3-4 Systems of Equations in Three Variables

Solve each system of equations.

$-3a - 4b + 2c = 28$

1. $a + 3b - 4c = -31$

$2a + 3c = 11$

ANSWER:

(–2, –3, 5)

$3x + 6y - 2z = -6$

3. $2x + y + 4z = 19$

$-5x - 2y + 8z = 62$

ANSWER:

(–4, 3, 6)

$3x + 5y - z = 12$

5. $-2x - 3y + 5z = 14$

$4x + 7y + 3z = 38$

ANSWER:

Infinite solutions

7. **DOWNLOADING** Heather downloaded some television shows. A sitcom uses 0.3 gigabyte of memory; a drama, 0.6 gigabyte; and a talk show, 0.6 gigabyte. She downloaded 7 programs totaling 3.6 gigabytes. There were twice as many episodes of the drama as the sitcom.

a. Write a system of equations for the number of episodes of each type of show.

b. How many episodes of each show did she download?

ANSWER:

a. $s + d + t = 7, d = 2s, 0.3s + 0.6d + 0.6t = 3.6$

b. 2 sitcoms, 4 dramas, 1 talk show

Solve each system of equations.

$4a + 5b - 6c = 2$

9. $-3a - 2b + 7c = -15$

$-a + 4b + 2c = -13$

ANSWER:

(–3, –2, –4)

$4r + 6s - t = -18$

11. $3r + 2s - 4t = -24$

$-5r + 3s + 2t = 15$

ANSWER:

(–2, –1, 4)

$4x + 2y + 6z = 13$

13. $-12x + 3y - 5z = 8$

$-4x + 7y + 7z = 34$

ANSWER:

Infinite solutions

$-6x - 5y + 4z = 53$

15. $5x + 3y + 2z = -11$

$8x - 6y + 5z = 4$

ANSWER:

(–4, –1, 6)

$2x - y + z = 1$

17. $x + 2y - 4z = 3$

$4x + 3y - 7z = -8$

ANSWER:

No solution

$r - 3s + t = 4$

19. $3r - 6s + 9t = 5$

$4r - 9s + 10t = 9$

ANSWER:

Infinite solutions

21. **AMUSEMENT PARKS** Nick goes to the amusement park to ride roller coasters, bumper cars, and water slides. The wait for the roller coasters is 1 hour, the wait for the bumper cars is 20 minutes long, and the wait for the water slides is only 15 minutes long. Nick rode 10 total rides during his visit. Because he enjoys roller coasters the most, the number of times he rode the roller coasters was the sum of the times he rode the other two rides. If Nick waited in line for a total of 6 hours and 20 minutes, how many of each ride did he go on?

ANSWER:

roller coasters: 5; bumper cars: 1; water slides: 4

23. **FINANCIAL LITERACY** Kate invested \$100,000 in three different accounts. If she invested \$30,000 more in account A than account C and is expected to earn \$6300 in interest, how much did she invest in each account?

Account	Expected Interest
A	4%
B	8%
C	10%

ANSWER:

A: \$55,000; B: \$20,000; C: \$25,000

25. **CHALLENGE** The general form of an equation for a parabola is $y = ax^2 + bx + c$, where (x, y) is a point on the parabola. If three points on a parabola are $(2, -10)$, $(-5, -101)$, and $(6, -90)$, determine the values of a, b, and c and write the general form of the equation.

ANSWER:

$y = -3x^2 + 4x - 6; a = -3, b = 4, c = -6$

27. **OPEN ENDED** Write a system of three linear equations that has a solution of $(-5, -2, 6)$. Show that the ordered triple satisfies all three equations.

ANSWER:

Sample answer:

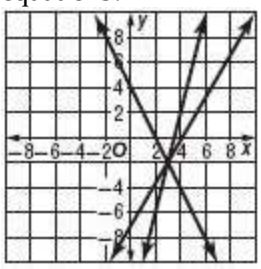
$$3x + 4y + z = -17$$
$$3(-5) + 4(-2) + 6 \stackrel{?}{=} -17$$
$$-15 + (-8) + 6 \stackrel{?}{=} -17$$
$$-23 + 6 \stackrel{?}{=} -17$$
$$-17 = -17 \checkmark$$
$$2x - 5y - 3z = -18$$
$$2(-5) - 5(-2) - 3(6) \stackrel{?}{=} -18$$
$$-10 + 10 - 18 \stackrel{?}{=} -18$$
$$-18 = -18 \checkmark$$
$$-x + 3y + 8z = 47$$
$$-(-5) + 3(-2) + 8(6) \stackrel{?}{=} 47$$
$$5 - 6 + 48 \stackrel{?}{=} 47$$
$$-1 + 48 \stackrel{?}{=} 47$$
$$47 = 47 \checkmark$$

29. **WRITING IN MATH** Use your knowledge of solving a system of three linear equations with three variables to explain how to solve a system of four equations with four variables.

ANSWER:

Sample answer: First, combine two of the original equations using elimination to form a new equation with three variables. Next, combine a different pair of the original equations using elimination to eliminate the same variable and form a second equation with three variables. Do the same thing with a third pair of the original equations. You now have a system of three equations with three variables. Follow the same procedure you learned in this section. Once you find the three variables, you need to use them to find the eliminated variable.

31. **ACT/SAT** The graph shows which system of equations?



A
$$y + 14 = 4x$$
$$y = 4 - 2x$$

B
$$-7 = y - \frac{5}{3}x$$
$$y - 14x = 4$$
$$2x = 4 + y$$
$$7 = y - \frac{5}{3}x$$

C
$$y - 14 = 4x$$
$$y = 4 + 2x$$
$$-7 = y + \frac{5}{3}x$$
$$y - 4x = 14$$
$$y = 2x + 4$$
$$7 = y + \frac{5}{3}x$$

D
$$y + 14x = 4$$
$$-2y = 4 + y$$
$$-7 = y - \frac{5}{3}x$$

E

ANSWER:
A

33. Which of the following represents a correct procedure for solving each equation?

F
$$-3(x - 7) = -16$$
$$-3x - 21 = -16$$
$$-3x = 5$$
$$x = -\frac{5}{3}$$

G
$$7 - 4x = 3x + 27$$
$$7 - 7x = 27$$
$$-7x = -\frac{20}{7}$$
$$x = 20$$

H
$$2(x - 4) = 20$$
$$2x - 8 = 20$$
$$2x = 12$$
$$x = 6$$

J
$$6(2x + 1) = 30$$
$$12x + 6 = 30$$
$$12x = 24$$
$$x = 2$$

ANSWER:
J

A feasible region has vertices at (–3, 2), (1, 3), (6, 1), and (2, –2). Find the maximum and minimum values of each function.

35. $f(x, y) = x + 5y$

ANSWER:
16; –8

37. $f(x, y) = -x + 3y$

ANSWER:
9; –8

Solve each system of equations.

39. $x = y + 5$
$3x + y = 19$

ANSWER:
(6, 1)

41. $5x + 3y = 25$
$4x + 7y = -3$

ANSWER:
(8, −5)