LESSON 2.3

Solving Absolute Value Inequalities

Common Core Math Standards

The student is expected to:

COMMON CORE **A-CED.A.1**

Create equations and inequalities in one variable and use them to solve problems. Also A-REI.B.3, F-IF.C.7b

Mathematical Practices

COMMON CORE **MP.4 Modeling**

Language Objective

Match absolute value equations and inequalities with their graphs, explaining and justifying reasoning.

ENGAGE

Essential Question: What are two ways to solve an absolute value inequality?

Possible answer: You can solve an absolute value inequality graphically or algebraically. For a graphical solution, treat each side of the inequality as a function and graph the two functions. Use the inequality symbol to determine the intervals on the *x*-axis where one graph lies above or below the other. For an algebraic solution, isolate the absolute value expression and rewrite the inequality as a compound inequality that doesn't involve absolute value so that you can finish solving the inequality.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss the names of the planets, the elliptical path the planets follow, and why the distance a planet is from the sun might relate to absolute value inequalities. Then preview the Lesson Performance Task.

Name ______ Class ______ Date ______

2.3 Solving Absolute Value Inequalities

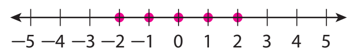
Essential Question: What are two ways to solve an absolute value inequality?

Resource Locker

Explore Visualizing the Solution Set of an Absolute Value Inequality

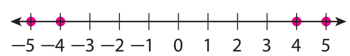
You know that when solving an absolute value equation, it's possible to get two solutions. Here, you will explore what happens when you solve absolute value inequalities.

(A) Determine whether each of the integers from −5 to 5 is a solution of the inequality $|x| + 2 < 5$. Write *yes* or *no* for each number in the table. If a number is a solution, plot it on the number line.

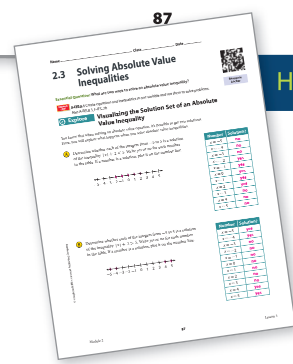


Number	Solution?
$x = -5$	no
$x = -4$	no
$x = -3$	no
$x = -2$	yes
$x = -1$	yes
$x = 0$	yes
$x = 1$	yes
$x = 2$	yes
$x = 3$	no
$x = 4$	no
$x = 5$	no

(B) Determine whether each of the integers from −5 to 5 is a solution of the inequality $|x| + 2 > 5$. Write *yes* or *no* for each number in the table. If a number is a solution, plot it on the number line.



Number	Solution?
$x = -5$	yes
$x = -4$	yes
$x = -3$	no
$x = -2$	no
$x = -1$	no
$x = 0$	no
$x = 1$	no
$x = 2$	no
$x = 3$	no
$x = 4$	yes
$x = 5$	yes



HARDCOVER PAGES 63–70

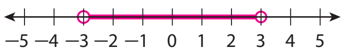
Turn to these pages to find this lesson in the hardcover student edition.

Ⓒ State the solutions of the equation $|x| + 2 = 5$ and relate them to the solutions you found for the inequalities in Steps A and B.

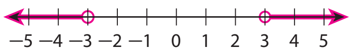
The solutions are -3 and 3. These are the only numbers that are not solutions of the inequalities $|x| + 2 < 5$ and $|x| + 2 > 5$.

Ⓓ If x is any real number and not just an integer, graph the solutions of $|x| + 2 < 5$ and $|x| + 2 > 5$.

Graph of all real solutions of $|x| + 2 < 5$:



Graph of all real solutions of $|x| + 2 > 5$:



Reflect

1. It's possible to describe the solutions of $|x| + 2 < 5$ and $|x| + 2 > 5$ using inequalities that don't involve absolute value. For instance, you can write the solutions of $|x| + 2 < 5$ as $x > -3$ and $x < 3$. Notice that the word *and* is used because x must be both greater than -3 and less than 3. How would you write the solutions of $|x| + 2 > 5$? Explain.
   Write the solutions of $|x| + 2 > 5$ as $x < -3$ or $x > 3$. Use the word *or* because x must be either less than -3 or greater than 3; it can't be both.

2. Describe the solutions of $|x| + 2 \leq 5$ and $|x| + 2 \geq 5$ using inequalities that don't involve absolute value.
   The solutions of $|x| + 2 \leq 5$ are the values of x for which $x \geq -3$ and $x \leq 3$. The solutions of $|x| + 2 \geq 5$ are the values of x for which $x \leq -3$ or $x \geq 3$.

