LESSON 2.2

Solving Absolute Value Equations

Common Core Math Standards

The student is expected to:

COMMON CORE **A-CED.A.1**

Create equations and inequalities in one variable and use them to solve problems. Also A-REI.B.3, A-REI.D.11

Mathematical Practices

COMMON CORE **MP.6 Precision**

Language Objective

Explain to a partner why solutions to a variety of absolute value equations make sense and contain more than one solution, one solution, or no solution.

ENGAGE

Essential Question: How can you solve an absolute value equation?

Possible answer: Isolate the absolute value expression, then write two related equations with a disjunction, also known as an "or" statement.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss the photo and why this situation can be represented by a V-shaped path and an absolute value equation. Then preview the Lesson Performance Task.

Name ______________ Class ______________ Date ______________

2.2 Solving Absolute Value Equations

Essential Question: How can you solve an absolute value equation?

Resource Locker

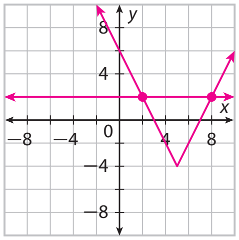
Explore Solving Absolute Value Equations Graphically

Absolute value equations differ from linear equations in that they may have two solutions. This is indicated with a **disjunction**, a mathematical statement created by a connecting two other statements with the word "or." To see why there can be two solutions, you can solve an absolute value equation using graphs.

(A) Solve the equation $2|x - 5| - 4 = 2$.

Plot the function $f(x) = 2|x - 5| - 4$ on the grid. Then plot the function $g(x) = 2$ as a horizontal line on the same grid, and mark the points where the graphs intersect.

The points are (2, 2) and (8, 2).



(B) Write the solution to this equation as a disjunction:

$x =$ **2** or $x =$ **8**

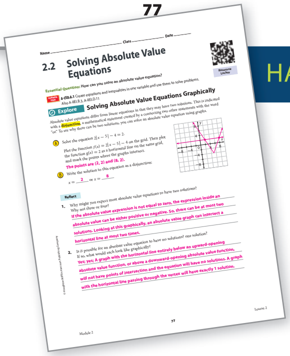
Reflect

1. Why might you expect most absolute value equations to have two solutions? Why not three or four?

If the absolute value expression is not equal to zero, the expression inside an absolute value can be either positive or negative. So, there can be at most two solutions. Looking at this graphically, an absolute value graph can intersect a horizontal line at most two times.

2. Is it possible for an absolute value equation to have no solutions? one solution? If so, what would each look like graphically?

Yes; yes; A graph with the horizontal line entirely below an upward-opening absolute value function, or above a downward-opening absolute value function, will not have points of intersection and the equation will have no solutions. A graph with the horizontal line passing through the vertex will have exactly 1 solution.



HARDCOVER PAGES 57–62

Turn to these pages to find this lesson in the hardcover student edition.

Explain 1 Solving Absolute Value Equations Algebraically

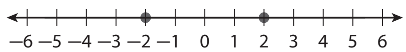
To solve absolute value equations algebraically, first isolate the absolute value expression on one side of the equation the same way you would isolate a variable. Then use the rule:

If $|x| = a$ (where a is a positive number), then $x = a$ OR $x = -a$.

Notice the use of a **disjunction** here in the rule for values of x. You cannot know from the original equation whether the expression inside the absolute value bars is positive or negative, so you must work through both possibilities to finish isolating x.

Example 1 **Solve each absolute value equation algebraically. Graph the solutions on a number line.**

(A) $|3x| + 2 = 8$



Subtract 2 from both sides. $|3x| = 6$

Rewrite as two equations. $3x = 6$ or $3x = -6$

Solve for x. $x = 2$ or $x = -2$

(B) $3|4x - 5| - 2 = 19$

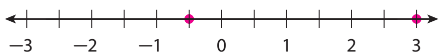
Add 2 to both sides. $3|4x - 5| = 21$

Divide both sides by 3. $|4x - 5| = 7$

Rewrite as two equations. $4x - 5 = 7$ or $4x - 5 = -7$

Add 5 to all four sides. $4x = 12$ or $4x = -2$

Solve for x. $x = 3$ or $x = -\frac{1}{2}$



Your Turn

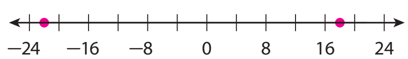
Solve each absolute value equation algebraically. Graph the solutions on a number line.

