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There Must Be a Rational Explanation
Adding and Subtracting Rational Expressions

Problem Set

Calculate the least common denominator (LCD) for each sum and difference. Describe any restriction(s) for the value of x.

1. $\frac{x}{3} + \frac{x+1}{15}$

   The LCD is 3(5), or 15.

   No restrictions for x

   $\frac{x}{3} + \frac{x+1}{3(5)}$

2. $\frac{7x}{12} + \frac{x-2}{6} - \frac{x^2}{3}$

   The LCD is $3(2^2)$, or 12.

   No restrictions for x

   $\frac{7x}{3(2^2)} + \frac{x-2}{2(3)} - \frac{x^2}{3}$

3. $\frac{x+1}{x} - \frac{x-1}{x^2+x}$

   The LCD is $x(x+1)$, or $x^2 + x$, and $x \neq -1, 0$.

   $\frac{x+1}{x} - \frac{x-1}{x(x+1)}$

4. $\frac{3}{2x} + \frac{x^2+1}{4x^2+8}$

   The LCD is $(2^2)(x)(x^2+2)$, or $4x^3 + 8x$, and $x \neq 0$.

   $\frac{3}{2x} + \frac{x^2+1}{(2^2)(x^2+2)}$

5. $\frac{3x+4}{x} - \frac{5}{6x} + \frac{9}{2x}$

   The LCD is $2(3)(x)$, or $6x$, and $x \neq 0$.

   $\frac{3x+4}{x} - \frac{5}{2(3)x} + \frac{9}{2x}$

6. $\frac{x}{x^2-1} - \frac{x-3}{x-1}$

   The LCD is $(x-1)(x+1)$, or $x^2 - 1$, and $x \neq \pm 1$.

   $\frac{x}{(x-1)(x+1)} - \frac{x-3}{x-1}$

7. $\frac{3x}{x-2} + \frac{x}{2x+4} + \frac{5}{x}$

   The LCD is $2x(x-2)(x+2)$, or $2x^3 - 8x$, and $x \neq -2, 0, 2$.

   $\frac{3x}{x-2} + \frac{x}{2(x+2)} + \frac{5}{x}$

8. $\frac{x-3}{2x^2+7x+6} - \frac{x}{2x+3}$

   The LCD is $(x+2)(2x+3)$, or $2x^2 + 7x + 6$, and $x \neq -2, -\frac{3}{2}$.

   $\frac{x-3}{(x+2)(2x+3)} - \frac{x}{2x+3}$

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Calculate each sum and difference. Simplify the answer when possible.

9. $\frac{x}{2} + \frac{7x}{6}$

$$\frac{x}{2} + \frac{7x}{6} = \frac{x(3)}{2(3)} + \frac{7x}{6}$$
$$= \frac{3x}{6} + \frac{7x}{6}$$
$$= \frac{10x}{6}$$
$$= \frac{5x}{3}$$

10. $\frac{x + 2}{4} - \frac{z}{10}$

$$\frac{x + 2}{4} - \frac{z}{10} = \frac{(x + 2)(5)}{4(5)} - \frac{z(2)}{10(2)}$$
$$= \frac{5x + 10}{20} - \frac{2z}{20}$$
$$= \frac{5x - 2z + 10}{20}$$

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11. $\frac{-2x}{5} - \frac{y}{10} + z$

$$\frac{-2x}{5} - \frac{y}{10} + z = \frac{-2x(2)}{5(2)} - \frac{y}{10} + \frac{z(10)}{10}$$

$$= \frac{-4x}{10} - \frac{y}{10} + \frac{10z}{10}$$

$$= \frac{-4x - y + 10z}{10}$$

12. $\frac{x - 3}{15} - \frac{x - 3}{10}$

$$\frac{x - 3}{15} - \frac{x - 3}{10} = \frac{(x - 3)(2)}{15(2)} - \frac{(x - 3)(3)}{10(3)}$$

$$= \frac{2x - 6}{30} - \frac{3x - 9}{30}$$

$$= \frac{-x + 3}{30}$$

13. $\frac{2x}{7} - \frac{y}{2} + \frac{x+1}{3}$

$$\frac{2x}{7} - \frac{y}{2} + \frac{x+1}{3} = \frac{2x(2)(3)}{7(2)(3)} - \frac{y(3)(7)}{2(3)(7)} + \frac{(x+1)(2)(7)}{3(2)(7)}$$

$$= \frac{12x}{42} - \frac{21y}{42} + \frac{14x+14}{42}$$

$$= \frac{26x - 21y + 14}{42}$$

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14. $\frac{x-1}{4} + \frac{x+2}{2} - \frac{x^2}{8}$

$$\frac{x-1}{4} + \frac{x+2}{2} - \frac{x^2}{8} = \frac{(x-1)(2)}{4(2)} + \frac{(x+2)(4)}{2(4)} - \frac{x^2}{8}$$

$$= \frac{2x-2}{8} + \frac{4x+8}{8} - \frac{x^2}{8}$$

$$= \frac{-x^2 + 6x + 6}{8}$$

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15. $\frac{2x}{5} + \frac{2x^2 - 1}{10} - \frac{4x + 1}{6}$

$$\frac{2x}{5} + \frac{2x^2 - 1}{10} - \frac{4x + 1}{6} = \frac{2x(6)}{5(6)} + \frac{(2x^2 - 1)(3)}{10(3)} - \frac{(4x + 1)(5)}{6(5)}$$

$$= \frac{12x}{30} + \frac{6x^2 - 3}{30} - \frac{20x + 5}{30}$$

$$= \frac{6x^2 - 8x - 8}{30}$$

$$= \frac{2(3x^2 - 4x - 4)}{30}$$

$$= \frac{3x^2 - 4x - 4}{15}$$

16. $\frac{x + 4}{2} - \frac{x + 5}{3} + \frac{x + 6}{21}$

$$\frac{x + 4}{2} - \frac{x + 5}{3} + \frac{x + 6}{21} = \frac{(x + 4)(3)(7)}{2(3)(7)} - \frac{(x + 5)(2)(7)}{3(2)(7)} + \frac{(x + 6)(2)}{21(2)}$$

$$= \frac{21x + 84}{42} - \frac{14x + 70}{42} + \frac{2x + 12}{42}$$

$$= \frac{9x + 26}{42}$$

Calculate each sum and difference. Describe any restriction(s) for the value of *x* and simplify the answer when possible.

17. $\frac{3}{x} + \frac{1}{x + 1}$

$$\frac{3}{x} + \frac{1}{x + 1} = \frac{3(x + 1)}{x(x + 1)} + \frac{1(x)}{(x + 1)(x)}$$

$$= \frac{3x + 3}{x(x + 1)} + \frac{x}{x(x + 1)}$$

$$= \frac{4x + 3}{x(x + 1)};\ x \neq -1, 0$$

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18. $\frac{2}{x - 2} - \frac{5}{x + 3}$

$$\frac{2}{x - 2} - \frac{5}{x + 3} = \frac{2(x + 3)}{(x - 2)(x + 3)} - \frac{5(x - 2)}{(x + 3)(x - 2)}$$

$$= \frac{2x + 6}{(x - 2)(x + 3)} - \frac{5x - 10}{(x - 2)(x + 3)}$$

$$= \frac{-3x + 16}{(x - 2)(x + 3)};\ x \neq -3, 2$$

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19. $\dfrac{x}{2x-1} + \dfrac{x+2}{x}$

$$\frac{x}{2x-1} + \frac{x+2}{x} = \frac{x(x)}{(2x-1)(x)} + \frac{(x+2)(2x-1)}{(x)(2x-1)}$$

$$= \frac{x^2}{(2x-1)(x)} + \frac{2x^2+3x-2}{(2x-1)(x)}$$

$$= \frac{3x^2+3x-2}{(2x-1)(x)};\ x \neq 0, \frac{1}{2}$$

20. $\dfrac{1}{x+3} - \dfrac{1}{x-3}$

$$\frac{1}{x+3} - \frac{1}{x-3} = \frac{1(x-3)}{(x+3)(x-3)} - \frac{1(x+3)}{(x-3)(x+3)}$$

$$= \frac{x-3}{(x+3)(x-3)} - \frac{x+3}{(x+3)(x-3)}$$

$$= \frac{-6}{(x+3)(x-3)};\ x \neq \pm 3$$

21. $\frac{1}{x^2 - 4} - \frac{1}{x - 2}$

$$\frac{1}{x^2 - 4} - \frac{1}{x - 2} = \frac{1}{(x - 2)(x + 2)} - \frac{1}{x - 2}$$
$$= \frac{1}{(x - 2)(x + 2)} - \frac{1(x + 2)}{(x - 2)(x + 2)}$$
$$= \frac{1}{(x - 2)(x + 2)} - \frac{x + 2}{(x - 2)(x + 2)}$$
$$= \frac{-x - 1}{(x - 2)(x + 2)}; x \neq \pm 2$$

