Calculus III (One Semester Course)
Overview

Calculus III is a course designed for students who have successfully completed AP Calculus BC, a course covering the content included in a traditional college Calculus I/Calculus II sequence. Calculus III presents a study of differentiation and integration of functions of several variables, parametric curves and surfaces in a three-dimensional setting, and the calculus of vector fields. Calculus III sits outside the Alabama state course of study; topics and standards below are from UAB, through which this course is offered as a dual enrollment course.

Topics in Calculus III include, but are not limited to:

- multivariable vector functions in two and three dimensions, their geometric and algebraic representation, dot product, and cross product
- vector functions, including continuity, derivatives, and integrals
- parametric curves and surfaces
- polar, cylindrical, and spherical coordinates
- velocity, acceleration, arc length, and curvature
- functions of several variables, including continuity, partial derivatives, gradient, and directional derivatives
- linear approximations and tangent planes
- the chain rule
- optimization, including Lagrange multipliers
- double and triple integrals
- iterated integrals
- integration using polar, cylindrical, and spherical coordinates
- change of variables
- line and surface integrals, including surface area
- curl and divergence
- integration theorems, including Green's Theorem, Stokes's Theorem, and Divergence Theorem

Upon successful completion of the course, a student will be able to:

- understand how coordinates and vectors are used in the treatment of three-space problems
- apply one-dimensional calculus techniques to vector-valued functions
- apply the calculus of vector-valued functions to treat motion problems
- understand basic concepts and applications of multi-variable calculus
- solve standard optimization problems
- use different coordinate systems to solve two- and three-dimensional integration problems
- know when and how to apply important concepts from vector analysis

Calculus III Content Units

Unit 1 – Vectors and the Geometry of Space
- Three-Dimensional Coordinate Systems
- Vectors and their Operations
- Line and Plane Equations
- Functions of Multiple Variables
- Cylindrical and Spherical Coordinates

Unit 2 – Vector Functions
- Vector Functions and Space Curves
- Derivatives and Integrals of Vector Functions
- Arc Length and Curvature
- Motion in Space
- Parametric Surfaces

Unit 3 – Differentiation and Applications of Multivariable Functions
- Limits & Continuity
- Partial Derivatives
- Tangent Planes and Linear Approximations
- The Chain Rule
- Directional Derivatives and the Gradient Vector
- Maximum and Minimum Values
- Lagrange Multipliers

Unit 4 – Multiple Integrals
- Double Integrals Over Rectangles
- Iterated Integrals
- Double Integrals Over General Regions
- Double Integrals in Polar Coordinates
- Applications of Double Integrals, including Surface Area
- Triple Integrals
- Triple Integrals in Cylindrical and Spherical Coordinates
- Change of Variables in Multiple Integrals

Unit 5 – Vector Calculus
- Vector Fields
- Line Integrals
- The Fundamental Theorem for Line Integrals
- Green's Theorem
- Curl and Divergence
- Surface Integrals
- Stokes' Theorem
- The Divergence Theorem