3. $\frac{1}{2}|x + 2| = 10$

$|x + 2| = 20$

$x + 2 = 20$ or $x + 2 = -20$

$x = 18$ or $x = -22$



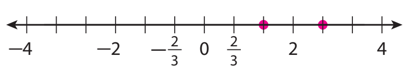
4. $-2|3x - 6| + 5 = 1$

$-2|3x - 6| = -4$

$|3x - 6| = 2$

$3x - 6 = 2$ or $3x - 6 = -2$

$x = \frac{8}{3}$ or $x = \frac{4}{3}$



PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices

This lesson provides an opportunity to address Mathematical Practice **MP.6**, which calls for students to "attend to precision" and communicate precisely. Students find the solutions to absolute value equations both by graphing them, with and without technology, and through algebra. Students learn that a *disjunction* is often used to express the solutions to absolute value equations, and they use the properties of algebra to accurately and efficiently find the solutions to various types of absolute value equations.

EXPLORE

Solving Absolute Value Equations Graphically

INTEGRATE TECHNOLOGY

Students have the option of completing the graphing activity either in the book or online.

QUESTIONING STRATEGIES

? How do you solve an absolute value equation graphically? **Plot each side as if it were a separate function of x, and find the x-coordinates of the intersection points.**

? Why do you write the solutions to the absolute value equation as a disjunction? **If two values of the variable both satisfy an equation, then one *or* the other can be correct.**

EXPLAIN 1

Solving Absolute Value Equations Algebraically

AVOID COMMON ERRORS

Some students may not isolate the absolute value expression on one side of the equation as a first step when solving the equation. Stress the importance of this step so that the equation is in the form $|x| = a$, which has the solution $x = a$ or $x = -a$.

QUESTIONING STRATEGIES

? How do you interpret the solutions to an absolute value equation like $|x| = a$ on a number line? **Sample answer: The solutions are the same distance from 0 on either side of the number line.**

? Why is it important to isolate the absolute value expression when solving an absolute value equation? **So you can remove the absolute value bars and rewrite the expression as a disjunction.**

EXPLAIN 2

Absolute Value Equations with Fewer than Two Solutions

QUESTIONING STRATEGIES

When does an absolute value equation have fewer than two solutions? **when the absolute value expression is equal to zero or equal to a negative number**

In the absolute value expression $d|ax+b|-c=-c$ for nonzero variables, how does d affect the solution? **It does not affect it. The first step is to add c to both sides to get $d|ax+b|=0$. Because the product of a number and 0 is 0, you can divide both sides by d to get $|ax+b|=0$.**

INTEGRATE TECHNOLOGY

A graphing calculator can be used to check the number of solutions to an absolute value equation. Graph each side of the equation as a function and then count the number of intersection points.

AVOID COMMON ERRORS

Some students may think that if an absolute value equation does not have two solutions, then there must be no solution. Explain to students that when the absolute value expression equals zero, there will be one solution. For example, $|3x+6|=0$ has one solution, $x=-2$, because 0 is neither positive nor negative.

Explain 2 Absolute Value Equations with Fewer than Two Solutions

You have seen that absolute value equations have two solutions when the isolated absolute value expression is equal to a positive number. When the absolute value is equal to zero, there is a single solution because zero is its own opposite. When the absolute value expression is equal to a negative number, there is no solution because absolute value is never negative.

Example 2 **Isolate the absolute value expression in each equation to determine if the equation can be solved. If so, finish the solution. If not, write "no solution."**

(A) $-5|x+1|+2=12$

Subtract 2 from both sides.	$-5|x+1|=10$
Divide both sides by -5.	$|x+1|=-2$
Absolute values are never negative.	No Solution

(B) $\frac{3}{5}|2x-4|-3=-3$

Add 3 to both sides.	$\frac{3}{5}|2x-4|=$ **0**
Multiply both sides by $\frac{5}{3}$.	$|2x-4|=$ **0**
Rewrite as one equation.	$2x-4=$ **0**
Add 4 to both sides.	$2x=$ **4**
Divide both sides by 2.	$x=$ **2**

Your Turn

Isolate the absolute value expression in each equation to determine if the equation can be solved. If so, finish the solution. If not, write "no solution."