22. $\frac{x + 3}{x - 1} + \frac{x - 4}{x + 2}$

$$\frac{x + 3}{x - 1} + \frac{x - 4}{x + 2} = \frac{(x + 3)(x + 2)}{(x - 1)(x + 2)} + \frac{(x - 4)(x - 1)}{(x + 2)(x - 1)}$$
$$= \frac{x^2 + 5x + 6}{(x - 1)(x + 2)} + \frac{x^2 - 5x + 4}{(x - 1)(x + 2)}$$
$$= \frac{2x^2 + 10}{(x - 1)(x + 2)}; x \neq -2, 1$$

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23. $\dfrac{x+1}{x^2-16} - \dfrac{x}{x^2+7x+12}$

$$\frac{x+1}{x^2-16} - \frac{x}{x^2+7x+12} = \frac{x+1}{(x-4)(x+4)} - \frac{x}{(x+3)(x+4)}$$
$$= \frac{(x+1)(x+3)}{(x-4)(x+4)(x+3)} - \frac{x(x-4)}{(x+3)(x+4)(x-4)}$$
$$= \frac{x^2+4x+3}{(x-4)(x+4)(x+3)} - \frac{x^2-4x}{(x-4)(x+4)(x+3)}$$
$$= \frac{8x+3}{(x-4)(x+4)(x+3)};\ x \neq -4, -3, 4$$

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24. $\dfrac{1}{x-4} - \dfrac{x}{x+2} + \dfrac{x^2}{x-1}$

$$\frac{1}{x-4} - \frac{x}{x+2} + \frac{x^2}{x-1} = \frac{1(x+2)(x-1)}{(x-4)(x+2)(x-1)} - \frac{x(x-4)(x-1)}{(x+2)(x-4)(x-1)} + \frac{x^2(x-4)(x+2)}{(x-1)(x-4)(x+2)}$$
$$= \frac{x^2+x-2}{(x-4)(x+2)(x-1)} - \frac{x^3-5x^2+4x}{(x-4)(x+2)(x-1)} + \frac{x^4-2x^3-8x^2}{(x-4)(x+2)\,(x-1)}$$
$$= \frac{x^4-3x^3-2x^2-3x-2}{(x-4)(x+2)(x-1)};\ x \neq -2, 1, 4$$

25. $\frac{x+1}{x^2-3x-4} + \frac{x-3}{x-2}$

$$\frac{x+1}{x^2-3x-4} + \frac{x-3}{x-2} = \frac{x+1}{(x+1)(x-4)} + \frac{x-3}{x-2}$$

$$= \frac{1}{x-4} + \frac{x-3}{x-2}$$

$$= \frac{1(x-2)}{(x-4)(x-2)} + \frac{(x-3)(x-4)}{(x-2)(x-4)}$$

$$= \frac{x-2}{(x-4)(x-2)} + \frac{x^2-7x+12}{(x-4)(x-2)}$$

$$= \frac{x^2-6x+10}{(x-4)(x-2)};\ x \neq -1, 2, 4$$

26. $\frac{x+2}{2x-2} - \frac{-2x-1}{x^2-4x+3}$

$$\frac{x+2}{2x-2} - \frac{-2x-1}{x^2-4x+3} = \frac{x+2}{2(x-1)} - \frac{-2x-1}{(x-1)(x-3)}$$

$$= \frac{(x+2)(x-3)}{2(x-1)(x-3)} - \frac{(-2x-1)(2)}{(x-1)(x-3)(2)}$$

$$= \frac{x^2-x-6}{2(x-1)(x-3)} - \frac{-4x-2}{(2)(x-1)(x-3)}$$

$$= \frac{x^2+3x-4}{2(x-1)(x-3)}$$

$$= \frac{(x-1)(x+4)}{2(x-1)(x-3)}$$

$$= \frac{x+4}{2(x-3)};\ x \neq 1, 3$$

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Different Client, Same Deal

Multiplying and Dividing Rational Expressions

Problem Set

Perform the indicated operation. Simplify the answer when possible.

1. $\frac{2}{21} \cdot \frac{3}{4}$

$$\frac{2}{21} \cdot \frac{3}{4} = \frac{\overset{1}{\cancel{2}}}{\underset{7}{\cancel{21}}} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} = \frac{1}{14}$$

2. $\frac{15}{22} \cdot \frac{8}{15}$

$$\frac{15}{22} \cdot \frac{8}{15} = \frac{\overset{1}{\cancel{15}}}{\underset{11}{\cancel{22}}} \cdot \frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{15}}} = \frac{4}{11}$$

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3. $\frac{27}{32} \cdot \frac{1}{8} \cdot \frac{16}{9}$

$$\frac{27}{32} \cdot \frac{1}{8} \cdot \frac{16}{9} = \frac{\overset{3}{\cancel{27}}}{\underset{2}{\cancel{32}}} \cdot \frac{1}{8} \cdot \frac{\overset{1}{\cancel{16}}}{\underset{1}{\cancel{9}}} = \frac{3}{16}$$

4. $\frac{8}{9} \div \frac{2}{3}$

$$\frac{8}{9} \div \frac{2}{3} = \frac{8}{9} \cdot \frac{3}{2} = \frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{9}}} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} = \frac{4}{3}$$

5. $\frac{4}{21} \div \frac{12}{49}$

$$\frac{4}{21} \div \frac{12}{49} = \frac{4}{21} \cdot \frac{49}{12} = \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{21}}} \cdot \frac{\overset{7}{\cancel{49}}}{\underset{3}{\cancel{12}}} = \frac{7}{9}$$

6. $\frac{1}{8} \div \frac{7}{4} \div \frac{1}{14}$

$$\frac{1}{8} \div \frac{7}{4} \div \frac{1}{14} = \frac{1}{8} \cdot \frac{4}{7} \cdot \frac{14}{1} = \frac{1}{\underset{1}{\cancel{\underset{2}{\cancel{8}}}}} \cdot \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{7}}} \cdot \frac{\overset{1}{\cancel{\overset{2}{\cancel{14}}}}}{1} = 1$$

Multiply each expression. Describe any restriction(s) for the variables and simplify the answer when possible.

7. $\frac{5x^2}{7} \cdot \frac{14}{3x}$

$$\frac{5x^2}{7} \cdot \frac{14}{3x} = \frac{5\cancel{x^2}^{\,x}}{\cancel{7}_{1}} \cdot \frac{\cancel{14}^{\,2}}{3\cancel{x}_{1}}$$

$$= \frac{10x}{3};\ x \neq 0$$

8. $\frac{2ab^2}{5c^3} \cdot \frac{15c}{4a}$

$$\frac{2ab^2}{5c^3} \cdot \frac{15c}{4a} = \frac{\cancel{2a}^{\,1}b^2}{\cancel{5c^3}_{c^2}} \cdot \frac{\cancel{15c}^{\,3}}{\cancel{4a}_{2}}$$

$$= \frac{3b^2}{2c^2};\ a, c \neq 0$$

9. $\frac{3mn^2}{10} \cdot \frac{m^2}{8n} \cdot \frac{20}{3n^2}$

$$\frac{3mn^2}{10} \cdot \frac{m^2}{8n} \cdot \frac{20}{3n^2} = \frac{\cancel{3}^{\,1}m\,\cancel{n^2}^{\,1}}{\cancel{10}_{1}} \cdot \frac{m^2}{\cancel{8}_{4}\,n} \cdot \frac{\cancel{\cancel{20}^{\,2}}^{\,1}}{\cancel{3n^2}_{1}}$$

$$= \frac{m^3}{4n};\ n \neq 0$$

10. $\frac{x+1}{x} \cdot \frac{x^2}{2x+2}$

$$\frac{x+1}{x} \cdot \frac{x^2}{2x+2} = \frac{x+1}{x} \cdot \frac{x^2}{2(x+1)}$$

$$= \frac{\cancel{x+1}^{\,1}}{\cancel{x}_{1}} \cdot \frac{\cancel{x^2}^{\,x}}{2\cancel{(x+1)}_{1}}$$

$$= \frac{x}{2};\ x \neq -1, 0$$

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11. $\frac{x^2 - 4}{x + 5} \cdot \frac{x + 5}{x - 2}$

$$\frac{x^2 - 4}{x + 5} \cdot \frac{x + 5}{x - 2} = \frac{(x - 2)(x + 2)}{x + 5} \cdot \frac{x + 5}{x - 2}$$

$$= \frac{\overset{1}{\cancel{(x - 2)}}(x + 2)}{\underset{1}{\cancel{x + 5}}} \cdot \frac{\overset{1}{\cancel{x + 5}}}{\underset{1}{\cancel{x - 2}}}$$

