution involves recognizing that the number pattern of the units digit generated by the powers of 7 repeats every 4 terms. Using that structure, the student can figure out where the 50th term fits into the pattern, which can be connected to the remainder when 50 is divided by 4. Answer option E is the correct answer.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.

Explain why there are no solutions to the system of inequalities given below.

$$\begin{cases} y < -\frac{1}{2}x - 3\\ y > -\frac{1}{2}x + 2 \end{cases}$$

Question type		CCSSM topic	Correct respon	se	
Justification & Ex (Constructed Res	xplanation sponse)	A-REI.D, MP3, MP7, Solve systems of two linear equations (A 28–32)	See explanatior	۱.	
Appropriate grade level(s)	Foundatio	on and Grade Level Progress reporting categori	es JE level	Modeling	DOK level
EHS	Grade Le	vel Progress	3	No	3

This task elicits an explanation of why a system of linear inequalities has no solution. Logical flow is a key element to creating a viable argument that justifies the statement (MP3). Using appropriate definitions to support claims leads to a correct conclusion and successfully creates a complex argument. The content here is solving systems of inequalities—a high school algebra topic—and uses ACT CCRS "Solve systems of two linear equations." The content is in CCSSM as A-REI.D, but another important element is using the structure of lines with similar equations to deduce something about the lines (MP7).

For Early High School, this task would be a part of the Grade Level Progress reporting category. This task requires JE level 3 reasoning and is part of the JE reporting category. It is a DOK level 3 task.

Correct Response

The reasoning in the following sample response is within reach of an EHS student and would receive full credit.

y = x/2 - 3 and y = x/2 + 2 have the same slope and so are parallel and never intersect. The first line is 5 units below the second line when x = 0. Because the lines are parallel, it is always below the second line. The solutions of y < x/2 - 3 are the points in the plane below the first line. The solutions of y > x/2 + 2 are points above the second line. Because the solutions to a system of linear inequalities are the points that satisfy all of the inequalities, there are no solutions because no point can be both above y = x/2 + 2 and below y = x/2 - 3.

The main justification aspect of this response is captured by the JE statement "Use two or more Claims to draw a Conclusion and provide Support for at least one Claim—at least one Claim or Support must be General." The response also demonstrates direct evidence of "State definitions, theorems, formulas, or axioms," "State a property or classification of an object," and "State a relationship between two or more objects." This type of response provides direct evidence for all three levels of justification.

The sample response would receive full credit for successfully creating an argument that uses mathematical claims and definitions to properly explain why the system has no solutions. The response synthesizes the given information with outside knowledge as evidenced by the explanation of what it means to be parallel and what it means to be a solution of a system of linear inequalities. Additionally, the response demonstrates understanding of the given information, uses logically consistent reasons to support mathematical claims, and expresses the argument in a clear, organized manner.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Justification & Explanation	9	Understand mathematical derivations and justification in your textbooks. Know the concepts behind math terms and why procedures work.	Read a proof you found in a college mathematics textbook or on the internet.
Justification & Explanation	10	Understand not just what to do but why that works. Create reasoning based on different cases, being sure to cover all of the cases and then summarizing the result.	Read a proof you found in a college mathematics textbook or on the internet.
Grade Level Progress	EHS	Ask questions when you don't understand the lesson. Start a group with classmates to study for quizzes and tests.	Try completing the "challenge" questions in your textbook for your current math work.

A student thinks that the sum of 4.3 and 8.4 is 12.7 because 4 + 8 = 12 and 3 + 4 = 7. The student then adds 3.7 and 2.6 and gets 5.13 because 3 + 2 = 5 and 6 + 7 = 13. Identify the mistake in the student's procedure, and explain why this procedure won't *always* work.

Question type		CCSSM topic	Correct respons	e	
Justification & Ex (Constructed Res	planation ponse)	5.NBT.B, MP3, Perform one-operation computation with whole numbers and decimals (N 13-15)	See explanation.		
Appropriate grade level(s)	Foundatio	on and Grade Level Progress reporting categorie	s JE level	Modeling	DOK level
5	Grade Le	vel Progress	3	No	3
6–8	Foundatio	n	3	No	3
EHS	Foundatio	on	2	No	3

This task elicits an explanation of why a procedure is not always effective. When learning about decimals, students often make the mistake of just adding the digits instead of adding the value of the numbers. This task is crafted carefully so that successful students must identify the misconception and explain why it is incorrect by appealing to a general mathematical concept. The content here is place value, a topic from CCSSM grade 5 (5.NBT.B.7). The focus is on mathematical justification, captured by CCSSM in MP3: "Create viable arguments and critique the reasoning of others."

