

# 4th Grade Mathematics

Fractions- Unit 3 Curriculum Map January 6th – March 7th



ORANGE PUBLIC SCHOOLS  
OFFICE OF CURRICULUM AND INSTRUCTION  
OFFICE OF MATHEMATICS

# Table of Contents

I.	Common Core Fluency	p. 3
II.	Unit Overview	p. 4
III.	Important Dates and Calendar	p. 5
IV.	Pacing Guide	p. 6
V.	Structure of the Modules	p. 7
VI.	Common core Standards – Review Content	p. 8-11
VII.	Teaching Multiple Representation – Review Content	p. 12
VIII.	4.OA.1 -2 Module	p. 13-39
IX.	4.OA.3 Module	p. 40-72
X.	NBT Review Content	p. 73-78
XI.	New Content CCSS Standards	p. 79
XII.	Essential Concepts and Questions	p. 80
XIII.	Math Practices	p. 81
XV.	Visual Vocabulary	p. 82-84
XVI.	Student Misconceptions	p. 85
XVII.	Teaching to Multiple Representations - Fractions	p. 86
XVIII.	4.NF.1-2 Module	p. 87-113
XIX.	4.NF 3-4	p. 114-119
XX.	Associated Illustrative Math Tasks	p. 120-125
XXI.	Assessment Checks	p. 126-131
XXII.	Extensions and Sources	p. 132-135

# Common Core Fluency

REVIEW OF GRADE 3 FLUENCIES	
<b><u>3.OA.7</u></b>	Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.
<b><u>3.NBT.2</u></b>	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

EXPECTED GRADE 4 FLUENCIES	
<b><u>4.NBT.4</u></b>	Fluently add and subtract multi-digit whole numbers using the standard algorithm. Add and Subtract within 1,000,000

# Unit Overview

## In this unit, students will ....

By the conclusion of this unit, students should be able to demonstrate the following review competencies:

- Estimate and find the products of a 2-digit number multiplied by a 2-digit number.
- Represent multiplication and division using a rectangular area model.
- Understand that multiplication may be used in problem contexts involving equal groups, rectangular arrays/area models, or rate.
- Multiply up to a 4-digit number by a 1-digit number using strategies.
- Solve division problems using strategies.
- Divide whole-numbers quotients and remainders with up to four-digit dividends and remainders with up to four-digit dividends and one-digit divisors.

By the conclusion of this unit, students should be able to demonstrate the following new competencies:

- Build on informal understanding of sharing and proportionality to develop initial fraction concept
- Understand that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward
- Understand why procedures for computations with fractions make sense
- Understand representations of simple equivalent fractions
- Compare fractions with different numerators and different denominators
- Identify visual and written representations of fractions
- Understand representations of simple equivalent fractions
- Understand the concept of mixed numbers with common denominators to 12
- Add and subtract fractions with common denominators
- Add and subtract mixed numbers with common denominators
- Convert mixed numbers to improper fractions and improper fractions to mixed fractions
- Understand a fraction  $a/b$  as a multiple of  $1/b$ . (for example: model the product of  $3/4$  as  $3 \times 1/4$  ).
- Understand a multiple of  $a/b$  as a multiple of  $1/b$ , and use this understanding to multiply a fraction by a whole number.
- Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.
- Multiply a whole number by a fraction

# Important Dates and Calendar

Week of ...	Monday	Tuesday	Wednesday	Thursday	Friday
1/6-1/10	REVIEW MODULES				
1/13-1/17					
1/21-1/24	No School				Checkpoint
1/27-1/31	UNIT 3 NEW CONTENT				1/2 Day
2/3-2/7					
2/10-2/14					
2/17-2/21	NO SCHOOL				
2/24-2/28	UNIT 3 NEW CONTENT				
3/3-3/7	Assessment Week				

## IMPORTANT DATES

Monday, Jan 20th	MLK Day
Friday, Jan 24th	Checkpoint 2 Grades 6-7
Friday, Jan 31st	1/2 Day
Week of Feb 17th	VACATION
Friday, March 14th	Data Due
Friday, March 21st	Data Returned to Principals

# Pacing Guide

## Review Content

Activity	Common Core Standards/SLO	Teaching Notes	Estimated Time
Common Core Review Modules	4.OA.1, 4.OA.2, 4.OA.3		10 days
The algorithm: Making Sense	4.NBT.4, 4.NBT.5	p. 73-75	2 days
Analyzing Multiplication & Division Expressions	4.NBT.5/6	p. 76-78	1 day
Assessment Check 1	Review Content	p. 127-129	1 day
Checkpoint #2 (Friday, January 24 <sup>th</sup> )	SGO Standards Fluency Only		1 day

## New Content

<b>Fraction Module</b>	4.NF.1 – 2	p. 88	10 days
<b>Tiles</b>	4.NF.3	p. 117	2 days
<b>Fraction Student Olympics</b>	4.NF.3	p. 118	2 days
<b>Illustrative Math Tasks</b>  Comparing Sums of Unit Fractions Making 22 Seventeenths in Different Ways Writing a Mixed Numbers as an Equivalent Fraction	4.NF.3a 4.NF.3b  4.NF.3b	p. 124 p. 125  p. 125	2 days
<b>Illustrative Math Tasks</b>  Peaches Plastic Building Blocks Cynthia's Perfect Punch	4.NF.3b	p. 126	2 days
Assessment Check 2	Fractions	p. 130-132	½ day
Selected Review	<i>Based on Assessment Checks</i>		2 days
Unit 2 Assessment	4.NF.1, 4.N.2, 4.NF.3 a-d		2 days

# Structure of the Modules

The Modules embody 3 integrated frameworks that promote the development of conceptual and problems solving skills and computational fluency. The **conceptual framework** of the Modules builds from the concrete to the pictorial to the abstract (and the constant blending of each) to help students develop a deeper understanding of mathematics. The Modules also reference a **multiple representations framework** that encourages teachers to present content in multiple modalities to support flexible thinking. These frameworks go beyond concrete representation (i.e. manipulatives) to promote the realistic representation of concepts addressed in multiple settings. Lastly, the Modules embody a **'gradual release' framework** that encourages teachers to progress from whole group to collaborative and finally to an independent practice format.

## OVERVIEW

Each module begins with an overview. The overview provides the standards, goals, prerequisites,

## INTRODUCTORY TASKS

The Introductory Tasks serve as the starting point for the referenced standard and are typically either diagnostic, prerequisite or anticipatory in nature.

## GUIDED PRACTICE

Serves for additional teacher guided instruction for students who need the additional help. The tasks can be modeled with students.

## COLLABORATIVE PRACTICE

Serve as small group, or partnered work. The work should promote student discourse, which allows students to make sense of problems and persevere in solving them (MP.1). Through teacher-facilitated, whole group discussion, students will have the opportunity to critique the reasoning of others (MP.3).

## JOURNAL QUESTIONS

Provide the opportunity to individual, independent reflection and practice. This independent format encourages students to construct viable arguments (MP.3) and to reason abstractly/quantitatively (MP.2).

## HOMEWORK

Can be used as additional in-class practice, Independent Practice, etc. This work should be reviewed and discussed. Procedural fluencies are reinforced within this section.

## GOLDEN PROBLEM

The Golden Problem is a performance task that reflects an amalgamation of the skills addressed within the Module. The Golden Problem assesses the student's ability to apply the skills learned in a new and non-routine context. More than one-step; problems usually require intermediate values before arriving at a solution (contextual applications). In the US, we see one step problems that require either recall or routine application of an algorithm.

# Common Core Standards – Review Content

<b>REVIEW CONTENT</b>	
4.NBT.4	<p>Fluently add and subtract multi-digit whole numbers using the standard algorithm.</p> <p>Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract.</p> <p>When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with the algorithm. Students should be able to explain why the algorithm works.</p> $\begin{array}{r} 3892 \\ + 1567 \\ \hline \end{array}$ <p>Student explanation for this problem:</p> <ol style="list-style-type: none"> <li>Two ones plus seven ones is nine ones.</li> <li>Nine tens plus six tens is 15 tens.</li> <li>I am going to write down five tens and think of the 10 tens as one more hundred. (notates with a 1 above the hundreds column)</li> <li>Eight hundreds plus five hundreds plus the extra hundred from adding the tens is 14 hundreds.</li> <li>I am going to write the four hundreds and think of the 10 hundreds as one more 1000. (notates with a 1 above the thousands column)</li> <li>Three thousands plus one thousand plus the extra thousand from the hundreds is five thousand.</li> </ol> $\begin{array}{r} 3546 \\ - 928 \\ \hline \end{array}$ <p>Student explanation for this problem:</p> <ol style="list-style-type: none"> <li>There are not enough ones to take 8 ones from 6 ones so I have to use one ten as 10 ones. Now I have 3 tens and 16 ones. (Marks through the 4 and notates with a 3 above the 4 and writes a 1 above the ones column to be represented as 16 ones.)</li> <li>Sixteen ones minus 8 ones is 8 ones. (Writes an 8 in the ones column of answer.)</li> <li>Three tens minus 2 tens is one ten. (Writes a 1 in the tens column of answer.)</li> <li>There are not enough hundreds to take 9 hundreds from 5 hundreds so I have to use one thousand as 10 hundreds. (Marks through the 3 and notates with a 2 above it. (Writes down a 1 above the hundreds column.) Now I have 2 thousand and 15 hundreds.</li> <li>Fifteen hundreds minus 9 hundreds is 6 hundreds. (Writes a 6 in the hundreds column of the answer).</li> <li>I have 2 thousands left since I did not have to take away any thousands. (Writes 2 in the thousands place of answer.)</li> </ol> <p>Note: Students should know that it is mathematically possible to subtract a larger number from a smaller number but that their work with whole numbers does not allow this as the difference would result in a negative number.</p>
4.NBT.5	<p>Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>



Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication is an expectation in the 5<sup>th</sup> grade.

Students may use digital tools to express their ideas.

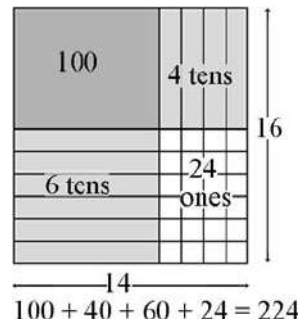
Use of place value and the distributive property are applied in the scaffold examples below.

- To illustrate  $154 \times 6$  students use base 10 blocks or use drawings to show 154 six times.

Seeing 154 six times will lead them to understand the distributive property,  $154 \times 6 = (100 + 50 + 4) \times 6 = (100 \times 6) + (50 \times 6) + (4 \times 6) = 600 + 300 + 24 = 924$ .

- The area model shows partial products.

$$14 \times 16 = 224$$



Using the area model, students first verbalize their understanding:

- $10 \times 10$  is 100
- $4 \times 10$  is 40
- $10 \times 6$  is 60, and
- $4 \times 6$  is 24.

4.NBT.6

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.

Examples:

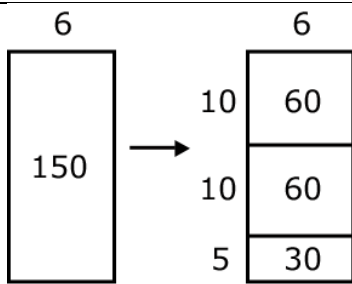
A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

- Using Base 10 Blocks:** Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.
- Using Place Value:**  $260 \div 4 = (200 \div 4) + (60 \div 4)$
- Using Multiplication:**  $4 \times 50 = 200$ ,  $4 \times 10 = 40$ ,  $4 \times 5 = 20$ ;  $50 + 10 + 5 = 65$ ; so  $260 \div 4 = 65$

Using an Open Array or Area Model

After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5<sup>th</sup> grade.

Example:  $150 \div 6$



Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150.

1. Students think, 6 times what number is a number close to 150? They recognize that  $6 \times 10$  is 60 so they record 10 as a factor and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.

2. Recognizing that there is another 60 in what is left they repeat the process above. They express that they have used 120 of the 150 so they have 30 left.

3. Knowing that  $6 \times 5$  is 30. They write 30 in the bottom area of the rectangle and record 5 as a factor.

4. Students express their calculations in various ways:

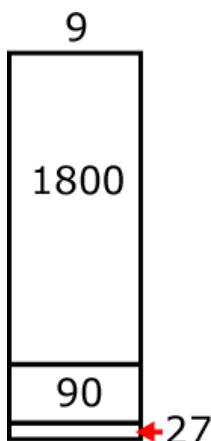
a.  $150 \div 6 = 10 + 10 + 5 = 25$

$$\begin{array}{r} - 60 \ (6 \times 10) \\ \hline 90 \\ - 60 \ (6 \times 10) \\ \hline 30 \\ - 30 \ (6 \times 5) \\ \hline 0 \end{array}$$

b.  $150 \div 6 = (60 \div 6) + (60 \div 6) + (30 \div 6) = 10 + 10 + 5 = 25$

Example 2:

$1917 \div 9$



A student's description of his or her thinking may be:

I need to find out how many 9s are in 1917. I know that  $200 \times 9$  is 1800. So if I use 1800 of the 1917, I have 117 left. I know that  $9 \times 10$  is 90. So if I have 10 more 9s, I will have 27 left. I can make 3 more 9s. I have 200 nines, 10 nines and 3 nines. So I made 213 nines.  
 $1917 \div 9 = 213$ .

4.OA.1

Interpret a multiplication equation as a comparison, e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

A *multiplicative comparison* is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., “*a* is *n* times as much as *b*”). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times.

4.OA.2

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive

comparison.1

When distinguishing multiplicative comparison from additive comparison, students should note that:

- additive comparisons focus on the difference between two quantities (e.g., Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have?). A simple way to remember this is, “How many more?”
- multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other (e.g., Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run?). A simple way to remember this is “How many times as much?” or “How many times as many?”

4.OA.3

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Students need many opportunities solving multistep story problems using all four operations.

An interactive whiteboard, , drawings, words, numbers, and/or objects may be used to help solve story problems.

Example:

Chris bought clothes for school. She bought 3 shirts for \$12 each and a skirt for \$15. How much money did Chris spend on her new school clothes?

$$3 \times \$12 + \$15 = a$$

In division problems, the remainder is the whole number left over when as large a multiple of the divisor as possible has been subtracted.

Example:

Kim is making candy bags. There will be 5 pieces of candy in each bag. She had 53 pieces of candy. She ate 14 pieces of candy. How many candy bags can Kim make now?  
(7 bags with 4 leftover)

Kim has 28 cookies. She wants to share them equally between herself and 3 friends. How many cookies will each person get?  
(7 cookies each)  $28 \div 4 = a$

There are 29 students in one class and 28 students in another class going on a field trip. Each car can hold 5 students. How many cars are needed to get all the students to the field trip?

(12 cars, one possible explanation is 11 cars holding 5 students and the 12<sup>th</sup> holding the remaining 2 students)  $29 + 28 = 11 \times 5 + 2$

Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to:

- front-end estimation with adjusting (using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts),
- clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate),
- rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values),
- using friendly or compatible numbers such as factors (students seek to fit numbers together - e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000),
- using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate).

# Teaching to Multiple Representations – Review Content

CONCRETE REPRESENTATIONS										
<ul style="list-style-type: none"> <li>Number Lines</li> </ul>	$243 - 87 =$ <p>A number line starting at 87 and ending at 243. Tick marks are at 87, 90, 100, 200, and 243. Dashed orange arrows show jumps: +3 from 87 to 90, +10 from 90 to 100, +100 from 100 to 200, and +43 from 200 to 243.</p>									
<ul style="list-style-type: none"> <li>color coin counters to represent whole numbers</li> <li>Number Lines</li> <li>Thermometers, rulers and other equally partitioned tools</li> </ul>	<p>Three coin counters: two red and one yellow. Below them is a number line from 0 to 6 with tick marks at every integer.</p>									
PICTORIAL REPRESENTATIONS										
<ul style="list-style-type: none"> <li>Bar Models</li> <li>Visual Representation</li> </ul>	<p>A bar model divided into two sections. The first section is labeled 120 and the second is labeled 190. A question mark is at the end of the bar. Below the bar model are four base ten blocks, each representing 10. To the right of the blocks is the number 40. Below the blocks are the equations <math>4 \times 10 =</math> and <math>4 \times 9 =</math>.</p>									
ABSTRACT REPRESENTATIONS										
<ul style="list-style-type: none"> <li>Applying the Operations</li> <li>Applying Properties of Numbers</li> <li>Applying the standard algorithms for addition, subtraction, multiplication, and division and strategies based on place value such as:</li> </ul>	<table border="1"> <thead> <tr> <th></th> <th>Possible Strategy #1</th> <th>Possible Strategy #2</th> </tr> </thead> <tbody> <tr> <td><math>29 + 8</math></td> <td>29 can become 30 and take 1 from 8 reducing it to 7.</td> <td>9 and 8 becomes 17 17 plus 20</td> </tr> <tr> <td><math>54 + 86</math></td> <td><math>50 + 80 + 10 =</math></td> <td>Add 6 to 54 to get 60. Then <math>60 + 80 = 140</math></td> </tr> </tbody> </table>		Possible Strategy #1	Possible Strategy #2	$29 + 8$	29 can become 30 and take 1 from 8 reducing it to 7.	9 and 8 becomes 17 17 plus 20	$54 + 86$	$50 + 80 + 10 =$	Add 6 to 54 to get 60. Then $60 + 80 = 140$
	Possible Strategy #1	Possible Strategy #2								
$29 + 8$	29 can become 30 and take 1 from 8 reducing it to 7.	9 and 8 becomes 17 17 plus 20								
$54 + 86$	$50 + 80 + 10 =$	Add 6 to 54 to get 60. Then $60 + 80 = 140$								

## Operations & Algebraic Thinking 4.OA.1-2

Use the four operations with whole numbers to solve word problems.

### Goal:

To interpret multiplication equations as comparisons and solve multiplication and division word problems involving multiplicative comparisons.

### Essential Questions:

- How does making effective comparisons help us in problem solving?
- Why is there an opposite for every mathematical operation?
- Is there one “best” way to approach problem solving?

### Prerequisites:

- Place Value concepts
- Solving problems using repeated addition
- Fluent with multiplication facts 0-12
- Skip counting to compute

### Embedded Mathematical Practices

**MP.1** Make sense of problems and persevere in solving them

**MP.2** Reason abstractly and quantitatively

**MP.3** Construct viable arguments and critique the reasoning of others

**MP.4** Model with mathematics

**MP.5** Use appropriate tools strategically

**MP.6** Attend to precision

**MP.7** Look for and make use of structure

**MP.8** Look for and express regularity in repeated reasoning

### Lesson 5 - Golden Problem

**4.OA.1-2** Using the four operations to solve word problems.

### Lesson 4 - Multiplier Unknown

**4.OA.2** Multiply or divide to solve word problems involving multiplicative Comparison.

### Lesson 3 - Set Size Unknown

**4.OA.2** Multiply or divide to solve word problems involving multiplicative Comparison.

### Lesson 2 - Product Unknown

**4.OA.2** Multiply or divide to solve word problems involving multiplicative Comparison.

### Lesson 1

**4.OA.1** Interpret a multiplication equation as a comparison

### Lesson Structure:

Assessment Task  
Prerequisite Skills  
Focus Questions  
Guided Practice  
Homework  
Journal Question

# Content Overview: Multiplicative Comparison

## Solving Multiplicative Comparison Word Problems

### Multiplication as Comparing

In multiplicative comparison problems, there are two different sets being compared. The first set contains a certain number of items. The second set contains multiple copies of the first set. Any two factors and their product can be read as a comparison. Let's look at a basic multiplication equation:  $4 \times 2 = 8$ .

8 is the same as 4 sets of 2 or 2 sets of 4.

8 is 4 times as many as 2, and 2 times as many as 4

### Different Types of Problem

There are three kinds of multiplicative comparison word problems (see list below). Knowing which kind of problem you have in front of you will help you know how to solve it.

- Product Unknown Comparisons
- Set Size Unknown Comparisons
- Multiplier Unknown Comparisons

## Multiplicative Comparison Product Unknown

In some multiplicative comparison word problems, you are given the number of items in one set, and you are given the "multiplier" amount. The multiplier amount tells you how many times bigger (or more) the second set is than the first. "Bigger" can also mean "longer," or "wider," or "taller" in problems involving measurement, or "faster" in problems involving a rate of speed. These problems in which you know both the number in one set, and the multiplier are called "Product Unknown" comparisons, because the total is the part that is unknown. In order to answer the question you are being asked, you need to multiply the number in the set by the multiplier to find the product.

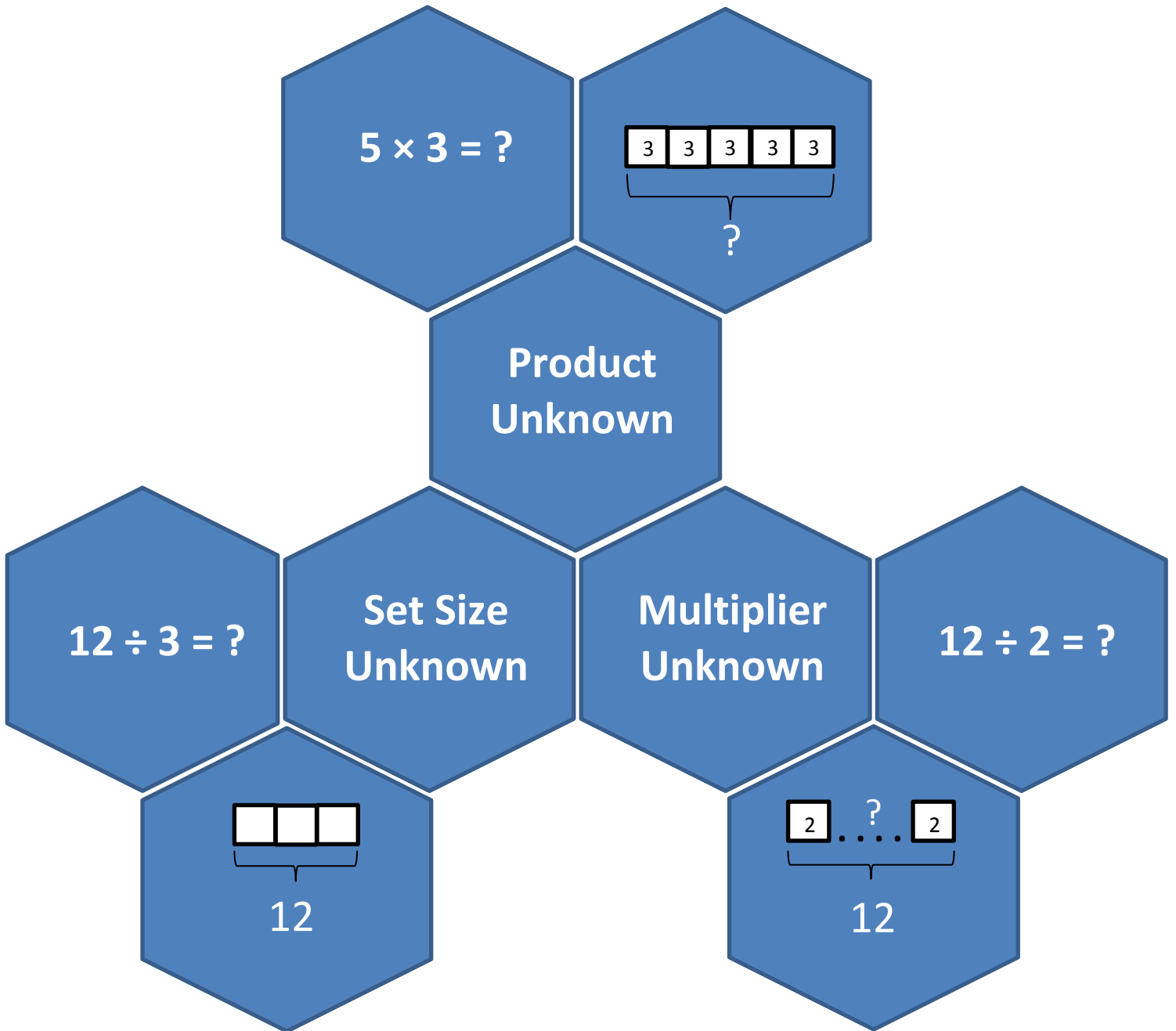
## Multiplicative Comparison Set Size Unknown

In some multiplicative comparison word problems, the part that is unknown is the number of items in one set. You are given the amount of the second set, which is a multiple of the unknown first set, and the "multiplier" amount, which tells you how many times bigger (or more) the second set is than the first. "Bigger" can also mean "longer," or "wider," "older," or "taller" in problems involving measurement, or "faster" in problems involving a rate of speed. These problems in which you know both the number in the second set, and the multiplier are called "Set Size Unknown" comparisons, because the number in one set is the part that is unknown. In order to answer the question you are being asked, you need to use the inverse operation of multiplication: division. This kind of division is "partition" or "sharing" division. Dividing the number in the second set by the multiplier will tell you the number in one set, which is the question you are being asked in this kind of problem.