$$= x + 2;\ x \neq -5, 2$$

12. $\frac{x^2 + 2x - 3}{x^2} \cdot \frac{x^3 + x^2}{x + 3}$

$$\frac{x^2 + 2x - 3}{x^2} \cdot \frac{x^3 + x^2}{x + 3} = \frac{(x - 1)(x + 3)}{x^2} \cdot \frac{x^2(x + 1)}{x + 3}$$

$$= \frac{(x - 1)\overset{1}{\cancel{(x + 3)}}}{\underset{1}{\cancel{x^2}}} \cdot \frac{\overset{1}{\cancel{x^2}}(x + 1)}{\underset{1}{\cancel{x + 3}}}$$

$$= x^2 - 1;\ x \neq -3, 0$$

13. $\frac{x^2 - 4x}{x - 2} \cdot \frac{2 - x}{x}$

$$\frac{x^2 - 4x}{x - 2} \cdot \frac{2 - x}{x} = \frac{x(x - 4)}{x - 2} \cdot \frac{-1(x - 2)}{x}$$

$$= \frac{\overset{1}{\cancel{x}}(x - 4)}{\underset{1}{\cancel{x - 2}}} \cdot \frac{-1\overset{1}{\cancel{(x - 2)}}}{\underset{1}{\cancel{x}}}$$

$$= 4 - x;\ x \neq 0, 2$$

14. $\frac{1}{2x^2 + 3x - 2} \cdot \frac{x^2 - 2x - 8}{x - 4}$

$$\frac{1}{2x^2 + 3x - 2} \cdot \frac{x^2 - 2x - 8}{x - 4} = \frac{1}{(x + 2)(2x - 1)} \cdot \frac{(x + 2)(x - 4)}{x - 4}$$

$$= \frac{1}{\underset{1}{\cancel{(x + 2)}}(2x - 1)} \cdot \frac{\overset{1}{\cancel{(x + 2)}}\overset{1}{\cancel{(x - 4)}}}{\underset{1}{\cancel{x - 4}}}$$

$$= \frac{1}{2x - 1};\ x \neq -2, \frac{1}{2}, 4$$

15. $\frac{x+3}{x-5} \cdot \frac{1}{x^2+6x+9} \cdot (x^2-25)$

$$\frac{x+3}{x-5} \cdot \frac{1}{x^2+6x+9} \cdot (x^2-25) = \frac{x+3}{x-5} \cdot \frac{1}{(x+3)^2} \cdot \frac{(x-5)(x+5)}{1}$$

$$= \frac{\cancel{x+3}}{\cancel{x-5}} \cdot \frac{1}{\cancel{(x+3)^2}} \cdot \frac{\cancel{(x-5)}(x+5)}{1}$$

$$= \frac{x+5}{x+3};\ x \neq -3, 5$$

16. $\frac{5x^2}{x+4} \cdot \frac{3x^2+12x}{7x-7} \cdot \frac{x^2-2x+1}{3}$

$$\frac{5x^2}{x+4} \cdot \frac{3x^2+12x}{7x-7} \cdot \frac{x^2-2x+1}{3} = \frac{5x^2}{x+4} \cdot \frac{3x(x+4)}{7(x-1)} \cdot \frac{(x-1)^2}{3}$$

$$= \frac{5x^2}{\cancel{x+4}} \cdot \frac{\cancel{3}x\cancel{(x+4)}}{7\cancel{(x-1)}} \cdot \frac{\cancel{(x-1)^2}}{\cancel{3}}$$

$$= \frac{5x^4-5x^3}{7};\ x \neq -4, 1$$

Determine the quotient of each expression. Describe any restriction(s) for the variables and simplify the answer when possible.

17. $\frac{3c^2}{5ab} \div \frac{9}{2a}$

$$\frac{3c^2}{5ab} \div \frac{9}{2a} = \frac{3c^2}{5ab} \cdot \frac{2a}{9}$$

$$= \frac{\cancel{3}c^2}{5\cancel{a}b} \cdot \frac{2\cancel{a}}{\cancel{9}}$$

$$= \frac{2c^2}{15b};\ a \neq 0, b \neq 0$$

18. $\frac{4x^2y}{5z^4} \div \frac{2x}{z} \div \frac{1}{2z}$

$$\frac{4x^2y}{5z^4} \div \frac{2x}{z} \div \frac{1}{2z} = \frac{4x^2y}{5z^4} \cdot \frac{z}{2x} \cdot \frac{2z}{1}$$

$$= \frac{4\cancel{x^2}y}{5\cancel{z^4}} \cdot \frac{\cancel{z}}{\cancel{2}\cancel{x}} \cdot \frac{\cancel{2}\cancel{z}}{1}$$

$$= \frac{4xy}{5z^2};\ x \neq 0, z \neq 0$$

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19. $\frac{x^2+1}{x} \div \frac{x^2-1}{2x}$

$$\frac{x^2+1}{x} \div \frac{x^2-1}{2x} = \frac{x^2+1}{x} \cdot \frac{2x}{x^2-1}$$

$$= \frac{x^2+1}{x} \cdot \frac{2x}{(x-1)(x+1)}$$

$$= \frac{x^2+1}{\cancel{x}_1} \cdot \frac{2\cancel{x}^1}{(x-1)(x+1)}$$

$$= \frac{2x^2+2}{x^2-1};\ x \neq -1, 0, 1$$

20. $\frac{x^2+6x-27}{x^2} \div \frac{x^2-3x}{9}$

$$\frac{x^2+6x-27}{x^2} \div \frac{x^2-3x}{9} = \frac{x^2+6x-27}{x^2} \cdot \frac{9}{x^2-3x}$$

$$= \frac{(x-3)(x+9)}{x^2} \cdot \frac{9}{x(x-3)}$$

$$= \frac{\cancel{(x-3)}^1(x+9)}{x^2} \cdot \frac{9}{x\cancel{(x-3)}_1}$$

$$= \frac{9x+81}{x^3};\ x \neq 0, 3$$

21. $\frac{x^2+6x+8}{3x+2} \div \frac{-x-4}{3x^2-x-2}$

$$\frac{x^2+6x+8}{3x+2} \div \frac{-x-4}{3x^2-x-2} = \frac{x^2+6x+8}{3x+2} \cdot \frac{3x^2-x-2}{-x-4}$$

$$= \frac{(x+4)(x+2)}{3x+2} \cdot \frac{(3x+2)(x-1)}{-1(x+4)}$$

$$= \frac{\cancel{(x+4)}^1(x+2)}{\cancel{3x+2}_1} \cdot \frac{\cancel{(3x+2)}^1(x-1)}{-1\cancel{(x+4)}_1}$$

$$= \frac{x^2+x-2}{-1}$$

$$= -x^2 - x + 2;\ x \neq -4, -\frac{2}{3}, 1$$

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22. $\frac{x^2 - 9}{x + 3} \div (x - 3)$

$$\frac{x^2 - 9}{x + 3} \div (x - 3) = \frac{x^2 - 9}{x + 3} \cdot \frac{1}{x - 3}$$

$$= \frac{(x - 3)(x + 3)}{x + 3} \cdot \frac{1}{x - 3}$$

$$= \frac{\cancel{(x - 3)}\cancel{(x + 3)}}{\cancel{x + 3}} \cdot \frac{1}{\cancel{x - 3}}$$

$$= 1;\ x \neq \pm 3$$

23. $\frac{2x^2 - 2x}{x^2 + 2x + 1} \div \frac{3x - 3}{2x + 2}$

$$\frac{2x^2 - 2x}{x^2 + 2x + 1} \div \frac{3x - 3}{2x + 2} = \frac{2x^2 - 2x}{x^2 + 2x + 1} \cdot \frac{2x + 2}{3x - 3}$$

$$= \frac{2x(x - 1)}{(x + 1)^2} \cdot \frac{2(x + 1)}{3(x - 1)}$$

$$= \frac{2x\cancel{(x - 1)}}{\cancel{(x + 1)^2}_{(x+1)}} \cdot \frac{2\cancel{(x + 1)}}{3\cancel{(x - 1)}}$$

$$= \frac{4x}{3x + 3};\ x \neq \pm 1$$

24. $\frac{x^2 + 4x + 3}{2x^2 - 11x + 5} \div \frac{x^2 + 3x}{2x - 1}$

$$\frac{x^2 + 4x + 3}{2x^2 - 11x + 5} \div \frac{x^2 + 3x}{2x - 1} = \frac{x^2 + 4x + 3}{2x^2 - 11x + 5} \cdot \frac{2x - 1}{x^2 + 3x}$$

$$= \frac{(x + 1)(x + 3)}{(2x - 1)(x - 5)} \cdot \frac{2x - 1}{x(x + 3)}$$

$$= \frac{(x + 1)\cancel{(x + 3)}}{\cancel{(2x - 1)}(x - 5)} \cdot \frac{\cancel{2x - 1}}{x\cancel{(x + 3)}}$$