Explain 1 Solving Absolute Value Inequalities Graphically

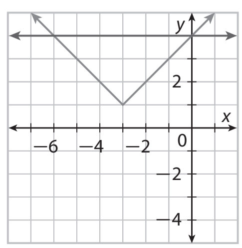
You can use a graph to solve an absolute value inequality of the form $f(x) > g(x)$ or $f(x) < g(x)$, where $f(x)$ is an absolute value function and $g(x)$ is a constant function. Graph each function separately on the same coordinate plane and determine the intervals on the x-axis where one graph lies above or below the other. For $f(x) > g(x)$, you want to find the x-values for which the graph $f(x)$ is above the graph of $g(x)$. For $f(x) < g(x)$, you want to find the x-values for which the graph of $f(x)$ is below the graph of $g(x)$.

Example 1 **Solve the inequality graphically.**

Ⓐ $|x + 3| + 1 > 4$

The inequality is of the form $f(x) > g(x)$, so determine the intervals on the x-axis where the graph of $f(x) = |x + 3| + 1$ lies above the graph of $g(x) = 4$.

The graph of $f(x) = |x + 3| + 1$ lies above the graph of $g(x) = 4$ to the left of $x = -6$ and to the right of $x = 0$, so the solution of $|x + 3| + 1 > 4$ is $x < -6$ or $x > 0$.



PROFESSIONAL DEVELOPMENT

Math Background

An absolute value inequality is often in the form $|ax + b| < c$ or $|ax + b| > c$. If the inequality is in the form $|ax + b| < c$, then it can be rewritten as the compound inequality $-c < ax + b < c$, and solved for x. The solution will be of the form $\frac{-c - b}{a} < x$ and $x < \frac{c - b}{a}$. If the inequality is in the form $|ax + b| > c$, it can be rewritten as the compound inequality $c < ax + b$ or $ax + b < -c$, and solved for x. The solution will be of the form $\frac{c - b}{a} < x$ or $x < \frac{-c - b}{a}$. The solutions are easily adjusted for $\leq$ and $\geq$.

EXPLORE

Visualizing the Solution Set of an Absolute Value Inequality

QUESTIONING STRATEGIES

? How would you characterize the solutions to an absolute value inequality? **Sample answer: They lie either between two values or everywhere else except between the two values, depending on the inequality.**

? Why do you write the solutions to the absolute value inequality as a compound inequality statement? **Sample answer: because the solution consists of the union or intersection of the solutions of the two related linear inequalities.**

EXPLAIN 1

Solving Absolute Value Inequalities Graphically

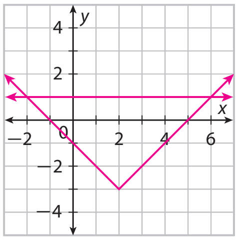
QUESTIONING STRATEGIES

? How do you interpret which points are solutions of the inequality? **Sample answer: If $f(x) > g(x)$, the graph of $f(x)$ must be "above" the graph of $g(x)$. The solutions are the x-values in the interval along the x-axis where $f(x)$ has y-values greater than $g(x)$. If $f(x) < g(x)$, the graph of $f(x)$ must be "below" the graph of $g(x)$. The solutions are the x-values in the interval along the x-axis where $f(x)$ has y-values less than $g(x)$.**

? How do you know when the endpoints of the solution interval on the x-axis are not included in the solution? **The original inequality is $<$ or $>$.**

Ⓑ $|x-2|-3<1$

The inequality is of the form $f(x) < g(x)$, so determine the intervals on the x-axis where the graph of $f(x) = |x-2|-3$ lies **below** the graph of $g(x) = 1$.



The graph of $f(x) = |x-2|-3$ lies **below** the graph of $g(x) = 1$ between $x =$ **−2** and $x =$ **6**, so the solution of $|x-2|-3<1$ is $x >$ **−2** *and* $x <$ **6**.

Reflect

3. Suppose the inequality in Part A is $|x+3|+1 \geq 4$ instead of $|x+3|+1>4$. How does the solution change?
The solution now includes the endpoints of the interval: $x \leq -6$ *or* $x \geq 0$.

4. In Part B, what is another way to write the solution $x > -2$ and $x < 6$?
$-2 < x < 6$

5. **Discussion** Suppose the graph of an absolute value function $f(x)$ lies entirely above the graph of the constant function $g(x)$. What is the solution of the inequality $f(x) > g(x)$? What is the solution of the inequality $f(x) < g(x)$?
The solution of $f(x) > g(x)$ is all real numbers, because every point on the graph of $f(x)$ is above the corresponding point on the graph of $g(x)$. The solution of $f(x) < g(x)$ is no real number, because no point on the graph of $f(x)$ is below the corresponding point on the graph of $g(x)$.

Your Turn

6. Solve $|x+1|-4 \leq -2$ graphically.
The inequality is of the form $f(x) \leq g(x)$, so determine the intervals on the x-axis where the graph of $f(x) = |x+1|-4$ intersects or lies below the graph of $g(x) = -2$.
The graph of $f(x) = |x+1|-4$ intersects the graph of $g(x) = -2$ at $x = -3$ and $x = 1$ and lies below the graph of $g(x) = -2$ between those x-values, so the solution of $|x+1|-4 \leq -2$ is $x \geq -3$ *and* $x \leq 1$.