## Multiplicative Comparison Multiplier Unknown

In some multiplicative comparison word problems, you are given the number of items in one set, and you are given the number of items in the second set, which is a multiple of the first set. The "multiplier" amount is the part that is unknown. The multiplier amount tells you how many times bigger (or more) the second set is than the first. "Bigger" can also be "longer," or "wider," or "older," or "taller" in problems involving measurement, or "faster" in problems involving a rate of speed. These problems in which you know both the number in one set, and the number in the second set are called "Multiplier Unknown" comparisons, because the multiplier is the part that is unknown. In order to answer the question you are being asked, you need to use the inverse (opposite) operation of multiplication: division. This kind of division is called "measurement" division.

# Multiple Representations to Multiplicative Comparisons



# Content Background

## Examples

### Multiplicative Comparison Product Unknown

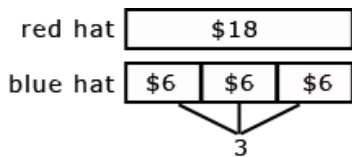
Students need many opportunities to solve contextual problems.

“A blue hat costs \$6. A red hat costs 3 times as much as the blue hat.

How much does the red hat cost?”

In solving this problem, the student should identify \$6 as the quantity that is being multiplied by 3. The student should write the problem using a symbol to represent the unknown.

$$(\$6 \times 3 = \square)$$

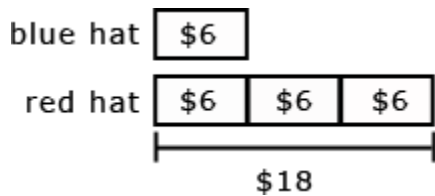


### Multiplicative Comparison Multiplier Unknown

A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?

In solving this problem, the student should identify \$18 as the quantity being divided into shares of \$6.

The student should write the problem using a symbol to represent the unknown. ( $\$18 \div \$6 = \square$ )





## 4.OA.1: Lesson 1

Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations.

*Introductory Task*

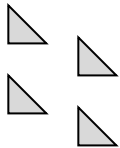
*Guided Practice*

*Collaborative*

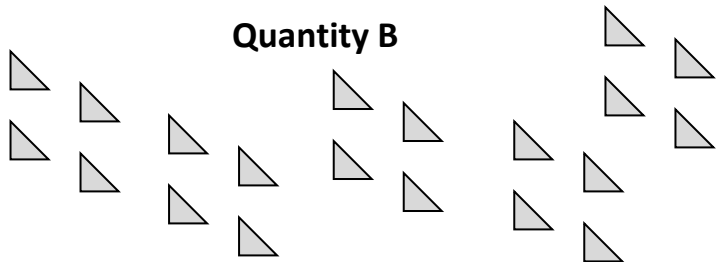
*Homework*

*Assessment*

**Quantity A**



**Quantity B**



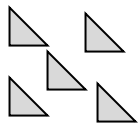
- Compare the two quantities above. Quantity A has a group of four triangles. How many times more triangles does Quantity B have than Quantity A?

**Quantity B has \_\_\_\_\_ times as many triangles as Quantity A.**

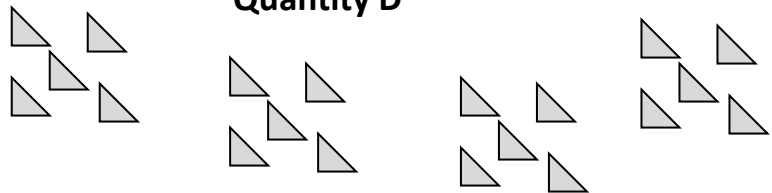
- Write an equation that shows how many triangles are in Quantity B.

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

**Quantity C**



**Quantity D**



- Compare the two quantities above. Quantity C has a group of five triangles. How many times more triangles does Quantity D have than Quantity C?

**Quantity D has \_\_\_\_\_ times as many triangles as Quantity C.**

- Write an equation that shows how many triangles are in Quantity C.

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

- How are your two equations alike? How are they different?

### Focus Questions

**Question 1:** How can we interpret a multiplication equation as a comparison?

### Journal Question

How are  $2 \times 6$  and  $6 \times 2$  different?

## 4.OA.1: Lesson 1

Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations.

*Introductory Task*

*Guided Practice*

*Collaborative*

*Homework*

*Assessment*

For each problem, write an equation and solve:

<p>1) There are 15 bags of crayons with 3 crayons in each bag. How many crayons are there in all?</p> <p><b>Equation:</b> _____</p> <p><b>Sentence answer:</b> _____</p>	<p>6) Rita has 6 packs of 7 pencils. She gives away 7 pencils. How many pencils does she have left?</p> <p><b>Equation:</b> _____</p> <p><b>Sentence answer:</b> _____</p>
<p>2) There are 7 bags of books with 3 books in each bag. How many books are there in all?</p> <p><b>Equation:</b> _____</p> <p><b>Sentence answer:</b> _____</p>	<p>7) Jackie has 8 packs of 8 crayons. She gives away 16 crayons. How many crayons does she have left?</p> <p><b>Equation:</b> _____</p> <p><b>Sentence answer:</b> _____</p>
<p>3) There are 9 packs of stickers with 4 stickers in each pack. How many stickers are there in all?</p> <p><b>Equation:</b> _____</p> <p><b>Sentence answer:</b> _____</p>	<p>8) There are 4 red cards and 3 blue cards on each table. How many cards are on 7 tables?</p> <p><b>Equation:</b> _____</p> <p><b>Sentence answer:</b> _____</p>
<p>4) You will collect 8 bugs each month for 5 months. How many bugs will you collect?</p> <p><b>Equation:</b> _____</p> <p><b>Sentence answer:</b> _____</p>	<p>9) There are 7 plastic chairs and 8 metal chairs in each room. How many chairs are in 9 rooms?</p> <p><b>Equation:</b> _____</p> <p><b>Sentence answer:</b> _____</p>
<p>5) There are 5 piles of coins with 15 coins in each pile. How many coins are there in all?</p> <p><b>Equation:</b> _____</p> <p><b>Sentence answer:</b> _____</p>	<p>10) There are 5 wood picture frames and 2 metal picture frames on each shelf. How many picture frames are on 4 shelves?</p> <p><b>Equation:</b> _____</p> <p><b>Sentence answer:</b> _____</p>

## 4.OA.1: Lesson 1

Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations.

*Introductory Task*

*Guided Practice*

*Collaborative*

*Homework*

*Assessment*

**For each figure, draw straight lines to complete the figure.**

1) There are 12 bags of crayons with 5 crayons in each bag. How many crayons are there in all?

**Equation:** \_\_\_\_\_

**Sentence answer:** \_\_\_\_\_

2) There are 9 bags of books with 4 books in each bag. How many books are there in all?

**Equation:** \_\_\_\_\_

**Sentence answer:** \_\_\_\_\_

3) There are 7 packs of stickers with 3 stickers in each pack. How many stickers are there in all?

**Equation:** \_\_\_\_\_

**Sentence answer:** \_\_\_\_\_

4) You will collect 3 bugs each month for 9 months. How many bugs will you collect?

**Equation:** \_\_\_\_\_

**Sentence answer:** \_\_\_\_\_

5) There are 4 piles of coins with 12 coins in each pile. How many coins are there in all?

**Equation:** \_\_\_\_\_

**Sentence answer:** \_\_\_\_\_

6) Rita has 4 packs of 9 pencils. She gives away 9 pencils. How many pencils does she have left?

**Equation:** \_\_\_\_\_

**Sentence answer:** \_\_\_\_\_

7) Jackie has 6 packs of 7 crayons. She gives away 14 crayons. How many crayons does she have left?

**Equation:** \_\_\_\_\_

**Sentence answer:** \_\_\_\_\_

8) There are 6 red cards and 8 blue cards on each table. How many cards are on 9 tables?

**Equation:** \_\_\_\_\_

**Sentence answer:** \_\_\_\_\_

9) There are 2 plastic chairs and 7 metal chairs in each room. How many chairs are in 8 rooms?

**Equation:** \_\_\_\_\_

**Sentence answer:** \_\_\_\_\_

10) There are 8 wood picture frames and 5 metal picture frames on each shelf. How many picture frames are on 6 shelves?

**Equation:** \_\_\_\_\_

**Sentence answer:** \_\_\_\_\_

## 4.OA.2: Lesson 2 - Product Unknown

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

*Introductory Task*

*Guided Practice*

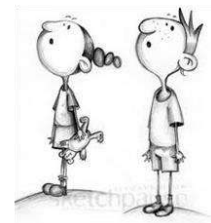
*Collaborative*

*Homework*

*Assessment*

**Solve the following problem:**

Gina and her brother Shawn are saving money to go to an amusement park. This month, Gina saved three times as much as she saved last month and Shawn saved twice as much as he saved last month. Last month, Gina saved \$12 and Shawn saved \$10.



- How much money did Gina save this month?

Write an equation →

Gina saved \_\_\_\_\_ this month.

- How much money did Shawn save this month?

Write an equation →

Shawn saved \_\_\_\_\_ this month.

- What information in the word problem tells you that this is a comparison problem?

- In Gina's case, what is the "set size" and which is the "multiplier"?

Set = \_\_\_\_\_

Multiplier = \_\_\_\_\_

- In Shawn's case, what is the "set size" and which is the "multiplier"?

Set = \_\_\_\_\_

Multiplier = \_\_\_\_\_

### Focus Questions

**Question 1:** How can you identify the multiplier?

**Question 2:** How can you identify the set that is being multiplied?

### Journal Question

What do we mean by "product" in math?

## 4.OA.2: Lesson 2 - Product Unknown

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

*Introductory Task*

*Guided Practice*

*Collaborative*

*Homework*

*Assessment*

### Multiplicative Comparison - Product Unknown

Solve each problem below by identifying the number in one set, and the multiplier. Multiply the number in one set by the multiplier. Go back to the problem to make sure you have answered the question being asked, and that your answer makes sense.

- 1) A rabbit can go two feet in one jump. A kangaroo can go five times as far as a rabbit. How far can a kangaroo go in one jump?

**The number in one set is \_\_\_\_\_. The multiplier is \_\_\_\_\_.**

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

The kangaroo can go \_\_\_\_\_ feet in one jump. Since you are multiplying a whole number by a whole number, the distance the kangaroo can go should be larger than the distance the rabbit can go. Is your answer reasonable?

- 2) Amanda grew two plants for the science fair. The first plant was eight centimeters tall. The second plant was three times as tall. How tall was the second plant?

**The number in one set is \_\_\_\_\_. The multiplier is \_\_\_\_\_.**

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

The second plant was \_\_\_\_\_ centimeters tall. Since you are multiplying a whole number by a whole number, the height of the second plant should be more than the height of the first plant. Is your answer reasonable?

- 3) Chris can go 10 miles per hour on his bicycle. Jill can go seven times that fast on her motorcycle. How fast can Jill go on her motorcycle?

**The number in one set is \_\_\_\_\_. The multiplier is \_\_\_\_\_.**

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

Jill can go \_\_\_\_\_ miles per hour on her motorcycle. Since you are multiplying a whole number by a whole number, the speed Jill can go on her motorcycle should be faster than the speed Chris can go on his bicycle. Is your answer reasonable?

4)

## 4.OA.2: Lesson 2 - Product Unknown

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

*Introductory Task*

*Guided Practice*

*Collaborative*

*Homework*

*Assessment*

### Multiplicative Comparison - Product Unknown

Solve each problem below by identifying the number in one set, and the multiplier. Multiply the number in one set by the multiplier. Go back to the problem to make sure you have answered the question being asked, and that your answer makes sense.

- 1) Sara can paint 4 paintings in one hour. Mary can paint three times as many paintings in one hour than Sara. How many paintings can Mary paint in one hour?

**The number in one set is \_\_\_\_\_. The multiplier is \_\_\_\_\_.**

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

Mary can paint \_\_\_\_\_ paintings in one hour. Since you are multiplying a whole number by a whole number, the number of paintings that Mary paints should be larger than the number of paintings Sara paints in one hour. Is your answer reasonable?

- 2) Nick has two bags of coins. The first bag has 6 coins. The second bag has 7 times as many coins. How many coins are in the second bag?

**The number in one set is \_\_\_\_\_. The multiplier is \_\_\_\_\_.**

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

The second bag had \_\_\_\_\_ coins. Since you are multiplying a whole number by a whole number, the number of coins in the second bag should be more than the number of coins in the first bag. Is your answer reasonable?

- 3) Malcolm has two dogs. Butkus is a 10 inch tall beagle. Fluffy, the German Sheppard is 3 times taller than Butkus. How tall is Fluffy?

**The number in one set is \_\_\_\_\_. The multiplier is \_\_\_\_\_.**

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

Fluffy is \_\_\_\_\_ tall. Since you are multiplying a whole number by a whole number, the height of Fluffy should be greater than the height of Butkus. Is your answer reasonable?

## 4.OA.2: Lesson 3 – Set Size Unknown

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

*Introductory Task*

*Guided Practice*

*Collaborative*

*Homework*

*Assessment*

Solve the following problem:

Student Council President Sheryl collected 48 aluminum cans for recycling during the month of May. She collected 4 times as many cans as Vice President Mike.



- How many cans did Vice President Mike collect?

Write an equation →

Mike collected \_\_\_\_\_ cans.

- What information in the word problem tells you that this is a comparison problem?

- In this problem, what is the “multiplier”?

Multiplier is \_\_\_\_\_

- In this problem, what is the amount in the known set?

Known set is \_\_\_\_\_

### Focus Questions

**Question 1:** What do we mean by Set Size?

**Question 2:** What does inverse operation mean?

### Journal Question

Explain why division is the opposite of multiplication.

## 4.OA.2: Lesson 3 – Set Size Unknown

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

*Introductory Task*

**Guided Practice**

*Collaborative*

*Homework*

*Assessment*

### Multiplicative Comparison - Set Size Unknown

Solve each problem below by identifying the multiplier, and the number in the second set, which is a multiple of the first, unknown set. Divide the second set by the multiplier to find your answer. Go back to the problem to make sure you have answered the question being asked, and that your answer makes sense.

- 1) It costs \$500 to take a bus tour of Europe. This is four times as much as it costs to take a bus tour of Mexico City. How much is the bus tour of Mexico City?

**The multiplier is \_\_\_\_\_. The number in the second set is \_\_\_\_\_.**

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

The tour of Mexico City costs \$ \_\_\_\_\_. If you multiply the multiplier by your answer, you should get the cost of the European tour. Is your answer reasonable?

- 2) There are 256 students outside on the playground. This is eight times as many students as there are inside the cafeteria. How many students are inside the cafeteria?

**The multiplier is \_\_\_\_\_. The number in the second set is \_\_\_\_\_.**

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

There are \_\_\_\_\_ students inside the cafeteria. If you multiply the multiplier by your answer, you should get the number of students who are outside on the playground. Is your answer reasonable?

- 3) Franklin Middle School has 1,593 students. It has three times as many students as the nearest elementary school, Roosevelt Elementary. How many students attend Roosevelt Elementary School?

**The multiplier is \_\_\_\_\_. The number in the second set is \_\_\_\_\_.**

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

There are \_\_\_\_\_ students attending Roosevelt Elementary School. If you multiply the multiplier by your answer, you should get the number of students who are attending Franklin Middle school. Is your answer reasonable?



## 4.OA.2: Lesson 3 – Set Size Unknown

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

*Introductory Task*

*Guided Practice*

*Collaborative*

*Homework*

*Assessment*

### Multiplicative Comparison - Set Size Unknown

Solve each problem below by identifying the multiplier, and the number in the second set, which is a multiple of the first, unknown set. Divide the second set by the multiplier to find your answer. Go back to the problem to make sure you have answered the question being asked, and that your answer makes sense.

- 1) A jacket costs \$120 at the Jackets Plus store. This is three times as much as it costs at the Newark Jackets store. How much is the jacket at Newark Jackets?

**The multiplier is \_\_\_\_\_. The number in the second set is \_\_\_\_\_.**

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

The jacket at Newark Jackets costs \$ \_\_\_\_\_. If you multiply the multiplier by your answer, you should get the cost of the jacket at Jackets Plus store. Is your answer reasonable?

- 2) There are 360 people outside the gym waiting for a game to start. This is nine times as many people as there are inside the gym. How many people are inside the gym?

**The multiplier is \_\_\_\_\_. The number in the second set is \_\_\_\_\_.**

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

There are \_\_\_\_\_ people inside the gym. If you multiply the multiplier by your answer, you should get the number of people who are outside the gym. Is your answer reasonable?

- 3) The Newark Video Hut had 1,240 customers in the afternoon. This was four times as many customers as there were in the morning. How many customers were there in the morning?

**The multiplier is \_\_\_\_\_. The number in the second set is \_\_\_\_\_.**

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

There were \_\_\_\_\_ customers in the morning at the Newark Video Hut. If you multiply the multiplier by your answer, you should get the number of customers there were in the afternoon. Is your answer reasonable?

## 4.OA.2: Lesson 4 – Multiplier Unknown

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

*Introductory Task*

*Guided Practice*

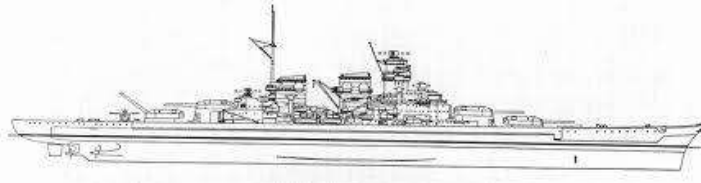
*Collaborative*

*Homework*

*Assessment*



Christopher Columbus' The Pinta



United States Battleship

Christopher Columbus' The Pinta's top speed was 5 knots. A United States Battleship's top speed is 25 knots. How many times faster is than The Pinta is a Battleship?

- How many times faster than The Pinta is a Battleship?

Write an equation →

The Battleship is \_\_\_\_\_ times faster.

- What information in the word problem tells you that this is a comparison problem?

- In this problem, what is the first set size?

First set size is \_\_\_\_\_

- In this problem, what is the second set size?

Second set size is \_\_\_\_\_

### Focus Questions

**Question 1:** How can we determine how much larger one set is when compared to another set?

### Journal Question

What other things about ships can we compare by measuring?

## 4.OA.2: Lesson 4 - Multiplier Unknown

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

*Introductory Task*

*Guided Practice*

*Collaborative*

*Homework*

*Assessment*

### Multiplicative Comparison - Multiplier Unknown

Solve each problem below by identifying the number in one set, and the number in the second set, which is a multiple of the first. Divide the second set by the first set. Go back to the problem to make sure you have answered the question being asked, and that your answer makes sense.

- 1) The plane goes 700 miles an hour. The car goes 50 miles an hour. How many times faster than the car is the plane?

**The number in one set is \_\_\_\_\_. The number in the second set is \_\_\_\_\_.**

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

The plane is \_\_\_\_\_ times faster than the car. If you multiply the speed of the car by your answer, you should get the speed of the plane. Is your answer reasonable?

- 2) Eric has 9 video games. Bryan has 54 video games. How many times more video games does Bryan have than Eric?

**The number in one set is \_\_\_\_\_. The number in the second set is \_\_\_\_\_.**

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

Bryan has \_\_\_\_\_ times as many video games as Eric. If you multiply the number of video games that Eric has by your answer, you should get the number of video games that Bryan has. Is your answer reasonable?

- 3) Shannon is 37 inches tall. Her teenaged brother, Rick, is 74 inches tall. How many times as tall as Shannon is Rick?

**The number in one set is \_\_\_\_\_. The number in the second set is \_\_\_\_\_.**

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

Rick is \_\_\_\_\_ times as tall as Shannon. If you multiply the number of inches in Shannon's height by your answer, you should get the number of inches in Rick's height. Is your answer reasonable?

## 4.OA.2: Lesson 4 - Multiplier Unknown

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

*Introductory Task*

*Guided Practice*

*Collaborative*

*Homework*

*Assessment*

### Multiplicative Comparison - Multiplier Unknown

Solve each problem below by identifying the number in one set, and the number in the second set, which is a multiple of the first. Divide the second set by the first set. Go back to the problem to make sure you have answered the question being asked, and that your answer makes sense.

- 1) A truck holds 440 jugs of milk. A crate holds 8 jugs of milk. How many times more jugs does the truck hold than a crate?

The number in one set is \_\_\_\_\_. The number in the second set is \_\_\_\_\_.

$$\underline{\quad} \div \underline{\quad} = \underline{\quad}$$

The truck holds \_\_\_\_\_ times more jugs of milk than the crate. If you multiply number jugs in a crate by your answer, you should get the number of jugs a truck holds. Is your answer reasonable?

- 2) Newark Central has won 8 games. Newark West has won 56 games. How many times more games has Newark West won than Newark Central?

The number in one set is \_\_\_\_\_. The number in the second set is \_\_\_\_\_.

$$\underline{\quad} \div \underline{\quad} = \underline{\quad}$$

Newark West has won \_\_\_\_\_ times as many games as Newark Central. If you multiply the number of games that Newark Central won by your answer, you should get the number of games that Newark West won. Is your answer reasonable?

- 3) Anthony is 12 years old. His great-uncle, George, is 84 years old. How many times as old as Anthony is George?

The number in one set is \_\_\_\_\_. The number in the second set is \_\_\_\_\_.

$$\underline{\quad} \div \underline{\quad} = \underline{\quad}$$

George is \_\_\_\_\_ times as old as Anthony. If you multiply Anthony's age by your answer, you should get George's age. Is your answer reasonable?

## 4.OA.1-2: Lesson 5 – Golden Problem

Use the four operations with whole numbers to solve problems.

*Introductory Task*

*Guided Practice*

*Collaborative*

*Homework*

*Assessment*



There were 3 pills in *Bottle 1*. The pharmacist increased the number of pills in *Bottle 1* by 4 times. How many pills are there now in *Bottle 1*?

**Write an equation:**

\_\_\_\_\_

**Answer:** \_\_\_\_\_



The pharmacist put 3 times as many pills in *Bottle 2* than in *Bottle 1*. How many pills are there now in *Bottle 2*?

**Write an equation:**

\_\_\_\_\_

**Answer:** \_\_\_\_\_



*Bottle 3* ended up with 48 pills. How many times more pills are in *Bottle 3* than in *Bottle 1*?

**Write an equation:**

\_\_\_\_\_

**Answer:** \_\_\_\_\_



Besides having different numbers and answers, in your own words, describe one major way that the three problems above are different and also explain what they have in common? Use additional paper to answer this question.

### Focus Questions

**Question 1:** How many different types of multiplicative comparison problems are there?

**Question 2:** How does division help to solve a multiplication problem?

### Journal Question

Do you think it is possible for a multiplier to be a fraction? What do we do if it is?

# Golden TASK RUBRIC

## Mathematical Problem Solving: Thinking and Applying

### **SCORE POINT = 3**

**Part 1:** The student correctly provides equations for all three bottles

Bottle 1:  $3 \times 4 = 12$

Bottle 2:  $3 \times 12 = 36$

Bottle 3:  $48 \div 12 = 4$

AND

**Part 2:** Correctly determines how many pills are in each of the three bottles

Bottle 1: answer: 12 pills

Bottle 2:, answer: 36 pills

Bottle 3:, answer: 4 times

AND

**Part 3:** Describes one way the three problems are alike and one way they are different.

\*Note: In order to preserve the Common Core's expectation of rigor, the descriptions that are expected here need to go beyond the trivial and/or obvious differences and similarities, i.e., "the numbers are different", "all the bottles have pills", etc.. Additionally, the descriptions need to include vocabulary found within the lesson such as "multiplier", "sets", "groups", etc..

### **SCORE POINT = 2**

The student correctly solves two of the three parts. Explanation or steps must be shown on how the student arrives at the answers. The explanation may not be clear.

### **SCORE POINT = 1**

The student correctly solves one of the three parts. However, the student shows incomplete explanation.

### **SCORE POINT = 0**

The response shows insufficient understanding of the problem's essential mathematical concepts.

The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation.

## Fluency Practice

Name \_\_\_\_\_ Date \_\_\_\_\_

### Addition without regrouping

Fill in the blanks:

1. We say \_\_\_\_\_ and \_\_\_\_\_ are inverse or opposite operations.
2. The putting together of two groups is called \_\_\_\_\_.
3. The answer to an addition problem is called the \_\_\_\_\_.

Work the problems:

$$\begin{array}{r} 4. \quad 183 \\ + 616 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 307 \\ + 791 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 230 \\ + 969 \\ \hline \end{array}$$

When adding numbers that are written across or horizontal. It is important to line up the numbers in the correct place values. Recopy these problems correctly and add:

$$7. \quad 100 + 22 + 31 + 2 =$$

$$8. \quad 203 + 4 + 80 + 13 =$$

Fill in the blanks:

9. When we add numbers together we start to the right in the \_\_\_\_\_ column and work left.
10. The numbers we add together are called the \_\_\_\_\_.