5. $3\left|\frac{1}{2}x+5\right|+7=5$

$3\left|\frac{1}{2}x+5\right|=-2$

$\left|\frac{1}{2}x+5\right|=-\frac{2}{3}$

No solution

6. $9\left|\frac{4}{3}x-2\right|+7=7$

$9\left|\frac{4}{3}x-2\right|=0$

$\left|\frac{4}{3}x-2\right|=0$

$x=\frac{3}{2}$

COLLABORATIVE LEARNING

Peer-to-Peer Activity

Have students work in pairs to brainstorm types of absolute value equations that have two solutions, one solution, or no solution. For example, instruct one student to write a conjecture about what type of absolute value equation has no solutions, and give an example. Then have the other student solve the example and write an explanation about whether the conjecture is correct or incorrect. Have students switch roles and repeat the exercise using an equation that has a different number of solutions.

Elaborate

7. Why is important to solve both equations in the disjunction arising from an absolute value equation? Why not just pick one and solve it, knowing the solution for the variable will work when plugged backed into the equation?

The solution to a mathematical equation is not simply any value of the variable that makes the equation true. Supplying only one value that works in the equation implies that it is the only value that works, which is incorrect.

8. **Discussion** Discuss how the range of the absolute value function differs from the range of a linear function. Graphically, how does this explain why a linear equation always has exactly one solution while an absolute value equation can have one, two, or no solutions?

The range of a non-constant linear function is all real numbers. The range of an absolute value function is $y \geq k$ if the function opens upward and $y \leq k$ if the function opens downward. Because the graph of a linear function is a line, a horizontal line will intersect it only once. Because the graph of an absolute value function is a V, a horizontal line can intersect it once, twice, or not at all.

9. **Essential Question Check-In** Describe, in your own words, the basic steps to solving absolute value equations and how many solutions to expect.

Isolate the absolute value expression. If the absolute value expression is equal to a positive number, solve for both the positive and negative case. If the absolute value expression is equal to zero, then remove the absolute value bars and solve the equation. There is one solution. If the absolute value expression is equal to a negative number, then there is no solution.

DIFFERENTIATE INSTRUCTION

Critical Thinking

Some students may need help in deciding whether absolute value equations have no solutions, one solution, or two solutions. You may want to suggest that they *always* follow this solving plan: (1) Write the original equation; then (2) isolate the absolute value expression on one side of the equal sign. It will have the form $|ax + b| = c$. (3) Rewrite the equation as two equations of the form $ax + b = c$ and $ax + b = -c$; and (4) solve each equation for x. There may be 0, 1, or 2 solutions. (5) If there are two solutions, write the answer using "or." (6) Check the solution(s) in the original problem.

ELABORATE

INTEGRATE MATHEMATICAL PRACTICES

Focus on Patterns

MP.8 Discuss with students how to solve an absolute value equation of the form $|ax + b| = c$. Students should routinely rewrite the next step as a disjunction, or a compound equation of the form $ax + b = c$ or $ax + b = -c$ and then solve each part of the equation.

QUESTIONING STRATEGIES

? How is the process of solving a linear absolute value equation like the process of solving a regular linear equation? **Both processes are similar initially, except that you isolate the absolute value in one case, but isolate the variable in the case of the linear equation. From there, the process is the same for each part of the disjunction of the two linear equations for the absolute value equation.**

PEER-TO-PEER ACTIVITY

Have students work in pairs. Have one student write an absolute value equation and have the partner solve it. The partner then explains why the solution(s) makes sense. Students switch roles and repeat the process. Encourage students to use the phrase "distance from zero" and the statement "This negative/positive integer makes the equation true."

SUMMARIZE THE LESSON

? How do you solve a linear absolute value equation? **Isolate the absolute value expression; write resulting equation as the disjunction of two linear equations; and solve each equation.**

EVALUATE

ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Solving Absolute Value Equations Graphically	Exercise 1–4
Example 1 Solving Absolute Value Equations Algebraically	Exercises 5–8
Example 2 Absolute Value Equations with Fewer than Two Solutions	Exercises 9–16

INTEGRATE MATHEMATICAL PRACTICES

Focus on Reasoning

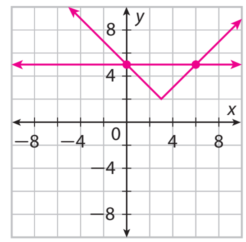
MP.2 Remind students to check their solutions by substituting the values into the original equation and verifying that both solutions make the equation true. When solving equations graphically, remind students that the x-value of an intersection point is a solution to the original equation.