COLLABORATIVE LEARNING

Small Group Activity

Have students work in groups to make a flowchart that explains how to solve each of the four types of an absolute value inequality of the form $|ax+b| \square\, c$ or $|ax-b| \square\, c$, where the box represents the inequality symbol. For example:

$|2x+3| \leq 6$ $\quad |2x+3| < 6$ $\quad |2x+3| > 6$ $\quad |2x+3| \geq 6$

Ask each student in a group to finish one branch of the flowchart. Then have the group collate the branches to make the entire flowchart.

Explain 2 Solving Absolute Value Inequalities Algebraically

To solve an absolute value inequality algebraically, start by isolating the absolute value expression. When the absolute value expression is by itself on one side of the inequality, apply one of the following rules to finish solving the inequality for the variable.

Solving Absolute Value Inequalities Algebraically
1. If $|x| > a$ where a is a positive number, then $x < -a$ or $x > a$.
2. If $|x| < a$ where a is a positive number, then $-a < x < a$.

Example 2 **Solve the inequality algebraically. Graph the solution on a number line.**

Ⓐ $|4 - x| + 15 > 21$

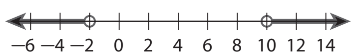
$|4 - x| > 6$

$4 - x < -6$ or $4 - x > 6$

$-x < -10$ or $-x > 2$

$x > 10$ or $x < -2$

The solution is $x > 10$ or $x < -2$.



Ⓑ $|x + 4| - 10 \leq -2$

$|x + 4| \leq$ **8**

$x + 4 \geq$ **−8** and $x + 4 \leq$ **8**

$x \geq$ **−12** and $x \leq$ **4**

The solution is $x \geq$ **−12** and $x \leq$ **4**,

or **−12** $\leq x \leq$ **4**.

Reflect

7. In Part A, suppose the inequality were $|4 - x| + 15 > 14$ instead of $|4 - x| + 15 > 21$. How would the solution change? Explain.

The first step in solving would be to subtract 15 from both sides and get $|4 - x| > -1$. At this point, the solving process can stop, because the absolute value of every number is greater than −1. So, the solution is all real numbers.

8. In Part B, suppose the inequality were $|x + 4| - 10 \leq -11$ instead of $|x + 4| - 10 \leq -2$. How would the solution change? Explain.

The first step in solving would be to add 10 to both sides and get $|x + 4| \leq -1$. At this point, the solving process can stop, because there are no real numbers whose absolute value is less than or equal to −1. So, there is no solution.

EXPLAIN 2

Solving Absolute Value Inequalities Algebraically

QUESTIONING STRATEGIES

When does the graph of the solution to an inequality include the endpoints? **when the original inequality is ≤ or ≥**

When does the graph of the solution include the points between the endpoints found in the solution? **When the original inequality is < or ≤, the compound inequality is an "and" statement, so its graph includes the intersection of two graphs. These graphs intersect between the endpoints.**

INTEGRATE TECHNOLOGY

A graphing calculator can be used to check the solution graph for an inequality. For example, you would graph $y = |4 - x| + 15 > 21$ to check the solution for $|4 - x| + 15 > 21$. The graph will be a broken horizontal line above the x-axis with endpoints at −2 and 10. You must interpret the graph to be open at the endpoints. So, the solution graph is $x < -2$ or $x > 10$.

AVOID COMMON ERRORS

Some students confuse when to use *or* and *and* when rewriting an absolute value inequality. Remind students that when the inequality is $|ax + b| < c$ or $|ax + b| \leq c$, they should rewrite the inequality using *and*. When the inequality is $|ax + b| > c$ or $|ax + b| \geq c$, they should rewrite the inequality using *or*. Emphasize the importance of checking some of the solutions in the original inequality to help avoid this error.

DIFFERENTIATE INSTRUCTION

Visual Cues

You may want students to use visual models to help them understand some simple inequalities as well as some real-world inequalities. For simple inequalities of the form $|x| < a$ or $|x| > a$, constructing a graph of possible solutions on a number line as a first step may be helpful. For a real-world problem, involving allowable tolerance, for example, drawing an unlabeled conjunction graph centered at the middle value with the endpoints representing the minimum and maximum tolerance values may be most helpful. This should help students visualize how to construct an inequality based on the graph.

EXPLAIN 3

Solving a Real-World Problem with Absolute Value Inequalities

QUESTIONING STRATEGIES

When does an absolute value inequality apply to a real-world situation? **Sample answer: When a model for the real-world situation includes a range of values where the sign of the difference between the values doesn't matter, you can write an absolute value model that will apply. For example, if the model is $|\ell - 3.25| \le 0.02$, solving the inequality gives $3.23 \le \ell \le 3.27$, which gives possible values on either side of ℓ.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Reasoning

MP.2 Encourage students to solve the absolute value inequality for a real-world problem in the standard way: (1) Write a compound inequality using and or *or*, depending on the original problem; (2) solve each inequality; (3) rewrite the solution as a compound inequality using *and* or *or*; (4) graph the compound inequality if needed; and (5) check some values from the solution in the original problem to see if they make sense.