Work the problems:

11. 
$$\begin{array}{r} 633 \\ + 148 \\ \hline \end{array}$$

12. 
$$\begin{array}{r} 801 \\ + 617 \\ \hline \end{array}$$

13. 
$$\begin{array}{r} 216 \\ + 523 \\ \hline \end{array}$$

14.  $481 + 204 =$

15.  $103 + 555 + 31 =$

16. Valerie's puppy weighed 25 pounds when she adopted it from the animal shelter. The puppy has since gained 32 pounds. What does the puppy weigh now?

17. Three friends play a video game. Their scores are 304 points, 231 points and 352 points. What is the total number of points they earned?



Name \_\_\_\_\_ Date \_\_\_\_\_

### Addition with regrouping

Work the problems:

18. 954

$$\begin{array}{r} 954 \\ + 657 \\ \hline \end{array}$$

19. 469

$$\begin{array}{r} 469 \\ + 539 \\ \hline \end{array}$$

20. 395

$$\begin{array}{r} 395 \\ + 405 \\ \hline \end{array}$$

21.  $524 + 239 + 217 =$

22.  $845 + 17 + 72 =$

23. You decide to buy three turkeys from the supermarket. They weigh 17 pounds, 23 pounds and 15 pounds. What is the total number of pounds purchased?

24. A pizza delivery driver traveled 143 miles on Monday, 208 miles on Tuesday and 177 miles on Friday. What is the total number of miles driven on the three days.

Find the sum:

$$\begin{array}{r} 25. \quad 1097 \\ +4603 \\ \hline \end{array}$$

$$\begin{array}{r} 26. \quad 9115 \\ + \quad 235 \\ \hline \end{array}$$

$$\begin{array}{r} 27. \quad 1345 \\ + 7291 \\ \hline \end{array}$$

$$28. \quad 467 + 34 + 1042 =$$

$$29. \quad 636 + 727 =$$

$$30. \quad 603 + 338 + 29 =$$

$$31. \quad 538 + 538 =$$

32. When Gunnar bought a used motorcycle last year, the odometer read 17,237 miles. Since his purchase he has driven another 6,168 miles. What does the odometer read now?

Name \_\_\_\_\_ Date \_\_\_\_\_

### Subtraction without regrouping

33. When we see the words “how many are left” or “how many more or less” we should realize we have to \_\_\_\_\_ the numbers given.

34. The answer to a subtraction problem is called the \_\_\_\_\_.

Work these problems:

$$\begin{array}{r} 35. \quad 89 \\ - \quad 27 \\ \hline \end{array}$$

$$\begin{array}{r} 36. \quad 805 \\ - \quad 103 \\ \hline \end{array}$$

$$\begin{array}{r} 37. \quad 1299 \\ - \quad 107 \\ \hline \end{array}$$

$$38. \quad 659 - 236 =$$

$$39. \quad 8,261 - 4,030 =$$

40. For their vacation, a family needs to drive 438 miles. If they drive 316 miles the first day, how many more miles do they have left to drive?

$$\begin{array}{r} 41. \quad 399 \\ - \quad 269 \\ \hline \end{array}$$

$$\begin{array}{r} 42. \quad 605 \\ - \quad 203 \\ \hline \end{array}$$

$$\begin{array}{r} 43. \quad 1859 \\ - \quad 721 \\ \hline \end{array}$$

44.  $4,348 - 237 =$

45.  $599 - 222 =$

46.  $637 - 136 =$

47.  $542 - 310 =$

48. Jessie wants to buy a bike. It will cost \$174. If she has saved \$141, how much more money does she need to save?

Name \_\_\_\_\_ Date \_\_\_\_\_

### Subtraction with regrouping

49) How do you know when you need to regroup with subtraction?

---

---

50) Solve:

$634 - 472 =$

$534 - 76 =$

$7,527 - 716 =$

$645 - 578 =$

51) The class sold 754 plants as a fundraiser this year. Last year they only sold 499 plants. How many more plants did they sell this year?

52) To drive to your destination for vacation it is 784 miles. If you drive 495 miles on the first day, how many more miles do you need to drive?

53) Solve:

$$657 - 438 =$$

$$3,463 - 574 =$$

$$6,273 - 5,195 =$$

$$8,340 - 274 =$$

$$7,432 - 2,375 =$$

$$684 - 99 =$$

$$5,327 - 1,482 =$$

$$953 - 874 =$$

54) At Friday night's football game, 648 people were in attendance. Last week 829 people were in attendance. How many more people attended last week?

Name \_\_\_\_\_

## Review of Subtraction with and without regrouping

Fill in the blanks:

55) We say \_\_\_\_\_ and \_\_\_\_\_ are inverse or opposite operations.

56) The answer to a subtraction problem is called the \_\_\_\_\_.

Work these problems:

$$\begin{array}{r} 57) \quad 983 \\ - \quad 611 \\ \hline \end{array}$$

$$\begin{array}{r} 58) \quad 897 \\ - \quad 491 \\ \hline \end{array}$$

$$\begin{array}{r} 59) \quad 437 \\ - \quad 417 \\ \hline \end{array}$$

When subtracting numbers that are written across or horizontal. It is important to line up the numbers in the correct place values. Recopy these problems correctly and subtract:

$$60) \quad 854 - 311 =$$

$$61) \quad 583 - 260 =$$

Work the problems:

$$\begin{array}{r} 62) \quad 984 \\ - \quad 539 \\ \hline \end{array}$$

$$\begin{array}{r} 63) \quad 762 \\ - \quad 405 \\ \hline \end{array}$$



## Operations and Algebraic Thinking 4.OA.3

Solve multistep word problems posed with whole numbers and having whole number answers using the four operations.

### Goal:

Students will solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Student will also represent these problems using equations with a letter standing for the unknown quantity, and assess the reasonableness of answers using mental computation and estimation strategies including rounding.

### Essential Question(s):

How do you identify the important information?  
How are remainders interpreted?  
What does a reasonable answer look like?

### Prerequisites:

Whole Numbers  
Addition  
Subtraction  
Multiplication  
Division  
Factors

### Embedded Mathematical Practice(s)

- MP.1 Make sense of problems and persevere in solving them
- MP.2 Reason abstractly and quantitatively
- MP.3 Construct viable arguments and critique the reasoning of others
- MP.4 Model with mathematics
- MP.5 Use appropriate tools strategically
- MP.6 Attend to precision
- MP.7 Look for and make use of structure
- MP.8 Look for and express regularity in repeated reasoning

**Lesson 1**  
4.OA.3 Multistep problems

**Lesson 2**  
4.OA.3 Interpreting remainders

**Lesson 3**  
4.OA.3 Representing problems with equations

**Lesson 4**  
4.OA.3  
Assessing reasonableness using mental computation and estimation

**Lesson 5**  
4.OA.3 Golden Problem:  
The Knapsack Problem

**Lesson Structure:**  
Introductory Task  
Prerequisite Skills  
Focus Questions  
Guided Practice  
Homework  
Journal Question



Operations and Algebraic Thinking 4.OA.3, 4.MP.1, 4.MP.2, 4.MP.3, 4.MP.4, 4.MP.5, 4.MP.6, 4.MP.7

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations. MP: Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure

## Lesson 1

### Introductory Task

*Introductory Task*

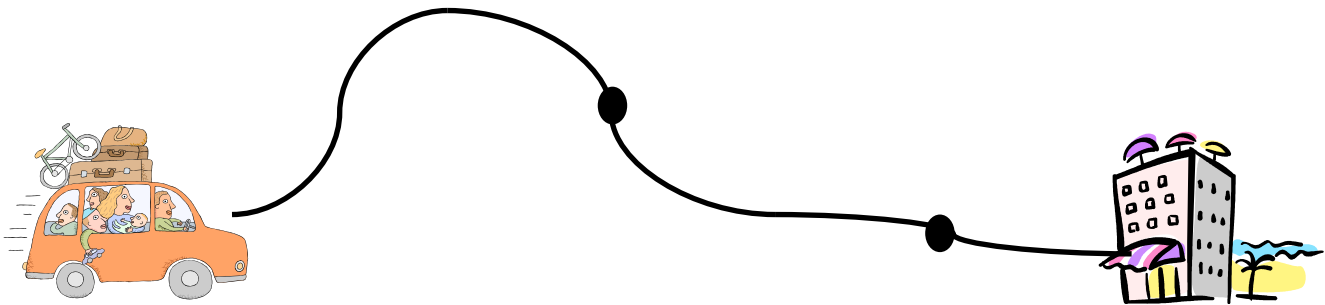
*Guided Practice*

*Homework*

*Assessment*

## Vacation Distance

On a vacation, your family travels 267 miles on the first day, 194 miles on the second day, and 34 more miles on the third day than on the second day in order to reach your destination. Then a week later you drive home. How many miles did you travel total?



### Focus Questions

**Question 1:** What pieces of the question tell you what functions to use in order to solve it correctly?

**Question 2:** What strategies can you use to keep your math organized during multistep problems?

### Journal Question

Describe a situation in your life, outside of school, when you might need to add, subtract, multiply, and/or divide several numbers to find an answer.

**Operations and Algebraic Thinking 4.OA.3, 4.MP.1, 4.MP.2, 4.MP.3, 4.MP.4, 4.MP.5, 4.MP.6, 4.MP.7**

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations. MP: Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure

## Lesson 1: Guided Practice

**Teachers model with  
students.**

Solve each problem below using addition, subtraction, multiplication, and/or division.

1. Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. How many bottles of water still need to be collected?
2. Mary had 48 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill?
3. Victor's mom is throwing him a birthday party and has invited 15 of his friends. She wants to make sure everyone gets one slice of pizza. If there are 8 slices on each whole pizza, how many pizzas should she order?
4. Gary, Stan, and Tom are comparing their collections of Pokémon cards. All together they have 700 cards. Stan has 250 cards and Gary has 300. How many cards does Tom have?

5. Justin can solve 900 math problems in 30 minutes. Steve can solve 400 math problems in 20 minutes. How many math problems can each solve in one minute?

6. Grace has three apple trees in her back yard. The first tree has 66 apples on it, the second has 42 apples, and the third has 12 apples. When she picks the apples she puts them in boxes of 24. How many boxes will she need?

7. Over the weekend Travis read part of "The Hunger Games." On Friday he read 66 pages, on Saturday he read 123 pages, and on Sunday he read 75 fewer pages than he read on Saturday. How many pages did he read in all?

8. There were 646 people riding the train. 324 more people got on the train at the next station. At the third station 524 people got off the train. How many people are left on the train?

9. Joanna is making her famous chocolate chip cookies. Each cookie has exactly 14 chips on it. If she plans on making 40 cookies, how many chocolate chips does she need?

10. Tim and Kelly are printing brochures on their computers. Tim's printer can print 2 brochures every minute. Kelly's printer can print 3 brochures every minute. After 20 minutes of printing, how many brochures will they have printed?

11. There are 67 employees working in an office building. 30 of them are leaving for lunch, and 5 of them are going home for the day. How many people will be left in the building?

12. There are 50 rows of seats in the school's auditorium. For graduation, parents are allowed to sit in the last 30 rows. The rest of the rows are divided equally between the five graduating classes. How many rows does each class have for seating?

Operations and Algebraic Thinking 4.OA.3, 4.MP.1, 4.MP.2, 4.MP.3, 4.MP.4, 4.MP.5, 4.MP.6, 4.MP.7

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations. MP: Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure

## Lesson 1: Homework

Students practice  
skills at home.

Name \_\_\_\_\_

Date \_\_\_\_\_

Solve each problem below using addition, subtraction, multiplication, and/or division.

1. The sum of two numbers is 340. If the bigger number is 210. What is the value of the smaller number?

2. Melvin has twice as many pencils as Ali. Zachary has four times as many pencils as Ali. If Ali has 3 pencils, how many pencils do the three boys have in all?

3. Li Wen bought some lollipops. He gave 4 lollipops to his friend. He then put the rest of the lollipops into 3 bags. There were 5 lollipops in each bag. How many lollipops did Li Wen buy?

4. Find the quotient when 160 and 20 is divided by 6.

5. A bottle holds 260 ml of juice when it is a quarter full. How much juice can it hold when it is completely full?

6. Five years ago Eric was 16 years old. How old will he be in 2 years' time?

7. There were 423 people riding the train. 273 more people got on the train, and 93 got off at the next station. At the third station half of all of the people got off the train. How many people are left on the train?

8. Kevin has \$1,498 in his savings account. He has \$529 more than Albert. How much money do they have altogether?

9. A jug of orange juice can fill 4 large mugs. A large mug can fill 5 small cups. How many jugs of orange juice are needed to fill 100 small cups?

10. Bill has \$20. He goes to the deli and spends \$5 on a sandwich, \$2 on a drink, and \$1 on a bag of chips. How much money does Bill have left?

Operations and Algebraic Thinking 4.OA.3, 4.MP.1, 4.MP.2, 4.MP.3, 4.MP.4, 4.MP.5, 4.MP.6, 4.MP.7

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, and interpret remainders. MP: Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure

## Lesson 2

### Introductory Task

## Pouring Milk

A gallon of milk contains 128 fl oz. How many 6 fl oz. glasses can you pour from 5 gallons of milk? Explain your answer and mathematical thinking.



### Focus Questions

**Question 1:** What do you do with a remainder in a problem?

**Question 2:** What clues in the scenario tell you when you drop the remainder or add it into the answer?

### Journal Question

You and two friends are sharing a whole pizza. The pizza has 8 slices. How can you share it equally?



**Operations and Algebraic Thinking 4.OA.3, 4.MP.1, 4.MP.2, 4.MP.3, 4.MP.4, 4.MP.5, 4.MP.6, 4.MP.7**

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, and interpret remainders. MP: Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure

**Lesson 2:  
Guided Practice****Teachers model with  
students.**

Solve each problem below using addition, subtraction, multiplication, and/or division.

1. Mary has \$50 and she wants to buy boxes of chocolates as presents for her friends. Each box of chocolate costs \$6. How many boxes can Mary buy? Do you end up with a remainder? If so, what does it mean?
2. There are 43 fishermen entered in the 23<sup>rd</sup> Annual Lakeshore Fishing Contest. Each boat can hold 5 people. How many boats are needed to take all of the fishermen out onto the lake for the contest? Do you end up with a remainder? If so, what does it mean?
3. Dunkin Donuts sold 120 cups of coffee on Monday, 234 on Tuesday, and 112 on Wednesday. If the cups come in pack of 50, how many packs did they use? Do you end up with a remainder? If so, what does it mean?
4. There were 279 apples in a box. 23 apples had to be thrown away because they were rotten. The remaining apples were repacked into 4 bags equally. How many apples were in each bag? Do you end up with a remainder? If so, what does it mean?

5. Cay has 13 less beads than Betty. Betty has four times as many beads as Anna. Anna has 290 beads. How many beads do they have in all? Do you end up with a remainder? If so, what does it mean?

6. There are an unknown number of cars and motorcycles parked in a parking garage. The sensor at the gate counted 250 wheels in total. If there are 15 motorcycles in the garage, how many cars are there? Do you end up with a remainder? If so, what does it mean?

7. Over the weekend Travis began reading a 700 page book. On Friday he read 66 pages, on Saturday he read 123 pages, and on Sunday he read 75 pages. How many pages does he still need to read? Do you end up with a remainder? If so, what does it mean?

8. Andy can fit 12 eggs in every carton that goes down the assembly line. If he need to pack 244 eggs, how many cartons will he need? Do you end up with a remainder? If so, what does it mean?

9. Isabel is baking brownies to take to a party. The recipe calls for 40 oz. of flour. If each bag of flour contains 16 oz., how many bags will she need? Do you end up with a remainder? If so, what does it mean?

10. George, Julio, Tyrone, and Filipe are working together to win a math contest. The contest requires them to answer 34 problems. If the boys divide the problems between themselves, how many does each boy have to complete? Do you end up with a remainder? If so, what does it mean?

**Operations and Algebraic Thinking 4.OA.3, 4.MP.1, 4.MP.2, 4.MP.3, 4.MP.4, 4.MP.5, 4.MP.6, 4.MP.7**

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, and interpret remainders. MP: Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure

## **Lesson 2: Homework**

**Students practice  
skills at home.**

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Solve each problem below using addition, subtraction, multiplication, and/or division.

1. Mr. Jenson's 4<sup>th</sup> grade class is having a food drive to help less fortunate members of their community. Heidy brought in 5 cans, Joao brought in 7 cans, Leon brought in 4 cans, and Tara brought in 18 cans. If each box holds 10 cans, how many boxes will the class need to pack all of their donations? Do you end up with a remainder? If so, what does it mean?
2. Tim has a summer job retrieving golf balls from ponds on several different golf courses. He repackages the balls into boxes 12 and sells them on eBay. Tim found 300 golf balls over the summer. How many boxes of balls did he make from the balls he found? Do you end up with a remainder? If so, what does it mean?
3. Tony is going to the store to stock up his hot dog truck. He buys 124 hot dogs. Buns come in pack of 8. How many pack will he need to buy to make sure he has enough for all of his hot dogs? Do you end up with a remainder? If so, what does it mean?

4. Jamal is sending out invitations to his birthday party. He needs to invite 40 people. Invitations come in packs of 8, and envelopes come in packs of 15. How many packs of each will he need to buy? Do you end up with a remainder? If so, what does it mean?

6. If a frog can jump 3 ft every time it hops, how many times will it need to jump to travel 40 ft? Do you end up with a remainder? If so, what does it mean?

7. You want to buy a TV (\$900), Blu-Ray player (\$100), and a PS3 (\$250) from Best Buy. They are going to let you pay off the total cost over the next 12 months. How much do you need to pay each month to pay off the cost of the items? Do you end up with a remainder? If so, what does it mean?

8. Andy can fit 12 eggs in every carton that goes down the assembly line. If he need to pack 100 eggs, how many cartons will he need? Do you end up with a remainder? If so, what does it mean?

9. Isabel is making pancakes for her family. The recipe calls for 6 oz of batter per pancake. She needs to make 20 pancakes. If each box of pancake mix makes 50 oz of batter, how many boxes does she need? Do you end up with a remainder? If so, what does it mean?

10. We are going on a class trip to the zoo. Each bus holds 54 people. There are 125 students and 20 chaperones. How many buses do we need for the trip? Do you end up with a remainder? If so, what does it mean?

Operations and Algebraic Thinking 4.OA.3, 4.MP.1, 4.MP.2, 4.MP.3, 4.MP.4, 4.MP.5, 4.MP.6, 4.MP.7

Represent these problems using equations with a letter standing for the unknown quantity. MP: Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure

## Lesson 3

### Introductory Task

### Mystery Symbols

$$\text{☺} + \text{☀} + \text{🌸} = 34$$

$$\text{☺} + \text{☀} = 18$$

$$\text{🌸} - \text{☀} = 10$$

$$\text{☺} = \underline{\hspace{2cm}}$$

$$\text{☀} = \underline{\hspace{2cm}}$$

$$\text{🌸} = \underline{\hspace{2cm}}$$

### Focus Questions

**Question 1:** What does a variable take the place of?

**Question 2:** Do variables always represent the same value?

### Journal Question

$$2 + \text{☺} = 22$$

$$5 \times \text{👉} = 100$$

Are 👉 and ☺ equal to the same value?

**Operations and Algebraic Thinking 4.OA.3, 4.MP.1, 4.MP.2, 4.MP.3, 4.MP.4, 4.MP.5, 4.MP.6, 4.MP.7**

Represent these problems using equations with a letter standing for the unknown quantity. MP: Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure

## Lesson 3: Guided Practice

**Teachers model with  
students.**

In the problems below, solve for the letter or symbol in each. Show your work.

<p>1. <math>98 + A = 124</math></p> <p style="text-align: right;">A= _____</p>	<p>2. <math>f - 77 = 103</math></p> <p style="text-align: right;">f= _____</p>
<p>3. <math>2 \times m = 44</math></p> <p style="text-align: right;">m= _____</p>	<p>4. <math>h \div 3 = 8</math></p> <p style="text-align: right;">h= _____</p>
<p>5. <math>123 + p + 22 = 276</math></p> <p style="text-align: right;">p= _____</p>	<p>5. <math>4 \times D = 36</math></p> <p style="text-align: right;">D= _____</p>
<p>7. <math>1,342 - k = 700</math></p> <p style="text-align: right;">k = _____</p>	<p>8. <math>S + S = 100</math></p> <p style="text-align: right;">S= _____</p>



<p>9. <math>L \times L = 144</math></p> <p style="text-align: right;">L= _____</p>	<p>10. <math>500 \div x = 5</math></p> <p style="text-align: right;">x= _____</p>
<p>11. <math>m \times 3 = 60</math></p> <p style="text-align: right;">m= _____</p>	<p>12. <math>h \div 12 = 4</math></p> <p style="text-align: right;">h= _____</p>
<p>13. <math>14 - p = 8</math> <math>p + s = 36</math></p> <p style="text-align: right;">s= _____</p>	<p>14. <math>4 \times D = 36</math> <math>D - E = 8</math></p> <p style="text-align: right;">E= _____</p>
<p>15. <math>1,342 - k = 700</math> <math>k + v = 900</math></p> <p style="text-align: right;">v = _____</p>	<p>16. <math>20 \div z = 5</math> <math>z \times q = 120</math></p> <p style="text-align: right;">q= _____</p>

**Operations and Algebraic Thinking 4.OA.3, 4.MP.1, 4.MP.2, 4.MP.3, 4.MP.4, 4.MP.5, 4.MP.6, 4.MP.7**

Represent these problems using equations with a letter standing for the unknown quantity. MP: Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure

**Lesson 3:  
Homework**  
Students practice  
skills at home.

Name \_\_\_\_\_

Date \_\_\_\_\_

In the problems below, solve for the letter or symbol in each. Show your work.

<p>1. <math>68 + \text{☺} = 100</math></p> <p style="text-align: right; margin-top: 100px;"><math>\text{☺} = \underline{\hspace{2cm}}</math></p>	<p>2. <math>\text{☎} - 57 = 78</math></p> <p style="text-align: right; margin-top: 100px;"><math>\text{☎} = \underline{\hspace{2cm}}</math></p>
<p>3. <math>4 \times m = 44</math></p> <p style="text-align: right; margin-top: 100px;"><math>m = \underline{\hspace{2cm}}</math></p>	<p>4. <math>h \div 9 = 8</math></p> <p style="text-align: right; margin-top: 100px;"><math>h = \underline{\hspace{2cm}}</math></p>
<p>5. <math>126 + p + 26 = 361</math></p> <p style="text-align: right; margin-top: 100px;"><math>p = \underline{\hspace{2cm}}</math></p>	<p>5. <math>4 \times D = 36</math> <math>D \times 3 = F</math></p> <p style="text-align: right; margin-top: 100px;"><math>F = \underline{\hspace{2cm}}</math></p>
<p>7. <math>1,222 - k = 511</math></p> <p style="text-align: right; margin-top: 100px;"><math>k = \underline{\hspace{2cm}}</math></p>	<p>8. <math>\square + \square = 150</math></p> <p style="text-align: right; margin-top: 100px;"><math>\square = \underline{\hspace{2cm}}</math></p>

<p>9. <math>66 - J = L</math> <math>20 + 14 = J</math></p> <p style="text-align: right;">L= _____</p>	<p>10. <math>50 \div x = 5</math></p> <p style="text-align: right;">x= _____</p>
<p>11. <math>m \times 3 = 12</math> <math>12 - m - t</math></p> <p style="text-align: right;">t= _____</p>	<p>12. <math>h \div 12 = 4 + 3</math></p> <p style="text-align: right;">h= _____</p>
<p>13. <math>14 - s = 8</math> <math>s + p = 30</math></p> <p style="text-align: right;">p= _____</p>	<p>14. <math>4 \times D = 32</math> <math>D - E = 1</math></p> <p style="text-align: right;">E= _____</p>
<p>15. <math>1,400 - k = 600</math> <math>k + v = 827</math></p> <p style="text-align: right;">v= _____</p>	<p>16. <math>45 \div z = 5</math> <math>z \times q = 72</math></p> <p style="text-align: right;">q= _____</p>

<b>Introductory Task</b>	<b>Guided Practice</b>	<b>Homework</b>	<b>Assessment</b>
--------------------------	------------------------	-----------------	-------------------

**Operations and Algebraic Thinking 4.OA.3, 4.MP.1, 4.MP.2, 4.MP.3, 4.MP.4, 4.MP.5, 4.MP.6, 4.MP.7**

Assess the reasonableness of answers using mental computation and estimation strategies including rounding. MP: Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure

## Lesson 4

### Introductory Task

## Recycling Campaign

Your school has begun a recycling campaign to help protect the environment. Each class is trying to bring in as many aluminum cans as they can. If the entire school can bring in 3,000 cans, Principal Smith says she will buy everyone ice cream. Look at the totals below and estimate to determine about how many more cans are needed to reach the goal.