For grade 5 students, this task would be a part of the Grade Level Progress reporting category. This task would also be appropriate for the Grades 6, 7, 8, and Early High School tests as a part of the Foundation reporting category. At grades 5–8, this task is a part of the Justification and Explanation reporting category, requires JE level 3 reasoning, and is DOK level 3. For the Early High School test, this would be considered JE level 2.

Correct Response

A student could receive full credit for the following sample response:

The student didn't pay attention to place value and added the tenths place wrong. 0.6 + 0.7 = 1.3. The student's procedure won't always work because if you add numbers by place value and get a number greater than 9, you must carry to the next largest place value.

The heart of the justification in this response is captured by the JE statements "Indicate an error occurred" and "Indicate an error and use a mathematical concept (definition, theorem, or axiom) to explain why an error occurred." The student uses a general mathematical concept in the response ("if you add numbers by place value and get a number greater than 9, you must carry to the next largest place value"), a skill captured by the JE statement "State a definition, theorem, formula, or axiom." This response also provides direct evidence of "Provide a computation," "State a relationship between two or more objects," and "Use a General Statement to draw a Conclusion or Provide General Support for a Statement."

The response successfully completes the task assigned, telling why the student in the problem is incorrect and thoroughly explaining why the procedure won't always work. The response demonstrates understanding of the given information, uses logically consistent reasons to support mathematical claims, and expresses the argument in a clear, organized manner.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Justification & Explanation	9	Understand mathematical derivations and justification in your textbooks. Know the concepts behind math terms and why procedures work.	Read a proof you found in a college mathematics textbook or on the internet.
Justification & Explanation	10	Understand not just what to do but why that works. Create reasoning based on different cases, being sure to cover all of the cases and then summarizing the result.	Read a proof you found in a college mathematics textbook or on the internet.
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.

The table below represents the number of episodes of a certain television series Jessica, Kev, and Miriam watched last weekend.

Name	Total number of episodes watched
Jessica	1
Kev	k
Miriam	m

Kev watched 2 fewer episodes than Jessica. Miriam watched $\frac{1}{2}$ as many episodes as Kev. One of the following statements is NOT true. Which one?

- A. Jessica watched k+2 episodes.
- **B.** Miriam watched $k \times \frac{1}{2}$ episodes.
- C. Miriam watched $k \times 2$ episodes.
- D. Kev watched m × 2 episodes.
- E. Kev watched j 2 episodes.

Question type	CCSSM topic	Correc	t response	
Selected Respon	se 6.EE.B, MP2, Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$) (A 13–15)	С		
Appropriate grade level(s)	Foundation and Grade Level Progress reporting categ	gories	Modeling	DOK level
6	Grade Level Progress > Expressions & Equations		No	3
7	Foundation		No	3
8, EHS	Foundation		No	2

This question measures a student's ability to reason with and find relationships between variables, skills that are a part of CCSSM Mathematical Practice 2 (MP2) and CCSSM.6.EE.B.6. This question would be part of the Expressions and Equations reporting category within the Grade Level Progress reporting category for the Grade 6 test and the Foundation reporting category for the Grade 7 and 8 and Early High School tests. The analysis required to determine which of the five statements is not true coupled with the abstract reasoning required by the use of variables makes this question a DOK level 3 skill when used on the Grades 6 and 7 tests. Since students grow more comfortable using variables at higher grades, this question would be DOK level 2 for the Grade 8 and Early High School tests.

Correct Response

A successful response requires the student to synthesize all the given information and use that information to decide which statement is not true. Because it was given that Miriam watched half as many shows as Kev, and because the expression $k \times 2$ represents twice as many shows watched as Kev, the statement "Miriam watched $k \times 2$ episodes" is not true, which means answer option C is correct.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.

On a given day on a certain poultry farm, the relative frequencies of chickens, ducks, and geese that laid an egg or did not lay an egg are given in the table below. Two of the relative frequencies are given by letters.

	Laid an egg	Did not lay an egg	Total
Chickens	а	0.33	1.00
Ducks	0.45	0.55	1.00
Geese	0.48	b	1.00

If it can be determined, what is the sum of the values of *a* and *b* in this table?