Your Turn

Solve the inequality algebraically. Graph the solution on a number line.

9. $3|x-7| \ge 9$

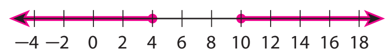
$3|x-7| \ge 9$

$|x-7| \ge 3$

$x - 7 \le -3$ or $x - 7 \ge 3$

$x \le 4$ or $x \ge 10$

The solution is $x \le 4$ or $x \ge 10$.



10. $|2x+3| < 5$

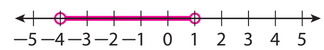
$|2x+3| < 5$

$2x + 3 > -5$ and $2x + 3 < 5$

$2x > -8$ and $2x < 2$

$x > -4$ and $x < 1$

The solution is $-4 < x < 1$.



Explain 3 Solving a Real-World Problem with Absolute Value Inequalities

Absolute value inequalities are often used to model real-world situations involving a margin of error or *tolerance*. Tolerance is the allowable amount of variation in a quantity.

Example 3

A machine at a lumber mill cuts boards that are 3.25 meters long. It is acceptable for the length to differ from this value by at most 0.02 meters. Write and solve an absolute value inequality to find the range of acceptable lengths.

Analyze Information

Identify the important information.

- The boards being cut are **3.25** meters long.
- The length can differ by at most 0.02 meters.

Formulate a Plan

Let the length of a board be ℓ. Since the sign of the difference between ℓ and 3.25 doesn't matter, take the absolute value of the difference. Since the absolute value of the difference can be at most 0.02, the inequality that models the situation is $|\ell - \mathbf{3.25}| \le \mathbf{0.02}$.

Solve

$|\ell - 3.25| \le 0.02$

$\ell - 3.25 \ge -0.02$ and $\ell - 3.25 \le 0.02$

$\ell \ge \mathbf{3.23}$ and $\ell \le \mathbf{3.27}$

So, the range of acceptable lengths is $\mathbf{3.23} \le \ell \le \mathbf{3.27}$.

LANGUAGE SUPPORT EL

Connect Vocabulary

Help students understand the term *compound inequality* as it is used to solve an absolute value inequality. Have them recall that a *disjunction* is a compound statement joined by the word *or*. A disjunction applies to all inequalities of the form $|x| > a$ or $|x| \ge a$. Inequalities of the form $|x| < a$ or $|x| \le a$ can be solved using a *conjunction*, a compound statement joined by the word *and*. Have students use note cards to write examples of all four types of inequalities they may see in the lesson, and ask them to write the associated compound inequality with a graph of the solution set for each example.

Justify and Evaluate

The bounds of the range are positive and close to **3.25**, so this is a reasonable answer.

The answer is correct since **3.23** $+ 0.02 = 3.25$ and **3.27** $- 0.02 = 3.25$.

Your Turn

11. A box of cereal is supposed to weigh 13.8 oz, but it's acceptable for the weight to vary as much as 0.1 oz. Write and solve an absolute value inequality to find the range of acceptable weights.

Let the weight of the cereal be w. The sign of the difference between w and 13.8 doesn't matter, so take the absolute value of the difference. Since the absolute value of the difference can be as much as 0.1, the inequality that models the situation is $|w - 13.8| \leq 0.1$.

Solve:

$|w - 13.8| \leq 0.1$

$w - 13.8 \geq -0.1$ and $w - 13.8 \leq 0.1$

$w \geq 13.7$ and $w \leq 13.9$

So, the range of acceptable weights of the cereal (in ounces) is $13.7 \leq w \leq 13.9$.

Elaborate

12. Describe the values of x that satisfy the inequalities $|x| < a$ and $|x| > a$ where a is a positive constant.
For $|x| < a$, the solutions are values of x between $-a$ and a. For $|x| > a$, the solutions are the values of x beyond $-a$ and a (that is, the values of x less than $-a$ or the values of x greater than a).

13. How do you algebraically solve an absolute value inequality?
Isolate the absolute value expression. Then rewrite the inequality as a compound inequality that uses either *and* or *or* and that doesn't involve absolute value. Finish solving for the variable.

14. Explain why the solution of $|x| > a$ is all real numbers if a is a negative number.
Since the absolute value of any number is always nonnegative, it is always greater than any negative number. So, all real numbers satisfy the inequality.

15. **Essential Question Check-In** How do you solve an absolute value inequality graphically?
Treat each side of the inequality as a function and graph the two functions. Use the inequality symbol to determine the intervals on the x-axis where one graph lies above or below the other.

ELABORATE

INTEGRATE MATHEMATICAL PRACTICES

Focus on Patterns

MP.8 Discuss with students how to solve an absolute value inequality of the form $|ax + b| < c$ and $|ax + b| > c$. Students should routinely rewrite the next step as a compound inequality using *and* or *or,* and then solve each part of the inequality.