<b>Class</b>	<b># of cans</b>
<b>Mr. Jones</b>	245
<b>Ms. Piret</b>	175
<b>Mrs. Owens</b>	439
<b>Ms. Tucker</b>	322
<b>Mr. Tavares</b>	109
<b>Mrs. Franklin</b>	211
<b>Ms. Pinho</b>	399

### Focus Questions

**Question 1:** How do we estimate and round numbers?

**Question 2:** What are some scenarios when estimating and rounding are appropriate?

### Journal Question

Citi Field can hold 41,922 fans for a game. Yankee stadium can hold 50,086. About how many more seats does Yankee Stadium have than Citi Field?

**Operations and Algebraic Thinking 4.OA.3, 4.MP.1, 4.MP.2, 4.MP.3, 4.MP.4, 4.MP.5, 4.MP.6, 4.MP.7**

Assess the reasonableness of answers using mental computation and estimation strategies including rounding. MP: Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure

## **Lesson 4: Guided Practice**

**Teachers model with  
students.**

Use estimation and rounding to help you answer the questions below.

1. Ron and Amanda collected 497 buttons, but they used 177 of them while fixing some jackets. About how many buttons do they have now? Choose the best estimate.  
A 500  
B 400  
C 300  
D 100
2. Mom made 510 cupcakes for the bake sale. 226 sold in two days. About how many cupcakes are left?  
A 300  
B 700  
C 800  
D 837
3. Tim, Caleb, and Michael collected 989 buttons, but 165 got lost in the couch. About how many buttons do they have now? Choose the better estimate.  
A 400  
B 800  
C 900  
D 1,000
4. 237 of Lizzy's 609 screws got lost in the garage. About how many screws does she have now? Choose the best estimate.  
A 100  
B 200  
C 300  
D 400

5. 882 passengers are in the train station. 660 get on a train. About how many are left?
- A 200
  - B 900
  - C 1000
  - D 100
6. Liz and Mary went to a candy store and bought 959 pieces of candy. 152 were bought by Liz. About how many did Mary buy?
- A 400
  - B 800
  - C 1,000
  - D 1,000
7. Sam loves marbles. 220 of his 615 marbles rolled away. About how many does Sam have now?
- A 800
  - B 600
  - C 400
  - D 200
8. Ronny, Katy, and Wally had 354 plastic balls, but then 259 rolled away down a big hill. About how many plastic balls do they have now? Choose the better estimate.
- A 100
  - B 150
  - C 200
  - D 300
9. Trish and Aaron had 717 raisins, but 185 fell on the ground and had to be thrown out. About how many do they have now?
- A 300
  - B 400
  - C 500
  - D 800
10. Kelly spends 125 minutes on the phone on Monday, 123 minutes, on Tuesday, 63 minutes on Wednesday, 89 minutes on Thursday, 191 minutes on Friday, and 356 minutes over the weekend. About how many minutes did she spend on the phone all week?
- A 800
  - B 900
  - C 1,000
  - D 1,100

<i>Introductory Task</i>	<i>Guided Practice</i>	<i>Homework</i>	<i>Assessment</i>
--------------------------	------------------------	-----------------	-------------------

**Operations and Algebraic Thinking 4.OA.3, 4.MP.1, 4.MP.2, 4.MP.3, 4.MP.4, 4.MP.5, 4.MP.6, 4.MP.7**

Assess the reasonableness of answers using mental computation and estimation strategies including rounding. MP: Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure

## **Lesson 4: Homework**

**Students practice skills at home.**

Name \_\_\_\_\_

Date \_\_\_\_\_

Use estimation and rounding to help you answer the questions below.

1. Ladona is writing a report on the fishing industry. She found that last year one company caught a combined total of 9,679 fish in the Atlantic and Pacific Oceans. If the company caught 1,520 in the Pacific Ocean, about how many fish did they catch in all? Choose the better estimate.
  - A. 10,000
  - B. 11,000
  - C. 12,000
  - D. 13,000
2. A cell phone company has a total of 5,208 customers across the world. If 1,214 of its customers live in England, about how many more customers live in other countries than in England? Choose the better estimate.
  - A. 4,000
  - B. 3,000
  - C. 2,000
  - D. 1,000

3. A team of engineers built a dam over a large river. In the process, they created a lake covering a combined total of 701 acres of farmland and forest. If the lake covers 292 acres of farmland, about how many acres of forest does it cover? Choose the better estimate.
- A. 400
  - B. 300
  - C. 200
  - D. 100
4. Roberto bought his condo for \$86,819. A few years later, he sold it for \$35,943 more. About how much did Roberto sell his condo for? Choose the better estimate.
- A. 120,000
  - B. 130,000
  - C. 140,000
  - D. 150,000
5. An oil spill washed onto the coast and coated many birds with oil. So far, a team of volunteers has cleaned 604 birds, but 767 birds are still dirty. About how many birds did the oil spill affect? Choose the better estimate.
- A. 1,400
  - B. 1,300
  - C. 1,200
  - D. 1,100
6. Param jumped on the couch 165 times, and his friend Miriam jumped on it 625 times. Then the couch broke and they both got in big trouble. About how many times did they jump on the couch altogether? Choose the better estimate.
- A. 800
  - B. 900
  - C. 700
  - D. 600



7. A monument has a combined total of 6,973 sandstone and limestone blocks. If 5,048 limestone blocks are used in the monument, about how many sandstone blocks are used? Choose the better estimate.
- A. 1,000
  - B. 2,000
  - C. 3,000
  - D. 4,000
8. Since the company started, Nature's Best Water has sold a combined total of 4,175 gallons of plain bottled water and sparkling bottled water. If they have sold 2,154 gallons of plain bottled water, about how many gallons of sparkling bottled water has the company sold? Choose the better estimate.
- A. 2,000
  - B. 6,000
  - C. 8,000
  - D. 10,000
9. Last year at Westford's airport, 76,073 passengers landed on time. Unfortunately, 74,768 passengers landed late. In all, about how many passengers landed in Westford? Choose the better estimate.
- A. 150,000
  - B. 140,000
  - C. 130,000
  - D. 120,000

<i>Introductory Task</i>	<i>Guided Practice</i>	<i>Homework</i>	<i>Assessment</i>
--------------------------	------------------------	-----------------	-------------------

**Operations and Algebraic Thinking 4.OA.3, 4.MP.1, 4.MP.2, 4.MP.3, 4.MP.4, 4.MP.5, 4.MP.6, 4.MP.7**

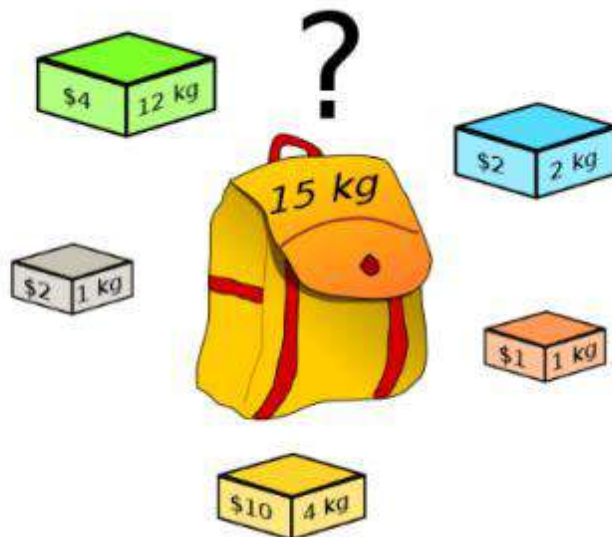
Students will solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Student will also represent these problems using equations with a letter standing for the unknown quantity, and assess the reasonableness of answers using mental computation and estimation strategies including rounding. MP: Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure

**Lesson 5  
Golden Problem**

**The Knapsack Problem**

Vivian has a knapsack that can only hold 15kg of weight. The diagram shows all of the possible items that Vivian had to choose from to pack her knapsack.

1. Find three possible ways that Vivian packed her knapsack that reached but did not exceed the weight capacity. What was the total dollar value for each of those ways?
2. Find one possible way that Vivian packed her knapsack whose dollar value exceeded each of the three ways you found in the first question. Was there a weight remainder? Why?



## GOLDEN PROBLEM RUBRIC

Score	Description
3	<p>The student correctly determines three different ways that Vivian could have packed her knapsack. The student clearly shows the sum of the weights and their total values</p> <p style="text-align: center;">AND</p> <p>The student correctly determines one way that Vivian could have packed her knapsack whose total value exceeds each of the previous three ways the student had determined. The student further provides explanations about any remaining weight capacity and its meaning. The student clearly shows the sum of the weights and their total values</p> <p style="text-align: center;">AND</p> <p>The student provides a clear mathematical explanations of their reasoning of the problem showing calculations or explanations to support their answers.</p>
2	<p>The student correctly determines three different ways that Vivian could have packed her knapsack. The student clearly shows the sum of the weights and their total values</p> <p style="text-align: center;">AND</p> <p>The student correctly determines one way that Vivian could have packed her knapsack whose total value exceeds each of the previous three ways the student had determined. The student further provides explanations about any remaining weight capacity and its meaning. The student clearly shows the sum of the weights and their total values</p> <p style="text-align: center;">However,</p> <p>The student does not provide clear mathematical explanations of their reasoning of the problem or fails to show calculations or work to support their answers leading the teacher to make inferences as to how the student arrived at the answers.</p>
1	<p>The student correctly determines three different ways that Vivian could have packed her knapsack. The student clearly shows the sum of the weights and their total values</p> <p>The student's response to the second part of the problem is wrong or incomplete. Some supporting work is present.</p>
0	<p>The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation to the solution or the reader may not be able to understand the explanation.</p>

# Fourth Grade CCSSM Fluencies

## Skills

Add/subtract within 1,000,000

1. Subtraction Worksheet  
Answer Key
2. Addition Worksheet  
Answer Key

## Four-Digit Minus Three-Digit Subtraction (A)

$$\begin{array}{r} 7989 \\ - 382 \\ \hline \end{array}$$
$$\begin{array}{r} 4211 \\ - 388 \\ \hline \end{array}$$
$$\begin{array}{r} 9158 \\ - 232 \\ \hline \end{array}$$
$$\begin{array}{r} 5841 \\ - 847 \\ \hline \end{array}$$
$$\begin{array}{r} 8622 \\ - 719 \\ \hline \end{array}$$
$$\begin{array}{r} 7095 \\ - 742 \\ \hline \end{array}$$

$$\begin{array}{r} 4313 \\ - 384 \\ \hline \end{array}$$
$$\begin{array}{r} 2148 \\ - 681 \\ \hline \end{array}$$
$$\begin{array}{r} 4959 \\ - 876 \\ \hline \end{array}$$
$$\begin{array}{r} 9577 \\ - 599 \\ \hline \end{array}$$
$$\begin{array}{r} 9074 \\ - 164 \\ \hline \end{array}$$
$$\begin{array}{r} 6528 \\ - 449 \\ \hline \end{array}$$

$$\begin{array}{r} 4626 \\ - 177 \\ \hline \end{array}$$
$$\begin{array}{r} 3304 \\ - 673 \\ \hline \end{array}$$
$$\begin{array}{r} 2079 \\ - 733 \\ \hline \end{array}$$
$$\begin{array}{r} 4839 \\ - 494 \\ \hline \end{array}$$
$$\begin{array}{r} 8283 \\ - 683 \\ \hline \end{array}$$
$$\begin{array}{r} 3345 \\ - 653 \\ \hline \end{array}$$

$$\begin{array}{r} 9158 \\ - 523 \\ \hline \end{array}$$
$$\begin{array}{r} 1117 \\ - 854 \\ \hline \end{array}$$
$$\begin{array}{r} 6329 \\ - 351 \\ \hline \end{array}$$
$$\begin{array}{r} 2314 \\ - 481 \\ \hline \end{array}$$
$$\begin{array}{r} 2321 \\ - 809 \\ \hline \end{array}$$
$$\begin{array}{r} 1419 \\ - 292 \\ \hline \end{array}$$

$$\begin{array}{r} 2405 \\ - 957 \\ \hline \end{array}$$
$$\begin{array}{r} 8722 \\ - 118 \\ \hline \end{array}$$
$$\begin{array}{r} 1905 \\ - 215 \\ \hline \end{array}$$
$$\begin{array}{r} 1104 \\ - 874 \\ \hline \end{array}$$
$$\begin{array}{r} 8316 \\ - 143 \\ \hline \end{array}$$
$$\begin{array}{r} 9584 \\ - 266 \\ \hline \end{array}$$

$$\begin{array}{r} 6461 \\ - 996 \\ \hline \end{array}$$
$$\begin{array}{r} 5860 \\ - 117 \\ \hline \end{array}$$
$$\begin{array}{r} 6041 \\ - 603 \\ \hline \end{array}$$
$$\begin{array}{r} 3645 \\ - 865 \\ \hline \end{array}$$
$$\begin{array}{r} 5047 \\ - 562 \\ \hline \end{array}$$
$$\begin{array}{r} 2724 \\ - 630 \\ \hline \end{array}$$

## Four-Digit Minus Three-Digit Subtraction (A) **Answers**

$$\begin{array}{r} 7989 \\ - 382 \\ \hline 7607 \end{array}$$

$$\begin{array}{r} 4211 \\ - 388 \\ \hline 3823 \end{array}$$

$$\begin{array}{r} 9158 \\ - 232 \\ \hline 8926 \end{array}$$

$$\begin{array}{r} 5841 \\ - 847 \\ \hline 4994 \end{array}$$

$$\begin{array}{r} 8622 \\ - 719 \\ \hline 7903 \end{array}$$

$$\begin{array}{r} 7095 \\ - 742 \\ \hline 6353 \end{array}$$

$$\begin{array}{r} 4313 \\ - 384 \\ \hline 3929 \end{array}$$

$$\begin{array}{r} 2148 \\ - 681 \\ \hline 1467 \end{array}$$

$$\begin{array}{r} 4959 \\ - 876 \\ \hline 4083 \end{array}$$

$$\begin{array}{r} 9577 \\ - 599 \\ \hline 8978 \end{array}$$

$$\begin{array}{r} 9074 \\ - 164 \\ \hline 8910 \end{array}$$

$$\begin{array}{r} 6528 \\ - 449 \\ \hline 6079 \end{array}$$

$$\begin{array}{r} 4626 \\ - 177 \\ \hline 4449 \end{array}$$

$$\begin{array}{r} 3304 \\ - 673 \\ \hline 2631 \end{array}$$

$$\begin{array}{r} 2079 \\ - 733 \\ \hline 1346 \end{array}$$

$$\begin{array}{r} 4839 \\ - 494 \\ \hline 4345 \end{array}$$

$$\begin{array}{r} 8283 \\ - 683 \\ \hline 7600 \end{array}$$

$$\begin{array}{r} 3345 \\ - 653 \\ \hline 2692 \end{array}$$

$$\begin{array}{r} 9158 \\ - 523 \\ \hline 8635 \end{array}$$

$$\begin{array}{r} 1117 \\ - 854 \\ \hline 263 \end{array}$$

$$\begin{array}{r} 6329 \\ - 351 \\ \hline 5978 \end{array}$$

$$\begin{array}{r} 2314 \\ - 481 \\ \hline 1833 \end{array}$$

$$\begin{array}{r} 2321 \\ - 809 \\ \hline 1512 \end{array}$$

$$\begin{array}{r} 1419 \\ - 292 \\ \hline 1127 \end{array}$$

$$\begin{array}{r} 2405 \\ - 957 \\ \hline 1448 \end{array}$$

$$\begin{array}{r} 8722 \\ - 118 \\ \hline 8604 \end{array}$$

$$\begin{array}{r} 1905 \\ - 215 \\ \hline 1690 \end{array}$$

$$\begin{array}{r} 1104 \\ - 874 \\ \hline 230 \end{array}$$

$$\begin{array}{r} 8316 \\ - 143 \\ \hline 8173 \end{array}$$

$$\begin{array}{r} 9584 \\ - 266 \\ \hline 9318 \end{array}$$

$$\begin{array}{r} 6461 \\ - 996 \\ \hline 5465 \end{array}$$

$$\begin{array}{r} 5860 \\ - 117 \\ \hline 5743 \end{array}$$

$$\begin{array}{r} 6041 \\ - 603 \\ \hline 5438 \end{array}$$

$$\begin{array}{r} 3645 \\ - 865 \\ \hline 2780 \end{array}$$

$$\begin{array}{r} 5047 \\ - 562 \\ \hline 4485 \end{array}$$

$$\begin{array}{r} 2724 \\ - 630 \\ \hline 2094 \end{array}$$

## Four-Digit Plus Four-Digit Addition

$$\begin{array}{r} 8498 \\ + 5366 \\ \hline \end{array}$$
$$\begin{array}{r} 5701 \\ + 8780 \\ \hline \end{array}$$
$$\begin{array}{r} 1373 \\ + 5811 \\ \hline \end{array}$$
$$\begin{array}{r} 6665 \\ + 7233 \\ \hline \end{array}$$
$$\begin{array}{r} 9399 \\ + 6165 \\ \hline \end{array}$$
$$\begin{array}{r} 5463 \\ + 3789 \\ \hline \end{array}$$

$$\begin{array}{r} 5048 \\ + 2690 \\ \hline \end{array}$$
$$\begin{array}{r} 4358 \\ + 6246 \\ \hline \end{array}$$
$$\begin{array}{r} 8275 \\ + 6493 \\ \hline \end{array}$$
$$\begin{array}{r} 3454 \\ + 6111 \\ \hline \end{array}$$
$$\begin{array}{r} 7995 \\ + 2873 \\ \hline \end{array}$$
$$\begin{array}{r} 2010 \\ + 7998 \\ \hline \end{array}$$

$$\begin{array}{r} 4031 \\ + 6099 \\ \hline \end{array}$$
$$\begin{array}{r} 6136 \\ + 8630 \\ \hline \end{array}$$
$$\begin{array}{r} 5097 \\ + 1915 \\ \hline \end{array}$$
$$\begin{array}{r} 7001 \\ + 7172 \\ \hline \end{array}$$
$$\begin{array}{r} 7308 \\ + 3537 \\ \hline \end{array}$$
$$\begin{array}{r} 4337 \\ + 8039 \\ \hline \end{array}$$

$$\begin{array}{r} 8010 \\ + 3429 \\ \hline \end{array}$$
$$\begin{array}{r} 8811 \\ + 1691 \\ \hline \end{array}$$
$$\begin{array}{r} 2814 \\ + 4876 \\ \hline \end{array}$$
$$\begin{array}{r} 7228 \\ + 9392 \\ \hline \end{array}$$
$$\begin{array}{r} 9804 \\ + 7978 \\ \hline \end{array}$$
$$\begin{array}{r} 4118 \\ + 5489 \\ \hline \end{array}$$

$$\begin{array}{r} 3193 \\ + 8419 \\ \hline \end{array}$$
$$\begin{array}{r} 8723 \\ + 5943 \\ \hline \end{array}$$
$$\begin{array}{r} 1769 \\ + 4037 \\ \hline \end{array}$$
$$\begin{array}{r} 1817 \\ + 5092 \\ \hline \end{array}$$
$$\begin{array}{r} 6791 \\ + 4188 \\ \hline \end{array}$$
$$\begin{array}{r} 5585 \\ + 4615 \\ \hline \end{array}$$

## Four-Digit Plus Four-Digit Addition (A) **Answers**

$$\begin{array}{r} 8498 \\ + \underline{5366} \\ \hline 13864 \end{array}$$

$$\begin{array}{r} 5701 \\ + \underline{8780} \\ \hline 14481 \end{array}$$

$$\begin{array}{r} 1373 \\ + \underline{5811} \\ \hline 7184 \end{array}$$

$$\begin{array}{r} 6665 \\ + \underline{7233} \\ \hline 13898 \end{array}$$

$$\begin{array}{r} 9399 \\ + \underline{6165} \\ \hline 15564 \end{array}$$

$$\begin{array}{r} 5463 \\ + \underline{3789} \\ \hline 9252 \end{array}$$

$$\begin{array}{r} 5048 \\ + \underline{2690} \\ \hline 7738 \end{array}$$

$$\begin{array}{r} 4358 \\ + \underline{6246} \\ \hline 10604 \end{array}$$

$$\begin{array}{r} 8275 \\ + \underline{6493} \\ \hline 14768 \end{array}$$

$$\begin{array}{r} 3454 \\ + \underline{6111} \\ \hline 9565 \end{array}$$

$$\begin{array}{r} 7995 \\ + \underline{2873} \\ \hline 10868 \end{array}$$

$$\begin{array}{r} 2010 \\ + \underline{7998} \\ \hline 10008 \end{array}$$

$$\begin{array}{r} 4031 \\ + \underline{6099} \\ \hline 10130 \end{array}$$

$$\begin{array}{r} 6136 \\ + \underline{8630} \\ \hline 14766 \end{array}$$

$$\begin{array}{r} 5097 \\ + \underline{1915} \\ \hline 7012 \end{array}$$

$$\begin{array}{r} 7001 \\ + \underline{7172} \\ \hline 14173 \end{array}$$

$$\begin{array}{r} 7308 \\ + \underline{3537} \\ \hline 10845 \end{array}$$

$$\begin{array}{r} 4337 \\ + \underline{8039} \\ \hline 12376 \end{array}$$

$$\begin{array}{r} 8010 \\ + \underline{3429} \\ \hline 11439 \end{array}$$

$$\begin{array}{r} 8811 \\ + \underline{1691} \\ \hline 10502 \end{array}$$

$$\begin{array}{r} 2814 \\ + \underline{4876} \\ \hline 7690 \end{array}$$

$$\begin{array}{r} 7228 \\ + \underline{9392} \\ \hline 16620 \end{array}$$

$$\begin{array}{r} 9804 \\ + \underline{7978} \\ \hline 17782 \end{array}$$

$$\begin{array}{r} 4118 \\ + \underline{5489} \\ \hline 9607 \end{array}$$

$$\begin{array}{r} 3193 \\ + \underline{8419} \\ \hline 11612 \end{array}$$

$$\begin{array}{r} 8723 \\ + \underline{5943} \\ \hline 14666 \end{array}$$

$$\begin{array}{r} 1769 \\ + \underline{4037} \\ \hline 5806 \end{array}$$

$$\begin{array}{r} 1817 \\ + \underline{5092} \\ \hline 6909 \end{array}$$

$$\begin{array}{r} 6791 \\ + \underline{4188} \\ \hline 10979 \end{array}$$

$$\begin{array}{r} 5585 \\ + \underline{4615} \\ \hline 10200 \end{array}$$



## The algorithm: Making Sense

### 4.NBT.4

#### BACKGROUND KNOWLEDGE

For students who are good at multi-digit addition and subtraction, learning a standard written subtraction is straightforward, provided they understand the core idea that the particular decomposition needed in a given subtraction depends on what is subtracted.

#### ESSENTIAL QUESTIONS

- What strategies can I use to help me make sense of a written algorithm?

#### MATERIALS

- Play money if needed
- Base-ten blocks
- The algorithm: Making Sense recording sheet

#### TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

This task allows students to make sense of the standard algorithm for subtraction. It is important you allow them to grapple with the strategies used by Mary. Through this grappling, students make sense of what Jane did to solve each problem. Through classroom discussion, student understanding will be shared and developed. Therefore, it is not necessary to work them through the methods presented in Mary's work.

After engaging in this task, students should know that it is mathematically possible to subtract a larger number from a smaller number but the difference would result in a negative number.

#### Task directions:

Students will follow the directions below from the "The algorithm: Making Sense" recording sheet.

Problems:

#### FORMATIVE ASSESSMENT QUESTIONS

- When you write the numbers in expanded form, what do you discover?
- What happens when one number has more or less tens than the other?
- Why do you think Mary rearranged the numbers before subtracting?

#### DIFFERENTIATION

##### Extension

- In each of these subtractions, explain how to split up 953 to solve the problem, then find the answers:  $953 - 234$ ;  $953 - 184$ ;  $953 - 594$ ;  $953 - 284$ ;  $953 - 388$  ...

##### Intervention

- Have students model Mary's methods using play money or base ten blocks

## The Algorithm: Making Sense

### Problems:

1. “To work out  $856 - 138$ , Mary rearranges 856 as  $800 + 40 + 16$ . Why does she do this?”

Explain, using play money or base ten blocks, if necessary.

2. “To work out  $856 - 162$ , Mary rearranges 856 as  $700 + 150 + 6$ . Why does she do this?”

Explain, using play money or base ten blocks, if necessary.

3. “To work out  $856 - 168$ , Mary rearranges 856 as  $700 + 140 + 16$ . Why does she do this?” Explain, using play money or base ten blocks, if necessary.

4. “To work out  $856 - 123$ , Mary does not have to rearrange 856 at all. Why not?”

Explain, using play money or base ten blocks, if necessary.