Evaluate: Homework and Practice

Personal Math Trainer
• Online Homework
• Hints and Help
• Extra Practice

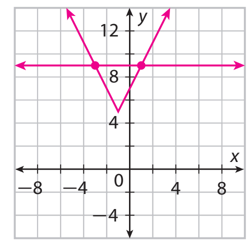
Solve the following absolute value equations by graphing.

1. $|x - 3| + 2 = 5$



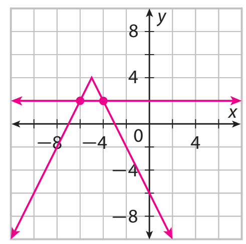
$x = 0$ or $x = 6$

2. $2|x + 1| + 5 = 9$



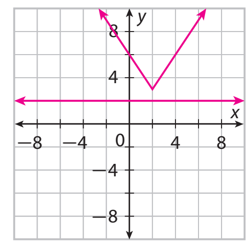
$x = -3$ or $x = 1$

3. $-2|x + 5| + 4 = 2$



$x = -4$ or $x = -6$

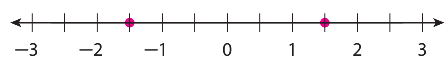
4. $\left|\frac{3}{2}(x - 2)\right| + 3 = 2$



No solution

Solve each absolute value equation algebraically. Graph the solutions on a number line.

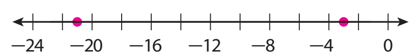
5. $|2x| = 3$



$2x = 3$ or $2x = -3$

$x = \frac{3}{2}$ or $x = -\frac{3}{2}$

6. $\left|\frac{1}{3}x + 4\right| = 3$



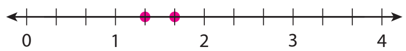
$\left(\frac{1}{3}\right)x + 4 = 3$ or $\left(\frac{1}{3}\right)x + 4 = -3$

$\left(\frac{1}{3}\right)x = -1$ or $\left(\frac{1}{3}\right)x = -7$

$x = -3$ or $x = -21$

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
1–4	**2** Skills/Concepts	**MP.5** Using Tools
5–16	**2** Skills/Concepts	**MP.6** Precision
17	**3** Strategic Thinking	**MP.4** Modeling
18, 21	**3** Strategic Thinking	**MP.4** Modeling
19	**3** Strategic Thinking	**MP.6** Precision
20	**2** Skills/Concepts	**MP.6** Precision
22	**3** Strategic Thinking H.O.T.	**MP.3** Logic
23–25	**3** Strategic Thinking H.O.T.	**MP.6** Precision

7. $3|2x - 3| + 2 = 3$



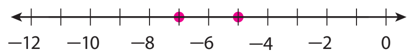
$2x - 3 = \frac{1}{3}$

$2x - 3 = \frac{1}{3}$ or $2x - 3 = \frac{-1}{3}$

$2x = \frac{10}{3}$ or $2x = \frac{8}{3}$

$x = \frac{5}{3}$ or $x = \frac{4}{3}$

8. $-8|-x - 6| + 10 = 2$



$-8|-x - 6| = -8$

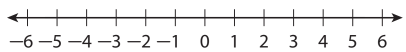
$|-x - 6| = 1$

$-x - 6 = 1$ or $-x - 6 = -1$

$x = -7$ or $x = -5$

Isolate the absolute value expressions in the following equations to determine if they can be solved. If so, find and graph the solution(s). If not, write "no solution".

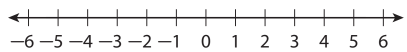
9. $\frac{1}{4}|x + 2| + 7 = 5$



$\frac{1}{4}|x + 2| = -2$

No solution

10. $-3|x - 3| + 3 = 6$

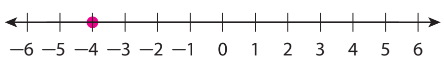


$-3|x - 3| = 3$

$|x - 3| = -1$

No solution

11. $2(|x + 4| + 3) = 6$



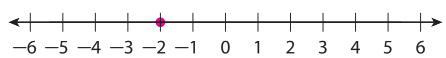
$2|x + 4| + 6 = 6$

$2|x + 4| = 0$

$|x + 4| = 0$

$x = -4$

12. $5|2x + 4| - 3 = -3$



$5|2x + 4| = 0$

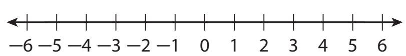
$|2x + 4| = 0$

$2x + 4 = 0$

$x = -2$

Solve the absolute value equations.