QUESTIONING STRATEGIES

How is solving an absolute value inequality like and different from solving an absolute value equation? **They are alike in that some of the solution steps are the same once a compound statement is written for the absolute value equation or inequality. They are different in that an absolute value inequality may be an *and* statement with infinitely many solutions as well as an *or* statement with infinitely many solutions, but an absolute value equation is only an *or* statement with two, one, or zero solutions.**

CONNECT VOCABULARY EL

Relate the word *conjunction* to its opposite, *disjunction* (discussed in the previous lesson). Explain that the prefix *con* means *to join.* For a conjunction to be true, all of its parts *(joined)* must be true.

SUMMARIZE THE LESSON

How do you solve a linear absolute value inequality? **Isolate the absolute value expression; write the absolute value inequality as a compound statement of two linear inequalities; solve each inequality; and rewrite the solution as a compound statement.**

EVALUATE

ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Visualizing the Solution Set of an Absolute Value Inequality	Exercises 1–2
Example 1 Solving Absolute Value Inequalities Graphically	Exercises 3–10
Example 2 Solving Absolute Value Inequalities Algebraically	Exercises 11–16
Example 3 Solving a Real-World Problem with Absolute Value Inequalities	Exercises 17–22

INTEGRATE MATHEMATICAL PRACTICES

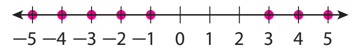
Focus on Reasoning

MP.2 Remind students to check their solutions by substituting some values into the original inequality and verifying that they make the inequality true. When solving inequalities graphically, remind students that the interval or intervals on the x-axis where $f(x) > g(x)$ or $f(x) < g(x)$ comprise the solution set.

Evaluate: Homework and Practice

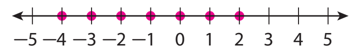
Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

1. Determine whether each of the integers from −5 to 5 is a solution of the inequality $|x - 1| + 3 \geq 5$. If a number is a solution, plot it on the number line.



The integers from −5 to 5 that satisfy the inequality are −5, −4, −3, −2, −1, 3, 4, and 5.

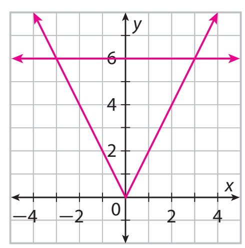
2. Determine whether each of the integers from −5 to 5 is a solution of the inequality $|x + 1| - 2 \leq 1$. If a number is a solution, plot it on the number line.



The integers from −5 to 5 that satisfy the inequality are −4, −3, −2, −1, 0, 1, and 2.

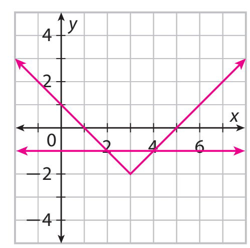
Solve each inequality graphically.

3. $2|x| \leq 6$



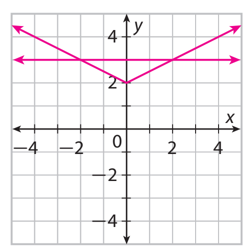
The solution is $-3 \leq x \leq 3$.

4. $|x - 3| - 2 > -1$



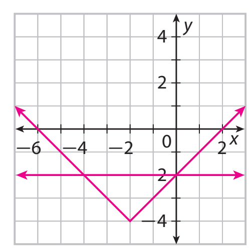
The solution is $x < 2$ or $x > 4$.

5. $\frac{1}{2}|x| + 2 < 3$



The solution is $-2 < x < 2$.

6. $|x + 2| - 4 \geq -2$



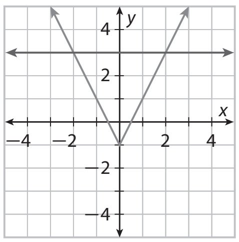
The solution is $x \leq -4$ or $x \geq 0$.

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
1–2	**1** Recall of Information	**MP.6** Precision
3–6	**1** Recall of Information	**MP.4** Modeling
7–16	**2** Skills/Concepts	**MP.2** Reasoning
17–21	**2** Skills/Concepts	**MP.1** Problem Solving
22	**2** Skills/Concepts H.O.T.	**MP.4** Modeling
23	**3** Strategic Thinking H.O.T.	**MP.6** Precision
24	**3** Strategic Thinking H.O.T.	**MP.2** Reasoning

Match each graph with the corresponding absolute value inequality. Then give the solution of the inequality.

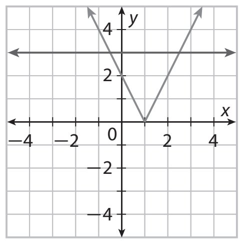
A. $2|x| + 1 > 3$ **B.** $2|x + 1| < 3$ **C.** $2|x| - 1 > 3$ **D.** $2|x - 1| < 3$

7.