Now establish a standard written form for subtraction. A good way to do this is to explain why  $546 - 278$  requires 546 to be renamed 4 hundreds + 13 tens and 16 ones and link this to the problem below.

$$\begin{array}{r} \phantom{0}13 \\ 4 \phantom{0}16 \\ \cancel{5}4\cancel{6} \\ -278 \\ \hline 268 \end{array}$$

# Analyzing Multiplication & Division Expressions

## 4.NBT.5/6

In this task, students analyze multiplication and division expressions to find patterns and make connections among division and multiplication problems.

### BACKGROUND KNOWLEDGE

In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.

Example: A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

Using Base 10 Blocks: Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.

Using Place Value:  $260 \div 4 = (200 \div 4) + (60 \div 4)$

Using Multiplication:  $4 \times 50 = 200$ ,  $4 \times 10 = 40$ ,  $4 \times 5 = 20$ ;  $50 + 10 + 5 = 65$ ; so  $260 \div 4 = 65$

Some students may need to use manipulatives and other strategies to model their thinking until they begin to “see” the patterns and understand what is happening. The teacher should not point these relationships out for the students. Instead, they should guide the thinking of the students through questions and allow students to discuss their thinking with peers; the teacher should act as a facilitator.

Example:

There are 592 students participating in the Trymathlon. They are put into teams of 8 for the competition. How many students are on each team?

Student 1	Student 2	Student 3
592 divided by 8 There are 70 8's in 560 $592 - 560 = 32$ There are 4 8's in 32 $70 + 4 = 74$	592 divided by 8 I know that 10 8's is 80 If I take out 50 8's that is 400 $592 - 400 = 192$ I can take out 20 more 8's which is 160 $192 - 160 = 32$ 8 goes into 32 4 times I have none left I took out 50, then 20 more, then 4 more That's 74	I want to get to 592 $8 \times 25 = 200$ $8 \times 25 = 200$ $8 \times 25 = 200$ $200 + 200 + 200 = 600$ $600 - 8 = 592$ I had 75 groups of 8 and took one away, so there are 74 teams

### ESSENTIAL QUESTIONS

- How are multiplication and division related to each other?
- What are some simple methods for solving multiplication and division problems?
- What patterns of multiplication and division can assist us in problem solving?

### MATERIALS

- “Analyzing Multiplication & Division Expressions” recording sheet

## **TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION**

In this task, students analyze multiplication and division expressions to find patterns and make connections among division and multiplication problems.

### **Comments**

It is critical for students to understand the relationship that exists between multiplication and division as well as the strong relationship between the dividend, divisor, and quotient. This task is designed to allow students to further explore these relationships.

### **Task Directions**

Students will follow the directions below from the “Analyzing Multiplication & Division Expressions” recording sheet.

### **FORMATIVE ASSESSMENT QUESTIONS**

- What patterns do you notice in the sets of numbers?
- How is each multiplication or division expression related to the others?
- What shortcuts do you think you can learn from analyzing these expressions?
- Can you think of other related multiplication or division problems?

## Analyzing Multiplication & Division Expressions

Use this problem to answer the following ones mentally.

$$1240 \div 4 = 310$$

1. Be able to explain the relationship of each problem to the one above.

$$\begin{aligned} 1240 \div 8 &= q \\ 620 \div d &= 155 \\ 155 \times 4 &= p \\ 620 \div 2 &= q \\ 310 \times 8 &= p \\ 620 \div d &= 310 \end{aligned}$$

2. Make up at least 3 more problems that are related to these.

\_\_\_\_\_ .

3. Swap with your partner and see if you can use only your brain to solve their related problems.

4. Be able to explain the relationship of each of your partner's problems, too.

# New Content

# Common Core Standards – Fractions

<b>GRADE 4 FRACTIONS</b>	
4.NF.1 (SLO 1)	Explain why a fraction $a/b$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
4.NF.2 (SLO 2)	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$ , $=$ , or $<$ , and justify the conclusions, e.g., by using a visual fraction model.
4.NF.3 a-d (SLO 3)	<p>Understand a fraction <math>a/b</math> with <math>a &gt; 1</math> as a sum of fractions <math>1/b</math>.            Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.</p> <p>b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. <i>Examples:</i> <math>3/8 = 1/8 + 1/8 + 1/8</math>; <math>3/8 = 1/8 + 2/8</math>; <math>2 \frac{1}{8} = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8</math>.</p> <p>c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.</p> <p>d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.</p>
<b>Standard below will be carried over into Unit 4</b>	
4.NF.4a -c (SLO 4)	<p>Apply and extend previous understanding of multiplication to multiply a fraction by a whole number.</p> <p>a. Understand a fraction <math>a/b</math> as a multiple of <math>1/b</math>.            For example, use a visual fraction model to represent <math>5/4</math> as the product <math>5 \times (1/4)</math>, recording the conclusion by the equation <math>5/4 = 5 \times (1/4)</math></p> <p>b. Understand a multiple of <math>a/b</math> as a multiple of <math>1/b</math>, and use this understanding to multiply a fraction by a whole number.            For example, use a visual fraction model to express <math>3 \times (2/5)</math> as <math>6 \times (1/5)</math>, recognizing this product as <math>6/5</math>. (In general, <math>n \times (a/b) = (n \times a)/b</math>.)</p> <p>c. Solve word problems involving multiplication of a fraction by a whole number, e.g. by using visual fraction models and equations to represent the problem.            For example: If each person at a party will eat <math>3/8</math> of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</p>
4.NF.5 (SLO 5)	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. <i>For example, express <math>3/10</math> as <math>30/100</math>, and add <math>3/10 + 4/100 = 34/100</math>.</i>

## Essential Concepts

- Visual fraction models can illustrate a principle for generating equivalent fractions. Attention must be paid to how the number and size of the parts differ even though the fractions are the same size.
- A fundamental property of equivalent fractions is: multiplying the numerator and denominator by the same non-zero whole number results in an equivalent fraction.
- Fractions can be compared by using benchmark fractions, and by creating common denominators or common numerators.
- Comparisons are only valid when the two fractions refer to the same whole.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers

## Essential Questions

- How can equivalencies among fractions be determined?
- Why are comparisons of fractions valid only when the two fractions refer to the same size whole?
- What models and strategies can you use to order and compare fractions?
- In what ways can you compare fractions with different denominators?
- How can benchmark fractions help with comparing fractions?
- Why does the whole matter when you compare fractions?
- How can we add and subtract fractions with like denominators?
- Does  $\frac{1}{4} + \frac{2}{4} = \frac{3}{8}$ ? Defend your thinking.
- What models and strategies can we use to add mixed numbers?
- What models and strategies can we use to multiply a fraction by a whole number?
- How is multiplication of fractions alike and different from multiplications of whole numbers?



# Connections to the Mathematical Practices

1	<p><b>Make sense of problems and persevere in solving them</b></p> <p>In <b>fourth</b> grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.</p>
2	<p><b>Reason abstractly and quantitatively</b></p> <p><b>Fourth</b> graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.</p>
3	<p><b>Construct viable arguments and critique the reasoning of others</b></p> <p>In <b>fourth</b> grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connection between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.</p>
4	<p><b>Model with mathematics</b></p> <p>Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.</p>
5	<p><b>Use appropriate tools strategically</b></p> <p><b>Fourth</b> graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.</p>
6	<p><b>Attend to precision</b></p> <p>As <b>fourth</b> graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.</p>
7	<p><b>Look for and make use of structure</b></p> <p>In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.</p>
8	<p><b>Look for and express regularity in repeated reasoning</b></p> <p>Students in fourth grade should notice repetitive actions in computation to make generalizations. Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.</p>

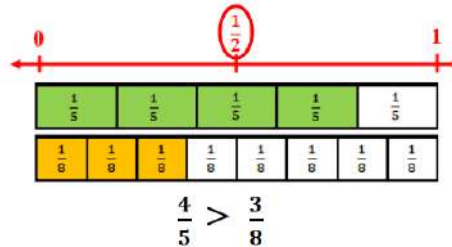
# Vocabulary

## Visual Definition

The terms below are for teacher reference only and are not to be memorized by students.

Teachers should first present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or use them with words, models, pictures, or numbers.

### benchmark fractions



Fractions that are commonly used for estimation. A benchmark fraction helps you compare two fractions.

### common denominator

**12 is a common denominator for**  
 $\frac{2}{3}$  and  $\frac{3}{4}$

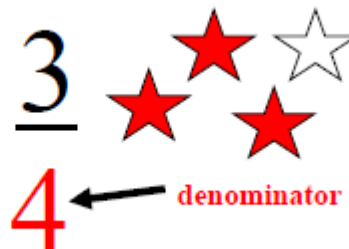
For two or more fractions, a common denominator is a common multiple of the denominators.

### common multiple

4, 8, 12, 16, 20, 24, 28, 32, 36...  
 6, 12, 18, 24, 30, 36, 42...  
 Common Multiples of 4 and 6:  
 12, 24, 36...

Any common multiple of two or more numbers.

### denominator



The quantity below the line in a fraction. It tells the number of equal parts into which a whole is divided.

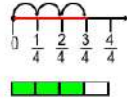
### equivalent fractions



Fractions that have the same value.

# fraction

Measurement Model



Bar Diagram  
(thickened number line)

Set Model



Area Model



A way to describe a part of a whole or a part of a group by using equal parts.

# fraction bar

$$\frac{2}{3} = 2 \div 3$$

A horizontal bar that separates the numerator and the denominator.

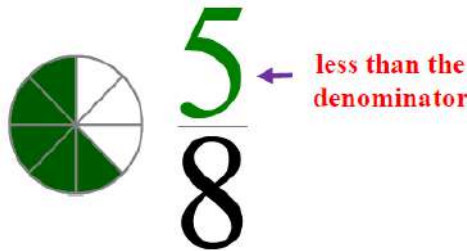
# fraction greater than one

$$\frac{7}{5}$$

← greater than denominator

A fraction where the numerator is greater than the denominator.

# fraction less than one



A fraction less than one. In a proper fraction the numerator is less than the denominator.

# like denominators

$$\frac{3}{8} \quad \frac{5}{8} \quad \frac{7}{8}$$

Denominators in two or more fractions that are the same.

# lowest terms

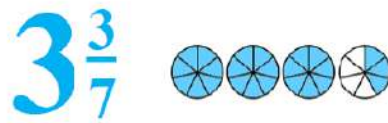


$\frac{4}{8}$  in lowest terms is  $\frac{1}{2}$ .

A fraction where the numerator and denominator have no common factor greater than 1.

**mixed  
number**

**Example:**



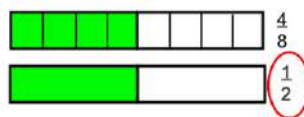
A number with an integer and a fraction part.

**numerator**



The number or expression written above the line in a fraction.

**simplest  
form**



A fraction in simplest form has the fewest possible pieces.

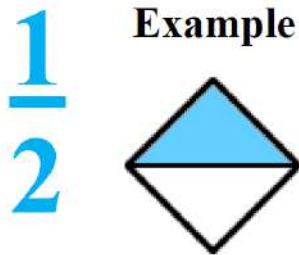
A fraction is in simplest form when the greatest common factor of the numerator and denominator is 1.

**simplify**



To express a fraction in simplest form.

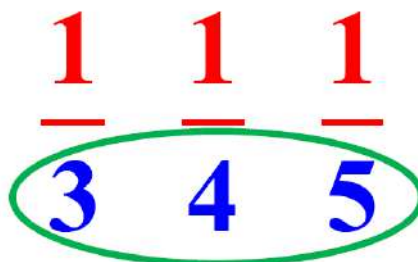
**unit  
fraction**



**Example**

A fraction with a numerator of 1.  
A unit fraction names 1 equal part of a whole.

**unlike  
denominators**

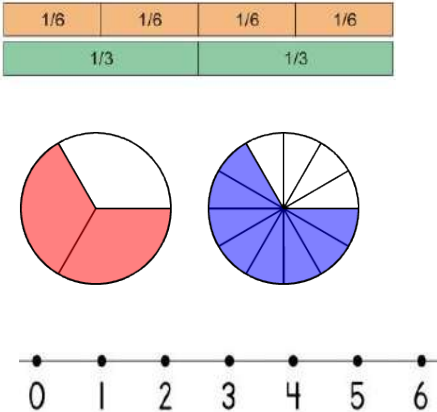
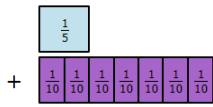

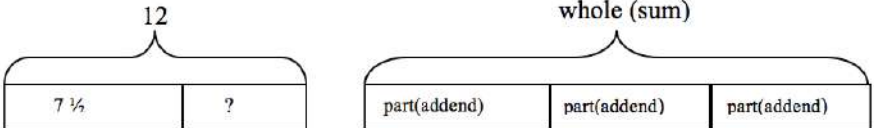
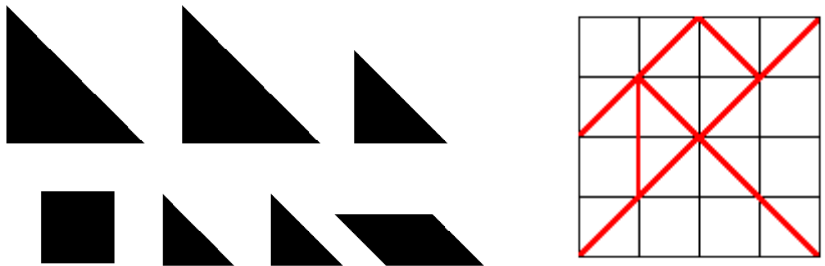


Denominators that are not equal.

# Potential Student Misconceptions

- **Students do not understand the difference between the numerator and denominator.**  
Fraction terminology is not intuitive. Have students count by fractions and highlight the relationship between the numerator and denominator. Continually connect the meaning of numerator and denominator to models.
- **Students believe that the larger the denominator, the larger the piece.**  
This can result from students incorrectly memorizing “the larger the denominator the smaller the piece.” Rather than simple memorization, have students make sense of this relationship themselves. For example, have students investigate whether they would prefer to eat one-hundredth of a 8-inch pizza or one-fourth of a 8-inch pizza. Have them defend their answer in terms of what you’ve heard other students say, that 100 is more than 4, so one-hundredth must be greater.
- **Students believe that the numerator alone determines the size of the fraction.**  
Fractions are a part-to-whole relationship. Have students create models of fractions, and associate the written fraction to the relationship between that part to its whole. Have students confront this relationship using a wide variety of fraction models. Continually connect the vocabulary for fraction names to models.
- **Students add fractions with unlike denominators by adding numerators together and adding denominators together.**  
For example, a student might say  $\frac{1}{2} + \frac{1}{4} = \frac{2}{6}$  when it really equals  $\frac{3}{4}$ . Have students use fraction models such as paper strips to model the problem and to verify their thinking.
- **Students may use the denominator as the decimal when converting fractions to decimals.**  
Students need many opportunities with models, e.g., hundred grids, money. Students will then be able to verify their thinking about  $\frac{1}{2}$  ; e.g., a student might say  $\frac{1}{2} = .2$  or  $.20$  instead of  $.5$  or  $.50$ , however using a model would resolve this misconception.
- **Students don’t understand that equivalent fractions are different names for the same amount.**  
Have students use fraction models to identify equivalent fractions and to verify their reasoning.

# Teaching Multiple Representations – Major Work

<p><b>Equal Partitioning and Unitizing</b> <i>Using Visual Fraction Models</i></p> <ul style="list-style-type: none"> <li>• Fraction Strips</li> <li>• Fraction Circles</li> <li>• Number line</li> </ul>	 <p>Add:  <math>\frac{1}{5} + \frac{7}{10} = ?</math></p> 
<p><b>Bar Model</b></p> 	<p>Leticia read <math>7\frac{1}{2}</math> books for the read-a-thon. She wants to read 12 books in all. How many more books does she have to read?</p>  <p><math>12 - 7\frac{1}{2} = ?</math> or <math>7\frac{1}{2} + ? = 12</math> so Leticia needs to read <math>4\frac{1}{2}</math> more books.</p>
<p><b>Tangram Puzzle</b> Choosing each piece of the Tangram set, students are asked identify the size of the pieces based upon</p> <ul style="list-style-type: none"> <li>• The original square</li> <li>• The size of a select piece</li> <li>• When assigning a value to each piece, for example when the large right triangle is equal to 2.</li> </ul>	
<p><b>Equivalent Fractions</b></p>	<p>For example, <math>\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}</math>. (In general, <math>\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}</math>.)</p>
<p><b>Benchmark Fractions</b></p>	<p><math>\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}</math></p>
<p><b>Abstract Representations</b></p>	
<p><b>Basic Mathematical Properties</b></p>	<p><b>Additive Inverse</b> <b>Example:</b> <math>7 + (-7) = 0</math></p>
<p><b>Algorithm</b></p>	<p>In general, <math>\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}</math></p>



## Number and Operations 4.NF.1-2

Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.) Extend understanding of fraction equivalence and ordering.

### Goal:

Students will explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models. Solve word problems with equivalent fraction while compare two fractions with different numerators and different denominators or by comparing to a benchmark fraction such as  $1/2$ .

### Essential Questions:

- Why is it important to identify fractions (thirds, sixths, eighths, tenths) as representations of equal parts of a whole or of a set?
- What is a fraction?
- How do you know how many fractional parts make a whole?

### Prerequisites:

- Understand fractional parts must be equal-sized
- The number of equal parts tell how many make a whole
- As the number of equal pieces in the whole increases, the size of the fractional pieces decreases

### Embedded Mathematical Practices

**MP.1** Make sense of problems and persevere in solving them

**MP.2** Reason abstractly and quantitatively

**MP.3** Construct viable arguments and critique the reasoning of others

**MP.4** Model with mathematics

**MP.5** Use appropriate tools strategically

**MP.6** Attend to precision

**MP.7** Look for and make use of structure

**MP.8** Look for and express regularity in repeated reasoning

**Lesson 5 - Golden Problem**  
**4.NF.1-2** Extend understanding of fraction equivalence and ordering.

#### Lesson 4

**4.NF.2** Compare two fractions with different numerators and different denominators or by comparing to a benchmark fraction such as  $1/2$ .

#### Lesson 3

**4.NF.2** Compare two fractions with different numerators and different denominators or by comparing to a benchmark fraction such as  $1/2$ .

#### Lesson 2

**4.NF.1** Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models

#### Lesson 1

**4.NF.1** Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models

### Lesson Structure:

Assessment Task  
 Prerequisite Skills  
 Focus Questions  
 Guided Practice  
 Homework  
 Journal Question



# Content Overview: Fraction

Fourth graders expand their work with fractions to include representation of equivalent fractions. They use models to compare and order whole numbers and fractions, including improper fractions and mixed numbers. They are able to locate fractions on a number line. Fourth graders add and subtract fractions with like denominators and develop a rule for this action.

Fourth graders use their knowledge of fractions to read and write tenths and hundredths using fraction notation. They represent equivalent fractions using fraction models such as parts of a set, fraction circles, fraction strips, number lines and other manipulatives. Use the models to determine equivalent fractions.

Fourth graders need to be able to locate fractions on a number line. Use models to order and compare whole numbers and fractions, including mixed numbers and improper fractions. *For example:* Locate on a number line and give a comparison statement about these two fractions, such as "... is less than ..."

Use fraction models to add and subtract fractions with like denominators in real-world and mathematical situations. Develop a rule for addition and subtraction of fractions with like denominators.

Read and write tenths and hundredths in decimal and fraction notations using words and symbols; know the fraction and decimal equivalents for halves and fourths. *For example:*  $0.5 = 0.50$  and  $1\frac{3}{4} = 1.75$ , which can also be written as one and three-fourths or one and seventy-five hundredths.

## **What students should know and be able to do [at a mastery level] related to these benchmarks.**

Students will be able to:

- use fraction models, including the following, to represent and determine equivalent fractions
- parts of whole - fraction circles, fraction strips
- parts of a set
- number lines
- use models to compare and order whole numbers, fractions, including mixed numbers and improper fractions.
- place a variety of fractions (including mixed  $1\frac{1}{2}$  and improper  $\frac{3}{2}$ ) and whole numbers accurately on a number line given pre-placed benchmarks. For example: Place  $\frac{1}{2}$ ,  $\frac{3}{4}$ ,  $\frac{3}{2}$ , and  $1\frac{1}{4}$  on a number line.
- accurately add and subtract fractions with like denominators and describe the process for this computation.

## **Work from previous grades that supports this new learning**

- know fractions can represent parts of a set, parts of a whole, a point on a number line as well as distance on a number line
- understand the concept of numerator and denominator
- understand that the size of a fractional part is relative to the size of the whole (a half of a small pizza is smaller than a half of a large pizza but both represent one-half)
- compare and order unit fractions
- compare and order fractions with like denominators



# Understand Fractions

## Fractions

Fractions are numbers that are needed to solve certain kinds of division problems. Much as the subtraction problem

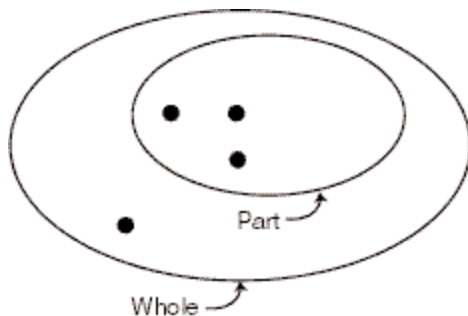
$$3 - 5 = -2$$

creates a need for numbers that are not positive, certain division problems create a need for numbers that are not integers. For example, fractions allow the solution to  $17 \div 3$  to be written as

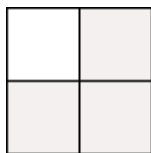
$$17 \div 3 = \frac{17}{3}.$$

When  $a$  and  $b$  are integers and  $b \neq 0$ , then the solution to the division problem  $a \div b$  can be expressed as a fraction  $\frac{a}{b}$ .

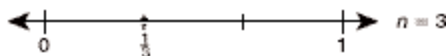
At this grade level, students should learn to identify fractions with models that convey their properties. Proper fractions can be modeled in terms of a part of a whole. The whole may be a group consisting of  $n$  objects where part of the group consists of  $k$  objects and  $k < n$ . The fraction  $\frac{3}{4}$  can be modeled as follows.



Equivalently, the whole may consist of a region that is divided into  $n$  congruent parts,  $k$  of which belong to a sub-region. For example, the fraction  $\frac{3}{4}$  can be identified as the shaded part of the region below.

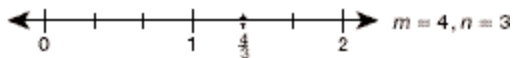


A unit fraction is a fraction with a numerator of 1 (for example,  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{n}$ ). The definition of a unit fraction,  $\frac{1}{n}$ , is to take one unit and divide it into  $n$  equal parts. One of these smaller parts is the amount represented by the unit fraction. On the number line, the unit fraction  $\frac{1}{n}$  represents the length of a segment when a unit interval on the number line is divided into  $n$  equal segments. The point located to the right of 0 on the number line at a distance  $\frac{1}{n}$  from 0 will be  $\frac{1}{n}$ .



The fraction  $\frac{m}{n}$  can represent the quotient of  $m$  and  $n$ , or  $m \div n$ . If the fraction  $\frac{m}{n}$  is defined in terms of the unit fraction  $\frac{1}{n}$ , the fraction  $\frac{m}{n}$  means  $m$  unit fractions  $\frac{1}{n}$ . In terms of distance along the number line, the fraction  $\frac{m}{n}$  means the length of  $m$  abutting segments each of length  $\frac{1}{n}$ . The point  $\frac{m}{n}$  is located to the right of 0 at a distance

$m \times \frac{1}{n}$  from 0. The numerator of the fraction tells how many segments. The denominator tells the size of each segment.



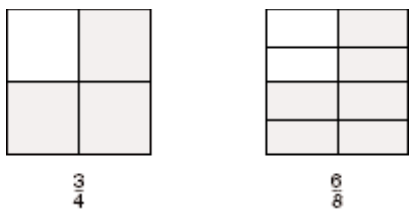
### Finding a Fractional Part of a Number

The word **of** is often used to pose problems involving the multiplication of a whole number by a fraction. At this level, students have not yet learned to multiply fractions. The problem of finding  $\frac{1}{3}$  of 6 can be modeled in terms of a group of 6 objects that has been separated into 3 smaller groups, each of which has 2 objects.



