Fourth Grade Quarter 3

Module 5: Fraction Equivalence, Ordering, and Operations – Part 2, Topics D-H

Approximately 25 days – Begin around January 4th

In this 45-day module, students build on their Grade 3 work with unit fractions as they explore fraction equivalence and extend this understanding to mixed numbers. This leads to the comparison of fractions and mixed numbers and the representation of both in a variety of models. Benchmark fractions play an important part in students' ability to generalize and reason about relative fraction and mixed number sizes. Students then have the opportunity to apply what they know to be true for whole number operations to the new concepts of fraction and mixed number operations.

Major Clusters:	**4.NF.A – Extend understanding of fraction equivalence and ordering.** **4.NF.B – Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**
Supporting Clusters:	4.OA.C – Generate and analyze patterns. 4.MD.B – Represent and interpret data.
Vocabulary	Benchmark, common denominator, denominator, line plot, mixed number, numerator

Domain	Cluster	Standard	Arizona's College and Career Ready Standards	Explanations & Examples	Notes & Resources
4.OA	B	5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* (Students work with multiplication and apply it to area.) **(Q1, Q3)**	Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations. Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features. **Example**: Pattern | Rule | Feature(s)	**Engage NY** M5 Lesson 41 **enVision** Topic 1,2

<table>
<tr><th>Domain</th><th>Cluster</th><th>Standard</th><th>Arizona's College and Career Ready Standards</th><th>Explanations & Examples</th><th>Notes & Resources</th></tr>
<tr><td></td><td></td><td></td><td>4.MP.2. Reason abstractly and quantitatively.
4.MP.4. Model with mathematics.
4.MP.5. Use appropriate tools strategically.
4.MP.7. Look for and make use of structure.</td><td>
<table>
<tr><td>3, 8, 13, 18, 23, 28, …</td><td>Start with 3, add 5</td><td>The numbers alternately end with a 3 or 8</td></tr>
<tr><td>5, 10, 15, 20 …</td><td>Start with 5, add 5</td><td>The numbers are multiples of 5 and end with either 0 or 5. The numbers that end with 5 are products of 5 and an odd number.
The numbers that end in 0 are products of 5 and an even number.</td></tr>
</table>
After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.
Example:
• Rule: Starting at 1, create a pattern that starts at 1 and multiplies each number by 3. Stop when you have 6 numbers.
Students write 1, 3, 9, 27, 81, 243. Students notice that all the numbers are odd and that the sums of the digits of the 2 digit numbers are each 9. Some students might investigate this beyond 6 numbers. Another feature to investigate is the patterns in the differences of the numbers (3 - 1 = 2, 9 - 3 = 6, 27 - 9 = 18, etc.)</td><td></td></tr>
<tr><td>**4.NF**</td><td>**A**</td><td>**1**</td><td>Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.MP.2. Reason abstractly and quantitatively.
4.MP.4. Model with mathematics.</td><td>This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100).

Students can use visual models or applets to generate equivalent fractions.

All the models show 1/2. The second model shows 2/4 but also shows that 1/2 and 2/4 are equivalent fractions because their areas are equivalent. When a horizontal line is drawn through the center of the model, the number of equal parts doubles and size of the parts is halved. Students will begin to notice connections between the models and fractions in the way both the parts and wholes are counted and begin to</td><td>**Engage NY**
M5 Lessons 7-11, 16-28