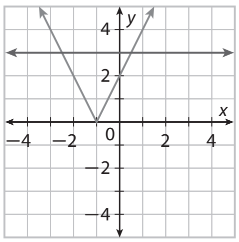
C. The solution is $x < -2$ or $x > 2$.

8.



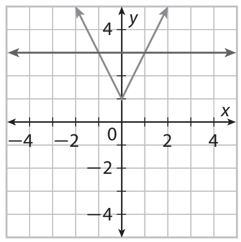
D. The solution is $-\frac{1}{2} < x < \frac{5}{2}$.

9.



B. The solution is $-\frac{5}{2} < x < \frac{1}{2}$.

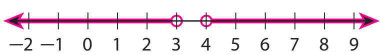
10.



A. The solution is $x < -1$ or $x > 1$.

Solve each absolute value inequality algebraically. Graph the solution on a number line.

11. $2\left|x - \frac{7}{2}\right| + 3 > 4$



$2\left|x - \frac{7}{2}\right| + 3 > 4$

$2\left|x - \frac{7}{2}\right| > 1$

$\left|x - \frac{7}{2}\right| > \frac{1}{2}$

$x - \frac{7}{2} < -\frac{1}{2}$ or $x - \frac{7}{2} > \frac{1}{2}$

$x < 3$ $\quad x > 4$

The solution is $x < 3$ or $x > 4$.

SMALL GROUP ACTIVITY

Have students work in small groups to make a poster showing how to apply the steps for solving an absolute value inequality. Give each group a different inequality to solve. Then have each group present its poster to the rest of the class, and ask for a volunteer from the group to explain each step.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 Point out that the concept *margin of error* is used in surveys to describe how many percentage points higher or lower a poll result can be and still be considered a “true” figure, meaning a result that is representative of the population.

12. $|2x + 1| - 4 < 5$

$|2x + 1| - 4 < 5$

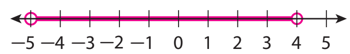
$|2x + 1| < 9$

$2x + 1 > -9$ and $2x + 1 < 9$

$2x > -10$ $\quad$ $2x < 8$

$x > -5$ $\quad$ $x < 4$

The solution is $-5 < x < 4$.



13. $3|x + 4| + 2 \geq 5$

$3|x + 4| + 2 \geq 5$

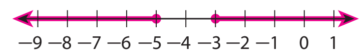
$3|x + 4| \geq 3$

$|x + 4| \geq 1$

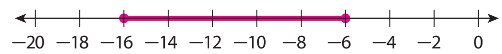
$x + 4 \leq -1$ or $x + 4 \geq 1$

$x \leq -5$ $\quad$ $x \geq -3$

The solution is $x \leq -5$ or $x \geq -3$.



14. $|x + 11| - 8 \leq -3$



$|x + 11| - 8 \leq -3$

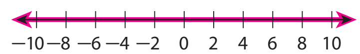
$|x + 11| \leq 5$

$x + 11 \geq -5$ and $x + 11 \leq 5$

$x \geq -16$ $\quad$ $x \leq -6$

The solution is $-16 \leq x \leq -6$.

15. $-5|x - 3| - 5 < 15$

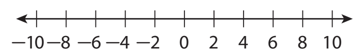


$-5|x - 3| < 20$

$|x - 3| > -4$

all real numbers

16. $8|x + 4| + 10 < 2$



$8|x + 4| < -8$

$|x + 4| < -1$

solution

Solve each problem using an absolute value inequality.

17. The thermostat for a house is set to 68 °F, but the actual temperature may vary by as much as 2 °F. What is the range of possible temperatures?

Let the temperature in the house be *T*. The sign of the difference between *T* and 68 doesn't matter, so take the absolute value of the difference. Since the absolute value of the difference can be as much as 2, the inequality that models the situation is $|T - 68| \leq 2$.

$|T - 68| \leq 2$

$|T - 68| \geq -2$ and $|T - 68| \leq 2$

$T \geq 66 \qquad T \leq 70$

The range of house temperatures (in degrees Fahrenheit) is $66 \leq T \leq 70$.

18. The balance of Jason's checking account is $320. The balance varies by as much as $80 each week. What are the possible balances of Jason's account?

Let the balance of Jason's account be *B*. The sign of the difference between *B* and 320 doesn't matter, so take the absolute value of the difference. Since the absolute value of the difference can be as much as 80, the inequality that models the situation is $|B - 320| \leq 80$.

$|B - 320| \leq 80$

$B - 320 \geq -80$ and $B - 320 \leq 80$

$B \geq 240 \qquad B \leq 400$

The range of possible balances (in dollars) is $240 \leq B \leq 400$.

19. On average, a squirrel lives to be 6.5 years old. The lifespan of a squirrel may vary by as much as 1.5 years. What is the range of ages that a squirrel lives?

Let the age of a squirrel be *a*. The sign of the difference between *a* and 6.5 doesn't matter, so take the absolute value of the difference. Since the absolute value of the difference can be as much as 1.5, the inequality that models the situation is $|a - 6.5| \leq 1.5$.