$$= \frac{x + 1}{x^2 - 5x};\ x \neq -3, 0, \frac{1}{2}, 5$$

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25. $\frac{x^2 - 121}{x^2 + x - 20} \div \frac{x^2 - 10x - 11}{x^2 - 25}$

$$\frac{x^2 - 121}{x^2 + x - 20} \div \frac{x^2 - 10x - 11}{x^2 - 25} = \frac{x^2 - 121}{x^2 + x - 20} \cdot \frac{x^2 - 25}{x^2 - 10x - 11}$$

$$= \frac{(x - 11)(x + 11)}{(x + 5)(x - 4)} \cdot \frac{(x + 5)(x - 5)}{(x - 11)(x + 1)}$$

$$= \frac{\cancel{(x - 11)}(x + 11)}{\cancel{(x + 5)}(x - 4)} \cdot \frac{\cancel{(x + 5)}(x - 5)}{\cancel{(x - 11)}(x + 1)}$$

$$= \frac{x^2 + 6x - 55}{x^2 - 3x - 4};\ x \neq -5, -1, 4, 5, 11$$

26. $\frac{(x - 5)^3}{(x + 2)^2(2x - 3)^4} \div \frac{(x - 5)^5}{(x + 2)(2x - 3)^2}$

$$\frac{(x - 5)^3}{(x + 2)^2(2x - 3)^4} \div \frac{(x - 5)^5}{(x + 2)(2x - 3)^2} = \frac{(x - 5)^3}{(x + 2)^2(2x - 3)^4} \cdot \frac{(x + 2)(2x - 3)^2}{(x - 5)^5}$$

$$= \frac{\cancel{(x - 5)^3}}{\underset{(x+2)}{\cancel{(x + 2)^2}}\underset{(2x-3)^2}{\cancel{(2x - 3)^4}}} \cdot \frac{\cancel{(x + 2)}\cancel{(2x - 3)^2}}{\underset{(x-5)^2}{\cancel{(x - 5)^5}}}$$

$$= \frac{1}{(x + 2)(2x - 3)^2(x - 5)^2};\ x \neq -2, \frac{3}{2}, 5$$

10

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Things Are Not Always as They Appear

Solving Rational Equations

Vocabulary

Write the term that best completes each sentence.

1. A(n) ___rational equation___ is an equation containing one or more rational expressions.

2. A(n) ___extraneous solution___ is a solution that results from the process of solving an equation; but is not a valid solution to the equation.

Problem Set

Solve each rational equation using cross multiplication. Describe any restrictions for the value of x. Check your answer(s) and identify any extraneous roots should they occur.

1. $\frac{x - 1}{x + 3} = \frac{x - 2}{x + 1}$

Restrictions: $x \neq -3, -1$

$(x - 1)(x + 1) = (x + 3)(x - 2)$

$x^2 - 1 = x^2 + x - 6$

$-1 = x - 6$

$x = 5$

Check $x = 5$.

$\frac{5 - 1}{5 + 3} \overset{?}{=} \frac{5 - 2}{5 + 1}$

$\frac{4}{8} \overset{?}{=} \frac{3}{6}$

$\frac{1}{2} = \frac{1}{2}$ ✓

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2. $\frac{x + 2}{x - 7} = \frac{x}{x - 3}$

Restrictions: $x \neq 3, 7$

$(x + 2)(x - 3) = x(x - 7)$

$x^2 - x - 6 = x^2 - 7x$

$-x - 6 = -7x$

$-6 = -6x$

$x = 1$

Check $x = 1$.

$\frac{1 + 2}{1 - 7} \stackrel{?}{=} \frac{1}{1 - 3}$

$\frac{3}{-6} \stackrel{?}{=} \frac{1}{-2}$

$-\frac{1}{2} = -\frac{1}{2}$ ✓

3. $\frac{2x - 1}{x + 1} = \frac{2x - 2}{x}$

Restrictions: $x \neq -1, 0$

$x(2x - 1) = (x + 1)(2x - 2)$

$2x^2 - x = 2x^2 - 2$

$-x = -2$

$x = 2$

Check $x = 2$.

$\frac{2(2) - 1}{2 + 1} \stackrel{?}{=} \frac{2(2) - 2}{2}$

$\frac{3}{3} \stackrel{?}{=} \frac{2}{2}$

$1 = 1$ ✓

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4. $\frac{x - 3}{x^2} = \frac{x - 3}{x^2 - 1}$

Restrictions: $x \neq -1, 0, 1$

$(x - 3)(x^2 - 1) = x^2(x - 3)$

$x^3 - 3x^2 - x + 3 = x^3 - 3x^2$

$-x + 3 = 0$

$-x = -3$

$x = 3$

Check $x = 3$.

$\frac{3 - 3}{(3)^2} \stackrel{?}{=} \frac{3 - 3}{(3)^2 - 1}$

$\frac{0}{9} \stackrel{?}{=} \frac{0}{8}$

$0 = 0$ ✓

5. $\frac{x^2 - 1}{x - 1} = \frac{x^2 + 1}{x + 1}$

Restrictions: $x \neq -1, 1$

$(x^2 - 1)(x + 1) = (x - 1)(x^2 + 1)$

$x^3 + x^2 - x - 1 = x^3 - x^2 + x - 1$

$x^2 - x = -x^2 + x$

$2x^2 - 2x = 0$

$2x(x - 1) = 0$

$x = 0$ or $x = 1$

However, $x \neq 1$ because it is a restriction on the variable and thus is an extraneous root, so only check $x = 0$.

Check $x = 0$.

$\frac{(0)^2 - 1}{0 - 1} \stackrel{?}{=} \frac{(0)^2 + 1}{0 + 1}$

$\frac{-1}{-1} \stackrel{?}{=} \frac{1}{1}$

$1 = 1$ ✓

6. $\frac{x + 5}{x - 4} = \frac{x + 4}{x - 5}$

Restrictions: $x \neq 4, 5$

$(x + 5)(x - 5) = (x - 4)(x + 4)$

$x^2 - 25 = x^2 - 16$

$-25 \neq -16$

This equation has no solution.

Solve each rational equation by multiplying both sides of the equation by the least common denominator. Describe any restrictions for the value of x. Check your answer(s) and identify any extraneous roots should they occur.

7. $\frac{2}{x} - \frac{3}{2x} = \frac{1}{x^2}$

Restriction: $x \neq 0$

$2x^2\left(\frac{2}{x} - \frac{3}{2x}\right) = 2x^2\left(\frac{1}{x^2}\right)$

$4x - 3x = 2$

$x = 2$

Check $x = 2$.

$\frac{2}{2} - \frac{3}{2(2)} \stackrel{?}{=} \frac{1}{2^2}$

$1 - \frac{3}{4} \stackrel{?}{=} \frac{1}{4}$

$\frac{1}{4} = \frac{1}{4}$ ✓

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8. $\frac{1}{x} + \frac{1}{x^2} = 2$

Restriction: $x \neq 0$

$$x^2\left(\frac{1}{x} + \frac{1}{x^2}\right) = x^2(2)$$

$$x + 1 = 2x^2$$

$$2x^2 - x - 1 = 0$$

$$(2x + 1)(x - 1) = 0$$

$$x = -\frac{1}{2} \text{ or } x = 1$$

Check $x = -\frac{1}{2}$.

$$\frac{1}{-\frac{1}{2}} + \frac{1}{\left(-\frac{1}{2}\right)^2} \stackrel{?}{=} 2$$

$$-2 + 4 \stackrel{?}{=} 2$$

$$2 = 2 \quad ✓$$

Check $x = 1$.

$$\frac{1}{1} + \frac{1}{1^2} \stackrel{?}{=} 2$$

$$1 + 1 \stackrel{?}{=} 2$$

$$2 = 2 \quad ✓$$

9. $\frac{5}{2x - 4} = \frac{15}{x^2 - 4}$

Restrictions: $x \neq -2, 2$

$$\frac{5}{2(x - 2)} = \frac{15}{(x - 2)(x + 2)}$$

$$2(x - 2)(x + 2)\left[\frac{5}{2(x - 2)}\right] = 2(x - 2)(x + 2)\left[\frac{15}{(x - 2)(x + 2)}\right]$$

$$5x + 10 = 30$$

$$x = 4$$

Check $x = 4$.

$$\frac{5}{2(4) - 4} \stackrel{?}{=} \frac{15}{4^2 - 4}$$

$$\frac{5}{4} \stackrel{?}{=} \frac{15}{12}$$

$$\frac{5}{4} = \frac{5}{4} \quad ✓$$

10

10. $\frac{2}{x+3} + \frac{6}{x^2+3x} = \frac{1}{x}$

Restrictions: $x \neq -3, 0$

$$\frac{2}{x+3} + \frac{6}{x(x+3)} = \frac{1}{x}$$

$$x(x+3)\left[\frac{2}{x+3} + \frac{6}{x(x+3)}\right] = x(x+3)\left[\frac{1}{x}\right]$$

$$2x + 6 = x + 3$$

$$x + 3 = 0$$

$$x = -3$$

However, $x \neq -3$ because it is a restriction on the variable and thus is an extraneous root. This equation has no solution.