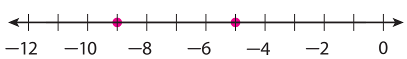
13. $|3x - 4| + 2 = 1$



$|3x - 4| = -1$

No solution

14. $7\left|\frac{1}{2}x + 3\frac{1}{2}\right| - 2 = 5$



$7\left|\frac{1}{2}x + \frac{7}{2}\right| = 7 \dashrightarrow \left|\frac{1}{2}x + \frac{7}{2}\right| = 1$

$\frac{1}{2}x + \frac{7}{2} = 1$ or $\frac{1}{2}x + \frac{7}{2} = -1$

$\frac{1}{2}x = -\frac{5}{2}$ or $\frac{1}{2}x = -\frac{9}{2}$

$x = -5$ or $x = -9$

AVOID COMMON ERRORS

Students may erroneously include points on the graph *between* the solution points when they graph solutions. Remind students that the solution process gives 0, 1, or 2 solutions to an absolute value equation, not infinitely many solutions.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 Ask students to give examples of absolute value equations that have no solutions. Suggest that they think about how a graph of an equation with no solution will look. This graph should not show any points, so it is an empty graph.

INTEGRATE MATHEMATICAL PRACTICES

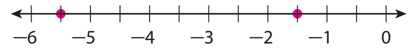
Focus on Modeling

MP.4 When modeling a problem in which an absolute value equation applies, have students start with a V-shaped diagram. This will help them remember that this type of function may have 0, 1, or 2 solutions to the associated equation, depending on the original real-world problem.

AVOID COMMON ERRORS

When solving absolute equations algebraically, watch for students who do not solve these equations by first rewriting them in the form $|ax + b| = c$. Remind them that the absolute value expression should be nonnegative before they proceed with the solution steps.

15. $|2(x+5) - 3| + 2 = 6$



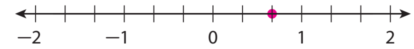
$|2x + 7| = 4$

$2x + 7 = 4$ or $2x + 7 = -4$

$2x = -3$ or $2x = -11$

$x = -\frac{3}{2}$ or $x = -\frac{11}{2}$

16. $-5|-3x + 2| - 2 = -2$



$-5|-3x + 2| = 0$

$|-3x + 2| = 0$

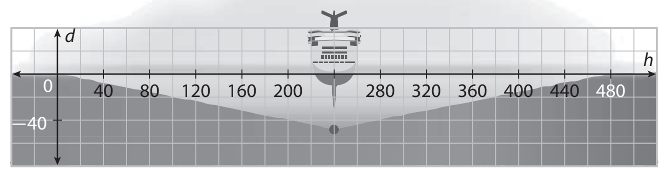
$-3x + 2 = 0$

$-3x = -2$

$x = \frac{2}{3}$

17. The bottom of a river makes a V-shape that can be modeled with the absolute value function, $d(h) = \frac{1}{5}|h - 240| - 48$, where d is the depth of the river bottom (in feet) and h is the horizontal distance to the left-hand shore (in feet).

A ship risks running aground if the bottom of its keel (its lowest point under the water) reaches down to the river bottom. Suppose you are the harbormaster and you want to place buoys where the river bottom is 30 feet below the surface. How far from the left-hand shore should you place the buoys?



$d(h) = -30$

$\frac{1}{5}|h - 240| - 48 = -30$

$h = 330$ or $h = 150$

The buoys should be placed at 150 ft and 330 ft from the left-hand shore.

18. A flock of geese is approaching a photographer, flying in formation. The photographer starts taking photographs when the lead goose is 300 feet horizontally from her, and continues taking photographs until it is 100 feet past. The flock is flying at a steady 30 feet per second. Write and solve an equation to find the times after the photographing begins that the lead goose is at a horizontal distance of 75 feet from the photographer.

The distance d in feet that the flock flies in t seconds after shooting begins is $d(t) = 30t$. The horizontal distance of the lead bird from the photographer is then $d(t) = |30t - 300|$. Find the times when this distance equals 75 feet.

