Calculus

Summer Packet

This packet of exercises reflects skills that the Math Department considers essential for your success in Calculus!

In this packet you will find the following:

- Questions on material previously learned in any of your math courses prior to calculus.
- Topics from Khan Academy referenced in the directions for each problem set. If you are having difficulty recalling how to do a specific type of problem, the Khan Academy videos are an excellent resource for re-teaching. Go to www.khanacademy.org, type in the phrase provided, and it will take you to a video(s) about the topic. Khan Academy also provides further practice on the topics that you can do for your own self-assessment.
- If Kahn Academy is not enough help, here are some other websites that may be of help.
 - Most algebra topics: http://www.purplemath.com/modules/index.htm
 - Trig Information http://www.mathematicshelpcentral.com/index.html
 Once in the site, go to lecture notes

Your Responsibility is to:

- Complete all problems and show all necessary work clearly and carefully
- Turn in the packet on **THE FIRST DAY OF SCHOOL**! It will be collected and checked for completion on the first day of school.

You will be tested on the material within the first two weeks of school.

Have a great summer!

Topic 1: Fractional & Negative Exponents:

Simplify using only positive exponents.

1.
$$2\left(\frac{2}{2-x}\right)\left[\frac{-2}{(2-x)^2}\right]_{\parallel}$$

$$2. \ \frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$$

Topic 2: Domain: (Khan Academy Topic: "Domain of advanced functions")

Find the domain of the following functions.

3.
$$y = log(2x - 12)$$

4.
$$y = \sqrt{3x - 15}$$

5.
$$y = \frac{x}{x^2 - 9}$$

Topic 3: Solving Inequalities (Absolute Value): (Khan Academy Topic: "Solving absolute value inequalities")

Solve the following absolute value inequalities.

6.
$$|x-3| > 12$$

7.
$$|x-3| \le 4$$

6.
$$|x-3| > 12$$
 7. $|x-3| \le 4$ 8. $|3x-4| > -2$ 9. $|x-6| < -8$

9.
$$|x-6| < -8$$

Topic 4: Solving inequalities (with Quadratics): (Khan Academy Topic: "Quadratic inequalities")

Solve the following by factoring and making appropriate sign charts.

10.
$$x^2 - 16 > 0$$

11.
$$x^2 - 3x \ge 10$$

Topic 5: Special Factorization: (Khan Academy Topic: "Advanced polynomial factorization methods")

Factor completely.

12.
$$27x^3 - 125y^3$$

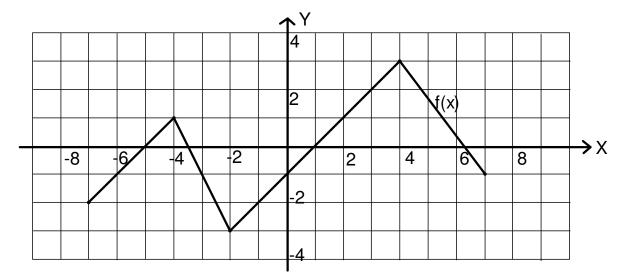
13.
$$2x^2 + 50y^2 - 20xy$$

15.
$$x^3 - xy^2 + x^2y - y^3$$

16.
$$(x-3)^3(2x+1)^3 + (x-3)^4(2x+1)^2$$

Topic 6: Evaluating Functions:

Use the graph of y = f(x), to evaluate the value of f(x).



17b.
$$3f(3) =$$

18a. If
$$x = 5$$
, what

18b.
$$|f(-2)| =$$

does
$$f(x + 2) =$$

20.
$$f|-3|=$$

Topic 7: Even and Odd functions: (Khan Academy Topic: "Intro to function symmetry")

Show work to determine if the relation is even, odd, or neither.

21.
$$f(x) = 2x^2 - 7$$

21.
$$f(x) = 2x^2 - 7$$
 22. $f(x) = -4x^3 - 2x$

23.
$$f(x) = 4x^2 - 4x + 4$$

Topic 8: Asymptotes: (Khan Academy Topic: "Graphing rational functions according to asymptotes")

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptotes (if they exist).

24.
$$y = \frac{x^2 - 2x + 1}{x^2 - 3x - 4}$$

25.
$$y = \frac{2x^2 + 6x}{x^3 - 3x^2 - 4x}$$

Topic 9: Complex Fractions: (Khan Academy Topic: "Nested fractions")

Simplify the following.

26.
$$\frac{x^{-3} - x}{x^{-2} - 1}$$

$$\frac{\frac{x}{1-x} + \frac{1+x}{x}}{\frac{1-x}{x} + \frac{x}{1+x}}$$

Topic 10: Composition of Functions: (Khan Academy Topic: "Evaluating composite functions") If $f(x) = x^2$, g(x) = 2x - 1, and $h(x) = 2^x$, find the following

28.
$$g\left(\int_{f} h\left(\frac{1}{2}\right)\right)$$
29. $g(g(x))$

<u>Topic 11: Solving Rational (fractional) Equations:</u> (Khan Academy Topic: "Equations with rational expressions")

Solve each equation for x.

30.
$$\frac{x}{2x-6} - \frac{3}{x^2 - 6x + 9} = \frac{x-2}{3x-9}$$

Topic 12: Trigonometry (Khan Academy Topic: "Trigonometric Functions")

Find the exact value of the following without calculators:

31.
$$\sin^2 \frac{5\pi}{4} - \cos^2 \frac{5\pi}{3}$$

32.
$$6 \sec \pi - 4 \cot \frac{\pi}{2}$$

31.
$$\sin^2 \frac{5\pi}{4} - \cos^2 \frac{5\pi}{3}$$
 32. $6\sec \pi - 4\cot \frac{\pi}{2}$ 33. $\left(4\cos \frac{\pi}{6} - 6\sin \frac{2\pi}{3}\right)^{-2}$

34.
$$\sin \frac{\pi}{3}$$

35.
$$\tan \frac{4\pi}{3}$$

34.
$$\sin \frac{\pi}{3}$$
 35. $\tan \frac{4\pi}{3}$ 36. $\csc \frac{5\pi}{6}$ 37. $\tan \frac{3\pi}{4}$

37.
$$\tan \frac{3\pi}{4}$$

Evaluate the inverse trigonometric function

38.
$$\arcsin\left(-\frac{\sqrt{3}}{2}\right)$$
 39. $\arccos\left(+\frac{1}{\sqrt{2}}\right)$ 40. $\arctan(\sqrt{3})$ 41. $arc \csc(2)$

39.
$$\arccos\left(+\frac{1}{\sqrt{2}}\right)$$

40.
$$\arctan(\sqrt{3})$$

<u>Topic 13: Logarithmic Rules:</u> (Khan Academy Topic: "Using the properties of logarithms: multiple steps"

Simplify the expression using rules for logarithms.

42.
$$\log_2 x + \log_2 (x - 2)$$

43.
$$3 \ln x^2 - \frac{1}{2} \ln(x+3) + \ln 5$$

Use log rules to expand the expression.

44.
$$\log_2 \frac{x^4}{\sqrt{2x-5}}$$

45.
$$\ln \frac{3x^5\sqrt[3]{(x+5)^2}}{(2x+7)^4}$$