### Equivalent Fractions

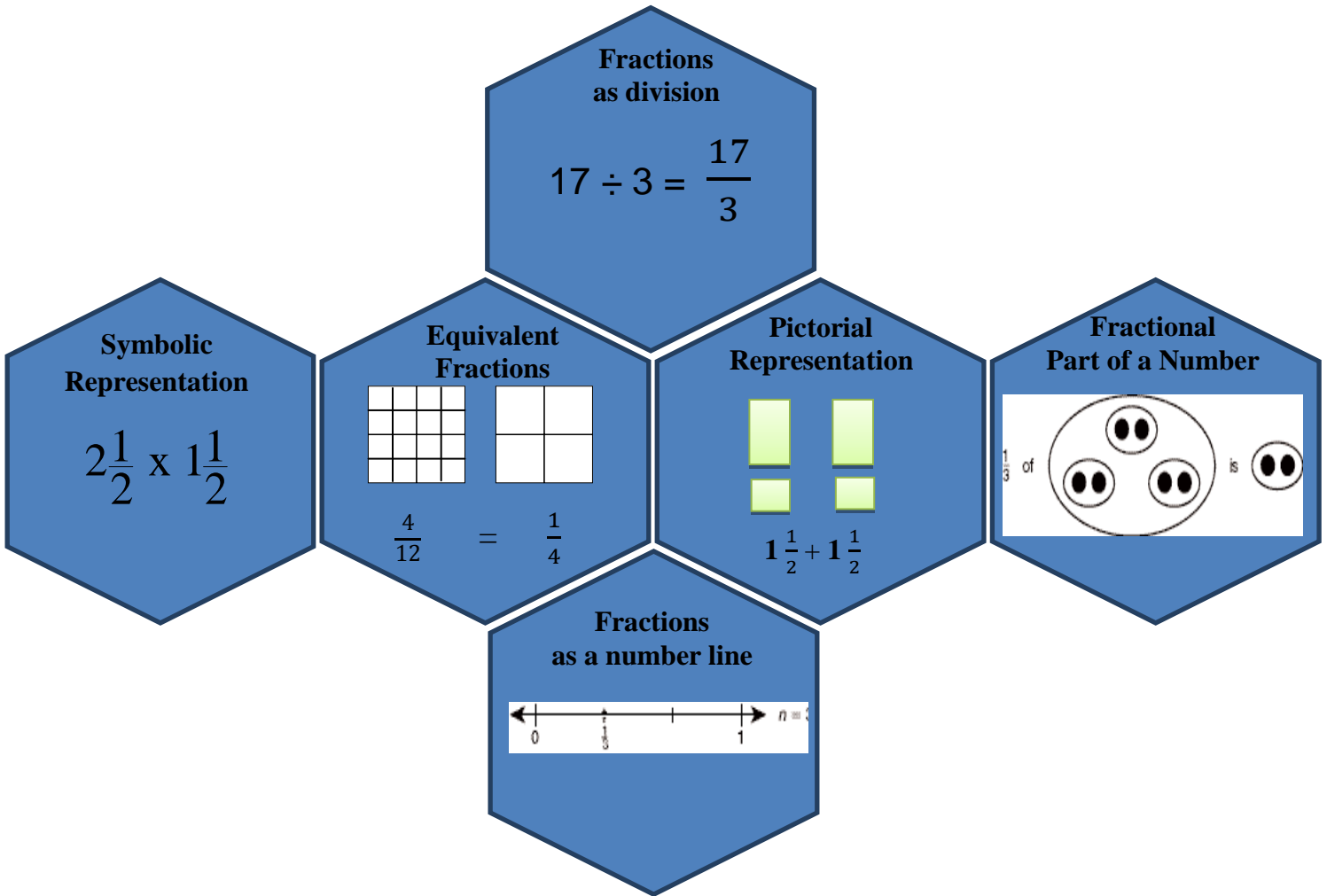
Geometrically, this concept can be conveyed in terms of a picture in which there are two ways of representing the same part of the whole. The fact that  $\frac{3}{4}$  is equivalent to  $\frac{6}{8}$  can be shown as follows.



Because equivalent fractions represent the same number, they are referred to as equal.

A fraction is in simplest form if the numerators and denominators are as small as possible. A more formal way of stating this is to say that in a simplest form fraction, the numerator and denominator have no common factors other than 1.

# Multiple Representations of Fractions



## 4.NF.1: Lesson 1

Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

*Introductory Task*

*Guided Practice*

*Collaborative*

*Homework*

*Assessment*

My mom left  $\frac{8}{12}$  of a pizza pie on the counter. The doorbell rang and one of my sister's friends came over. If they, the two girls, cut what's left into equal parts, what fraction of the whole pizza pie did they each eat?

### Focus Questions

**Question 1:** What do the parts of a fraction tell about its numerator and denominator?

### Journal Question

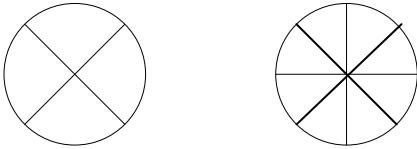
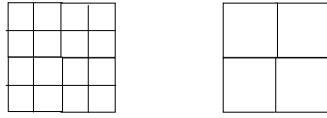
How would you explain to a 3<sup>rd</sup> grader fractions?

## 4.NF.1: Lesson 1

Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Introductory Task	Guided Practice	Collaborative	Homework	Assessment
-------------------	-----------------	---------------	----------	------------

Solve the problems below.

<p>5) Three of 4 equal pieces is the same as 9 of _____ equal pieces.</p>	<p>11) Four of 12 equal pieces is the same as _____ of 3 equal pieces.</p>
<p>6) Split and shade the bars below to show that <math>\frac{3}{4}</math> is equal to <math>\frac{6}{8}</math>.</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="border: 1px solid black; width: 150px; height: 30px;"></div> <div style="border: 1px solid black; width: 150px; height: 30px;"></div> </div>	<p>12) Split and shade the bars below to show that <math>\frac{4}{8}</math> is equal to <math>\frac{1}{2}</math>.</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="border: 1px solid black; width: 150px; height: 30px;"></div> <div style="border: 1px solid black; width: 150px; height: 30px;"></div> </div>
<p>7) Use the circles below to shade an equivalent fraction.</p> <div style="display: flex; justify-content: center; margin-top: 20px;">  </div>	<p>13) Use the squares below to shade an equivalent fraction.</p> <div style="display: flex; justify-content: center; margin-top: 20px;">  </div>
<p>8) Two of ten equal pieces is the same as _____ of 100 equal pieces.</p>	<p>14) Two of three equal pieces is the same as 6 of _____ equal pieces.</p>
<p>6) Nikki gets <math>\frac{2}{5}</math> of a bag of jelly beans. Complete the diagram below to show how many tenths Ari must get so that she gets the same amount of jelly beans as Nikki.</p> $\frac{2}{5} = \frac{\square}{10}$	<p>15) Caleb gets <math>\frac{6}{8}</math> of a ribbon. Complete the diagram below to show how many fourths John must get so that he gets the same amount of ribbon as Caleb.</p> $\frac{6}{8} = \frac{\square}{4}$

## 4.NF.1: Lesson 1

Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

*Introductory Task*

*Guided Practice*

*Collaborative*

*Homework*

*Assessment*

Solve the problems below.

1) 8 of 12 equal pieces is the same as 2 of \_\_\_\_\_ equal pieces.

6) 6 of 10 equal pieces is the same as \_\_\_\_\_ of 5 equal pieces.

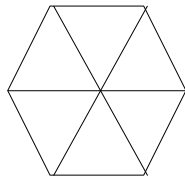
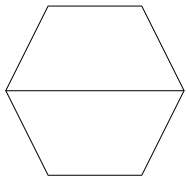
2) Split and shade the bars below to show that  $\frac{2}{3}$  is equal to  $\frac{4}{6}$ .



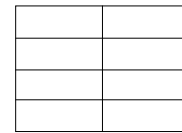
7) Split and shade the bars below to show that  $\frac{10}{12}$  is equal to  $\frac{2}{3}$ .



3) Use the hexagons below to shade an equivalent fraction.



8) Use the rectangles below to shade an equivalent fraction.



4) 6 of ten equal pieces is the same as \_\_\_\_\_ of 100 equal pieces.

9) 1 of three equal pieces is the same as 3 of \_\_\_\_\_ equal pieces.

5) Suzy gets  $\frac{2}{4}$  of an apple pie. Complete the diagram below to show how many tenths Alex must get so that he gets the same amount of apple pie as Suzy.

$$\frac{2}{4} = \frac{\square}{10}$$

10) Carlos gets  $\frac{8}{12}$  of his math problems right. Complete the diagram below to show how many thirds Jose must get so that he gets the same amount of math problems right as Carlos.

$$\frac{8}{12} = \frac{\square}{3}$$

## 4.NF.1: Lesson 2

Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

<i>Introductory Task</i>	<i>Guided Practice</i>	<i>Collaborative</i>	<i>Homework</i>	<i>Assessment</i>
--------------------------	------------------------	----------------------	-----------------	-------------------

Caleb and two friends are sharing three pizzas. Caleb ate  $\frac{5}{10}$  of the plain pizza. His friend Bill ate  $\frac{2}{4}$  of the mushroom pizza and John ate  $\frac{4}{8}$  of the bacon pizza. Do all three friends eat the same amount of pizza? Draw diagrams below to show what fraction of the pizzas each friend eats.

### Focus Questions

**Question 1:** What occurs to a fraction when the denominator increases?

**Question 2:** How can you model fractions?

### Journal Question

How do you determine if a fraction is equivalent to another fraction?

## 4.NF.1: Lesson 2

Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

*Introductory Task*

*Guided Practice*

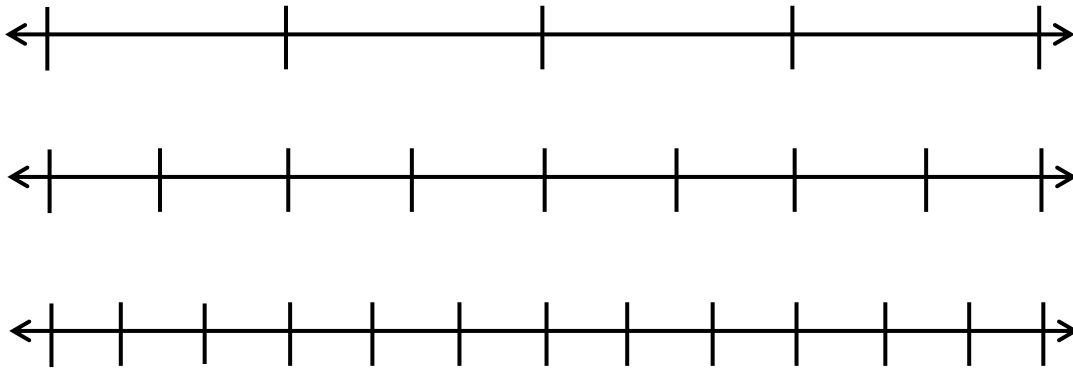
*Collaborative*

*Homework*

*Assessment*

Solve the problems below by splitting and shade the bars, number lines or shapes into equal parts.

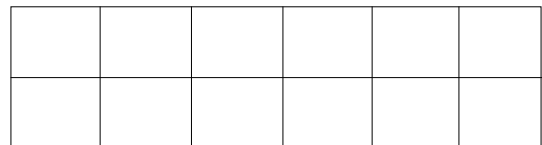
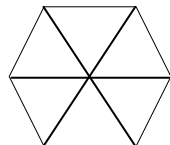
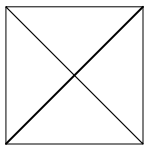
- 1) One of 4 equal pieces is the same as 2 of \_\_\_\_\_ equal pieces and 3 of \_\_\_\_\_ equal pieces.



- 2) Split and shade the bars below to show that  $\frac{4}{4}$  is equal to both  $\frac{8}{8}$  and  $\frac{10}{10}$ .

Three empty rectangular bars are stacked vertically, intended for students to split and shade to demonstrate equivalent fractions.

- 3) Use the shapes below to shade equivalent fractions.





## 4.NF.1: Lesson 2

Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

*Introductory Task*

*Guided Practice*

*Collaborative*

*Homework*

*Assessment*

**Solve the problems below by splitting and shade the bars, number lines or shapes into equal parts.**

4) Six of 10 equal pieces is the same as \_\_\_\_\_ of 100 equal pieces and \_\_\_\_\_ of 5 equal pieces.

5) Pat had a big bag of jelly beans. She took  $\frac{8}{12}$  of the bag for herself. Then shared the rest of the jelly beans with two other friends Tina and Victoria. Victoria wanted  $\frac{2}{3}$  of the jelly beans and Tina wanted  $\frac{4}{6}$ . Draw a model that shows the amount of jelly beans for each person.

6) 2 of 3 equal pieces is the same as 4 of \_\_\_\_\_ equal pieces and 6 of \_\_\_\_\_ equal pieces.

## 4.NF.1: Lesson 2

Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

*Introductory Task*

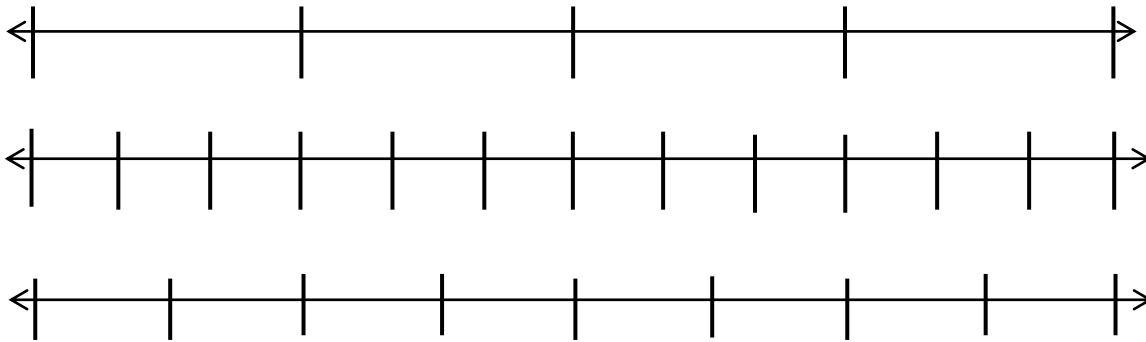
*Guided Practice*

*Collaborative*

*Homework*

*Assessment*

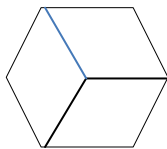
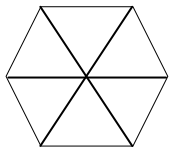
7) Split the number line below to show that  $\frac{3}{4}$  is equal to  $\frac{9}{12}$  is equal to  $\frac{6}{8}$ .



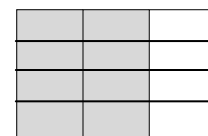
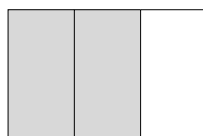
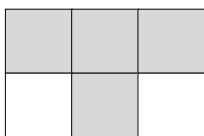
8) Complete the fraction equation below.

$$\frac{1}{5} = \frac{2}{10} = \frac{20}{\square}$$

9) 3 of 6 equal pieces is the same as  $\frac{\square}{3}$  and \_\_\_\_\_ of 12 equal pieces.



10) Use the non-shaded parts of the rectangles to write a fraction equation.



## 4.NF.1: Lesson 2

Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

*Introductory Task*

*Guided Practice*

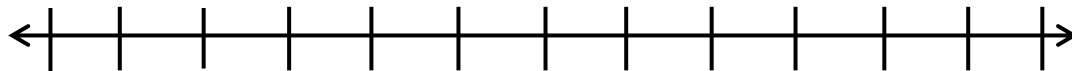
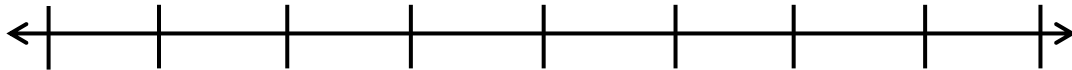
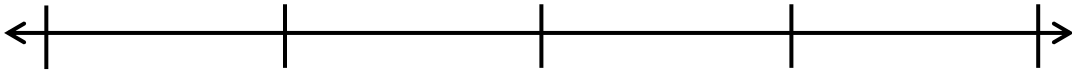
*Collaborative*

*Homework*

*Assessment*

Solve the problems below by splitting and shade the bars, number lines or shapes into equal parts.

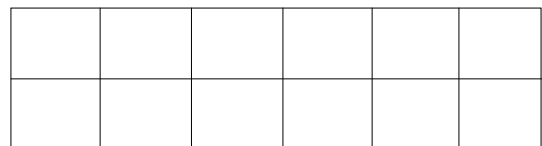
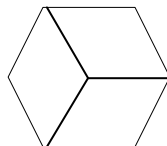
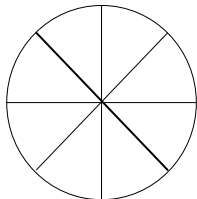
- 1) 8 of 12 equal pieces is the same as 3 of \_\_\_\_\_ equal pieces and 6 of \_\_\_\_\_ equal pieces.



- 2) Split and shade the bars below to show that  $\frac{5}{5}$  is equal to both  $\frac{4}{4}$  and  $\frac{10}{10}$ .



- 3) Use the shapes below to shade equivalent fractions.



## 4.NF.1: Lesson 2

Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

<i>Introductory Task</i>	<i>Guided Practice</i>	<i>Collaborative</i>	<i>Homework</i>	<i>Assessment</i>
--------------------------	------------------------	----------------------	-----------------	-------------------

Solve the problems below by splitting and shade the bars, number lines or shapes into equal parts.

4) 4 of 10 equal pieces is the same as \_\_\_\_\_ of 100 equal pieces and \_\_\_\_\_ of 5 equal pieces.

5) Sam had few boxes of cookies. She took  $\frac{2}{10}$  of the box for herself. Then shared the rest of the cookies with two other friends Katty and Nicole. Katty wanted  $\frac{1}{5}$  of a box and Nicole wanted  $\frac{20}{100}$ . Draw a model that shows the amount of jelly beans for each person.

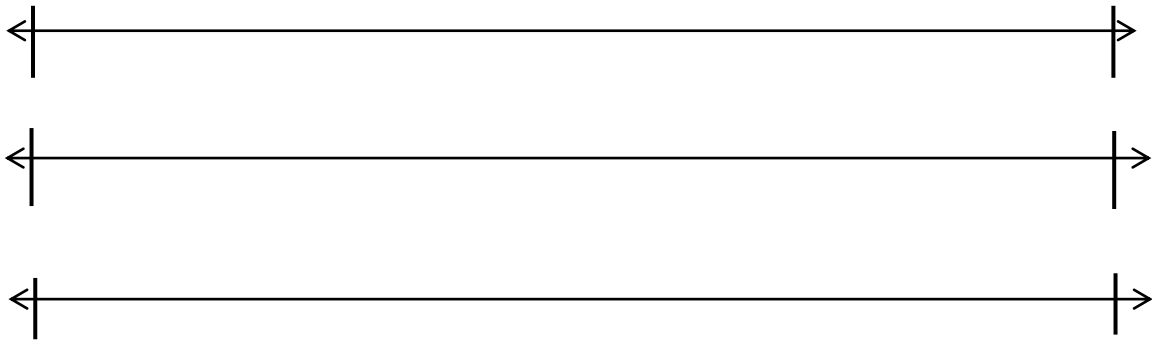
6) 1 of 3 equal pieces is the same as 4 of \_\_\_\_\_ equal pieces and 2 of \_\_\_\_\_ equal pieces.

## 4.NF.1: Lesson 2

Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Introductory Task	Guided Practice	Collaborative	Homework	Assessment
-------------------	-----------------	---------------	----------	------------

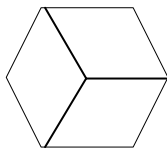
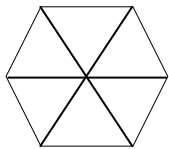
- 7) Split the number line below to show that  $\frac{4}{4}$  is equal to  $\frac{12}{12}$  is equal to  $\frac{8}{8}$ .



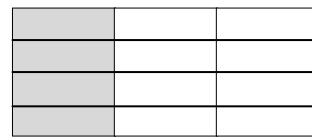
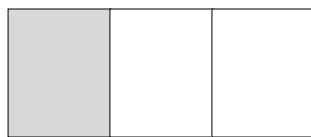
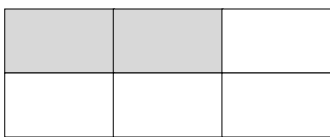
- 8) Complete the fraction equation below.

$$\frac{1}{4} = \frac{2}{\square} = \frac{25}{\square}$$

- 9) 2 of 6 equal pieces is the same as  $\frac{\square}{3}$  and \_\_\_\_\_ of 12 equal pieces.



- 10) Use the non-shaded parts of the rectangles to write a fraction equation.



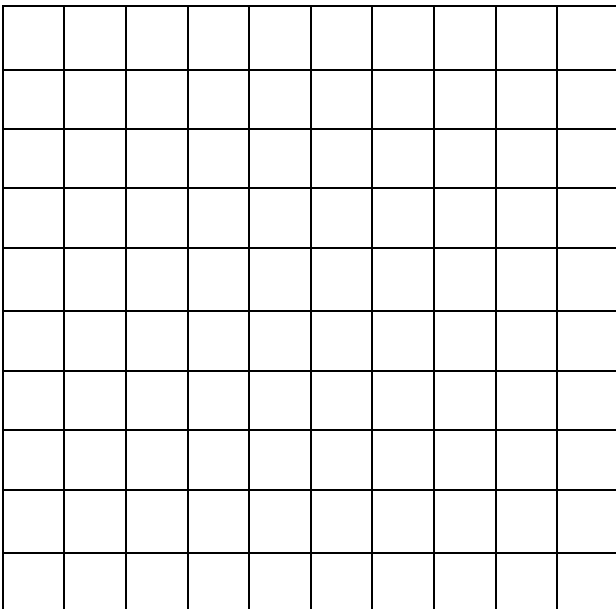
### 4.NF.2: Lesson 3

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

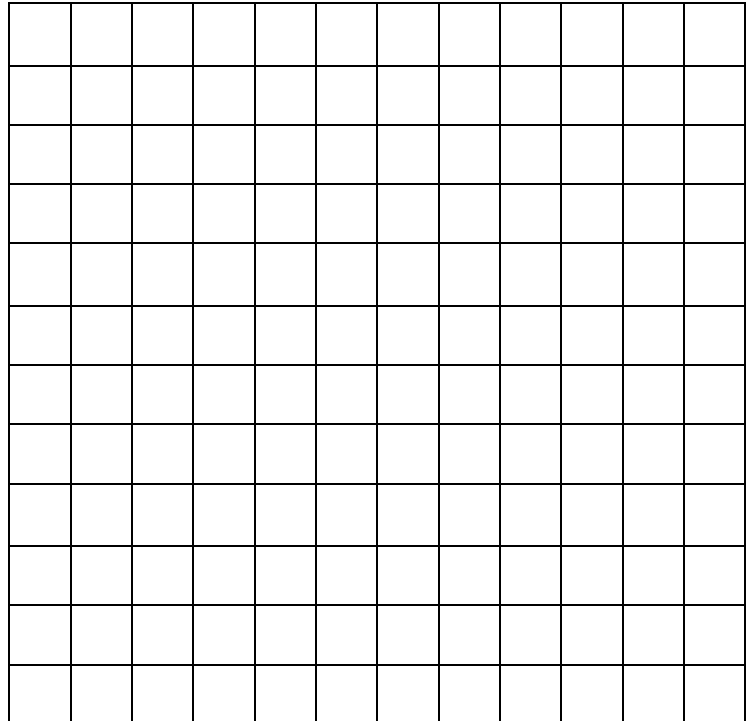
<i>Introductory Task</i>	<i>Guided Practice</i>	<i>Collaborative</i>	<i>Homework</i>	<i>Assessment</i>
--------------------------	------------------------	----------------------	-----------------	-------------------

Mary used a 12 x 12 grid to represent 1 and Janet used a 10 x 10 grid to represent 1. Each girl shaded  $\frac{1}{4}$ . How many grid squares did Mary shade? How many grid squares did Janet shade? Why did they need to shade different numbers of grid squares?

Janet



Mary



#### Focus Questions

**Question 1:** How does changing the denominator in a fraction change the size of the fraction?

**Question 2:** Is taking  $\frac{1}{4}$  of something always the same?

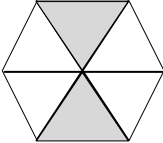
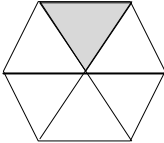
#### Journal Question

Which is closer to a  $\frac{1}{2}$ , is it  $\frac{2}{6}$  or  $\frac{3}{8}$ ?

## 4.NF.2: Lesson 3

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

Solve the problems below by splitting and shade the bars, number lines or shapes into equal parts.