$|30t - 300| = 75$

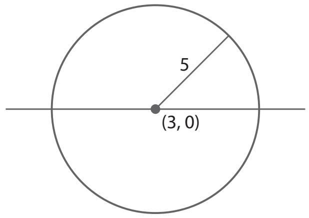
$30t - 300 = 75$ or $30t - 300 = -75$

$30t = 375$ or $30t = 225$

$t = 12.5$ or $t = 7.5$

The lead goose is at va horizontal distance of 75 feet from the photographer after 7.5 seconds and again after 12.5 seconds.

19. **Geometry** Find the points where a circle centered at (3, 0) with a radius of 5 crosses the x-axis. Use an absolute value equation and the fact that all points on a circle are the same distance (the radius) from the center.



The points on the x-axis that are a distance of 5 from the center of the circle at $x = 3$ are given by $|x - 3| = 5$.

Solving $|x - 3| = 5$ gives $x = 8$ or $x = -2$.

The points are $(-2, 0)$ and $(8, 0)$.

20. Select the value or values of x that satisfy the equation $-\frac{1}{2}|3x - 3| + 2 = 1$.

(A.) $x = \frac{5}{3}$ B. $x = -\frac{5}{3}$

(C.) $x = \frac{1}{3}$ D. $x = -\frac{1}{3}$

E. $x = 3$ F. $x = -3$

G. $x = 1$ H. $x = -1$

$$-\frac{1}{2}|3x - 3| = -1$$
$$|3x - 3| = 2$$
$$3x - 3 = 2 \quad \text{or} \quad 3x - 3 = -2$$
$$3x = 5 \quad \text{or} \quad 3x = 1$$
$$x = \frac{5}{3} \quad \text{or} \quad x = \frac{1}{3}$$

21. Terry is trying to place a satellite dish on the roof of his house at the recommended height of 30 feet. His house is 32 feet wide, and the height of the roof can be described by the function $h(x) = -\frac{3}{2}|x - 16| + 24$, where x is the distance along the width of the house. Where should Terry place the dish?

Use the model function to solve for x when $h(x) = 30$ feet.

$$-\frac{3}{2}|x - 16| + 24 = 30$$
$$|x - 16| = -4$$

No Solution. Terry does not have a spot on his roof that is 30 feet high.

H.O.T. Focus on Higher Order Thinking

22. **Explain the Error** While attempting to solve the equation $-3|x - 4| - 4 = 3$, a student came up with the following results. Explain the error and find the correct solution:

$$-3|x - 4| - 4 = 3$$
$$-3|x - 4| = 7$$
$$|x - 4| = -\frac{7}{3}$$
$$x - 4 = -\frac{7}{3} \quad \text{or} \quad x - 4 = \frac{7}{3}$$
$$x = \frac{5}{3} \quad \text{or} \quad x = \frac{19}{3}$$

The student tried to replace the absolute value equation with two equations using the positive and negative values of the number on the other side of the equal sign. However, this number was negative and cannot be treated like a positive number. The isolated absolute value expression is equal to a negative number and therefore this equation has no solution.

SMALL GROUP ACTIVITY

Have students work in small groups to make a poster showing how to apply the steps for solving an absolute value equation. Give each group a different equation to solve. Then have each group present its poster to the rest of the class, and ask for a volunteer from the group to explain each step.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Reasoning

MP.2 When solving absolute value equations of the form $-|ax + b| = c$, where $c > 0$, students should recognize that the equation states that a negative absolute value is equal to a positive number. This is not possible because of the definition of absolute value, so while "solving the equation" gives numerical answers, these answers are not solutions to the original equation. You may want students to verify this by graphing. There will be no intersection points.

AVOID COMMON ERRORS

Watch for students who are confused by nested absolute value equations. Remind students to carefully write disjunctions for each part of the solution, as appropriate, using the same solution process they use for a single absolute value equation.

PEER-TO-PEER DISCUSSION

Ask students to discuss with a partner what the solution to $|ax + b| = c$ means in terms of the graph of the related functions $f(x) = |ax + b|$ and $g(x) = c$. Then ask students to make conjectures about the solutions to $|ax + b| = c$ and the graphs of their related functions. Conjectures should include the possible number of intersection points and how the graph of the function looks. **The solutions to $|ax + b| = c$ are the x-coordinates of the intersection points of the related functions. Based on this, conjectures should include that the graphs of $f(x) = |ax + b|$ and $g(x) = c$ can have two, one, or no intersection points, and that the graph of $f(x)$ is V-shaped and this graph can intersect a line in two or fewer places.**

JOURNAL

Have students compare and contrast the methods they have learned for solving absolute value equations.

