Mathematical Modeling
Overview

Mathematical Modeling is a newly-designed, specialized mathematics course developed to expand on and reinforce the concepts introduced in *Geometry with Data Analysis*, *Algebra I with Probability*, and *Algebra II with Statistics* by applying them in the context of mathematical modeling to represent and analyze data and make predictions regarding real-world phenomena. *Mathematical Modeling* is designed to engage students in doing, thinking about, and discussing mathematics, statistics, and modeling in everyday life. It allows students to experience mathematics and its applications in a variety of ways that promote financial literacy and data-based decision-making skills. This course also provides a solid foundation for students who are entering a range of fields involving quantitative reasoning, whether or not they require calculus.

In this course, students explore decision-making for financial planning and management, design in three dimensions, interpreting statistical studies, and creating functions to model change in the environment and society. Measurements are taken from the real world, and technology is used extensively for computation, with an emphasis on students' interpretation and explanation of results in context. Students will develop and use both the Mathematical Modeling Cycle and the Statistical Problem-Solving Cycle in this specialized course to further develop authentic decision-making skills.

It is essential for students to use technology and other mathematical tools such as graphing calculators, online graphing software, and spreadsheets to explore application-based, real-world problems; to develop their mathematical decision-making skills; and to increase precision in complex calculations throughout the mathematical modeling process.

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics, both in the classroom and in everyday life. **The Student Mathematical Practices are standards to be incorporated across all grades.**

Student Mathematical Practices	
1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

The standards in this course extend beyond the essential concepts described in the overview. The standards indicating what students should know or be able to do are listed in the right columns of the content area tables below. Important topics within these content areas are described in the left columns.

Mathematical Modeling Content Standards

Each content standard completes the stem *"Students will…"*

Modeling

Mathematical modeling and statistical problem-solving are extensive, cyclical processes that can be used to answer significant real-world problems.	1. Use the full Mathematical Modeling Cycle or Statistical Problem-Solving Cycle to answer a real-world problem of particular student interest, incorporating standards from across the course. *Examples: Use a mathematical model to design a three-dimensional structure and determine whether particular design constraints are met; to decide under what conditions the purchase of an electric vehicle will save money; to predict the extent to which the level of the ocean will rise due to the melting polar ice caps; or to interpret the claims of a statistical study regarding the economy.*

Financial Planning and Management

Mathematical models involving growth and decay are useful in solving real-world problems involving borrowing and investing; spreadsheets are a frequently-used and powerful tool to assist with modeling financial situations.	2. Use elements of the Mathematical Modeling Cycle to solve real-world problems involving finances. 3. Organize and display financial information using arithmetic sequences to represent simple interest and straight-line depreciation. 4. Organize and display financial information using geometric sequences to represent compound interest and proportional depreciation, including periodic (yearly, monthly, weekly) and continuous compounding. a. Explain the relationship between annual percentage yield (APY) and annual percentage rate (APR) as values for r in the formulas $A=P(1+r)^t$ and $A=Pe^{rt}$. 5. Compare simple and compound interest, and straight-line and proportional depreciation. 6. Investigate growth and reduction of credit card debt using spreadsheets, including variables such as beginning balance, payment structures, credits, interest rates, new purchases, finance charges, and fees.

	7. Compare and contrast housing finance options including renting, leasing to purchase, purchasing with a mortgage, and purchasing with cash. a. Research and evaluate various mortgage products available to consumers. b. Compare monthly mortgage payments for different terms, interest rates, and down payments. c. Analyze the financial consequence of buying a home (mortgage payments vs. potentially increasing resale value) versus investing the money saved when renting, assuming that renting is the less expensive option. 8. Investigate the advantages and disadvantages of various means of paying for an automobile, including leasing, purchasing by cash, and purchasing by loan.

Design in Three Dimensions	
Two- and three-dimensional representations, coordinates systems, geometric transformations, and scale models are useful tools in planning, designing, and constructing solutions to real-world problems.	9. Use the Mathematical Modeling Cycle to solve real-world problems involving the design of three-dimensional objects. 10. Construct a two-dimensional visual representation of a three-dimensional object or structure. a. Determine the level of precision and the appropriate tools for taking the measurements in constructing a two-dimensional visual representation of a three-dimensional object or structure. b. Create an elevation drawing to represent a given solid structure, using technology where appropriate. c. Determine which measurements cannot be taken directly and must be calculated based on other measurements when constructing a two-dimensional visual representation of a three-dimensional object or structure. d. Determine an appropriate means to visually represent an object or structure, such as drawings on paper or graphics on computer screens.