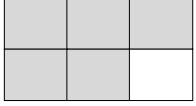
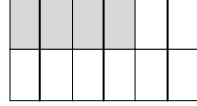
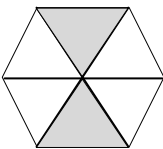
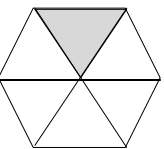
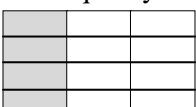
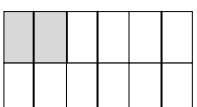
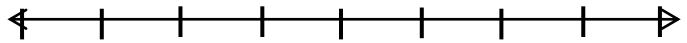

<p>1) There are two cakes on the counter that are the same size. The first cake has <math>\frac{1}{2}</math> of it left. The second cake has <math>\frac{5}{12}</math> left. Which cake has more left?</p>	<p>6) Two friends are debating who has done more homework. John said he has done <math>\frac{3}{8}</math> and Tim said he has done <math>\frac{3}{10}</math>. Who has more homework complete?</p>
<p>2) Which fraction is larger <math>\frac{4}{6}</math> or <math>\frac{5}{8}</math> show your answer by using the bars below.</p> <div style="display: flex; gap: 20px;"> <div style="border: 1px solid black; width: 400px; height: 20px;"></div> <div style="border: 1px solid black; width: 400px; height: 20px;"></div> </div>	<p>7) Split and shade the bars to show that <math>\frac{4}{8} &lt; \frac{1}{4}</math>.</p> <div style="display: flex; gap: 20px;"> <div style="border: 1px solid black; width: 400px; height: 20px;"></div> <div style="border: 1px solid black; width: 400px; height: 20px;"></div> </div>
<p>3) Use <math>&gt;</math>, <math>&lt;</math> or <math>=</math> to complete the fraction inequality.</p> $\frac{6}{8} \square \frac{6}{12}$	<p>8) Use the shaded part of the rectangles to write a fraction inequality.</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; width: 100px; height: 60px; background-color: #cccccc; position: relative;"> <span style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; background-color: white; border: 1px solid black;"></span> </div> <div style="border: 1px solid black; width: 100px; height: 60px; background-color: #cccccc; position: relative;"> <span style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; background-color: white; border: 1px solid black;"></span> </div> </div>
<p>4) Use the unshaded parts of the hexagons to write a fraction inequality.</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div>	<p>9) Use <math>&gt;</math>, <math>&lt;</math> or <math>=</math> to complete the fraction inequality.</p> $\frac{4}{5} \square \frac{5}{5}$
<p>5) Use the shaded part of the rectangles to write a fraction inequality.</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; width: 100px; height: 60px; background-color: #cccccc; position: relative;"> <span style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; background-color: white; border: 1px solid black;"></span> </div> <div style="border: 1px solid black; width: 100px; height: 60px; background-color: #cccccc; position: relative;"> <span style="position: absolute; top: 0; left: 0; right: 0; bottom: 0; background-color: white; border: 1px solid black;"></span> </div> </div>	<p>10) Shade the line graph to show that <math>\frac{5}{8} &gt; \frac{1}{4}</math>.</p> <div style="display: flex; flex-direction: column; gap: 10px;"> <div style="border-top: 1px solid black; width: 100%; position: relative;"> <span style="position: absolute; left: 0; top: -5px;">←</span> <span style="position: absolute; right: 0; top: -5px;">→</span> <div style="position: absolute; top: 5px; left: 0; right: 0; border-bottom: 1px solid black;"> <span style="position: absolute; left: 20%; top: -5px;"> </span> <span style="position: absolute; left: 30%; top: -5px;"> </span> <span style="position: absolute; left: 40%; top: -5px;"> </span> <span style="position: absolute; left: 50%; top: -5px;"> </span> <span style="position: absolute; left: 60%; top: -5px;"> </span> <span style="position: absolute; left: 70%; top: -5px;"> </span> </div> </div> <div style="border-top: 1px solid black; width: 100%; position: relative;"> <span style="position: absolute; left: 0; top: -5px;">←</span> <span style="position: absolute; right: 0; top: -5px;">→</span> <div style="position: absolute; top: 5px; left: 0; right: 0; border-bottom: 1px solid black;"> <span style="position: absolute; left: 30%; top: -5px;"> </span> <span style="position: absolute; left: 60%; top: -5px;"> </span> </div> </div> </div>

## 4.NF.2: Lesson 3

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

Introductory Task	Guided Practice	Collaborative	Homework	Assessment
-------------------	-----------------	---------------	----------	------------

Solve the problems below by splitting and shade the bars, number lines or shapes into equal parts.

<p>1) There are two brownie pans on the counter that are the same size. The first cake has <math>\frac{4}{5}</math> of it left. The second cake has <math>\frac{9}{12}</math> left. Which cake has more left?</p>	<p>6) Two friends are debating who can run the greatest distance Bob said he has <math>\frac{8}{10}</math> of a mile and Ben said he can run <math>\frac{8}{12}</math> of a mile. Who runs the longest distance?</p>
<p>2) Which fraction is larger <math>\frac{3}{6}</math> or <math>\frac{3}{8}</math> show your answer by using the bars below.</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="border: 1px solid black; width: 150px; height: 20px;"></div> <div style="border: 1px solid black; width: 150px; height: 20px;"></div> </div>	<p>7) Split and shade the bars to show that <math>\frac{1}{2} &lt; \frac{5}{6}</math>.</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="border: 1px solid black; width: 150px; height: 20px;"></div> <div style="border: 1px solid black; width: 150px; height: 20px;"></div> </div>
<p>3) Use <math>&gt;</math>, <math>&lt;</math> or <math>=</math> to complete the fraction inequality.</p> $\frac{6}{10} \square \frac{6}{12}$	<p>8) Use the unshaded part of the rectangles to write a fraction inequality.</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;">   </div>
<p>4) Use the shaded parts of the hexagons to write a fraction inequality.</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;">   </div>	<p>9) Use <math>&gt;</math>, <math>&lt;</math> or <math>=</math> to complete the fraction inequality.</p> $\frac{10}{12} \square \frac{40}{100}$
<p>5) Use the unshaded part of the rectangles to write a fraction inequality.</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;">   </div>	<p>10) Shade the line graph to show that <math>\frac{7}{8} &gt; \frac{3}{4}</math>.</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;">   </div>



## 4.NF.2: Lesson 4

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

<i>Introductory Task</i>	<i>Guided Practice</i>	<i>Collaborative</i>	<i>Homework</i>	<i>Assessment</i>
--------------------------	------------------------	----------------------	-----------------	-------------------

Three brothers who own diners are always competing. The brother who owns the Middletown Diner sold  $\frac{6}{8}$  of his brownies on Monday night. The brother who owns the Red Bank Diner sold  $\frac{6}{12}$  of his brownies and the brother who owns the Red Oak Diner sold  $\frac{5}{6}$  of his brownies. Which brother sold the most brownies? Draw and label the three pans of brownies to help determine the winner of Monday's Night Brownie sale.

### Focus Questions

**Question 1:** Using benchmarks how can we determine how much larger a fraction is from another fraction?

**Question 2:** Why do equivalent fractions have the same value?

### Journal Question

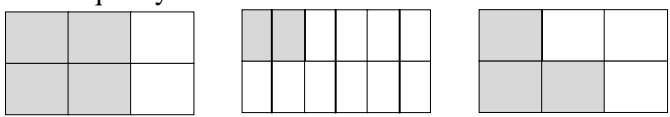
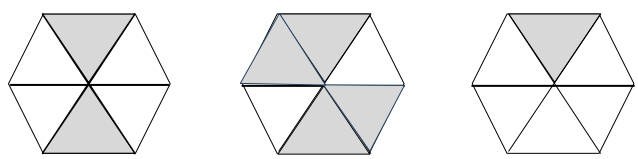
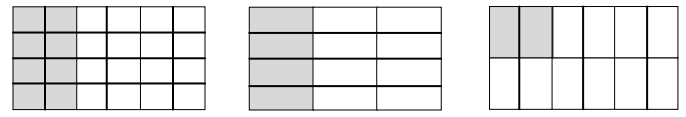
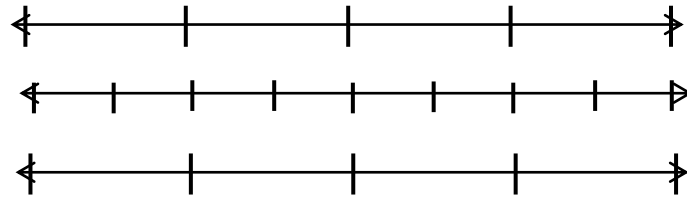
Someone said fractions live between all numbers not just 0 and 1. What do you think this person meant?

## 4.NF.2: Lesson 4

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

Introductory Task	Guided Practice	Collaborative	Homework	Assessment
-------------------	-----------------	---------------	----------	------------

Solve the problems below by splitting and shade the bars, number lines or shapes into equal parts.

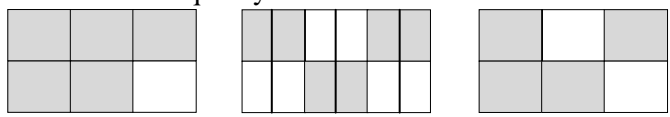
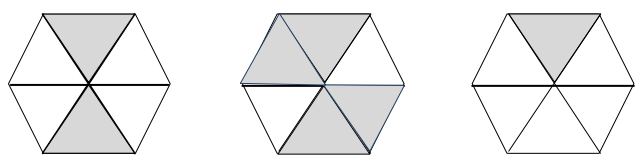
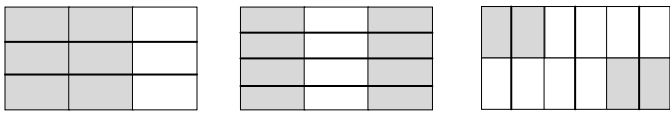
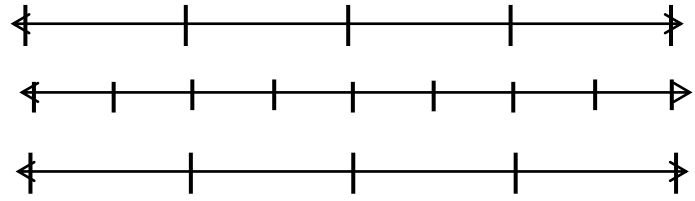
<p>1) There are three cakes on the counter that are the same size. Someone ate <math>\frac{1}{2}</math> of the first cake <math>\frac{2}{3}</math> of the second cake and <math>\frac{10}{12}</math> of the third. Which cake has more left over?</p>	<p>6) Three friends are debating who has most homework complete. John said he has done <math>\frac{3}{5}</math>, Tim said he has done <math>\frac{4}{5}</math> and Jose said he has done <math>\frac{6}{8}</math>. Who has more homework to do?</p>
<p>2) Which fraction is larger <math>\frac{4}{6}</math>, <math>\frac{5}{8}</math> or <math>\frac{2}{4}</math> show your answer by using the bars below.</p> <div style="border: 1px solid black; height: 20px; width: 100%; margin-bottom: 5px;"></div> <div style="border: 1px solid black; height: 20px; width: 100%; margin-bottom: 5px;"></div> <div style="border: 1px solid black; height: 20px; width: 100%;"></div>	<p>7) Split and shade the bars to show that <math>\frac{4}{8} &lt; \frac{1}{4} &lt; \frac{1}{8}</math>.</p> <div style="border: 1px solid black; height: 20px; width: 100%; margin-bottom: 5px;"></div> <div style="border: 1px solid black; height: 20px; width: 100%; margin-bottom: 5px;"></div> <div style="border: 1px solid black; height: 20px; width: 100%;"></div>
<p>3) Use <math>&gt;</math>, <math>&lt;</math> or <math>=</math> to complete the fraction inequality.</p> $\frac{6}{8} \square \frac{6}{12} \square \frac{6}{100}$	<p>8) Use the shaded part of the rectangles to write a fraction inequality.</p> 
<p>4) Use the unshaded parts of the hexagons to write a fraction inequality.</p> 	<p>9) Use <math>&gt;</math>, <math>&lt;</math> or <math>=</math> to complete the fraction inequality.</p> $\frac{2}{5} \square \frac{3}{5} \square \frac{4}{5}$
<p>5) Use the shaded part of the rectangles to write a fraction inequality.</p> 	<p>10) Shade the line graph to show that <math>\frac{3}{4} &gt; \frac{5}{8} &gt; \frac{1}{4}</math>.</p> 

## 4.NF.2: Lesson 4

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

Introductory Task	Guided Practice	Collaborative	Homework	Assessment
-------------------	-----------------	---------------	----------	------------

Solve the problems below by splitting and shade the bars, number lines or shapes into equal parts.

<p>1) There are three cakes on the counter that are the same size. Someone ate <math>\frac{3}{4}</math> of the first cake <math>\frac{2}{3}</math> of the second cake and <math>\frac{4}{5}</math> of the third. Which cake has more left over?</p>	<p>6) Three friends are debating who has the most homework. John said he has done <math>\frac{4}{5}</math>, Tim said he has done <math>\frac{4}{6}</math> and Jose said he has done <math>\frac{4}{8}</math>. Who has more homework to do?</p>
<p>2) Which fraction is larger <math>\frac{4}{6}</math>, <math>\frac{5}{6}</math> or <math>\frac{6}{6}</math> show your answer by using the bars below.</p> <div style="border: 1px solid black; height: 20px; width: 100%; margin-bottom: 5px;"></div> <div style="border: 1px solid black; height: 20px; width: 100%; margin-bottom: 5px;"></div> <div style="border: 1px solid black; height: 20px; width: 100%;"></div>	<p>7) Split and shade the bars to show that <math>\frac{1}{3} &gt; \frac{1}{4} &gt; \frac{1}{5}</math>.</p> <div style="border: 1px solid black; height: 20px; width: 100%; margin-bottom: 5px;"></div> <div style="border: 1px solid black; height: 20px; width: 100%; margin-bottom: 5px;"></div> <div style="border: 1px solid black; height: 20px; width: 100%;"></div>
<p>3) Use <math>&gt;</math>, <math>&lt;</math> or <math>=</math> to complete the fraction inequality.</p> $\frac{6}{10} \square \frac{7}{10} \square \frac{8}{10}$	<p>8) Use the unshaded part of the rectangles to write a fraction inequality.</p> 
<p>4) Use the shaded parts of the hexagons to write a fraction inequality.</p> 	<p>9) Use <math>&gt;</math>, <math>&lt;</math> or <math>=</math> to complete the fraction inequality.</p> $\frac{4}{5} \square \frac{4}{6} \square \frac{4}{10}$
<p>5) Use the shaded part of the rectangles to write a fraction inequality.</p> 	<p>10) Shade the line graph to show that <math>\frac{2}{4} &gt; \frac{3}{8} &gt; \frac{1}{4}</math>.</p> 

## 4.NF.1-2: Lesson 5

Extend understanding of fraction equivalence and ordering.

<i>Introductory Task</i>	<i>Guided Practice</i>	<i>Collaborative</i>	<i>Homework</i>	<i>Assessment</i>
--------------------------	------------------------	----------------------	-----------------	-------------------

The bicycle, track and band clubs are all trying to raise money for new uniforms. The principal wants to make sure all the clubs get an equal amount of money from the school. The principal has decided to give money to each club based on the number of students they have participating in the club. The bicycle club will get a total of  $\frac{2}{5}$  of the money. The track club will get  $\frac{4}{10}$  and the band club will get  $\frac{30}{100}$ . Did the principal share the money equally among all three clubs why or why not? Solve the problem by using either bars, number lines or shapes to show the fractional parts.

# Golden TASK RUBRIC

Score	Description
3	Students used representation to find equivalents fractional benchmarks. Students were able to use benchmarks to help estimate the size of the number and compare fractions to see if they were equal. Students were able to develop and use benchmarks that relates to different forms of representation of rational numbers (for example, 25 out of 100 is the same as $\frac{1}{4}$ ). By doing so, students were able to determine that two out of the three fractions were equal and 30/100 would give the band club less money. Students showed their work and gave a clear explanation of the answer to their problem.
2	Students did not use benchmarks to solve the problem, however, they were able to determine that two out of the three fractions were equal and 30/100 would give the band club less money. Students showed their work and gave a clear explanation of the answer to their problem.
1	Students attempted to compare the fractions using representation; however, their answer did not come up with the correct solution. An understanding of using benchmark fractions was not evident in their work.
0	Does not address task, unresponsive, unrelated or inappropriate.

# Fluency Practice

## Worksheet 1

1) $8,915 - 2,596 = \underline{\hspace{2cm}}$	7) $5,653 - 4,517 = \underline{\hspace{2cm}}$
2) $3,578 + 2,216 = \underline{\hspace{2cm}}$	8) Harold has 53,543 marbles. He gives Steve 16,897. How many marbles does Harold have in all?
3) Andrea collects 73,999 Skittles. Andrea's father gives Andrea 26,587 more. How many Skittles does Andrea have?	9) Joshua has 620,876 cards. Christina has 64,456 cards. If Christina gives all of her cards to Joshua, how many cards will Joshua have?
4) If there are 668,895 pencils in a case and Bridget puts 44,444 more pencils inside, how many pencils are in the case?	10) $8,253 + 8,210 = \underline{\hspace{2cm}}$
7) Joan weighs 96,987 pounds on Jupiter. Teresa weighs 34,564 pounds on Jupiter. How much heavier is Joan than Teresa on Jupiter?	11) If there are 79,867 blocks in a box and Christine puts 15,890 more blocks inside, how many blocks are in the box?
8) If there are 41,568 erasers in a box and Stephanie puts 668,983 more erasers inside, how many erasers are in the box?	12) If there are 79,856 blocks in a box and Christine removes 15,567 blocks, how many blocks are in the box?

# Fluency Practice

## Worksheet 2

1) $89,915 + 112,596 = \underline{\hspace{2cm}}$	7) $555,653 - 49,517 = \underline{\hspace{2cm}}$
2) $398,578 + 287,216 = \underline{\hspace{2cm}}$	8) Harold has 513,543 marbles. He gives Steve 116,897. How many marbles does Harold have in all?
3) Andrea collects 773,999 Skittles. Andrea's father gives Andrea 26,587 more. How many Skittles does Andrea have?	9) Joshua has 640,876 cards. Christina has 44,456 cards. If Christina gives all of her cards to Joshua, how many cards will Joshua have?
4) If there are 548,895 pencils in a case and Bridget puts 44,567 more pencils inside, how many pencils are in the case?	10) $809,253 + 8,210 = \underline{\hspace{2cm}}$
5) Joan weighs 196,987 pounds on Jupiter. Teresa weighs 134,564 pounds on Jupiter. How much heavier is Joan than Teresa on Jupiter?	11) If there are 779,867 blocks in a box and Christine puts 15,886 more blocks inside, how many blocks are in the box?
6) If there are 241,568 erasers in a box and Stephanie puts 68,983 more erasers inside, how many erasers are in the box?	12) If there are 979,856 blocks in a box and Christine removes 15,567 blocks, how many blocks are in the box?

# Fluency Practice

## Worksheet 3

1) $989,915 - 112,596 = \underline{\hspace{2cm}}$	7) $555,653 + 249,517 = \underline{\hspace{2cm}}$
2) $698,578 - 287,216 = \underline{\hspace{2cm}}$	8) Harold has 999,543 marbles. He gives Steve 116,897. How many marbles does Harold have in all?
3) Andrea collects 835,989 Skittles. Andrea's father gives Andrea 26,547 more. How many Skittles does Andrea have?	9) Joshua has 650,876 cards. Christina has 244,456 cards. If Christina gives all of her cards to Joshua, how many cards will Joshua have?
4) If there are 548,895 pencils in a case and Bridget puts 441,000 more pencils inside, how many pencils are in the case?	10) $899,953 - 123,210 = \underline{\hspace{2cm}}$
5) Joan weighs 96,657 pounds on Jupiter. Teresa weighs 34,587 pounds on Jupiter. How much heavier is Joan than Teresa on Jupiter?	11) If there are 649,855 blocks in a box and Christine puts 15,555 more blocks inside, how many blocks are in the box?
6) If there are 541,568 erasers in a box and Stephanie puts 68,983 more erasers inside, how many erasers are in the box?	12) If there are 979,856 blocks in a box and Christine removes 125,577 blocks, how many blocks are in the box?



# Fluency Practice

## Worksheet 4

1) $689,923 + 112,545 = \underline{\hspace{2cm}}$	7) $545,653 - 429,517 = \underline{\hspace{2cm}}$
2) $598,578 + 287,216 = \underline{\hspace{2cm}}$	8) Harold has 763,543 marbles. He gives Steve 216,897. How many marbles does Harold have in all?
3) Andrea collects 653,959 Skittles. Andrea's father gives Andrea 36,587 more. How many Skittles does Andrea have?	9) Joshua has 640,876 cards. Christina has 54,436 cards. If Christina gives all of her cards to Joshua, how many cards will Joshua have?
4) If there are 448,844 pencils in a case and Bridget puts 48,567 more pencils inside, how many pencils are in the case?	10) $899,253 + 48,240 = \underline{\hspace{2cm}}$
5) Joan weighs 16,997 pounds on Jupiter. Teresa weighs 14,594 pounds on Jupiter. How much heavier is Joan than Teresa on Jupiter?	11) If there are 766,967 blocks in a box and Christine puts 13,986 more blocks inside, how many blocks are in the box?
6) If there are 641,578 erasers in a box and Stephanie puts 65,983 more erasers inside, how many erasers are in the box?	12) If there are 979,856 blocks in a box and Christine removes 315,576 blocks, how many blocks are in the box?

# Tiles

## 4.NF.3

Students will add and subtract fractions with common denominators.

### BACKGROUND KNOWLEDGE

Students will have experienced seeing fractions as both a bar and as a set. This activity will have students see fractions as portions of an area. By having students create multiple designs with the same criteria they will be forced to verify their results repeatedly, as well as show the cost of each design. Students will also be able to copy their colored tile designs on to grid paper, however they may need their colored tiles to be rearranged to help them determine their fractional worth. For example a student could make the design below but have a difficult time determining what fraction of each color he or she used. Tiles can easily be rearranged to aid the students' fractional understanding.

By allowing students to make several designs they will be forced into verifying their answers as well as thinking critically about what looks artistically pleasing while keeping the cost of each tile in mind. Before asking students to work on this task, be sure students are able to identify the number of equal pieces needed to cover one whole as the denominator, be comfortable with different size "wholes" such as 12 in a dozen, show equivalent fractions with an area model, record on the student sheet equivalent fractions or fraction sets (either by coloring or gluing die cut yellow and red circles), write an equation which shows the equivalent fractions, and write an equation that shows addition of fractions with like denominators.

### ESSENTIAL QUESTIONS

- What is a fraction and how can it be represented?
- How can a fraction represent parts of a set?
- How can I represent fractions in different ways?
- How can I find equivalent fractions?
- How can I add and subtract fractions of a given set?

### MATERIALS

- Colored tiles
- Tiles recording sheet
- Crayons or colored pencils

### TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task students are asked to design tiled coffee tables for a local furniture store. It allows for a lot of creativity, but the tiles cost different amounts so some designs are not profitable. Students will need to design several colorful coffee tables. However, some tiles cost more than others.

### Task Directions

Students will follow directions below from the Fraction Clues task sheet. .

- Obtain a set of colored tiles.
- Work with a partner to make several designs and record it on their activity sheet.
- Keep record of fractional values as well as cost.
- Determine which of their designs is the most cost effective and artistic.

## **FORMATIVE ASSESSMENT QUESTIONS**

- Tell me about your design.
- What tiles are you using most frequently?
- What fraction of the total are your blue tiles? Red tiles? Etc.
- Could you make this design a different way? If so, would it be cheaper or more expensive?

## **DIFFERENTIATION**

### **Extension**

- Once students have completed the task above, this lesson can be extended to have students make a slightly larger coffee table that is perhaps four by eight or even four by nine tiles in area.
- Students could be asked to determine the perimeter of their coffee tables if they were to use standard four inch square ceramic tiles.
- Students could be asked to determine the cost of putting molding around the tiles given a certain cost per foot.
- Adjust the values of the tiles to include money values in cents as well as dollars. For example: yellow \$3.50, red \$1.75, blue \$2.50, green \$1.25

### **Intervention**

- If necessary students could begin this activity with a smaller set
- Also if students are struggling they could attempt with activity with only three colors instead of using all four colored tiles.

Name \_\_\_\_\_ Date \_\_\_\_\_

## Tiles

### Part 1

Julie's Small Furniture Store is selling tiled coffee tables.

The tables have four inch tiles on them in an assortment of colors; yellow, red, green and blue. The store is selling coffee tables that are 4 tiles wide and 6 tiles long. Julie needs your help to design some coffee tables. He wants each table top to have some of each color, and of course he wants it to look great. However, some tiles cost more than others, and yellow tiles are very expensive. Help Julie out by designing 3 table tops of your own. Make sure to include ALL the colors, and pay attention to the price!

Your job has several parts:

1. Use colored tiles and the grid paper below to design at least three coffee tables.
2. Tally the number of each colored tile,
3. Find what fraction that number is out of a total of 24 tiles.
4. Find the cost of each set of colored tile
4. And finally, determine the total cost of your design.

# Fraction Student Olympics

## 4.NF.3

Students will solve story problems with involving mixed numbers.

### ESSENTIAL QUESTIONS

- How do we add/subtract fractions?
- What is an improper fraction and how can it be represented?
- What is a mixed number and how can it be represented?

### MATERIALS

“Fraction Student Olympics” student recording sheet

### TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

#### Comments

Students can be encouraged to participate in Student Olympics and add their scores to find the total score as required in this task.

