THE 12 FRIENDLY TRANSFORMATIONS

Each year in calculus we do a lot of work with transformations of graphs. AP Calculus teachers make the implicit (though dangerous) assumption that each student entering their course will have quick mental images of several functions and the ability to do transformations of those functions. This assignment is meant to insure that these assumptions are indeed true.

Transformations of functions are the processes of shifting, stretching, compressing, reflecting, etc., existing functions to obtain new functions. There are:

(1) translations (shifting of original function either horizontally or vertically, keeping the shape the same),
(2) transformations (stretching or compressing the original function either horizontally or vertically which changes the shape of the original function),
(3) reflections (a.k.a. flips) (reflecting the original function over the *x*- or *y*-axis), and
(4) absolute value (alters the shape of the graph considerably).

One purpose of transformations is to make graphing easier. If you know the general shape of the original function, then you will be able to quickly shift, stretch, compress, reflect, etc., the original function to get the function that you desire. The 12 friendly transformations are:

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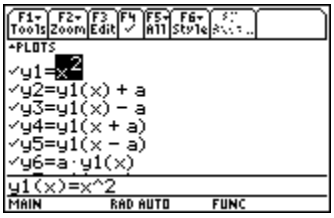
<u>Translations</u>		
Vertical shifts	$f(x)+a$	shifts graph up *a* units
	$f(x)-a$	shifts graph down *a* units
Horizontal shifts	$f(x+a)$	shifts graph left *a* units
	$f(x-a)$	shifts graph right *a* units
<u>Stretches and Compressions</u>		
Vertical stretch	$a \cdot f(x)$	steeper, x-intercepts same
Horizontal compression	$f(a \cdot x)$	squinched, y-intercepts same
Vertical compression	$\frac{f(x)}{a}$	wider, x-intercepts same
Horizontal stretch	$f(\frac{x}{a})$	stretched, y-intercepts same
<u>Reflections</u>		
	$f(-x)$	reflected over y-axis
	$-f(x)$	reflected over x-axis

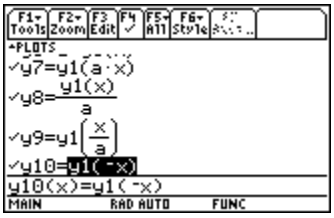
Absolute Value

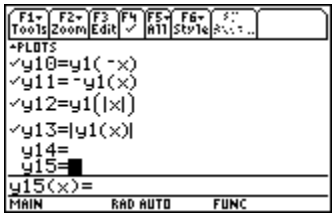
$f(|x|)$ ignore negative x's, keep graph of positive x's the same, reflect that graph over y-axis

$|f(x)|$ keep positive y's the same, flip negative y's up over x-axis

The TI-89 graphing calculator can assist you in graphing these functions. First, put your original function in **y1**. Then type the following functions in the "**y =**" menu. An example follows:







Then, go back to the home screen and store your value for a. Use **a = 2** for all these graphs. Go back to the "**y =**"menu and leave **y1** on and turn on only one at a time. Use **F4** to turn on or off any equation. You should then be able to graph all the transformations for your first function. Then, for the second function, only change the y1 entry; all the rest should remain the same. When you finish this part of the activity, go back and analyze each transformation. There is a writing part entitled "Friendly Translations" that you will have to complete.

Another activity in this assignment involves cutting out and matching cards. You will have 16 sets of 3 cards when you are finished. In each set, you should have an equation, a graph, and a description of the graph when transformed from the graph of $y = x^2$. Try to do this activity without using your calculator and only using my descriptions. Also, try to time yourself and get to the point where you can do this activity in three minutes.

Below you will four functions that you should graph on each side of the grid paper, one per side. For each of these, you will be sketching the function and 12 transformations. You should use the graph paper provided. On each of the 12 grids on the paper, graph the original function in one color. Then, in another color, graph the transformation on the same set of axes.

1. $f(x) = x^2 + 2x$
2. $f(x) = |x|$
3. $f(x) = 2^x$
4. $f(x) = \dfrac{1}{x}$