A. 0.19

B. 0.67

C. 1.19

- **D.** 1.80
- E. Cannot be determined from the given information

Question type	CCSSM topic	Correct	t response	
Selected Respon	se 8.SP.A, MP4, Manipulate data from tables and charts (S 24–27)	С		
Appropriate grade level(s)	Foundation and Grade Level Progress reporting cate	gories	Modeling	DOK level
8, EHS	Grade Level Progress > Statistics and Probability		No	2

This question assesses a student's ability to read and interpret relative frequencies in a twoway table—a fundamental skill when exploring early statistics. The content of this question can best be mapped to CCSSM.8.SP.B.4. Because there are multiple ways for a student to misread a two-way table, and because there exists the possibility that a student might choose the answer option that corresponds to only one of the unknown values instead of the sum of the unknown values, success at this question shows evidence of MP4. This question could appear on the Grade 8 or Early High School tests; for both tests, it is part of the Statistics and Probability reporting category within Grade Level Progress. Because a student must find two unknowns by correctly reading the two-way table and then find the sum of those unknowns, this question measures a DOK level 2 skill.

Correct Response

In this two-way relative frequency table, the rows must add to 1.00. Since the relative frequency of chickens that did not lay an egg is 0.33, *a* must be 0.67. Likewise, *b* must be 0.52. Therefore, the sum of *a* and *b* is 1.19, which makes answer option C correct.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Grade Level Progress	EHS	Ask questions when you don't understand the lesson. Start a group with classmates to study for quizzes and tests.	Try completing the "challenge" questions in your textbook for your current math work.
Statistics and Probability	EHS	Work on comparing distributions and interpreting the differences. Do you understand the difference between finding the probability of repeated events with replacement versus without?	What could you do if you gave a survey but some people didn't answer the question? Would it matter?

The volume, $\pi \approx 3.14$, <i>r</i> i and height are cubic inches?	V, of a right circular cylinder may be calculated using the formula $V = \pi r^2 h$, where is the radius, and h is the height. In the right circular cylinder shown below, the radius e labeled in inches. Which of the following numbers is closest to the cylinder's volume, in
	12
\square	
 A. 113 B. 226 C. 339 D. 452 E. 1,360 	

Question type	CCSSM topic	Correc	t response	
Selected Respor	se 8.G.C, Use geometric formulas when all necessary information is given (G 20–23)	С		
Appropriate grade level(s)	Foundation and Grade Level Progress reporting ca	ategories	Modeling	DOK level
8	Grade Level Progress > Geometry		No	2
EHS	Grade Level Progress > Geometry		No	1

Students are expected to identify the parts of the pictured cylinder and calculate its volume in this question, which falls under the Geometry reporting category within the Grade Level Progress reporting category on the Grade 8 and EHS tests (8.G.C.9). Grade 8 students will likely be unfamiliar with volume formulas for nonpolygonal figures. In the Early High School grades students are more familiar with evaluation of such formulas. When the student is provided with such a formula that is appropriate for a particular scenario, this is a DOK level 1 skill for the Early High School test.

Correct Response

The radius of the cylinder is 3 inches, and the height is the cylinder is 12 inches, as shown in the figure. When these values, along with the approximation for pi, are substituted into the given equation, the volume of the cylinder is approximately 339.12 cubic inches. Rounding this value to the nearest cubic inch yields the correct response, answer option C.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Grade Level Progress	EHS	Ask questions when you don't understand the lesson. Start a group with classmates to study for quizzes and tests.	Try completing the "challenge" questions in your textbook for your current math work.
Geometry	EHS	Work on explaining geometric reasoning related to lines and angles. Derive the formula for the volume of a pyramid.	Make a list of at least 6 real-world objects and the 3-dimensional objects that model each of them.

A doctor surveys her patients to determine whether they have had back pain in the last 3 months. She also records whether they are shorter than 6 feet, or 6 feet tall or taller. Her results are summarized in the two-way table below.

	Had back pain	Did not have back pain	Total
Shorter than 6 feet	54	81	135
6 feet tall or taller	25	37	62
Total	79	118	197

One of the following conclusions is supported by the data in the table. Which one?

A. People 6 feet tall or taller always have a higher frequency of back pain than people shorter than 6 feet.

B. People shorter than 6 feet always have a higher frequency of back pain than people 6 feet tall or taller.

C. People 6 feet tall or taller are more likely than not to have a higher frequency of back pain than people shorter than 6 feet.

D. People shorter than 6 feet are more likely than not to have a higher frequency of back pain than people 6 feet tall or taller.