11. $\frac{2}{x^2-x} - \frac{1}{x-1} = 0$

Restrictions: $x \neq 0, 1$

$$\frac{2}{x(x-1)} - \frac{1}{x-1} = 0$$

$$x(x-1)\left[\frac{2}{x(x-1)} - \frac{1}{x-1}\right] = x(x-1)(0)$$

$$2 - x = 0$$

$$x = 2$$

Check $x = 2$.

$$\frac{2}{(2)^2-2} - \frac{1}{2-1} \stackrel{?}{=} 0$$

$$\frac{2}{2} - \frac{1}{1} \stackrel{?}{=} 0$$

$$0 = 0 \quad ✓$$

10

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12. $\frac{x}{x+2} + \frac{4x+6}{2x^2+5x+3} = \frac{x-1}{2x+4}$

Restrictions: $x \neq -2, -\frac{3}{2}, -1$

$$\frac{x}{x+2} + \frac{2(2x+3)}{(x+1)(2x+3)} = \frac{x-1}{2(x+2)}$$

$$\frac{x}{x+2} + \frac{2}{(x+1)} = \frac{x-1}{2(x+2)}$$

$$2(x+1)(x+2)\left[\frac{x}{x+2} + \frac{2}{(x+1)}\right] = 2(x+1)(x+2)\left[\frac{x-1}{2(x+2)}\right]$$

$$2(x+1)x + 2(x+2)(2) = (x+1)(x-1)$$

$$2x^2 + 6x + 8 = x^2 - 1$$

$$x^2 + 6x + 9 = 0$$

$$(x+3)(x+3) = 0$$

$$x = -3$$

Check $x = -3$.

$$\frac{-3}{-3+2} + \frac{4(-3)+6}{2(-3)^2+5(-3)+3} \stackrel{?}{=} \frac{-3-1}{2(-3)+4}$$

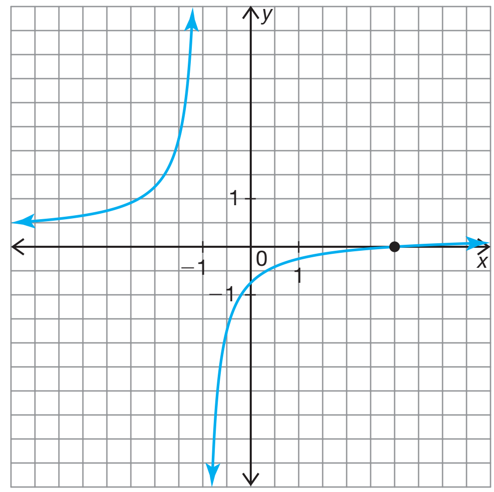
$$\frac{-3}{-1} + \frac{-6}{6} \stackrel{?}{=} \frac{-4}{-2}$$

$$2 = 2 \quad ✓$$

10

Solve each rational equation using a graphing calculator. Sketch the graph. Describe any restrictions for the value of x. Check your answer(s).

13. $\frac{x}{x+1} = \frac{3}{4}$



Rewrite the equation so that one side equals 0, then graph $y = \frac{x}{x+1} - \frac{3}{4}$.

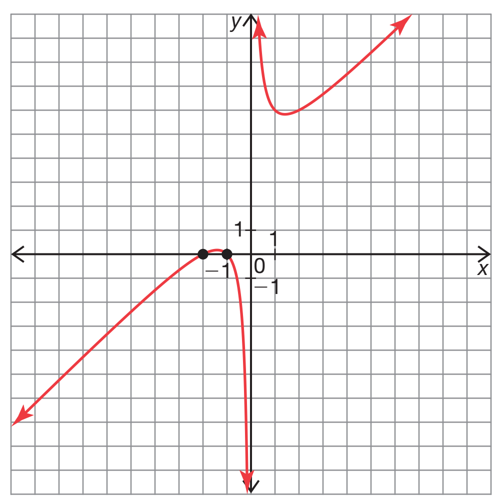
The graph shows that $x = -1$ is the location of a vertical asymptote and thus represents a restriction on the variable. The graph also shows that $x = 3$ is a possible solution to the original rational equation.

Check $x = 3$.

$\frac{3}{3+1} \stackrel{?}{=} \frac{3}{4}$

$\frac{3}{4} = \frac{3}{4}$ ✓

14. $x + 3 = \frac{-2}{x}$



Rewrite the equation so that one side equals 0, then graph $y = x + 3 + \frac{2}{x}$.

The graph shows that $x = 0$ is the location of a vertical asymptote and thus represents a restriction on the variable. The graph also shows that $x = -2$ and $x = -1$ are possible solutions to the original rational equation.

Check $x = -2$.

$-2 + 3 \stackrel{?}{=} \frac{-2}{-2}$

$1 = 1$ ✓

Check $x = -1$.

$-1 + 3 \stackrel{?}{=} \frac{-2}{-1}$

$2 = 2$ ✓

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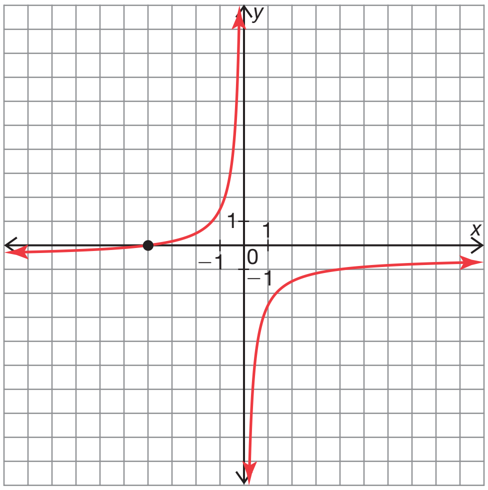
15. $\frac{2}{x} - \frac{1}{2} = \frac{4}{x}$

Rewrite the equation so that one side equals 0, then graph $y = \frac{2}{x} - \frac{1}{2} - \frac{4}{x}$.



The graph shows that $x = 0$ is the location of a vertical asymptote and thus represents a restriction on the variable. The graph also shows that $x = -4$ is a possible solution to the original rational equation.

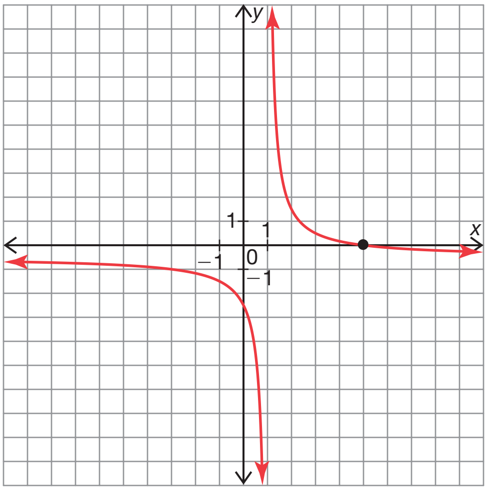
Check $x = -4$.

$\frac{2}{-4} - \frac{1}{2} \stackrel{?}{=} \frac{4}{-4}$

$-1 = -1$ ✓

16. $\frac{1}{2} + \frac{4}{x-1} = \frac{x+1}{x-1}$

Rewrite the equation so that one side equals 0, then graph $y = \frac{1}{2} + \frac{4}{x-1} - \frac{x+1}{x-1}$.



The graph shows that $x = 1$ is the location of a vertical asymptote and thus represents a restriction on the variable. The graph also shows that $x = 5$ is a possible solution to the original rational equation.

Check $x = 5$.

$\frac{1}{2} + \frac{4}{5-1} \stackrel{?}{=} \frac{5+1}{5-1}$

$\frac{1}{2} + \frac{4}{4} \stackrel{?}{=} \frac{6}{4}$

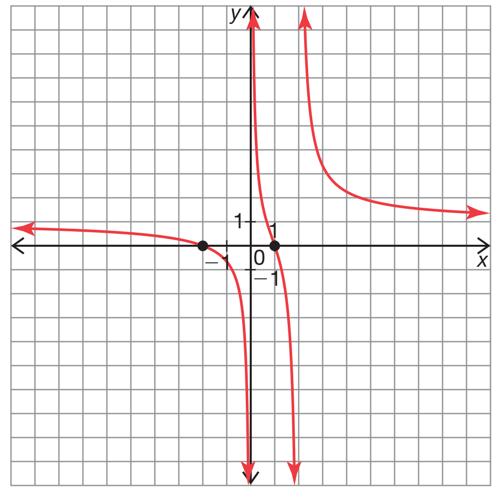
$\frac{3}{2} = \frac{3}{2}$ ✓

10

17. $\frac{x}{x-2} = \frac{-1}{x}$

Rewrite the equation so that one side equals 0, then graph $y = \frac{x}{x-2} + \frac{1}{x}$.