	11. Plot coordinates on a three-dimensional Cartesian coordinate system and use relationships between coordinates to solve design problems. a. Describe the features of a three-dimensional Cartesian coordinate system and use them to graph points. b. Graph a point in space as the vertex of a right prism drawn in the appropriate octant with edges along the *x*, *y*, and *z* axes. *c.* Find the distance between two objects in space given the coordinates of each. *Examples: Determine whether two aircraft are flying far enough apart to be safe; find how long a zipline cable would need to be to connect two platforms at different heights on two trees.* *d.* Find the midpoint between two objects in space given the coordinates of each. *Example: If two asteroids in space are traveling toward each other at the same speed, find where they will collide.* 12. Use technology and other tools to explore the results of simple transformations using three-dimensional coordinates, including translations in the *x, y*, and/or *z* directions; rotations of 90º, 180º, or 270º about the *x, y*, and *z* axes; reflections over the *xy*, *yz*, and *xy* planes; and dilations from the origin. *Example: Given the coordinates of the corners of a room in a house, find the coordinates of the same room facing a different direction.* 13. Create a scale model of a complex three-dimensional structure based on observed measurements and indirect measurements, using translations, reflections, rotations, and dilations of its components. *Example: Develop a plan for a bridge structure using geometric properties of its parts to determine unknown measures and represent the plan in three dimensions.*

Creating Functions to Model Change in the Environment and Society	
Functions can be used to represent general trends in conditions that change over time and to predict future conditions based on present observations.	14. Use elements of the Mathematical Modeling Cycle to make predictions based on measurements that change over time, including motion, growth, decay, and cycling. 15. Use regression with statistical graphing technology to determine an equation that best fits a set of bivariate data, including nonlinear patterns. *Examples: global temperatures, stock market values, hours of daylight, animal population, carbon dating measurements, online streaming viewership* a. Create a scatter plot with a sufficient number of data points to predict a pattern. b. Describe the overall relationship between two quantitative variables (increase, decrease, linearity, concavity, extrema, inflection) or pattern of change. c. Make a prediction based upon patterns. 16. Create a linear representation of non-linear data and interpret solutions, using technology and the process of linearization with logarithms.

Modeling to Interpret Statistical Studies	
Statistical studies allow a conclusion to be drawn about a population that is too large to survey completely or about cause and effect in an experiment.	17. Use the Statistical Problem Solving Cycle to answer real-world questions. *18.* Construct a probability distribution based on empirical observations of a variable. *Example: Record the number of student absences in class each day and find the probability that each number of students will be absent on any future day.* a. Estimate the probability of each value for a random variable based on empirical observations or simulations, using technology. b. Represent a probability distribution by a relative frequency histogram and/or a cumulative relative frequency graph. c. Find the mean, standard deviation, median, and interquartile range of a probability distribution and make long-term predictions about future possibilities. Determine which measures are most appropriate based upon the shape of the distribution.

	19. Construct a sampling distribution for a random event or random sample. *Examples: How many times do we expect a fair coin to come up "heads" in 100 flips, and on average how far away from this expected value do we expect to be on a specific set of flips? What do we expect to be the average height for a random sample of students in a local high school given the mean and standard deviation of the heights of all students in the high school?* a. Use the binomial theorem to construct the sampling distribution for the number of successes in a binary event or the number of positive responses to a yes/no question in a random sample. b. Use the normal approximation of a proportion from a random event or sample when conditions are met. c. Use the central limit theorem to construct a normal sampling distribution for the sample mean when conditions are met. d. Find the long-term probability of a given range of outcomes from a random event or random sample. 20. Perform inference procedures based on the results of samples and experiments. a. Use a point estimator and margin of error to construct a confidence interval for a proportion or mean. b. Interpret a confidence interval in context and use it to make strategic decisions. *Example: short-term and long-term budget projections for a business* c. Perform a significance test for null and alternative hypotheses. d. Interpret the significance level of a test in the context of error probabilities, and use the results to make strategic decisions. *Example: How do you reduce the rate of human error on the floor of a manufacturing plant?* 21. Critique the validity of reported conclusions from statistical studies in terms of bias and random error probabilities.

	22. Conduct a randomized study on a topic of student interest (sample or experiment) and draw conclusions based upon the results. *Example: Record the heights of thirty randomly selected students at your high school. Construct a confidence interval to estimate the true average height of students at your high school. Question whether or not this data provides significant evidence that your school's average height is higher than the known national average, and discuss error probabilities.*