E. There is no relationship between a person's height and having back pain.

Question type	CCSSM topic Correct resp	onse
Selected Respor	ise 8.SP.A, MP3, MP4, MP6, Analyze and draw E conclusions based on information from tables and charts, including two-way frequency tables (S 33-36)	
Appropriate Grade Level(s)	Foundation and Grade Level Progress Reporting Categories Mod	leling DOK level
8. EHS	Grade Level Progress > Statistics and Probability Yes	3

This selected response question assesses a student's ability to analyze a two-way table to determine if a relationship exists between 2 categorical variables (CCSSM.8.SP.A.4). The skills needed to solve this problem map this question to the Statistics and Probability reporting category within the Grade Level Progress reporting category for both the Grade 8 and Early High School tests.

Undergraduate students use the chi-square statistic to reject or fail to reject the presence of such a relationship between two categorical variables. Students in grades 8–10 instead will first find what fraction an observed count (for a given row) is of its corresponding column total and then determine whether the observed fractions fit the model for unrelated variables; the fractions for each column by row are virtually equal. Because of this, this question is mapped to the Modeling reporting category.

Given that a student must judge whether each proposed conclusion is valid (MP3, MP4) by calculating and interpreting relevant values with a certain level of precision (MP6), a DOK level of 3 is appropriate for this question.

Correct Response

When expressed as a decimal, the fraction of all patients with back pain is approximately 0.684, and the fraction of all patients without back pain is approximately 0.686. Given that these fractions are nearly identical, evidence suggests that no association (or relationship) exists between a patient's height and back condition. Therefore, the correct answer is answer option E.

Students who select answer option D rather than B may recognize that these counts come from a sample and, given that these patients may not be representative of the population of all people, eschew absolute qualifiers like "always." However, these same students may incorrectly infer that an individual shorter than 6 feet must experience back pain more often than a taller individual (given the roughly 2:1 ratio found in the first column of the table). The table provides no information about the frequency of back pain experienced by an individual—only that certain individuals experienced back pain at all. Consequently, the only possible choice for a correct answer would be E.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Grade Level Progress	EHS	Ask questions when you don't understand the lesson. Start a group with classmates to study for quizzes and tests.	Try completing the "challenge" questions in your textbook for your current math work.
Statistics and Probability	EHS	Work on comparing distributions and interpreting the differences. Do you understand the difference between finding the probability of repeated events with replacement versus without?	What could you do if you gave a survey but some people didn't answer the question? Would it matter?

In sc Whic	cientific notation, the number 8,120 ch of the following gives these 3 ten	is written as 8.12×10 ³ . In a certain se ms in scientific notation?	equence, the first term is $0,120 \times 10$, the se	econd term is $8,120 \times 10^2$, and the third te	9777 is 8,120×10 ³
A.	8.12×10 ²				
	8.12×10 ¹				
	8.12×10 ⁰				
Β.	8.12×10 ³				
	0.812×10 ³				
	0.0812×10 ³				
2.	9.12-104				
	8.12×10				
	8.12×10 ⁴				
D.	8.12×10 ⁻				
	8.12×100 ⁸				
	8.12×1,000 ³				
	8.12×10,000 ³				
3	80.12×10 ⁸				
	800.12×10 ³				
	8,000.12×10 ³				

Question type	CCSSM topic Corr	ect response	
Selected Respon	se 8.EE.A, Work with scientific notation C (A 24–27)		
Appropriate grade level(s)	Foundation and Grade Level Progress reporting categories	Modeling	DOK level
8	Grade Level Progress > Expressions & Equations	No	2
EHS	Grade Level Progress > Algebra	No	2

The content of this question focuses on a student's ability to interpret and work with numbers in scientific notation. This content aligns with CCSSM.8.EE.A.4 and is considered a DOK level 2 skill because the students must interpret the sequence and then convert the terms to scientific notation. This question could appear on either the Grade 8 or the Early High School test. On the Grade 8 test, it would be part of the Expressions and Equations reporting category; on the Early High School test, it would contribute to the Algebra reporting category. Both categories lie within the Grade Level Progress reporting category for their respective tests.