The graph shows that $x = 0$ and $x = 2$ are the locations of vertical asymptotes and thus represent restrictions on the variable. The graph also shows that $x = -2$ and $x = 1$ are possible solutions to the original rational equation.



Check $x = -2$.

$\frac{-2}{-2-2} \stackrel{?}{=} \frac{-1}{-2}$

$\frac{-2}{-4} \stackrel{?}{=} \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$ ✓

Check $x = 1$.

$\frac{1}{1-2} \stackrel{?}{=} \frac{-1}{1}$

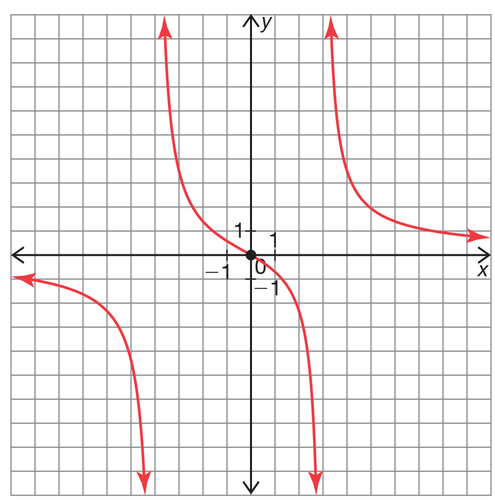
$\frac{1}{-1} \stackrel{?}{=} -1$

$-1 = -1$ ✓

18. $\frac{x}{x-3} = \frac{x}{x+4}$

Rewrite the equation so that one side equals 0, then graph $y = \frac{x}{x-3} - \frac{x}{x+4}$.

The graph shows that $x = -4$ and $x = 3$ are the locations of vertical asymptotes and thus represent restrictions on the variable. The graph also shows that $x = 0$ is a possible solution to the original rational equation.



Check $x = 0$.

$\frac{0}{0-3} \stackrel{?}{=} \frac{0}{0+4}$

$\frac{0}{-3} \stackrel{?}{=} \frac{0}{4}$

$0 = 0$ ✓

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Solve each rational equation without using a graphing calculator. Describe any restrictions for the value of x. Check your answer(s) and identify any extraneous roots should they occur.

19. $\frac{3}{x-1} = \frac{4}{3x+2}$

Restrictions: $x \neq -\frac{2}{3}, 1$

$$(x-1)(3x+2)\left(\frac{3}{x-1}\right) = (x-1)(3x+2)\left(\frac{4}{3x+2}\right)$$

$$9x + 6 = 4x - 4$$

$$5x = -10$$

$$x = -2$$

Check $x = -2$.

$$\frac{3}{-2-1} \stackrel{?}{=} \frac{4}{3(-2)+2}$$

$$\frac{3}{-3} \stackrel{?}{=} \frac{4}{-4}$$

$$-1 = -1 \quad ✓$$

20. $\frac{9}{x-3} = \frac{27}{x^2-3x} + \frac{6}{x}$

Restrictions: $x \neq 0, 3$

$$\frac{9}{x-3} = \frac{27}{x(x-3)} + \frac{6}{x}$$

$$x(x-3)\left[\frac{9}{x-3}\right] = x(x-3)\left[\frac{27}{x(x-3)} + \frac{6}{x}\right]$$

$$9x = 27 + 6x - 18$$

$$3x = 9$$

$$x = 3$$

However, $x \neq 3$ because it is a restriction on the variable and thus is an extraneous root. This equation has no solution.

21. $\frac{x+1}{x-2} = \frac{3x}{x-2} - \frac{2x+4}{x}$

Restrictions: $x \neq 0, 2$

$$x(x-2)\left[\frac{x+1}{x-2}\right] = x(x-2)\left[\frac{3x}{x-2} - \frac{2x+4}{x}\right]$$

$$x^2 + x = 3x^2 - (2x^2 - 8)$$

$$x^2 + x = x^2 + 8$$

$$x = 8$$

Check $x = 8$.

$$\frac{8+1}{8-2} \stackrel{?}{=} \frac{3(8)}{8-2} - \frac{2(8)+4}{8}$$

$$\frac{9}{6} \stackrel{?}{=} \frac{24}{6} - \frac{20}{8}$$

$$\frac{3}{2} = \frac{3}{2} \quad ✓$$

10

22. $\frac{-x}{2x+1} = \frac{5}{x-4}$

Restrictions: $x \neq -\frac{1}{2}, 4$

$$-x(x-4) = 5(2x+1)$$

$$-x^2 + 4x = 10x + 5$$

$$x^2 + 6x + 5 = 0$$

$$(x+5)(x+1) = 0$$

$$x = -5 \text{ or } = -1$$

Check $x = -5$.

$$\frac{-(-5)}{2(-5)+1} \stackrel{?}{=} \frac{5}{-5-4}$$

$$\frac{5}{-9} \stackrel{?}{=} \frac{5}{-9}$$

$$-\frac{5}{9} = -\frac{5}{9} \quad ✓$$

Check $x = -1$.

$$\frac{-(-1)}{2(-1)+1} \stackrel{?}{=} \frac{5}{-1-4}$$

$$\frac{1}{-1} \stackrel{?}{=} \frac{5}{-5}$$

$$-1 = -1 \quad ✓$$

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23. $1 + \frac{4}{x-4} = \frac{-3}{x^2-16}$

Restrictions: $x \neq -4, 4$

$$1 + \frac{4}{x-4} = \frac{-3}{(x-4)(x+4)}$$

$$(x-4)(x+4)\left[1 + \frac{4}{x-4}\right] = (x-4)(x+4)\left[\frac{-3}{(x-4)(x+4)}\right]$$

$$(x-4)(x+4) + 4(x+4) = -3$$

$$x^2 + 4x = -3$$

$$x^2 + 4x + 3 = 0$$

$$(x+3)(x+1) = 0$$

$$x = -3 \text{ or } x = -1$$

Check $x = -3$.

$$1 + \frac{4}{-3-4} \stackrel{?}{=} \frac{-3}{(-3)^2-16}$$

$$1 + \frac{4}{-7} \stackrel{?}{=} \frac{-3}{-7}$$

$$\frac{3}{7} = \frac{3}{7} \quad ✓$$

Check $x = -1$.

$$1 + \frac{4}{-1-4} \stackrel{?}{=} \frac{-3}{(-1)^2-16}$$

$$1 + \frac{4}{-5} \stackrel{?}{=} \frac{-3}{-15}$$

$$\frac{1}{5} = \frac{1}{5} \quad ✓$$

24. $\frac{5x}{x-2} - 7 = \frac{10}{x-2}$

Restriction: $x \neq 2$

$$(x-2)\left[\frac{5x}{x-2} - 7\right] = (x-2)\left[\frac{10}{x-2}\right]$$

$$5x - 7(x-2) = 10$$

$$-2x + 14 = 10$$

$$-2x = -4$$

$$x = 2$$

However, $x \neq 2$ because it is a restriction on the variable and thus is an extraneous root. This equation has no solution.

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Get to Work, Mix It Up, Go the Distance, and Lower the Cost!

Using Rational Equations to Solve Real-World Problems

Problem Set

Write an equation to model each work scenario. Then, solve each equation.

1. Cleo can paint a room in 8 hours; while Phil can paint the same room in 6 hours. If they paint the room together, how long will it take them to paint the room?

Working together, it will take Cleo and Phil $3\frac{3}{7}$ hours to paint the room.

Let x represent the number of hours it will take to paint the room while working together.

$$\frac{x}{8} + \frac{x}{6} = 1$$

$$24\left(\frac{x}{8} + \frac{x}{6}\right) = 24(1)$$

$$3x + 4x = 24$$

$$7x = 24$$

$$x = \frac{24}{7} \text{ or } 3\frac{3}{7}$$

2. Nyesha owns a lawn service company. Currently it takes her 50 hours a week to service all of her customers. To reduce the number of hours a week she needs to work, Nyesha hires Shantese to help her. While Nyesha was on vacation, Shantese was able to complete all of the work in 60 hours. If Shantese and Nyesha work together, after Nyesha returns from vacation, how long will it take them to service all their customers?

Working together, it will take Nyesha and Shantese $27\frac{3}{11}$ hours to service all of their customers.

Let x represent the number of hours it will take to service all of their customers while working together.

$$\frac{x}{50} + \frac{x}{60} = 1$$

$$300\left(\frac{x}{50} + \frac{x}{60}\right) = 300(1)$$

$$6x + 5x = 300$$

$$11x = 300$$

$$x = \frac{300}{11} \text{ or } 27\frac{3}{11}$$

3. Using a forklift, Rico can unload a box car in 90 minutes; while Ashkii takes twice as long to complete the same task. If Rico and Ashkii work together, how long will it take them to unload a box car?