$|a - 6.5| \leq 1.5$

$a - 6.5 \geq -1.5$ and $a - 6.5 \leq 1.5$

$a \geq 5 \qquad a \leq 8$

The range of ages (in years) that a squirrel lives is $5 \leq a \leq 8$.

PEER-TO-PEER DISCUSSION

Ask students to discuss with a partner what the solution to an inequality of the form $|ax + b| < c$ means in terms of the graph of the related functions $f(x) = |ax + b|$ and $g(x) = c$. Then ask students to make conjectures about the solutions to $|ax + b| < c$ based on the graphs of their related functions. Conjectures should include the interval along the *x*-axis that represents the solutions and how the graphs of these functions look. **The solutions to $|ax + b| < c$ are points in the interval along the *x*-axis for which $f(x) < g(x)$. Based on this, conjectures should include that the graphs of $f(x) = |ax + b|$ and $g(x) = c$ show the interval of points that satisfy the compound statement $-c < ax + b < c$.**

AVOID COMMON ERRORS

Watch for students who confuse which type of compound statement to use, *and* or *or*. Remind students that the solution process gives infinitely many solutions to the inequality, and that the type of compound statement determines whether the solution points are between the endpoints or everywhere but between the endpoints.

20. You are playing a history quiz game where you must give the years of historical events. In order to score any points at all for a question about the year in which a man first stepped on the moon, your answer must be no more than 3 years away from the correct answer, 1969. What is the range of answers that allow you to score points?

Let your answer be a. The sign of the difference between a and 1969 doesn't matter, so take the absolute value of the difference. Since the absolute value of the difference can be no more than 3, the inequality that models the situation is $|a - 1969| \leq 3$.

$|a - 1969| \leq 3$

$a - 1969 \geq -3$ **and** $a - 1969 \leq 3$

$a \geq 1966$ $\quad\quad$ $a \leq 1972$

The range of answers that allows you to score points is $1966 \leq a \leq 1972$.

21. The speed limit on a road is 30 miles per hour. Drivers on this road typically vary their speed around the limit by as much as 5 miles per hour. What is the range of typical speeds on this road?

Let the speed of a car be s. The sign of the difference between s and 30 doesn't matter, so take the absolute value of the difference. Since the absolute value of the difference can be as much as 5, the inequality that models the situation is $|s - 30| \leq 5$.

$|s - 30| \leq 5$

$s - 30 \geq -5$ **and** $s - 30 \leq 5$

$s \geq 25$ $\quad\quad$ $s \leq 35$

The range of typical speeds (in miles per hour) is $25 \leq s \leq 35$.

H.O.T. Focus on Higher Order Thinking

22. **Represent Real-World Problems** A poll of likely voters shows that the incumbent will get 51% of the vote in an upcoming election. Based on the number of voters polled, the results of the poll could be off by as much as 3 percentage points. What does this mean for the incumbent?

Let the incumbent's percentage of votes among all likely voters (not just those polled) be v. The sign of the difference between v and 51 doesn't matter, so take the absolute value of the difference. Since the absolute value of the difference can be as much as 3, the inequality that models the situation is $|v - 51| \leq 3$.

$|v - 51| \leq 3$

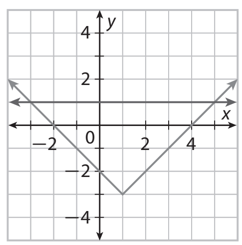
$v - 51 \geq -3$ **and** $v - 51 \leq 3$

$v \geq 48$ **and** $v \leq 54$

The range for the incumbent's percentage of votes among all likely voters is $48 \leq v \leq 54$, which means that the incumbent could still lose the election if the incumbent's percentage of votes among all likely voters (not just those polled) is less than 50%.

23. **Explain the Error** A student solved the inequality $|x - 1| - 3 > 1$ graphically. Identify and correct the student's error.

I graphed the functions $f(x) = |x - 1| - 3$ and $g(x) = 1$. Because the graph of $g(x)$ lies above the graph of $f(x)$ between $x = -3$ and $x = 5$, the solution of the inequality is $-3 < x < 5$.



The student identified where the graph of $g(x)$ lies above the graph of $f(x)$, but the student should have identified where the graph of $f(x)$ lies above the graph of $g(x)$ because the inequality has the form $f(x) > g(x)$. So, the solution of the inequality is $x < -3$ or $x > 5$.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 When modeling a problem in which an absolute value inequality involving tolerance applies, have students start with a number line diagram. Have them plot the starting value and then all points within the tolerance range greater than and less than the starting value. This will provide a visual connection to the inequality that applies, depending on the original real-world problem.

JOURNAL

Have students compare and contrast the methods they have learned for solving absolute value inequalities.

24. **Multi-Step** Recall that a literal equation or inequality is one in which the constants have been replaced by letters.

a. Solve $|ax + b| > c$ for x. Write the solution in terms of a, b, and c. Assume that $a > 0$ and $c \geq 0$.