23. **Communicate Mathematical Ideas** Solve this absolute value equation and explain what algebraic properties make it possible to do so.

$3|x - 2| = 5|x - 2| - 7$

$3\lvert x - 2\rvert - 5\lvert x - 2\rvert = -7$	**Subtraction Property of Equality**
$(3 - 5)\lvert x - 2\rvert = -7$	**Distributive Property**
$\lvert x - 2\rvert = \frac{7}{2}$	**Division Property of Equality**
$x - 2 = \frac{7}{2}$ or $x - 2 = -\frac{7}{2}$	**Definition of absolute value**
$x = \frac{11}{2}$ or $x = -\frac{3}{2}$	**Addition Property of Equality**

24. **Justify Your Reasoning** This absolute value equation has nested absolute values. Use your knowledge of solving absolute value equations to solve this equation. Justify the number of possible solutions.

$||2x + 5| - 3| = 10$

Follow each possible solution path and use more disjunctions if needed.

$||2x + 5| - 3| = 10$

$|2x + 5| - 3 = 10$ **or** $|2x + 5| - 3 = -10$

$|2x + 5| = 13$ **or** $|2x + 5| = -7$

$|2x + 5| = 13$ **or** **No solution**

$|2x + 5| = 13$

$2x + 5 = 13$ **or** $2x + 5 = -13$

$2x = 8$ **or** $2x = -18$

$x = 4$ **or** $x = -9$

There are two possible solutions because only one path produced solutions.

25. **Check for Reasonableness** For what type of real-world quantities would the negative answer for an absolute value equation not make sense?

Answers will vary. Sample answer: time, distance, height, length, speed

Lesson Performance Task

A snowball comes apart as a child throws it north, resulting in two halves traveling away from the child. The child is standing 12 feet south and 6 feet east of the school door, along an east-west wall. One fragment flies off to the northeast, moving 2 feet east for every 5 feet north of travel, and the other moves 2 feet west for every 5 feet north of travel. Write an absolute value function that describes the northward position, $n(e)$, of both fragments as a function of how far east of the school door they are. How far apart are the fragments when they strike the wall?

The fragments can be described as two lines originating at the child's coordinates, and then be replaced by a single absolute value function.

$$n(e) = \frac{5}{2}(e-6) \quad \text{or} \quad n(e) = -\frac{5}{2}(e-6)$$

$$n(e) = \frac{5}{2}|e-6|$$

To find where the fragments strike the school wall, solve for the eastward position when the fragments are 12 feet north of the child.

$$n(e) = \frac{5}{2}|e-6| = 12$$

$$|e-6| = \frac{24}{5}$$

$$e-6 = \frac{24}{5} \quad \text{or} \quad e-6 = -\frac{24}{5}$$

$$e = \frac{54}{5} \quad \text{or} \quad e = \frac{6}{5}$$

The fragments are $\left|\frac{54}{5} - \frac{6}{5}\right| = \left|\frac{48}{5}\right| = 9\frac{3}{5}$ **feet apart.**

EXTENSION ACTIVITY

Have students try to find an alternate solution method using the formula for the slope of a line. Student should find the coordinates for the snowball on the right to be $\left(10\frac{4}{5}, 12\right)$. Subtracting 6 from the value of x gives the distance from $(6, 12)$ to $(x, 12)$. That distance is $4\frac{4}{5}$ ft. The distance from the y-axis to the snowball on the left is $6 - 4\frac{4}{5} = 1\frac{1}{5}$ ft. So the fragments are $10\frac{4}{5} - 1\frac{1}{5} = 9\frac{3}{5}$ feet apart.

AVOID COMMON ERRORS

Some students may use the ratio $\frac{2}{5}$ in their equation instead of $\frac{5}{2}$. Explain that the snowball *rises* 5 feet north for every 2 feet west it *runs*. Thus, the ratio is $\frac{5}{2}$.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Discuss with students how solving for the eastward position solves for the distance of the snowball on the right, $e - 6 = \frac{24}{5}$, and the snowball on the left, $e - 6 = -\frac{24}{5}$. Then discuss how solving for e does not answer the problem. The difference between the two distances is equal to the distance the two are apart.

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.

0 points: Student does not demonstrate understanding of the problem.