Working together, it will take Rico and Ashkii 60 minutes to unload a box car.

Let x represent the number of minutes it will take to unload a box car while working together.

$$\frac{x}{90} + \frac{x}{2(90)} = 1$$

$$\frac{x}{90} + \frac{x}{180} = 1$$

$$180\left(\frac{x}{90} + \frac{x}{180}\right) = 180(1)$$

$$2x + x = 180$$

$$3x = 180$$

$$x = 60$$

4. Yu Jie can complete a 4 foot by 6 foot quilt in 16 days; while Mufeed can complete the same task in 12 days. If they solicit Mya's help, who can complete the task in 14 days by herself, how long will it take the three of them to complete a 4 foot by 6 foot quilt?

Working together, it will take Yu Jie, Mufeed, and Mya $4\frac{44}{73}$ days to complete a 4 foot by 6 foot quilt.

Let x represent the number of days it will take to complete a 4 foot by 6 foot quilt while working together.

$$\frac{x}{16} + \frac{x}{12} + \frac{x}{14} = 1$$

$$336\left(\frac{x}{16} + \frac{x}{12} + \frac{x}{14}\right) = 336(1)$$

$$21x + 28x + 24x = 336$$

$$73x = 336$$

$$x = 4\frac{44}{73}$$

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5. Kendall can wash 24 golf carts in a 4 hour shift. If Benny helps him they can get the job done in 2 hours. How long will it take Benny to do the job by himself?

Working by himself, it will take Benny 4 hours to wash 24 golf carts.

Let x represent the number of hours it will take Benny to wash 24 golf carts by himself.

Portion of the golf carts Benny can wash $= 2\left(\frac{1}{x}\right) = \frac{2}{x}$

Portion of the golf carts Kendall can wash $= 2\left(\frac{1}{4}\right) = \frac{2}{4}$

$$\frac{2}{4} + \frac{2}{x} = 1;\ x \neq 0$$

$$4x\left(\frac{2}{4} + \frac{2}{x}\right) = 4x(1)$$

$$2x + 8 = 4x$$

$$8 = 2x$$

$$x = 4$$

6. Felix and Oscar own a pastry shop. Working alone Felix can decorate 8 dozen cookies in 90 minutes. Oscar, on the other hand, needs 120 minutes to decorate 8 dozen cookies. If they work together, how long does it take them to decorate 16 dozen cookies?

Working together, it will take Felix and Oscar approximately 102.86 minutes to decorate 16 dozen cookies.

Let x represent the number of minutes it will take to decorate 16 dozen cookies while working together.

Felix can decorate 16 dozen cookies in 180 minutes.

Oscar can decorate 16 dozen cookies in 240 minutes.

$$\frac{x}{180} + \frac{x}{240} = 1$$

$$720\left(\frac{x}{180} + \frac{x}{240}\right) = 720(1)$$

$$4x + 3x = 720$$

$$7x = 720$$

$$x \approx 102.86$$

Write an equation to model each mixture scenario. Then, solve each equation.

7. Kaitlin knows that if she needs to add antifreeze to her car's radiator the mixture used must contain 50% antifreeze and 50% water. How many gallons of a mixture containing 80% antifreeze must be added to a 3 gallon mixture containing 40% antifreeze to obtain the mixture Kaitlin needs?

Kaitlin needs to add 1 gallon of the 80% antifreeze mixture to the 3 gallons of the 40% antifreeze mixture to obtain a mixture containing 50% antifreeze.

Let x represent the number of gallons of 80% antifreeze mixture needed.

$$\frac{0.4(3) + 0.8x}{3 + x} = 0.5; x \neq -3$$

$$\frac{1.2 + 0.8x}{3 + x} = 0.5$$

$$1.2 + 0.8x = 1.5 + 0.5x$$

$$0.3x = 0.3$$

$$x = 1$$

8. The directions on the back of a 2 quart bottle of a 60% orange concentrate says it needs to be mixed with water to obtain a 20% orange drink. How much water should Hector add to the concentrate to obtain a drink that is 20% orange concentrate?

Hector should add 4 quarts of water to obtain a drink that is 20% orange concentrate.

Let x represent the number of quarts of water needed.

$$\frac{0.6(2)}{2 + x} = 0.2; x \neq -2$$

$$\frac{1.2}{2 + x} = 0.2$$

$$1.2 = 0.4 + 0.2x$$

$$0.8 = 0.2x$$

$$x = 4$$

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9. A research scientist has 10 liters of a 40% acid solution. She needs to create a solution containing 35% acid by mixing the 10 liters with a second solution containing 20% acid. How much of this second solution should she use?

The research scientist needs to add $3\frac{1}{3}$ liters of the 20% acid solution to obtain a 35% acid solution.

Let x represent the number of liters of 20% acid solution needed.

$$\frac{0.4(10) + 0.2x}{10 + x} = 0.35;\ x \neq -10$$

$$\frac{4 + 0.2x}{10 + x} = 0.35$$

$$4 + 0.2x = 3.5 + 0.35x$$

$$0.5 = 0.15x$$

$$x = 3\frac{1}{3}$$

10. Rosa combines 2 gallons of 2% milk and 6 gallons of 4% milk. How many additional gallons of 6% milk must she add to obtain a 5% milk mixture?

Rosa needs to add 12 gallons of the 6% milk to obtain a 5% milk mixture.

Let x represent the number of additional gallons of 6% milk mixture needed.

$$\frac{0.02(2) + 0.04(6) + 0.06x}{2 + 6 + x} = 0.05;\ x \neq -8$$

$$\frac{0.28 + 0.06x}{8 + x} = 0.05$$

$$0.28 + 0.06x = 0.4 + 0.05x$$

$$0.01x = 0.12$$

$$x = 12$$

11. A jeweler has 30 ounces of an alloy consisting of 60% gold and 40% silver. How much of a second alloy containing 80% gold and 20% silver must be mixed with the first alloy to obtain an alloy containing 75% gold and 25% silver?

The jeweler needs to add 90 ounces of the second alloy to obtain an alloy that contains 75% gold and 25% silver.

The solution to this problem can be done in one of two ways. Consider either the percentage of gold needed in the final alloy or the percentage of silver needed in the final alloy. This solution considers the percentage of the silver needed in the final alloy.

Let x represent the number of ounces of the second alloy needed.

$$\frac{0.4(30) + 0.2x}{30 + x} = 0.25;\ x \neq -30$$

$$\frac{12 + 0.2x}{30 + x} = 0.25$$

$$12 + 0.2x = 7.5 + 0.25x$$

$$4.5 = 0.05x$$

$$x = 90$$

12. Keyon has 3 quarts of a 5% sugar solution. He wants to mix this with 2 quarts of an 8% sugar solution and x quarts of a 12% sugar to solution to make a 10% sugar solution. How many quarts of the 12% sugar solution should Keyon use?

Keyon needs to add 9.5 quarts of the 12% solution to obtain a 10% sugar solution.

Let x represent the number of quarts of 12% solution needed.

$$\frac{0.05(3) + 0.08(2) + 0.12x}{3 + 2 + x} = 0.1;\ x \neq -5$$

$$\frac{0.31 + 0.12x}{5 + x} = 0.1$$

$$0.31 + 0.12x = 0.5 + 0.1x$$

$$0.02x = 0.19$$

$$x = 9.5$$

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Write an equation to model each distance scenario. Then, solve each equation.

13. Kikki walked to the grocery store which was 2 miles away. Her walking rate on the way back was 0.75 of her walking rate on the way to the store because she was carrying a bag of groceries. If it took Kikki 1 hour to make the round trip, what was her walking rate on the way to the store?

Kikki's walking rate on the way to the store was $4\frac{2}{3}$ miles per hour.

Let r represent Kikki's walking rate on her way to the grocery store.

$$\frac{2}{r} + \frac{2}{0.75r} = 1; x \neq 0$$

$$0.75r\left(\frac{2}{r} + \frac{2}{0.75r}\right) = 0.75r(1)$$

$$1.5 + 2 = 0.75r$$

$$3.5 = 0.75r$$

$$r = 4\frac{2}{3}$$

14. Ricky paddles his canoe at a rate of 6 miles per hour in still water. Last weekend he canoed on Carver Creek going downstream with the current for 9 miles and then returning upstream against the current. If the trip took him 4 hours to complete, what was the speed of the current?

The speed of the current was 3 miles per hour.

Let r represent the speed of the current.

$$\frac{9}{6+r} + \frac{9}{6-r} = 4; x \neq \pm 6$$

$$(6+r)(6-r)\left(\frac{9}{6+r} + \frac{9}{6-r}\right) = (6+r)(6-r)(4)$$

$$9(6-r) + 9(6+r) = 4(36-r^2)$$

$$108 = 144 - 4r^2$$

$$-36 = -4r^2$$

$$9 = r^2$$

$$r = \pm 3; \text{ choose } r = 3$$

10

15. Suppose flying in calm air a robin can reach a speed of 25 kilometers per hour. Each day this robin flies from its nest to the nearest body of water 1 kilometer away. On one particular day it flew into a headwind and on its return trip the wind was at its back. If the total trip took $\frac{1}{10}$ of an hour, what was the speed of the wind?