$|ax + b| > c$

$ax + b < -c$ or $ax + b > c$

$ax < -c - b$ $\quad$ $ax > c - b$

$x < \frac{-c - b}{a}$ $\quad$ $x > \frac{c - b}{a}$

The solution is $x < \frac{-c - b}{a}$ or $x > \frac{c - b}{a}$.

b. Use the solution of the literal inequality to find the solution of $|10x + 21| > 14$.

Substitute 10 for a, 21 for b, and 14 for c in the solution of the literal inequality and simplify.

The solution is $x < \frac{-14 - 21}{10}$ or $x > \frac{14 - 21}{10}$, which simplifies to $x < -3.5$ or $x > -0.7$.

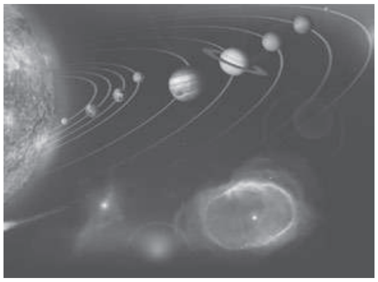
c. In Part a, explain how the restrictions $a > 0$ and $c \geq 0$ affect finding the solutions of the inequality.

If a is allowed to be 0, you obtain $|b| > c$, so you cannot solve for x. If $a < 0$, the solution process is the same, but when you divide both sides by a you must reverse the direction of the inequality, so you will get a different expression for the solution. If $c < 0$, you are left with the statement that the absolute value of a quantity is greater than a negative number, which is always true, so the solution is all real numbers.

Lesson Performance Task

The distance between the Sun and each planet in our solar system varies because the planets travel in elliptical orbits around the Sun. Here is a table of the average distance and the variation in the distance for the five innermost planets in our solar system.

	Average Distance	Variation
Mercury	36.0 million miles	7.39 million miles
Venus	67.2 million miles	0.43 million miles
Earth	93.0 million miles	1.55 million miles
Mars	142 million miles	13.2 million miles
Jupiter	484 million miles	23.2 million miles



a. Write and solve an inequality to represent the range of distances that can occur between the Sun and each planet.

b. Calculate the percentage variation (variation divided by average distance) in the orbit of each of the planets. Based on these percentages, which planet has the most elliptical orbit?

a. (Measurements are in millions of miles.)

Mercury $|x - 36.0| \leq 7.39$

$x - 36.0 \leq 7.39$ AND $x - 36.0 \geq -7.39$

$x \leq 43.39$ $\quad$ $x \geq 28.61$

Range for Mercury is $28.61 \leq x \leq 43.39$.

Venus $|x - 67.2| \leq 0.43$

$x - 67.2 \leq 0.43$ AND $x - 67.2 \geq -0.43$

$x \leq 67.63$ $\quad$ $x \geq 66.77$

Range for Venus is $66.77 \leq x \leq 67.63$.

Earth $|x - 93.0| \leq 1.55$

$x - 93.0 \leq 1.55$ AND $x - 93.0 \geq -1.55$

$x \leq 94.55$ $\quad$ $x \geq 91.45$

Range for Earth is $91.45 \leq x \leq 94.55$.

Mars $|x - 142| \leq 13.2$

$x - 142 \leq 13.2$ AND $x - 142 \geq -13.2$

$x \leq 155.2$ $\quad$ $x \geq 128.8$

Range for Mars is $128.8 \leq x \leq 155.2$.

Jupiter $|x - 484| \leq 23.2$

$x - 484 \leq 23.2$ AND $x - 484 \geq -23.2$

$x \leq 507.2$ $\quad$ $x \geq 460.8$

Range for Jupiter is $460.8 \leq x \leq 507.2$.

b. Mercury 21%, Venus 1%, Earth 2%, Mars 9%, Jupiter 5%;

Mercury has the most elliptical orbit.

EXTENSION ACTIVITY

Have students research the dwarf planet Pluto to find its average distance and its range of distance from the sun. Students will find that Pluto is approximately 39.5 AU (astronomical units) from the sun. Pluto's closest point to the sun is 29.7 AU. Pluto's farthest point away from the sun is 49.7 AU. 1 AU is equal to Earth's distance from the sun, or 1 AU is about 93 million miles.

CONNECT CONTEXT EL

Some students may not be familiar with the meanings of *average distance* and *variation*. Draw a horizontal line segment on the board. To the left of the line segment, some distance away, draw the sun. At the middle point of the line segment write "average distance". To the right and left of the middle point write "variation". Explain that the planet's location is at some point on the line segment. Its exact location varies, but the average of all of its possible locations is the middle point.

AVOID COMMON ERRORS

Some students may subtract x from the average distance. For example, for Mercury they may write $|36 - x| \leq 7.5$ instead of $|x - 36| \leq 7.5$. Because they are finding the absolute value of the expression, they will still get the correct values of x.

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.

0 points: Student does not demonstrate understanding of the problem.