# Mathematical Investigations

Mathematical Investigations is a course designed for students who have successfully completed the Algebra II with Trigonometry course and who choose not to continue mathematics study in the Precalculus or Analytical Mathematics courses. This course may be offered as an elective for students who have completed the four mathematics requirements for graduation.

Mathematical Investigations is intended to extend students' knowledge of mathematical development. Beginning with ancient numeration systems, students explore relationships between mathematics and nature, music, art, and architecture as well as the contributions of well-known mathematicians. It extends the scope of prerequisite courses, integrating topics with an emphasis on application-based problem solving. The wide range of topics and applied problems may lend itself to organizing the content into thematic units. The prerequisites for this course are Algebra I, Geometry, and Algebra II with Trigonometry.

Students will:

## Number and Quantity

Students will:

1. Critique ancient numeration systems and applications, including astronomy and the development and use of money and calendars.

   a. Determine relationships among mathematical achievements of ancient peoples, including the Sumerians, Babylonians, Egyptians, Mesopotamians, Chinese, Aztecs, and Incas.
   b. Explain origins of the Hindu-Arabic numeration system.

      Example: Perform addition and subtraction in both the Hindu-Arabic and the Roman numeration systems to compare place value and place holders.

2. Analyze mathematical relationships in music to interpret frequencies of musical notes and to compare mathematical structures of various musical instruments.

   Examples: Compare frequencies of notes exactly one octave apart on the musical scale; using frequencies and wave patterns of middle C, E above middle C, and G above middle C to explain why the C major chord is harmonious.

   a. Determining lengths of strings necessary to produce harmonic tones as in Pythagorean tuning.

3. Use special numbers, including $e$, $i$, $\pi$, and the golden ratio, to solve application-based problems.

   a. Identify transcendental numbers.

      Example: Calculate $e$ to ten decimal places using a summation with $\frac{1}{n!}$

4. Explain the development and uses of sets of numbers, including complex, real, rational, irrational, integer, whole, and natural numbers.

   a. Analyze contributions to the number system by well-known mathematicians, including Archimedes, John Napier, René Descartes, Sir Isaac Newton, Johann Carl Friedrich Gauss, and Julius Wilhelm Richard Dedekind.

      Example: Plot solutions to the polynomial equation, $x^2 - 6x + 11 = 0$, on the Gaussian plane

## Algebra

5. Identify beginnings of algebraic symbolism and structure through the works of European mathematicians.

   a. Create a Fibonacci sequence when given two initial integers.
   b. Investigate Tartaglia's formula for solving cubic equations.

6. Explain the development and applications of logarithms, including contributions of John Napier, Henry Briggs, and the Bernoulli family.

7. Justify the historical significance of the development of multiple perspectives in mathematics.

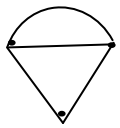
      Example: Relate the historical development of multiple perspectives to the works of Sir Isaac Newton and Gottfried Wilhelm von Leibniz in the foundations of calculus.

   a. Summarize the significance of René Descartes' Cartesian coordinate system.
   b. Interpret the foundation of analytic geometry with regard to geometric curves and algebraic relationships.

## Geometry

8. Solve problems from non-Euclidean geometry, including graph theory, networks, topology, and fractals.

      Examples: Observe the figure to the right to determine if it is traversable, and if it is, describe a path that will traverse it.
      Verify that two objects are topologically equivalent.
      Sketch four iterations of Sierpínski's triangle.



9. Analyze works of visual art and architecture for mathematical relationships.

      Examples: Use Leonardo da Vinci's *Vitruvian Man* to explore the golden ratio.
      Identify mathematical patterns in Maurits Cornelis Escher's drawings, including the use of tessellations in art, quilting, paintings, pottery, and architecture.

   a. Summarize the historical development of perspective in art and architecture.

10. Determine the mathematical impact of the ancient Greeks, including Archimedes, Eratosthenes, Euclid, Hypatia, Pythagoras, and the Pythagorean Society.

    Example: Use Euclid's proposition to inscribe a regular hexagon within a circle.

a. Construct multiple proofs of the Pythagorean Theorem.
b. Solve problems involving figurate numbers, including triangular and pentagonal numbers.

    Example: Write a sequence of the first 10 triangular numbers and hypothesize a formula for finding the $n^{th}$ triangular number.

11. Describe the development of mathematical tools and their applications.

    Examples: Use knotted ropes for counting; Napier's bones for multiplication; a slide rule for multiplying and calculating values of trigonometric, exponential, and logarithmic functions; and a graphing calculator for analyzing functions graphically and numerically.

## Statistics and Probability

12. Summarize the history of probability, including the works of Blaise Pascal; Pierre de Fermat; Abraham de Moivre; and Pierre-Simon, marquis de Laplace.

    Example: Discuss the impact of probability on gaming, economics, and insurance.