The speed of the wind was approximately 11.18 kilometers per hour.

Let r represent the speed of the wind.

$$\frac{1}{25+r} + \frac{1}{25-r} = \frac{1}{10};\ x \neq \pm 25$$

$$(25+r)(25-r)\left(\frac{1}{25+r} + \frac{1}{25-r}\right) = (25+r)(25-r)\left(\frac{1}{10}\right)$$

$$(25-r) + (25+r) = \frac{1}{10}(625 - r^2)$$

$$50 = \frac{1}{10}(625 - r^2)$$

$$500 = 625 - r^2$$

$$-125 = -r^2$$

$$125 = r^2$$

$$r \approx \pm 11.18;\ \text{choose } r = 11.18$$

16. Oni walked a half a mile to her sister's house to pick up her little brother and then walked back. The round trip took 60 minutes. If the rate at which she walked to her sister's house was 25% faster than the rate she walked while returning home, how fast did she walk on the way home?

Oni's rate walking home was 0.9 miles per hour.

Let r represent the rate at which Oni walked home.

$$\frac{0.5}{1.25r} + \frac{0.5}{r} = 1;\ x \neq 0$$

$$(1.25r)\left(\frac{0.5}{1.25r} + \frac{0.5}{r}\right) = (1.25r)(1)$$

$$0.5 + 0.625 = 1.25r$$

$$1.125 = 1.25r$$

$$r = 0.9$$

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17. An 8-man rowing crew rows at a speed of 10 miles per hour in still water. Every morning they practice by rowing 4 miles upstream and then 4 miles downstream. If it takes them $\frac{5}{6}$ of an hour to complete the trip, what is the speed of the current?

The speed of the current is 2 miles per hour.

Let r represent the speed of the current.

$$\frac{4}{10+r} + \frac{4}{10-r} = \frac{5}{6};\ x \neq \pm 10$$

$$(10+r)(10-r)\left(\frac{4}{10+r} + \frac{4}{10-r}\right) = (10+r)(10-r)\left(\frac{5}{6}\right)$$

$$4(10-r) + 4(10+r) = \frac{5}{6}(100 - r^2)$$

$$80 = \frac{5}{6}(100 - r^2)$$

$$96 = 100 - r^2$$

$$-4 = -r^2$$

$$\pm 2 = r;\ \text{choose } r = 2$$

10

18. Mildred runs the 100-meter dash in 15 seconds with no wind. Yesterday she ran the 100-meter dash twice. The first time she ran it in 18.75 seconds against the wind and the second time she ran it in 12.5 seconds with the wind. What was the speed of the wind?

The speed of the wind was $1\frac{1}{3}$ meters per second.

Let r represent the speed of the wind. Mildred's speed with no wind is $\frac{100}{15} = 6\frac{2}{3}$ meters per second.

$$\frac{100}{6\frac{2}{3}+r} + \frac{100}{6\frac{2}{3}-r} = 12.5 + 18.75;\ x \neq \pm 6\frac{2}{3}$$

$$\frac{100}{6\frac{2}{3}+r} + \frac{100}{6\frac{2}{3}-r} = 31\frac{1}{4}$$

$$\left(6\frac{2}{3}+r\right)\left(6\frac{2}{3}-r\right)\left(\frac{100}{6\frac{2}{3}+r} + \frac{100}{6\frac{2}{3}-r}\right) = \left(6\frac{2}{3}+r\right)\left(6\frac{2}{3}-r\right)\left(31\frac{1}{4}\right)$$

$$100\left(6\frac{2}{3}-r\right) + 100\left(6\frac{2}{3}+r\right) = 31\frac{1}{4}\left(44\frac{4}{9}-r^2\right)$$

$$1333\frac{1}{3} = 1388\frac{8}{9} - 31\frac{1}{4}r^2$$

$$-55\frac{5}{9} = -31\frac{1}{4}r^2$$

$$1\frac{7}{9} = r^2$$

$$r = \pm 1\frac{1}{3};\ \text{choose } r = 1\frac{1}{3}$$

10

Name ______________________________ Date ______________

Write an equation or inequality to model each cost scenario. Then, solve each equation or inequality.

19. Nicole purchased a power boat for \$15,000 and was told by the salesman that the yearly average cost to operate the boat was approximately \$900. In what year of ownership will Nicole's average annual cost of owning the boat be \$3900?

In the 5th year of ownership the average cost of owning the boat will be \$3900.

Let x be the year in which Nicole's average annual cost of owning the boat is \$3900.

$$\frac{15{,}000 + 900x}{x} = 3900;\ x \neq 0$$

$$15{,}000 + 900x = 3900x$$

$$15{,}000 = 3000x$$

$$x = 5$$

20. Remington purchased a new cell phone for \$350 and added an annual warranty plan that cost him \$35 dollars per year. In what year will Remington's average annual cost of owning the phone be \$122.50?

In the 4th year of ownership the average cost of owning the cell phone will be \$122.50.

Let x be the year in which Remington's average annual cost of owning the cell phone is \$122.50.

$$\frac{350 + 35x}{x} = 122.50;\ x \neq 0$$

$$350 + 35x = 122.50x$$

$$350 = 87.50x$$

$$x = 4$$

21. Luella always wanted a designer purse but they were too expensive to purchase. Recently she was introduced to a company that allowed individuals to rent one. The purse that Luella rented initially cost her \$80 with a \$15 monthly rental fee. She plans on renting a new purse until the average monthly cost of renting the purse is less than \$19. When will the average monthly cost of renting the purse drop to less than \$19?

In the 21st month Luella's average monthly cost of renting the purse will be less than \$19.

Let x be the month in which Luella's average monthly cost of renting the purse drops to less than \$19.

$$\frac{80 + 15x}{x} < 19;\ x \neq 0$$

$$80 + 15x < 19x$$

$$80 < 4x$$

$$x > 20$$

22. Shopping around for a freezer, Manuit finally settled on one with a purchase price of \$850. The annual cost of operating the freezer is \$40 dollar per year. When Manuit's average cost of owning the freezer is less than \$117, he plans to shop for a new freezer. When can Manuit shop for a new freezer?

In the 12th year Manuit can begin shopping for a new freezer.

Let x be the year in which Manuit's average annual cost of owning the freezer is less than \$117.

$$\frac{850 + 40x}{x} < 117;\ x \neq 0$$

$$850 + 40x < 117x$$

$$850 < 77x$$

$$x > 11.039$$

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23. Yuma purchased a flat screen TV for $700 and added an annual warranty plan that cost her $23 dollars per year. Reid purchased a similar TV for $650 and added an annual warranty plan that costs him $28 per year. In how many years will Yuma's average annual cost of owning her TV be less than Reid's annual cost of owning his TV?

In the 11th year, Yuma's average annual cost of owning her TV will be less than Reid's average annual cost of owning his TV.

Let x be the year in which Yuma's average annual cost of owning her TV is equal to Reid's average annual cost of owning his TV.

$$\frac{700 + 23x}{x} = \frac{650 + 28x}{x};\ x \neq 0$$

$$x(700 + 23x) = x(650 + 28x)$$

$$23x^2 + 700x = 28x^2 + 650x$$

$$5x^2 - 50x = 0$$

$$5x(x - 10) = 0$$

$$x = 0,\ x = 10$$

At the time of purchase, Reid's average annual cost for the TV is less than Yuma's. Reid's average annual cost will continue to be less than Yuma's until they are equal in the 10th year. Beginning in the 11th year, Yuma's average annual cost will be less than Reid's.

10

24. Gisela is considering purchasing one of two microwave ovens. The first microwave costs $300 and comes with an annual warranty plan that costs $20 per year. The second microwave costs $350 and comes with an annual warranty plan that costs $10 per year. When will the average annual cost of owning the second microwave be less than the average annual cost of owning the first microwave?

In the 6th year, the average annual cost of owning the second microwave will be less than the average annual cost of owning the first microwave.

Let x be the year in which the average annual cost of owning the second microwave is equal to the average annual cost of owning the first microwave.

$$\frac{300 + 20x}{x} = \frac{350 + 10x}{x};\ x \neq 0$$

$$x(300 + 20x) = x(350 + 10x)$$

$$20x^2 + 300x = 10x^2 + 350x$$

$$10x^2 - 50x = 0$$

$$10x(x - 5) = 0$$

$$x = 0, x = 5$$

At the time of purchase, the average annual cost for the first microwave is less than for the second microwave. The average annual cost of the first microwave will continue to be less than for the second microwave until they are equal in the 5th year. Beginning in the 6th year, the average annual cost of the second microwave will be less than for the first microwave.

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