Advanced Pre-Algebra Overview

Advanced Pre-Algebra content is organized into six domains of focused study as outlined below in the column to the left. The Advanced Pre-Algebra domains listed in bold print on the shaded bars are Ratios and Proportional Relationships, The Number System, Expressions and Equations, Functions, Geometry, and Statistics and Probability. Immediately following the domain and enclosed in brackets is an abbreviation denoting the domain. Identified below each domain are the clusters that serve to group related content standards. All Advanced Pre-Algebra content standards, grouped by domain and cluster, are located on the pages that follow.

The Standards for Mathematical Practice are listed below in the column to the right. These mathematical practice standards should be incorporated into classroom instruction of the content standards.

Content Standard Domains and Clusters

Ratios and Proportional Relationships (RP)

- Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System (NS)

- Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers.
- Know that there are numbers not rational, and approximate them by rational numbers.

Expressions and Equations (EE)

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
- Work with radicals and integer exponents.
- Understand the connections among proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions (F)

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Geometry (G)

- Draw, construct, and describe geometrical figures and describe the relationship between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Statistics and Probability (SP)

- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.
- Investigate patterns of association in bivariate data.

Advanced Pre-Algebra

In Advanced Pre-Algebra, instructional time should focus on seven critical areas. These areas are (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; (4) drawing inferences about populations based on samples; (5) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation and solving linear equations and systems of linear equations; (6) grasping the concept of a function and using functions to describe quantitative relationships; and (7) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence and understanding and applying the Pythagorean Theorem. Important information regarding these seven critical areas of instruction follows:

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. They use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals that have a finite or a repeating decimal representation, and percents as different representations of rational numbers. They extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction and multiplication and division. By applying these properties and by viewing negative numbers in terms of everyday contexts, such as amounts owed or temperature below zero, students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. Students use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

(5) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions such as $\frac{y}{x} = m$ or $y = mx$ as

special linear equations such as *y* = *mx* + *b*, understanding that the constant of proportionality, *m*, is the slope, and the graphs are lines through the origin. They understand that the slope, *m*, of a line is a constant rate of change, so that if the input, or *x*-coordinate changes by an amount *A*, the output, or *y*-coordinate changes by the amount *m*:*A*. Students also use linear equations to describe the association between two quantities in bivariate data such as the arm span versus height for students in a classroom. At this grade, fitting the model and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship, such as slope and y-intercept, in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. They solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(6) Students grasp the concept of function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. Students can translate among representations and partial representations of functions, while noting that tabular and graphical representations may be partial representations, and they can describe how aspects of the function are reflected in the different representations.

(7) Students use ideas about distance and angles, including how they behave under translations, rotations, reflections, and dilations and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. They show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distance between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders and spheres.

Students will:

Ratios and Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. (7RP-1)

   Example: If a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.

2. Recognize and represent proportional relationships between quantities. (7-RP2)

   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (7-RP2a)
   b. Identify the constant proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (7-RP2b)
   c. Represent proportional relationships by equations. (7-RP2c)

      Example: If total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

   d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate. (7-RP2d)

3. Use proportional relationships to solve multistep ratio and percent problems. (7-RP3)

   Example: Sample problems may involve simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increases and decrease, and percent error.

The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

4. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers, represent addition and subtraction on a horizontal or vertical number line diagram. (7-NS1)

   a. Describe situations in which opposite quantities combine to make 0. (7-NS1a)

      Example: A hydrogen atom has 0 charge because its two constituents are oppositely charged.

   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. (7-NS1b)

   c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. (7-NS1c)

   d. Apply properties of operations as strategies to add and subtract rational numbers. (7-NS1d)

5. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. (7-NS2)

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. (7-NS2a)
b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with nonzero divisor) is a rational number. If p and q are integers, then $-(\frac{p}{q}) = \frac{(-p)}{q} = \frac{p}{(-q)}$. Interpret quotients of rational numbers by describing real-world contexts. (7-NS2b)
c. Apply properties of operations as strategies to multiply and divide rational numbers. (7-NS2c)
d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. (7-NS2d)

6. Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) (7-NS3)

7. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. (8-NS1)

8. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., ^{2}). (8-NS2)

   Example: By truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Expressions and Equations

Use properties of operations to generate equivalent expressions.

9. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. (7-EE1)

10. Understand that rewriting an expression in different forms in a problem context can shed light on the problem, and how the quantities in it are related. (7-EE2)

    Example: $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

11. Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form, convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies. (7-EE3)

    Examples: If a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50.
    If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

12. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. (7-EE4)

    a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. (7-EE4a)

       Example: The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

    b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q > r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality, and interpret it in the context of the problem. (7-EE4b)

       Example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.

Work with radicals and integer exponents.

13. Know and apply the properties of integer exponents to generate equivalent numerical expressions. (8-EE1)

    Example: $3^2 \text{ x } 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.

14. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. (8-EE2)

15. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or small quantities, and to express how many times as much one is more than the other. (8-EE3)

    Example: Estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.

16. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading.) Interpret scientific notation that has been generated by technology. (8-EE4)

Understand the connections among proportional relationships, lines, and linear equations.

17. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. (8-EE5)

    Example: Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

18. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. (8-EE6)

Analyze and solve linear equations and pairs of simultaneous linear equations.

19. Solve linear equations in one variable. (8-EE7)

    a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). (8-EE7a)
    b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions, using the distributive property and collecting like terms. (8-EE7b)

20. Analyze and solve pairs of simultaneous linear equations. (8-EE8)

    a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersections of their graphs because points of intersection satisfy both equations simultaneously. (8-EE8a)
    b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. (8-EE8b)

       Example: $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

    c. Solve real-world and mathematical problems leading to two linear equations in two variables. (8-EE8c)

Example: Given coordinates for two pair of points, determine whether the line through the first pair of points intersects the line through the second pair.

Functions

Define, evaluate, and compare functions.

21. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is required in Grade 8.) (8-F1)

22. Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (8-F2)

    Example: Given a linear function represented by a table of values and linear function represented by an algebraic expression, determine which function has the greater rate of change.

23. Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear. (8-F3)

    Example: The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4), and (3,9), which are not on a straight line.

Use functions to model relationships between quantities.

24. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of linear function in terms of the situation it models and in terms of its graph or a table of values. (8-F4)

25. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (8-F5)

Geometry

Draw, construct, and describe geometrical figures and describe the relationships between them.

26. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (7-G1)

27. Draw (freehand, with ruler or protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. (7-G2)

28. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. (7-G3)

Solve real-world and mathematical problems involving angle measure, area, surface area, and volume.

29. Know the formulas for the area and circumference of a circle, and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. (7-G4)

30. Use facts about supplementary, complementary, vertical, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure. (7-G5)

31. Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (7-G6)

Understand congruence and similarity using physical models, transparencies, or geometry software.

32. Verify experimentally the properties of rotations, reflections, and translations: (8-G1)

    a. Lines are taken to lines, and line segments are taken to line segments of the same length. (8-G1a)
    b. Angles are taken to angles of the same measure. (8-G1b)
    c. Parallel lines are taken to parallel lines. (8-G1c)

32. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (8-G2)

33. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. (8-G3)

34. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (8-G4)

35. Use formal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. (8-G5)

    Example: Arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give argument in terms of transversals why this is so.

Understand and apply the Pythagorean Theorem.

36. Explain a proof of the Pythagorean Theorem and its converse. (8-G6)

37. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (8-G7)

38. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (8-G8)

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

39. Know the formulas for the volumes of cones, cylinders, and spheres, and use them to solve real-world and mathematical problems. (8-G9)

Statistics and Probability

Use random sampling to draw inferences about a population.

40. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. (7-SP1)

41. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. (7-SP2)

    Example: Estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Draw informal comparative inferences about two populations.

42. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. (7-SP3)

    Example: The mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

43. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. (7-SP4)

    Example: Decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter a fourth-grade science book.

Investigate chance processes and develop, use, and evaluate probability models.

44. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an even that is neither unlikely nor likely, and a probability near 1 indicates a likely event. (7-SP5)

45. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. (7-SP6)

    Example: When rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

46. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. (7-SP7)

    a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. (7-SP7a)

       Example: If a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

    b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. (7-SP7b)

       Example: Find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

47. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. (7-SP8)

    a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. (7-SP8a)
    b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. (7-SP8b)
    c. Design and use a simulation to generate frequencies for compound events. (7-SP8c)

       Example: Use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood.

Investigate patterns of association in bivariate data.

48. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. (8-SP1)

49. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (8-SP2)

50. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. (8-SP3)

    Example: In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

51. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. (8-SP4)

    Example: Collect data from students in your class on whether or not they have a curfew on school nights, and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?