Correct Response

To come to the correct response, a student must convert the three sequence numbers to equivalent numbers in scientific notation. 8,120 is 8.12×10^3 in scientific notation, and $8,120 \times 10$ is one order of magnitude larger than 8,120. So, in scientific notation, $8,120 \times 10$ must be equal to $8.12 \times 10^{3+1} = 8.12 \times 10^4$. Likewise, the second and third numbers in the sequence must be 8.12×10^5 and 8.12×10^6 , respectively. Therefore, answer option C is the correct response.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Grade Level Progress	EHS	Ask questions when you don't understand the lesson. Start a group with classmates to study for quizzes and tests.	Try completing the "challenge" questions in your textbook for your current math work.
Algebra	9	Work on performing operations on polynomials, solving linear equations and inequalities, and solving quadratic equations. Do algebraic expressions have meaning for you? Interpret expressions as they apply to the real world.	Practice a variety of methods to solve quadratic equations (e.g., completing the square, factoring, and applying the quadratic formula).
Algebra	10	Work on performing operations on polynomials, solving linear equations and inequalities, and solving quadratic equations. Can you identify when a system of equations has zero, one, or infinitely many solutions?	Practice a variety of methods to solve quadratic equations (e.g., completing the square, factoring, and applying the quadratic formula).

10

The graph below models a constant decrease in annual licorice sales for Licorice Company, Inc., from 1998 through 2000. The points have been connected to illustrate the trend. Which of the following values is closest to the amount, in dollars, of the decrease per year?

ution 25 (1998,20) (1999,15) (2000,1 (1999,15) (2000,1 (2000,1 (2000,1	0)
1998 1999 2000 year	
A. \$ 5,000 B. \$ 6,667 C. \$ 8,333 D. \$10,000 E. \$15,000	

Question type	CCSSM topic	Correc	t response	
Selected Respon	se S-ID.C, MP4, Interpret and use information from graphs in the coordinate plane (F 28 – 32)	A		
Appropriate grade level(s)	Foundation and Grade Level Progress reporting ca	tegories	Modeling	DOK level
EHS	Grade Level Progress > Statistics and Probability		No	3

This question assesses a student's ability to compute and interpret the slope of a linear model (MP4) for a scatterplot (CCSSM.S-ID.C.7) to solve this question, which is mapped to the Statistics and Probability reporting category within the Grade Level Progress reporting category. Answering this question correctly requires attention to subtle details, such as identifying the model needed to arrive at a solution and recognizing how the labeling of the scatterplot must influence the final answer. As such, the DOK for this question is 3.

Correct Response

The graph modeling the trend is a line because the decrease in the graph is characterized as constant. Therefore, finding the slope of the line passing through any two of the given points can be used to find the desired value. For example, the graph falls by five units between the years 1998 and 1999, so the slope of the line is 5. This slope is then multiplied by \$1,000, because the vertical axis label indicates that each of its values should be measured in thousands. Consequently, the annual decrease in licorice sales is \$5,000, and the correct response for this question is answer option A.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Grade Level Progress	EHS	Ask questions when you don't understand the lesson. Start a group with classmates to study for quizzes and tests.	Try completing the "challenge" questions in your textbook for your current math work.
Statistics and Probability	EHS	Work on comparing distributions and interpreting the differences. Do you understand the difference between finding the probability of repeated events with replacement versus without?	What could you do if you gave a survey but some people didn't answer the question? Would it matter?

A set is closed under a given arithmetic operation if performing the operation with any member(s) of the set always results in the same or another member of the set. For example, the set {0,1} is closed under multiplication, since all possible products (0×0 , 0×1 , 1×0 , and 1×1) are also members of the set. Under which of the following operations is the set {-1,1} closed?

- A. Addition and subtraction
- B. Addition and multiplication
- C. Subtraction and multiplication
- D. Subtraction and division
- E. Multiplication and division

Question type	CCSSM topic	CCSSM topic Correc		
Selected Respon	se N-RN.B, MP1, MP6, Analyze and draw conclusions based on number concepts (N 33–36)	E		
Appropriate grade level(s)	Foundation and Grade Level Progress reporting cate	egories	Modeling	DOK level
EHS	Grade Level Progress > Number & Quantity		No	2

This question allows a student to relate familiar, if informally understood, knowledge, such as the fact that a rational number multiplied by a rational number is a rational number (CCSSM.N-RN.B.3), to unfamiliar definitions—closure in this case. Because this question assesses a student's ability to understand and apply new definitions, both MP1 and MP6 are present. As part of an Early High School test, this question would contribute to the Number and Quantity reporting category within the Grade Level Progress reporting category. Though this question seems to involve little more than adding, subtracting, multiplying, and dividing rational numbers, the inclusion of a novel definition makes this a DOK level 2 skill.

