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What you'll Learn About

- Average Rates of Change
- A Definition of the Derivative

An object dropped from rest from the top of a tall building falls $y = 16t^2$ feet in the first t seconds. Find the **average speed/average rate of change** during the first 2 seconds of flight.

$t = 0$ sec $y = 0$ ft

$t = 2$ sec $y = 64$ ft

Avg Speed $= \frac{64-0}{2-0} = \frac{\Delta y}{\Delta t}$

32 ft/sec

Find the average rate of change of $f(x) = \sqrt{4x+1}$ over each interval

a) [0, 2]

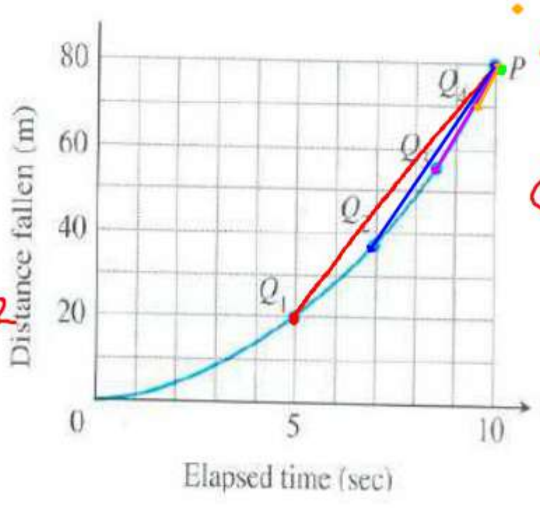
b) [10, 12] ← Interval

$x = 0 \quad f(0) = 1$

$x = 2 \quad f(2) = 3$

$A.R.O.C = \frac{3-1}{2-0} = \frac{2}{2} = 1$

Estimate the average rate of change by finding the slopes of each secant line. Indicate units of measure



Q1(5,20)
Q2(7,38)
Q3(8.5, 55)
Q4(9.5,71)
P(10,80)

PQ1= $\frac{80-20}{10-5} = \frac{60}{5} = 12$

PQ2= 14

PQ3= $16\frac{2}{3}$

PQ4= 18

Use the slopes of the secant lines to Estimate the instantaneous rate of change/slope at point P

$\lim_{x \to P} (\text{slopes}) = 20$

$\lim_{x \to P} \frac{\Delta y}{\Delta x} = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$

$\lim_{x \to a} \left(\frac{f(x)-f(a)}{x-a} \right)$ The definition of the derivative (slope) at a point (instantaneous rate of change)

Difference quotient ($x-a$ = Δx)

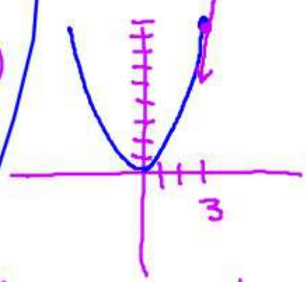
Using a definition of the derivative to find slope

A) Find the slope of $f(x) = x^2$ at the point (3,9)

(a = 3, f(a) = 9)

$\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$

$\lim_{x \to 3} \frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{x-3} = 6$ ← slope at the point (3,9)



B) Find the slope of $f(x) = \frac{2}{x}$ at $x = 4$

$(4, \frac{1}{2})$ (a, f(a))

$\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$

$\lim_{x \to 4} \frac{\frac{(2)2}{(2)x} - \frac{1(x)}{2(x)}}{x-4} = \frac{\left(\frac{4-x}{2x}\right)}{\left(\frac{x-4}{1}\right)} = \frac{(4-x)}{2x(x-4)} = \frac{-1}{2x} = -\frac{1}{8}$

C) Find the slope of $f(x) = \frac{1}{x-4}$ at $x = 7$

D) Find the slope of $f(x) = 9 - x^2$ at the point (-3,0)

What you'll Learn About

- Definition of the derivative
- Notation

Use the substitution h = x - a to create the definition of the derivative

A_1) Set-up a formula for the slope of $f(x) = x^2$ at x = -1

A_2) Use the substitution h = x - a to set-up the definition of the derivative

B_1) Set-up a formula for the slope of $f(x) = \frac{1}{x-2}$ at x = 4

B_2) Use the substitution h = x - a to set-up the definition of the derivative

Given a definition of the derivative(slope) find the function that you are taking the derivative of and the point you are finding the derivative(slope) at

A) $\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4}$

B) $\lim_{x \to 2} \frac{\ln x - \ln 2}{x-2}$

C) $\lim_{h \to 0} \frac{(2+h)^3 - 8}{h}$

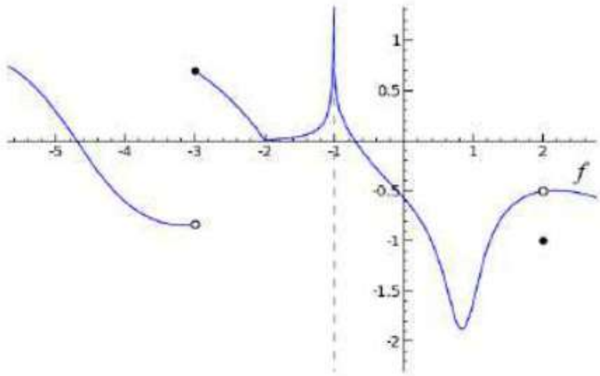
D) $\lim_{h \to 0} \frac{\frac{2}{3+h} - \frac{2}{3}}{h}$

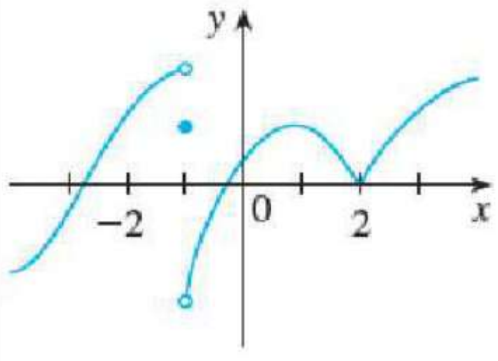
Another Definition: $\lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$

What you'll Learn About

- How the derivative might fail to exist
- Differentiability implies local linearity
- Differentiability implies Continuity

a. Find all points where the function, f(x), is differentiable.
b. Find all points where the function is continuous, but not differentiable.
c. Find all points where the graph is neither continuous nor differentiable.





3.2 Differentiability:

a. Find all points where the function, f(x), is differentiable.
b. Find all points where the function is continuous, but not differentiable.
c. Find all points where the graph is neither continuous nor differentiable.