enVision
Topic 11</td></tr>
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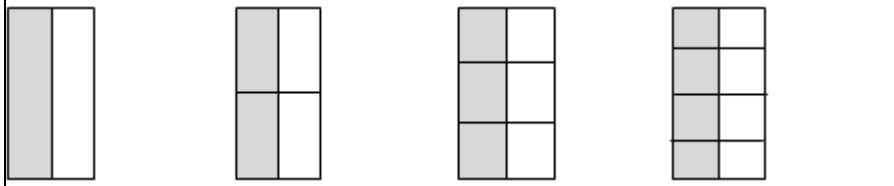
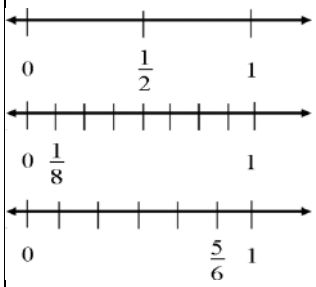
Domain	Cluster	Standard	Arizona's College and Career Ready Standards	Explanations & Examples	Notes & Resources
			4.MP.7. Look for and make use of structure. 4.MP.8. Look for and express regularity in repeated reasoning.	generate a rule for writing equivalent fractions. 1/2 x 2/2 = 2/4.  $\frac{1}{2}$ $\frac{2}{4} = \frac{2 \times 1}{2 \times 2}$ $\frac{3}{6} = \frac{3 \times 1}{3 \times 2}$ $\frac{4}{8} = \frac{4 \times 1}{4 \times 2}$ Technology Connection: http://illuminations.nctm.org/activitydetail.aspx?id=80	
4.NF	**A**	**2**	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. 4.MP.2. Reason abstractly and quantitatively. 4.MP.4. Model with mathematics. 4.MP.5. Use appropriate tools strategically. 4.MP.7. Look for and make use of structure.	Benchmark fractions include common fractions between 0 and 1 such as halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and hundredths. Fractions can be compared using benchmarks, common denominators, or common numerators. Symbols used to describe comparisons include <, >, =. • Fractions may be compared using $\frac{1}{2}$ as a benchmark.  Possible student thinking by using benchmarks: $\frac{1}{8}$ is smaller than $\frac{1}{2}$ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces.	**Engage NY** M5 Lessons 12-15, 22-28 **enVision** Topic 11

Domain	Cluster	Standard	Arizona's College and Career Ready Standards	Explanations & Examples	Notes & Resources
				Possible student thinking by creating common denominators: 5/6 ○ — > 1/2 because 3/6 = 1/2 and 5/6 > 3/6 Fractions with common denominators may be compared using the numerators as a guide. 2/6 ○ — < 3/6 < 5/6 Fractions with common numerators may be compared and ordered using the denominators as a guide. 3/10 ○ — < 3/8 < 3/4	
4.NF	**B**	**3 abc d**	Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8=1/8+1/8+1/8 ; 3/8=1/8+2/8; 2 1/8=1 + 1+1/8=8/8+8/8 +1/8.* *c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between*	A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as 2/3, they should be able to decompose the non-unit fraction into a combination of several unit fractions. **Examples**: Fraction Example 1: • 2/3 = 1/3 + 1/3 Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding. Fraction Example 2: • 1 ¼ - ¾ = □ 4/4 + ¼ = 5/4 5/4 – ¾ = 2/4 or ½ Word Problem Example 1: Mary and Lacey decide to share a pizza. Mary ate 3/6 and Lacey ate 2/6 of the pizza. How much of the pizza did the girls eat together?	**Engage NY** M5 Lessons 1-11, 16-28 **enVision** Topic 12

Domain	Cluster	Standard	Arizona's College and Career Ready Standards	Explanations & Examples	Notes & Resources
			addition and subtraction. *d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.* 4.MP.1. Make sense of problems and persevere in solving them. 4.MP.2. Reason abstractly and quantitatively. 4.MP.4. Model with mathematics. 4.MP.5. Use appropriate tools strategically. 4.MP.6. Attend to precision. 4.MP.7. Look for and make use of structure. 4.MP.8. Look for and express regularity in repeated reasoning.	Solution: The amount of pizza Mary ate can be thought of a 3/6 or 1/6 and 1/6 and 1/6. The amount of pizza Lacey ate can be thought of a 1/6 and 1/6. The total amount of pizza they ate is 1/6 + 1/6 + 1/6 + 1/6 + 1/6 or 5/6 of the whole pizza. A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions. Word Problem Example 2: • Susan and Maria need 8 3/8 feet of ribbon to package gift baskets. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not. The student thinks: I can add the ribbon Susan has to the ribbon Maria has to find out how much ribbon they have altogether. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. I can write this as 3 1/8 + 5 3/8. I know they have 8 feet of ribbon by adding the 3 and 5. They also have 1/8 and 3/8 which makes a total of 4/8 more. Altogether they have 8 4/8 feet of ribbon. 8 4/8 is larger than 8 3/8 so they will have enough ribbon to complete the project. They will even have a little extra ribbon left, 1/8 foot. Additional Example: • Trevor has 4 1/8 pizzas left over from his soccer party. After giving some pizza to his friend, he has 2 4/8 of a pizza left. How much pizza did Trevor give to his friend? Solution: Trevor had 4 1/8 pizzas to start. This is 33/8 of a pizza. The x's show the pizza he has left which is 2 4/8 pizzas or 20/8 pizzas. The shaded rectangles without the x's are the pizza he gave to his friend which is 13/8 or 1 5/8 pizzas.	