Correct Response

The set $\{-1,1\}$ is not closed under either addition or subtraction, because 1 + (-1) = 0, which is not in the set, and 1 - (-1) = 2, which is also not in the set. However, (1)(-1) = (-1)(1) = -1 and $\frac{1}{-1} = \frac{-1}{1} = -1$, so the set is closed with respect to both multiplication and division. Thus, answer option E is the correct response.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Grade Level Progress	EHS	Ask questions when you don't understand the lesson. Start a group with classmates to study for quizzes and tests.	Try completing the "challenge" questions in your textbook for your current math work.
Number & Quantity	9	Work on applying properties of exponents and rewriting radical expressions in terms of rational exponents. Do the properties of exponents make sense to you? If you make sense of them, they will be easier to use.	Explain how to solve a problem in a physics or chemistry textbook by using the units on the quantities as a guide.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Number & Quantity	10	Work on applying properties of exponents and rewriting radical expressions in terms of rational exponents. What does it mean that a number is the cube root of 3? Why should pi to the zero power equal 1?	Explain how to solve a problem in a physics or chemistry textbook by using the units on the quantities as a guide.



Question type		CCSSM topic	Correc	t response	
Technology Enhai	nced	4.MD.A, MP6, Compute the area and perimeter of triangles and rectangles in simple problems (G 20–23)	See ex	olanation.	
Appropriate	Found	dation and Grade Level Progress reporting cate	aories	Modeling	
7_8	Foun	dation	901100	No	3
7-0	roun			INU	0
EHS	Found	dation		No	2

This question is the first of three questions related to common information. These three questions are independent of each other in that it is not necessary to obtain the correct solution to one question in order to answer any of the other questions. Sets of questions like these require students to extract only the information needed to answer a particular question. This provides an additional cognitive demand on students that standalone questions may not and reinforces a skill that effective consumers of information have.

Correct Response

This free-response problem provides evidence of the student's ability to solve a word problem involving measurements given in decimal form (CCSSM.4.MD.A). In order to provide a correct response, the student must accurately calculate the answer to the requested precision (MP6). The student must find the area for each of the rooms and calculate the sum, obtaining a correct answer of 67.68. For students taking the Grades 7 or 8 tests, this problem would be considered nonroutine due to the fact that the student must find the missing dimensions in order to calculate the total area (DOK level 3). This problem is part of the Foundation reporting category for the Grades 7, 8, and Early High School tests.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.



Question type	CCSSM topic	Correct response
Technology Enhanced	7.G.A, Solve routine one-step arithmetic problems using positive rational numbers, such as single-step percent (A 16–19)	15 Family Room 18 12 Dining Room 9 3 Hall 6
Appropriate		

grade level(s)	Foundation and Grade Level Progress reporting categories	Modeling	DOK level
7	Grade Level Progress > Geometry	Yes	2
8, EHS	Foundation	No	2

This question is the second of three questions related to common information. This question is a DOK level 2 question for both the Grade 7 and Grade 8 tests. It is part of the Geometry reporting category within Grade Level Progress when placed on the Grade 7 test, and it is part of the Foundation reporting category if placed on the Grade 8 or Early High School tests. At grade 7, this problem would contribute to the Modeling reporting category. At grades 8 and above, producing this model should be automatic and therefore this question is not counted in the Modeling reporting category for the Grade 8 or Early High School tests.

Correct Response

In this question, students must determine actual lengths given information from a scale drawing (CCSSM.7.G.A.1). Students are given a conversion factor and must set up the correct proportions to determine the lengths of the rooms in the actual home, drawing on previous knowledge of proportional relationships (CCSSM.6.RP).

Moving clockwise around the figure, the missing lengths are 15, 18, 12, 9, 6, and 3. The process the student uses to determine one length must be repeatedly and consistently applied to determine all lengths.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.



Question type	CCSSM topic	Correc	t response	
Selected Respon	se 7.EE.A, MP2, Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$) (A 13–15)	E		
Appropriate grade level(s)	Foundation and Grade Level Progress reporting cat	egories	Modeling	DOK level
7	Grade Level Progress > Expressions & Equations		No	2
8, EHS	Foundation		No	2

This question is the third and final question in a set related to common information. This question is part of the Expressions and Equations reporting category within the Grade Level Progress reporting category for the Grade 7 test; it is part of the Foundation reporting category for the Grade 8 and the Early High School tests.

Correct Response

In this selected-response question, the student is required to identify an expression written in a form that demonstrates how quantities of interest can be represented in equivalent mathematical forms (CCSSM.7.EE.A), making sense of quantities in a problem situation (MP2). To arrive at the correct answer in answer option E, the student should recognize that 1.05 can be written as 1 + 0.05. The ability to represent a percent increase in various but equivalent forms is a DOK level 2 skill for grades 7 through EHS.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.