#### Task Directions

Students will follow the directions below from the “Fraction Student Olympics” student recording sheet. Orange Elementary School is having a field day! One of the events is the long jump. Participants of this event take a running start and then jump as far as they can. The winner is determined by adding the distances jumped in three trials. The highest total wins. Using the jump measures below, determine the winner of this year’s girls’ and boys’ long jump. Show all of your work on a separate sheet of paper.

### FORMATIVE ASSESSMENT QUESTIONS

- How did you find the sum of these mixed numbers?
- Is the fraction a proper or improper fraction? How do you know?
- If the fraction part of the mixed number is improper, what should you do?
- What do you know about the fraction?
- How can you rewrite that mixed number?
- Did you add the whole numbers? Did you add the fractions?
- What strategies helped you add and subtract mixed numbers?

### DIFFERENTIATION

#### Extension

- Challenge students to write and solve a problem based on the jumping distances provided. Then ask students to give the problem to a partner to solve.
- Students can add up all of the boys jumps to determine a total length, then do the same for the girl jumps and compare the two.
- Whenever applicable students can write equivalent fractions for their sums and differences.

Name \_\_\_\_\_ Date \_\_\_\_\_

### Fraction Student Olympics

Orange Elementary School is having a field day! One of the events is the long jump. Participants of this event take a running start and then jump as far as they can. The winner is determined by adding the distances jumped in three trials. The highest total wins. Using the jump measures below, determine the winner of this year's girls' and boys' long jump. Show all of your work on a separate sheet of paper.

1.

Name	1 <sup>st</sup> Jump	2 <sup>nd</sup> Jump	3 <sup>rd</sup> Jump	Total
Jane	$7\frac{2}{12}$ feet	$6\frac{11}{12}$ feet	$7\frac{1}{12}$ feet	
Susan	$6\frac{1}{12}$ feet	$5\frac{5}{12}$ feet	$6\frac{5}{12}$ feet	
Melissa	$6\frac{2}{12}$ feet	$7\frac{9}{12}$ feet	$7\frac{11}{12}$ feet	
Sara	$8\frac{2}{12}$ feet	$8\frac{1}{12}$ feet	$7\frac{10}{12}$ feet	
Meghan	$7\frac{10}{12}$ feet	$8\frac{2}{12}$ feet	$7\frac{10}{12}$ feet	

2. Who had the highest total score for the girls' long jump? \_\_\_\_\_

3.

Name	1 <sup>st</sup> Jump	2 <sup>nd</sup> Jump	3 <sup>rd</sup> Jump	Total
Justin	$8\frac{2}{12}$ feet	$7\frac{11}{12}$ feet	$8\frac{1}{12}$ feet	
Billy	$6\frac{1}{12}$ feet	$7\frac{5}{12}$ feet	$8\frac{5}{12}$ feet	
Mark	$8\frac{2}{12}$ feet	$9\frac{9}{12}$ feet	$8\frac{11}{12}$ feet	
Harry	$6\frac{2}{12}$ feet	$8\frac{1}{12}$ feet	$8\frac{10}{12}$ feet	
Lou	$7\frac{10}{12}$ feet	$7\frac{2}{12}$ feet	$7\frac{10}{12}$ feet	

4. Who had the highest total score for the boys' long jump? \_\_\_\_\_

5. Justin wants to find how long his 2<sup>nd</sup> jump needed to be in order to win the event. In order to score higher than the winner, how far did Carlos need to jump? Explain your thinking using words, numbers, and math pictures as needed.

6. Meghan wants to find how long her 2<sup>nd</sup> jump needed to be in order to win the event. In order to score higher than the winner, how far would Frieda need to jump? Explain your thinking using words, numbers, and math pictures as needed.

# Associated Illustrative Math Tasks

## Money in the Piggy (4.NF.A)

Alicia opened her piggy bank and counted the coins inside. Here is what she found:

- 22 pennies
- 5 nickels
- 5 dimes
- 8 quarters

- a. How many coins are in Alicia's piggy bank?
- b. What fraction of the coins in the piggy bank are dimes?
- c. What is the total value of the coins in the piggy bank? Give your answer in cents: for example \$2.35 is 235 cents.
- d. What fraction of the total value of the coins in the piggy bank is made up of dimes?

## Running Laps (4.NF.A)

Cruz and Erica were both getting ready for soccer.

- Cruz ran 1 lap around the school.
- Erica ran 3 laps around the playground.

Erica said,

*I ran more laps, so I ran farther.*

Cruz said,

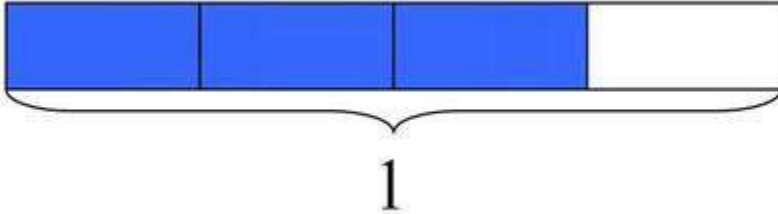
*4 laps around the school is 1 mile, but it takes 12 laps around the playground to go 1 mile. My laps are much longer, so I ran farther.*

Who is right? Draw a picture to help you explain your answer.

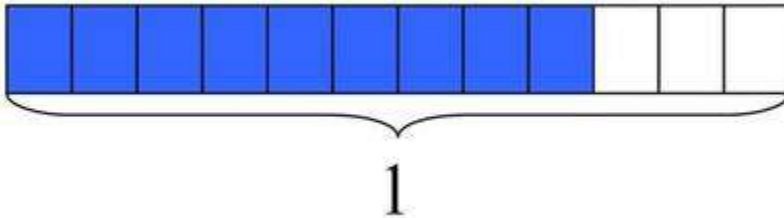


## Explaining Fraction Equivalence with Pictures (4.NF.1)

1. The rectangle below has length 1. What fraction does the shaded part represent?



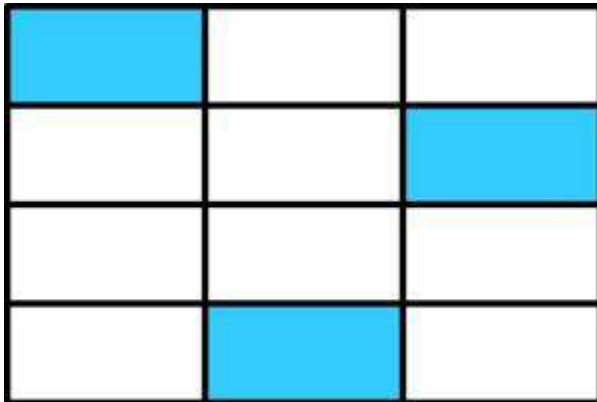
2. The rectangle below has the same length as the rectangle above. What fraction does the shaded part represent?



3. Use the pictures to explain why the two fractions represented above are equivalent.

## Fractions and Rectangles (4.NF.1)

1. What fraction of the rectangle below is shaded?



2. Laura says that  $\frac{1}{4}$  of the rectangle is shaded. Do you think she is correct? Explain why or why not by using the picture.

## Doubling Numerators and Denominators (4.NF.2)

1. How does the value of a fraction change if you double its numerator? Explain your answer.
2. How does the value of a fraction change if you double its denominator? Explain your answer.

## Listing Fractions in Increasing Size (4.NF.2)

Order the following fractions from smallest to largest:

$$\frac{3}{8}, \quad \frac{1}{3}, \quad \frac{5}{9}, \quad \frac{2}{5}$$

Explain your reasoning.

## Using Benchmarks to Compare Fractions (4.NF.2)

Melissa gives her classmates the following explanation for why  $\frac{1}{5} < \frac{2}{7}$ :

I can compare both  $\frac{1}{5}$  and  $\frac{2}{7}$  to  $\frac{1}{4}$ .

Since  $\frac{1}{5}$  and  $\frac{1}{4}$  are unit fractions and fifths are smaller than fourths, I know that  $\frac{1}{5} < \frac{1}{4}$ .

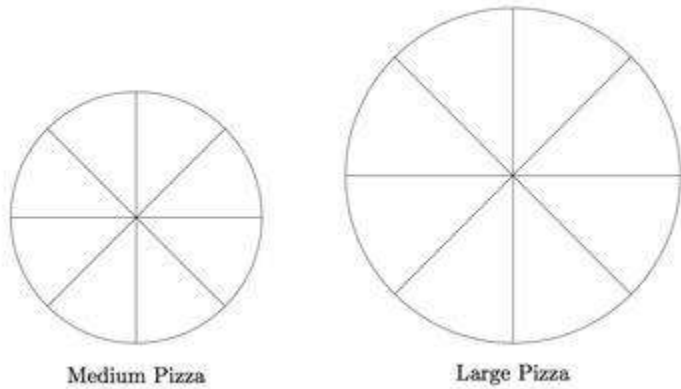
I also know that  $\frac{1}{4}$  is the same as  $\frac{2}{8}$ , so  $\frac{2}{7}$  is bigger than  $\frac{1}{4}$ .

Therefore  $\frac{1}{5} < \frac{2}{7}$ .

1. Explain each step in Melissa's reasoning. Is she correct?
2. Use Melissa's strategy to compare  $\frac{29}{60}$  and  $\frac{45}{88}$ , this time comparing both fractions with  $\frac{1}{2}$ .
3. Use Melissa's strategy to compare  $\frac{8}{25}$  and  $\frac{19}{45}$ . Explain which fraction you chose for comparison and why.

### Comparing Two Different Pizzas (4.NF.B)

Jessica and some friends have ordered two pizzas. One is a medium sized pizza while the other is a large.



Jessica eats two slices of the medium sized pizza. Has Jessica eaten  $\frac{2}{16}$  of the two pizzas? Explain your reasoning, and draw a picture to illustrate your explanation.

### Comparing Sums of Unit Fractions (4.NF.3a)

Use  $<$ ,  $=$ , or  $>$  to compare the following sums:

1.  $\frac{1}{2} + \frac{1}{4}$  \_\_\_\_\_  $\frac{1}{3} + \frac{1}{5}$

2.  $\frac{1}{3} + \frac{1}{2}$  \_\_\_\_\_  $\frac{1}{3} + \frac{1}{4}$

## Making 22 Seventeenths in Different Ways (4.NF.3b)

Which of the following sums are equal to  $\frac{22}{17}$ ?

a.  $\frac{5}{17} + \frac{4}{17} + \frac{3}{17} + \frac{10}{17}$

b.  $\frac{3}{17} + \frac{8}{17} + \frac{3}{17} + \frac{10}{17}$

c.  $\frac{6}{17} + \frac{4}{17} + \frac{3}{17} + \frac{5}{17} + \frac{2}{17} + \frac{2}{17}$

d.  $\frac{12}{17} + \frac{10}{17}$

e.  $\frac{1}{17} + \frac{1}{17} + \frac{9}{17} + \frac{3}{17}$

Find another way to write  $\frac{22}{17}$  as a sum of fractions.

## Writing a Mixed Number as an Equivalent Fraction (4.NF.3b)

Ben wrote a mixed number as the fraction  $7\frac{1}{3}$ . Here is his work:

$$7\frac{1}{3} = \frac{7}{1} + \frac{1}{3} \quad (\text{Step 1})$$

$$= \frac{(7 \times 3) + 1}{3} \quad (\text{Step 2})$$

$$= \frac{21 + 1}{3} \quad (\text{Step 3})$$

$$= \frac{22}{3} \quad (\text{Step 4})$$

Explain what Ben did in each step.



# Assessment Checks

## Assessment Check 1 - Review

1. Hannah was doing a report on animals' sleep habits. She made the charts below to show the number of hours certain animals usually sleep each day.

Animal	Bat	Mouse	Guinea Pig	Possum	Gray Seal
Hours of Sleep	20 hours	12 hours	9 hours	18 hours	6 hours

Animal	Tiger	Horse	Cheetah	Cow	Goat
Hours of Sleep	16 hours	3 hours	12 hours	4 hours	15 hours

a. Fill in the blanks to make the statements true.

- A possum sleeps \_\_\_\_\_ times as many hours a day as a guinea pig.
- A bat sleeps \_\_\_\_\_ times as many hours per day as a cow.

b. Write a multiplication equation to show the relationship between the length of time a gray seal sleeps and the length of time a possum sleeps.

\_\_\_\_\_ x \_\_\_\_\_ = \_\_\_\_\_

c. When Hannah was reading about donkeys, she said, "I can't believe that goats sleep 5 times as many hours per day as donkeys." Find the number of hours per day a donkey sleeps. Show your thinking below using words, numbers, and/or pictures.

A donkey sleeps _____ hours per day.
--------------------------------------

2. Sarah is 12 years old.  
 George is years old.  
 Sarah is 3 times as old as George.

For items a.-c., choose **Yes** or **No** to indicate whether each statement is true.

- a. George's age, in years, can be represented by the expression  $12 \div 3$ .  Yes  No  
 b. George is 15 years old.  Yes  No  
 c. George's age, in years, can be found by solving the equation  $12 = 3 \times g$ .  Yes  No
3. Every year a carnival comes to Hallie's town. The price of tickets to ride the rides has gone up every year.

Year	Ticket Price
2008	\$2.00
2009	\$2.50
2010	\$3.00
2011	\$3.50
2012	\$4.00

- a. In 2008, Hallie's allowance was \$9.00 a month. How many carnival tickets could she buy with one month's allowance?

\_\_\_\_\_

- b. If her allowance had stayed the same, \$9.00 a month, how many carnival tickets could she buy in 2012?

\_\_\_\_\_

- c. In 2012, Hallie's allowance was \$14.00 per month. How much did her monthly allowance increase between 2008 and 2012?

\_\_\_\_\_

- d. How much more did a carnival ticket cost in 2012 than it did in 2008?

\_\_\_\_\_

- e. Was Hallie able to buy more carnival tickets in 2008 or in 2012 with one month's allowance?

\_\_\_\_\_

- f. What would Hallie's allowance need to be in 2012 in order for her to be able to buy as many carnival tickets as she could in 2008?

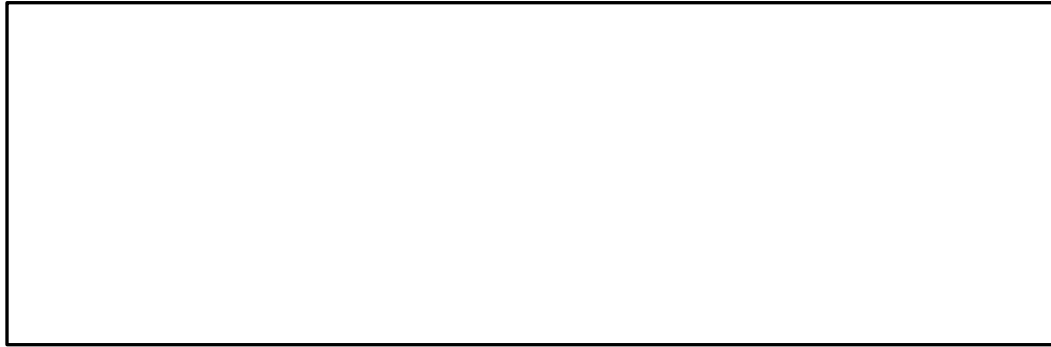
\_\_\_\_\_

- g. What happens to your ability to buy things if prices increase and your allowance doesn't increase?

---

4. Mr. Torres sold a total of 30 boxes of sports cards at his store on Monday. These boxes contained only baseball cards and football cards.
- Each box contained 25 sports cards.
  - He earned \$3 for each sports card he sold.
  - He earned a total of \$1134 from the football cards he sold.

What amount of money did Mr. Torres earn from the baseball cards he sold? In the space below, use pictures, numbers, and/or words to show how you got your answer.



5. Melanie has 819 green balloons. She gave Joan 334 of the balloons. How many green balloons does she now have?  
Show 2 **different** ways for solving the problem.

6. Aidan earned \$805 a month. How much money did he have by the end of the year? Show all your work.  
\$\_\_\_\_\_



## Assessment Check 2 – Fractions

1.



Aisha, Sara, and Brendan have 20 pencils. Aisha says 4 of the pencils are hers. Sara says  $\frac{1}{5}$  of the pencils are hers. Brendan says  $\frac{3}{5}$  of the pencils belong to him. Explain how they all could be right. Use words or drawings.

2. Which fraction is closest to  $\frac{3}{4}$ ? Answer this problem without a calculator.

a.  $\frac{5}{8}$

b.  $\frac{5}{4}$

c.  $\frac{1}{2}$

d.  $\frac{2}{3}$

3. Judy and Gregg are playing a game with fractions. They have to draw their own cards for each fraction. Whoever has the larger fraction wins both cards.

a. Draw the fraction card for each fraction below.



Judy gets the fraction

$$\frac{3}{4}$$



Gregg gets the fraction

$$\frac{5}{8}$$

b. Explain who wins the round and how you know it.

4. Decide whether each expression is True or False?

a.  $\frac{3}{8} < \frac{1}{4}$   True  False

b.  $\frac{1}{2} < \frac{3}{6}$   True  False

c.  $\frac{3}{5} = \frac{8}{10}$   True  False

d.  $\frac{2}{3} = \frac{4}{6}$   True  False

5. Which is larger?  $\frac{4}{6}$  or  $\frac{6}{9}$  ?

a.  $\frac{6}{9}$  because both numbers are greater than numbers in the other fraction. Yes No

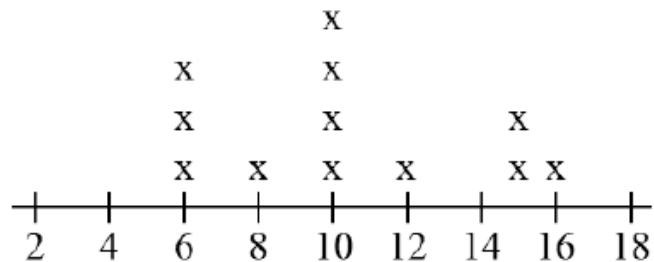
b. They are both the same because they each have a 6 in it. Yes No

c. They are both the same because they are equivalent fractions. Yes No

d.  $\frac{4}{6}$  because there is a smaller difference between numerator and Yes No

6. A zookeeper made this line plot to show the ages of all the monkeys at a zoo.

**Monkey Ages (in years)**



Part A

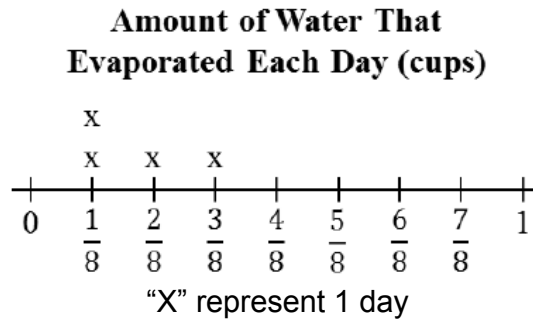
What fraction of the monkeys at this zoo are eight years old?

Part B

What fraction of all the monkeys at this zoo are **not** 8 years old?

7. Judy conducted an experiment. She put a total of  $2\frac{1}{8}$  cups of water into an empty container. Then, Judy recorded the amount of water that evaporated from the container each day for four days.

The plot line below shows the amount of water that evaporated from the container on each of the four days.



What mixed number represents the amount of water left in the container at the end of the fourth day?

8. A builder planned to build houses. Each house will be built on of an acre. How much land would be needed for 7 houses? Show your work.

The builder began with 10 acres of land. After 7 houses were built, how much land was left unused?

Show your work

# Extensions and Sources

## Online Resources

### Common Core Tools

<http://commoncoretools.me/>

<http://www.ccsstoolbox.com/>

<http://www.achievethecore.org/steal-these-tools>

### Manipulatives

<http://nlvm.usu.edu/en/nav/vlibrary.html>

<http://www.explorelarning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000>

<http://www.thinkingblocks.com/>

### Problem Solving Resources

#### \*Illustrative Math Project

<http://illustrativemathematics.org/standards/k8>

<http://illustrativemathematics.org/standards/hs>

The site contains sets of tasks that illustrate the expectations of various CCSS in grades K–8 grade and high school. More tasks will be appearing over the coming weeks. Eventually the sets of tasks will include elaborated teaching tasks with detailed information about using them for instructional purposes, rubrics, and student work.

#### \*Inside Mathematics

<http://www.insidemathematics.org/index.php/tools-for-teachers>

Inside Mathematics showcases multiple ways for educators to begin to transform their teaching practices. On this site, educators can find materials and tasks developed by grade level and content area.

### IXL

<http://www.ixl.com/>

### Sample Balance Math Tasks

<http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/>

### New York City Department of Education

<http://schools.nyc.gov/Academics/CommonCoreLibrary/SeeStudentWork/default.htm>

NYC educators and national experts developed Common Core-aligned tasks embedded in units of study to support schools in implementation of the CCSSM.

#### \*Georgia Department of Education

<https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx>

Georgia State Educator have created common core aligned units of study to support schools as they implement the Common Core State Standards.

### Gates Foundations Tasks

**Minnesota STEM Teachers' Center**

<http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships>

**Singapore Math Tests K-12**

<http://www.misskoh.com>

**Math Score:**

Math practices and assessments online developed by MIT graduates.

<http://www.mathscore.com/>

**Massachusetts Comprehensive Assessment System**

[www.doe.mass.edu/mcas/search](http://www.doe.mass.edu/mcas/search)

**Performance Assessment Links in Math (PALM)**

PALM is currently being developed as an on-line, standards-based, resource bank of mathematics performance assessment tasks indexed via the National Council of Teachers of Mathematics (NCTM).

<http://palm.sri.com/>

**Mathematics Vision Project**

<http://www.mathematicsvisionproject.org/>

**\*NCTM**

<http://illuminations.nctm.org/>

**Assessment Resources**

- \*Illustrative Math: <http://illustrativemathematics.org/>
- \*PARCC: <http://www.parcconline.org/samples/item-task-prototypes>
- NJDOE: <http://www.state.nj.us/education/modelcurriculum/math/> (username: model; password: curriculum)
- DANA: [http://www.ccsstoolbox.com/parcc/PARCCPrototype\\_main.html](http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html)
- New York: <http://www.p12.nysed.gov/assessment/common-core-sample-questions/>
- \*Delaware: <http://www.doe.k12.de.us/assessment/CCSS-comparison-docs.shtml>

PARCC Prototyping Project		
Elementary Tasks (ctrl+click)	Middle Level Tasks (ctrl+click)	High School Tasks (ctrl+click)
<ul style="list-style-type: none"> <li>● <b>Flower gardens (grade 3)</b></li> <li>● <b>Fractions on the number line (grade 3)</b></li> <li>● <b>Mariana's fractions (grade 3)</b></li> <li>● <b>School mural (grade 3)</b></li> <li>● <b>Buses, vans, and cars (grade 4)</b></li> <li>● <b>Deer in the park (grade 4)</b></li> <li>● <b>Numbers of stadium seats</b></li> </ul>	<ul style="list-style-type: none"> <li>● <b>Cake weighing (grade 6)</b></li> <li>● <b>Gasoline consumption (grade 6)</b></li> <li>● <b>Inches and centimeters (grade 6)</b></li> <li>● <b>Anne's family trip (grade 7)</b></li> <li>● <b>School supplies (grade 7)</b></li> <li>● <b>Spicy veggies (grade 7)</b></li> <li>● <b>TV sales (grade 7)</b></li> </ul>	<ul style="list-style-type: none"> <li>● <b><u>Cellular growth</u></b></li> <li>● <b><u>Golf balls in water</u></b></li> <li>● <b><u>Isabella's credit card</u></b></li> <li>● <b><u>Rabbit populations</u></b></li> <li>● <b><u>Transforming graphs of quadratic functions</u></b></li> </ul>

(grade 4) • Ordering juice drinks (grade 4)		
---	--	--

### **Professional Development Resources**

#### **Edmodo**

<http://www.edmodo.com>

Course: iibn34

#### **Clark County School District Wiki Teacher**

<http://www.wiki-teacher.com/wikiDevelopment/unwrappedSearch.php#contentAreaId=6&courseId=474>

#### **Learner Express Modules for Teaching and Learning**

[http://www.learner.org/series/modules/express/videos/video\\_clips.html?type=1&subject=math](http://www.learner.org/series/modules/express/videos/video_clips.html?type=1&subject=math)

#### **Additional Videos**

<http://www.achieve.org/achieving-common-core>; <http://www.youtube.com/user/TheHuntInstitute/videos>

#### **Mathematical Practices**

##### **Inside Mathematics**

<http://www.insidemathematics.org/index.php/common-core-math-intro>

Also see the *Tools for Educators*

##### **The Teaching Channel**

<https://www.teachingchannel.org>

##### **\*Learnzillion**

<https://www.learnzillion.com>

##### **Engage NY**

[http://www.engageny.org/video-library?f\[0\]=im\\_field\\_subject%3A19](http://www.engageny.org/video-library?f[0]=im_field_subject%3A19)

*\*Adaptations of the these resources has been included in various lessons.*