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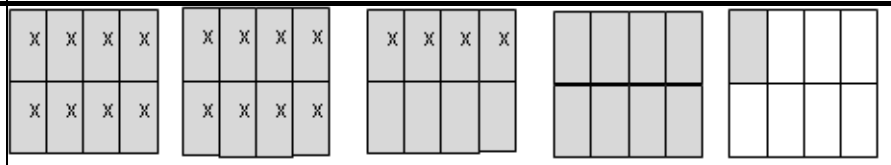
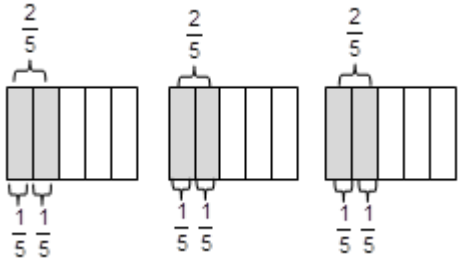
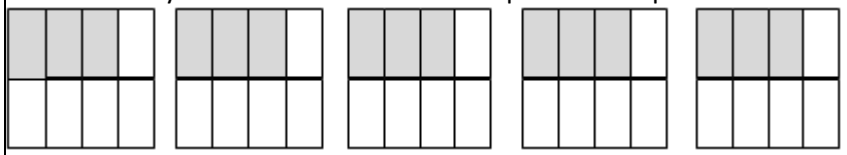
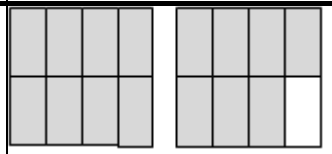
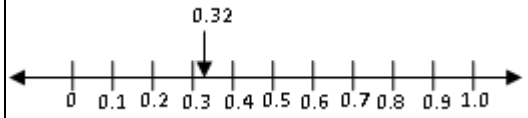
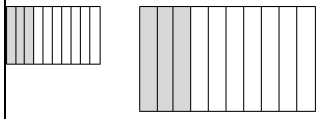
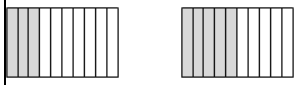
					
4.NF	**B**	**4 abc**	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.* b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)* c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns. **Examples**: • 3 x (2/5) = 6 x (1/5) = 6/5  • If each person at a party eats $3/8$ of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie? A student may build a fraction model to represent this problem.  3/8 3/8 3/8 3/8 3/8	**Engage NY** M5 Lessons 1-6, 22-28, 35-40 **enVision** Topic 13

Domain	Cluster	Standard	Arizona's College and Career Ready Standards	Explanations & Examples	Notes & Resources
			4.MP.1. Make sense of problems and persevere in solving them. *4.MP.2.* Reason abstractly and quantitatively. *4.MP.4.* Model with mathematics. *4.MP.5.* Use appropriate tools strategically. *4.MP.6.* Attend to precision. *4.MP.7.* Look for and make use of structure. *4.MP.8.* Look for and express regularity in repeated reasoning.	 3/8 + 3/8 + 3/8 + 3/8 + 3/8 = 15/8 = 1 7/8	
4.MD	B	4	Make a line plot to display a data set of measurements in fractions of a unit ($1/2$, $1/4$, $1/8$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.* 4.MP.2. Reason abstractly and quantitatively. 4.MP.4. Model with mathematics. 4.MP.5. Use appropriate tools strategically. 4.MP.6. Attend to precision. 4.MP.7. Look for and make use of structure.	• Ten students in Room 31 measured their pencils at the end of the day. They recorded their results on the line plot below. X X X X X X X X X X 3 ½" 4" 4 ¼" 5 1/8" 5 1/2" Possible questions: ○ What is the difference in length from the longest to the shortest pencil? ○ If you were to line up all the pencils, what would the total length be? If the 5 1/8" pencils are placed end to end, what would be their total length?	**Engage NY** M5 Lessons 22-28, 35-40 **enVision** Not Covered