Yes

2

Question type	CCSSM topic Co	rrect response	
Selected Respons	se 7.G.B, MP4, Compute the area and C circumference of circles after identifying necessary information (G 24–27)		
Appropriate grade level(s)	Foundation and Grade Level Progress reporting categorie	es Modeling	DOK level

This question is the first of four problems related to common information presented through text and a map. These four questions are independent of each other in that it is not necessary to obtain the correct solution to one question in order to correctly answer any of the other three questions.

Sets of questions like these can provide a richer context and the natural associated challenge in making sense of problems. Sets can provide greater opportunity to make connections within mathematics and to the real world.

Because this set contains questions assessing high school topics, this set would be used on the Early High School test, with this question contributing to the Foundation reporting category and the Modeling reporting category. This question refers to the signal range of a tower. The last sentence of the common information gives details about this range.

Correct Response

Foundation

EHS

To solve this question, students must know how to compute the area of a circle (CCSSM.7.G.B.4). More importantly, they must first understand that a circle is central to answering the question. When students attempt to interpret the phrase "10 miles in all directions," this is modeling (MP4). Students produce the model, a circle of radius 10 miles, and then they can find the area. Answer option C is the correct answer. This question has a DOK level of 2.

Incorrect answers often reflect specific misconceptions. Students who select answer option B, for example, may have computed the area of a square with side length 10 miles, thinking that "10 miles in all directions" meant 10 miles up and 10 miles across. Students who select answer option A may think that 10 miles is the distance across the circle or do not understand the formulas they are using.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Foundation	EHS	Continue to strengthen your skills by applying and integrating the mathematics you learned in previous grades.	Everybody makes mistakes. When you make one, think about what tipped you off to there being something wrong, and think about whether you could have noticed it sooner.
Modeling	9	Work on interpreting models. Identify the numbers and variables in the model and describe what they represent. Explain places where the model does not exactly reflect reality.	Find some real-world situations and create models to describe and predict information.
Modeling	10	Work on interpreting models. Identify the numbers and variables in the model and describe what they represent. Compare different models and explain trade-offs between accuracy and simplicity.	Find some real-world situations and create models to describe and predict information.



Question type	CCSSM topic	Correct	t response	
Selected Respon	se 8.G.B, Use the distance formula (G 28–32)	E		
Appropriate grade level(s)	Foundation and Grade Level Progress reporting cate	gories	Modeling	DOK level
EHS	Grade Level Progress > Geometry		No	2

This question is the second of four problems related to common information. This question asks students to find the distance between 2 points in the standard (x,y) coordinate plane (CCSSM.8.G.B.8). This question is part of the Grade Level Progress reporting category and the Geometry reporting category within that category.

Correct Response

Students can draw a right triangle whose hypotenuse is the line segment connecting the point representing Esteban's house, (10,-2), and the cell phone tower, (5,4), and then use the Pythagorean theorem to solve for the length of that line segment. An equivalent approach is to use the Pythagorean theorem as captured by the distance formula, substituting the values of the coordinates into the formula to evaluate the answer, which is given in E. Both approaches are DOK level 2 skills for the Early High School test.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Grade Level Progress	EHS	Ask questions when you don't understand the lesson. Start a group with classmates to study for quizzes and tests.	Try completing the "challenge" questions in your textbook for your current math work.
Geometry	EHS	Work on explaining geometric reasoning related to lines and angles. Derive the formula for the volume of a pyramid.	Make a list of at least 6 real-world objects and the 3-dimensional objects that model each of them.



Question type	CCSSM topic	Correct	t response	
Selected Respon	F-IFA, MP4, Evaluate linear and quadratic functions, expressed in function notation, at integer values (F 20–23)	С		
Appropriate grade level(s)	Foundation and Grade Level Progress reporting ca	tegories	Modeling	DOK level
EHS	Grade Level Progress > Functions		Yes	2

This question is the third of four problems related to common information. In this question, students interpret the given function in the context of the problem, which leads to evaluating the function by finding the appropriate values from the common information and substituting those values into the function (CCSSM.F-IF.A.2). This question is DOK level 2 for the Early High School test and is part of the Functions reporting category within the Grade Level Progress reporting category. This question is also part of the Modeling reporting category, assessing whether students correctly interpret the model in the real-world context.

