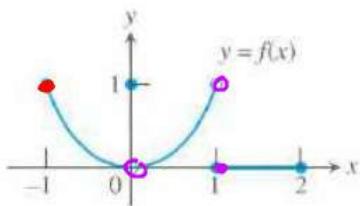


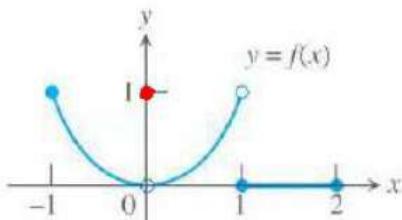
Page 12

What you'll Learn About

- Continuity at a point
- Continuous Functions
- Intermediate Value Theorem for Continuous Functions



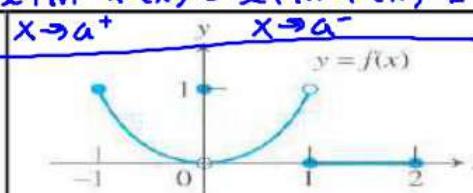
- 1a. Does $f(-1)$ exist? yes
- 1b. Does $\lim_{x \rightarrow -1^+} f(x)$ exist? yes
- 1c. Does $\lim_{x \rightarrow -1^+} f(x) = f(-1)$ yes
- 1d. Is f continuous at $x = -1$? yes



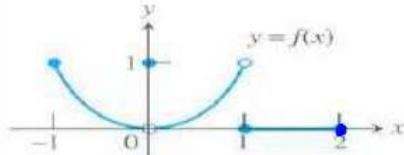
- 2a. Does $f(0)$ exist? yes
- 2b_i. Does $\lim_{x \rightarrow 0^+} f(x)$ exist? yes
- 2b_{ii}. Does $\lim_{x \rightarrow 0^-} f(x)$ exist? yes
- 2c. Does $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) \neq f(0)$ NO
- 2d. Is f continuous at $x = 0$? NO (Removable)

A function is continuous at $x=a$ if
 $\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x) = f(a)$

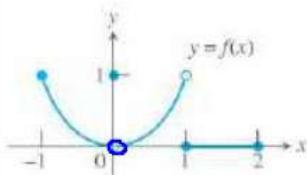
*



- 3a. Does $f(1)$ exist? 3b_i. Does $\lim_{x \rightarrow 1^+} f(x)$ exist?
 yes $f(1) = 0$ yes
- 3b_{ii}. Does $\lim_{x \rightarrow 1^-} f(x)$ exist? 1
 yes
- 3c. Does $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^-} f(x) = f(1)$ NO
- 3d. Is f continuous at $x=1$? NO Jump



- 4a. Does $f(2)$ exist? 4b. Does $\lim_{x \rightarrow 2^-} f(x)$ exist?
 yes yes
- 4c. Does $\lim_{x \rightarrow 2^-} f(x) = f(2)$ yes
 $0 = 0$
- 4d. Is f continuous at $x=2$? yes



5. For what values is the function continuous $x \neq 0, 1$

6a. Is it possible to extend f to be continuous at $x=0$? If so, what value should the extended function have? If not, why not? $f(0)=0$

6b. Is it possible to extend f to be continuous at $x=1$? If so, what value should the extended function have? If not, why not? Not a jump

Removable
Jump
Vertical asy

Determine the type of discontinuity

Left $\begin{cases} 3+x & x < 2 \\ 1 & x = 2 \end{cases}$ Right $\begin{cases} \frac{x}{2} & x > 2 \end{cases}$

~~Removable Jump (If limits are diff)~~

$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$

$5 \neq 1 = 1$

B) $f(x) = \begin{cases} \frac{1}{x-2} & x < 2 \\ x^2 + 5x & x > 2 \end{cases}$

$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$

$\frac{1}{0} \neq 14 \neq \text{DNE}$

C) $f(x) = \begin{cases} 9-x^2 & x \neq 3 \\ 5 & x = 3 \end{cases}$

$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3)$

Removable $0 = 0 \neq 5$

D) $f(x) = \begin{cases} 6-x & x < 3 \\ 2x-3 & x > 3 \end{cases}$

$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3)$

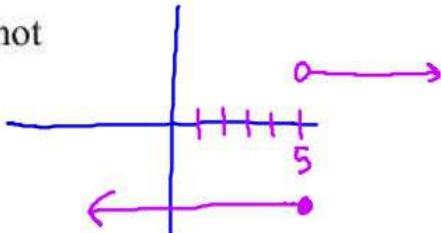
$3 = 3 \neq \text{DNE}$

Hole
Removable

Given the following information, sketch a graph of $f(x)$

A) $f(x)$ exists, but $\lim_{x \rightarrow 5}$ does not

Jump

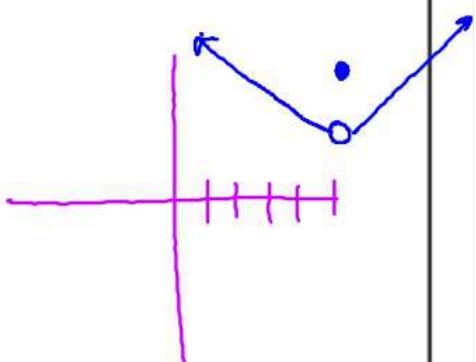


B) $f(5)$ exists

$\lim_{x \rightarrow 5}$ exists

Hole

f is not continuous at $x = 5$



Find a value for a so that the function is continuous

$$47) f(x) = \begin{cases} x^2 - 1 & x < 3 \\ 2ax & x \geq 3 \end{cases}$$

At $x = 3$ both functions
have to have the same y -value
(Both functions = to each other)

$$x = 3$$

$$x^2 - 1 = 2ax$$

$$9 - 1 = 6a$$

$$\frac{8}{6} = \frac{6a}{6}$$

$$\frac{4}{3} = a$$