Domain	Cluster	Standard	Arizona's College and Career Ready Standards	Explanations & Examples	Notes & Resources
Module 6: Decimal Fractions **Approximately 20 days – Begin around February 8th** This 20-day module gives students their first opportunity to explore decimal numbers via their relationship to decimal fractions, expressing a given quantity in both fraction and decimal forms. Utilizing the understanding of fractions developed throughout Unit 5, students apply the same reasoning to decimal numbers, building a solid foundation for Grade 5 work with decimal operations.					
Major Clusters:			**4.NF.C – Understand decimal notation for fractions, and compare decimal fractions.**		
Supporting Clusters:			4.MD.A – Solve problems involving measurement and conversion of measurements from a larger unit to a small unit.		
Vocabulary			Decimal number, decimal expanded form, decimal fraction, decimal point, fraction expanded form, hundredth, tenth		
Domain	**Cluster**	**Standard**	**Arizona's College and Career Ready Standards**	**Explanations & Examples**	**Notes & Resources**
4.NF	**C**	**5**	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)* 4.MP.2. Reason abstractly and quantitatively. 4.MP.4. Model with mathematics. 4.MP.5. Use appropriate tools strategically.	Students can use base ten blocks, graph paper, and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100. Students may represent 3/10 with 3 longs and may also write the fraction as 30/100 with the whole in this case being the flat (the flat represents one hundred units with each unit equal to one hundredth). Students begin to make connections to the place value chart as shown in 4.NF.6. This work in fourth grade lays the foundation for performing operations with decimal numbers in fifth grade.	**Engage NY** M6 Lessons 4-8, 12-16 **enVision** Topic 13

Domain	Cluster	Standard	Arizona's College and Career Ready Standards	Explanations & Examples	Notes & Resources
			4.MP.7. Look for and make use of structure.		
4.NF	**C**	**6**	Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* *4.MP.2.* Reason abstractly and quantitatively. 4.MP.4. Model with mathematics. *4.MP.5.* Use appropriate tools strategically. *4.MP.7.* Look for and make use of structure.	Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say 32/100 as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below. Hundreds | Tens | Ones | • | Tenths | Hundredths | | | • | 3 | 2 Students use the representations explored in 4.NF.5 to understand 32/100 can be expanded to 3/10 and 2/100. Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10). It is closer to 30/100 so it would be placed on the number line near that value. 	**Engage NY** M6 Lessons 1-8, 12-16 **enVision** Topic 13
4.NF	**C**	**7**	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. *4.MP.2.* Reason abstractly and quantitatively. *4.MP.4.* Model with mathematics. *4.MP.5.* Use appropriate tools strategically.	Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. • Each of the models below shows 3/10 but the whole on the right is much bigger than the whole on the left. They are both 3/10 but the model on the right is a much larger quantity than the model on the left.  When the wholes are the same, the decimals or fractions can be	**Engage NY** M6 Lessons 4-11 **enVision** Topic 13

Domain	Cluster	Standard	Arizona's College and Career Ready Standards	Explanations & Examples	Notes & Resources
			4.MP.7. Look for and make use of structure.	compared. **Example**: • Draw a model to show that 0.3 < 0.5. (Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths. 	
4.MD	A	2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and **money**, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. 4.MP.1. Make sense of problems and persevere in solving them. 4.MP.2. Reason abstractly and quantitatively. 4.MP.4. Model with mathematics. 4.MP.5. Use appropriate tools strategically. 4.MP.6. Attend to precision.	• <u>Addition:</u> Miguel had 1 dollar bill, 2 dimes, and 7 pennies. John had 2 dollar bills, 3 quarters, and 9 pennies. How much money did the two boys have in all? • <u>Subtraction:</u> A pound of apples costs $1.20. Rachel bought a pound and a half of apples. If she gave the clerk a $5.00 bill, how much change will she get back? • <u>Multiplication:</u> A pen costs $2.29. A calculator costs 3 times as much as a pen. How much do a pen and a calculator cost together? • Number line diagrams that feature a measurement scale can represent measurement quantities. Examples include: ruler, diagram marking off distance along a road with cities at various points, a timetable showing hours throughout the day, or a volume measure on the side of a container.	**Engage NY** M6 Lessons 9-12, 15-16 Also addressed in Module 7 **enVision** Topic 13,14,15