Correct Response

Since the jet is above Esteban's house, and the coordinates of his house are (10,-2), the value of f(a,b) for a = 10 and b = -2 gives the maximum height at which the plane would receive a signal. This is the value in answer option C. The skills needed for this question build on topics students learned starting in grade 6 (negative numbers) and grade 8 (exponents).

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Grade Level Progress	EHS	Ask questions when you don't understand the lesson. Start a group with classmates to study for quizzes and tests.	Try completing the "challenge" questions in your textbook for your current math work.
Functions	EHS	Work on interpreting and modeling with functions. Do you know the basic characteristics of exponential, quadratic, square-root, and absolute value functions?	Practice working with piecewise-defined functions; describe what the graph means in terms of a real-world situation.
Modeling	9	Work on interpreting models. Identify the numbers and variables in the model and describe what they represent. Explain places where the model does not exactly reflect reality.	Find some real-world situations and create models to describe and predict information.
Modeling	10	Work on interpreting models. Identify the numbers and variables in the model and describe what they represent. Compare different models and explain trade-offs between accuracy and simplicity.	Find some real-world situations and create models to describe and predict information.



Question type		CCSSM topic	Correct respons	е	
Justification & Explanation (Constructed Response)		G.GPE.A, MP1, MP3, MP4, Build functions and write expressions, equations, and inequalities for common algebra settings (e.g., distance to a point on a curve and profit for variable cost and demand) (A 28–32)	See explanation.		
Appropriate grade level(s)	Foundati	on and Grade Level Progress reporting categorie	s JE level	Modeling	DOK level
EHS	Grade Le	vel Progress	2	No	3

This problem is the fourth and final problem of a set of problems related to common information. This task prompts students to explain their reasoning and tie it to a real-world problem. The content here is the general equation of a circle in the standard (x,y) coordinate plane, a high school geometry topic (CCSSM.G.GPE.A). A successful student will make sense of the real-world problem and persevere in solving it, modeling a real-world construct (signal range) in the standard (x,y) coordinate plane (MP1 and MP4). The Justification and Explanation part of the task relates to MP3.

For the Early High School test, this task would be a part of the Grade Level Progress reporting category. This task is part of the Justification and Explanation reporting category and requires JE level 2 reasoning. It is a DOK level 3 task.

Correct Response

A student could receive full credit for the following sample response:

The center of the circle is the location of the point that represents the new cell phone tower. The *y*-coordinate of the center of the circle is 4 because the new cell phone tower lies on the same horizontal line as (5,4). To find the *x*-coordinate of the center, add 10, the distance the existing cell phone tower can broadcast, to 5, the location of the original cell phone tower, and then add 15 because this is the distance the new cell phone tower can broadcast, and then subtract 1 because of the overlap of the ranges. So the center is at (5 + 10 + 15 - 1, 4) or (29,4). The radius of the circle is the range of the new cell phone tower, which is 15. The resulting equation is $(x - 29)^2 + (y - 4)^2 = 15^2$.

This response is built around the JE statements "State one or more steps in a procedure" and "Explain why a step in a procedure is necessary." For example, a step in the procedure is "add 10" and the reason to add 10 is that this is "the distance the existing cell phone tower can broadcast." This response also demonstrates "State a property or classification of an object," State a relationship between two or more objects," and "Use two or more Specific Statements to draw a Conclusion and provide Specific Support for at least one of the Statements." This type of response provides direct evidence for JE level 1 and JE level 2, which is all that is required to successfully address the task. Some students may use an argument that involves JE level 3 reasoning, and for the purposes of scoring, this would be counted as equal to the response given above because JE level 3 reasoning is not required.

This response successfully accomplishes what the task requires: determining an equation of a circle whose interior represents the signal range of the new tower, and explaining the procedure used to identify the equation. This response shows understanding of the given information, uses logically consistent reasons to support mathematical claims, and expresses the argument in a clear, organized manner.

Reporting category	Grade	Low statement (scored below ACT Readiness Range)	High statement (scored at or above ACT Readiness Range)
Grade Level Progress	EHS	Ask questions when you don't understand the lesson. Start a group with classmates to study for quizzes and tests.	Try completing the "challenge" questions in your textbook for your current math work.
Justification & Explanation	9	Understand mathematical derivations and justification in your textbooks. Know the concepts behind math terms and why procedures work.	Read a proof you found in a college mathematics textbook or on the internet.
Justification & Explanation	10	Understand not just what to do but why that works. Create reasoning based on different cases, being sure to cover all of the cases and then summarizing the result.	Read a proof you found in a college mathematics textbook or on the internet.