12.6 HW

Assignment Guide

Core: 13–31 odd, 33–44
Enriched: 12–32 even, 33–44

Additional Answers

4. The exponents must add to 12, so the exponent of y is $12 - 7$ or 5.

To find the coefficient of the term use the formula

$$(x+y)^n = \sum_{r=0}^{n} \frac{n!}{r!(n-r)!} x^{n-r}y^r.$$

Evaluate the general term for $n = 12$ and $r = 5$. $\frac{12!}{5!7!} x^7y^5 = 792x^7y^5$.

6. $a^6 + 18a^5 + 135a^4 + 540a^3 + 1215a^2 + 1458a + 729$
7. $125 - 75y + 15y^2 - y^3$
8. $81p^4 - 216p^3q + 216p^2q^2 - 96pq^3 + 16q^4$
12. $a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8$
13. $n^6 - 24n^5 + 240n^4 - 1280n^3 + 3840n^2 - 6144n + 4096$
14. $81c^4 - 108c^3d + 54c^2d^2 - 12cd^3 + d^4$
16. $d^7 + 14d^6 + 84d^5 + 280d^4 + 560d^3 + 672d^2 + 448d + 128$
17. $243 - 405x + 270x^2 - 90x^3 + 15x^4 - x^5$
18. $256a^4 + 256a^3b + 96a^2b^2 + 16ab^3 + b^4$
19. $8x^3 - 36x^2y + 54xy^2 - 27y^3$
20. $81m^4 + 108\sqrt{2}m^3 + 108m^2 + 24\sqrt{2}m + 4$
21. $c^3 - 6c^2\sqrt{c} + 15c^2 - 20c\sqrt{c} + 15c - 6\sqrt{c} + 1$
22. $\frac{1}{32}n^5 + \frac{5}{8}n^4 + 5n^3 + 20n^2 + 40n + 32$
23. $81a^4 + 72a^3b + 24a^2b^2 + \frac{32}{9}ab^3 + \frac{16}{81}b^4$
24. $p^{16} + 8p^{14}q + 28p^{12}q^2 + 56p^{10}q^3 + 70p^8q^4 + 56p^6q^5 + 28p^4q^6 + 8p^2q^7 + q^8$

2c. Even-indexed terms are negative and odd-indexed terms are positive.

2. Examine the expansion of $(x - y)^n$ for $n = 3$, 4, and 5.
   a. **Identify** the sign of the second term for each expansion. **negative**
   b. **Identify** the sign of the third term for each expansion. **positive**
   c. **Explain** how to determine the sign of a term without writing out the entire expansion.
3. **Restate** what is meant by the observation that each term in a binomial expansion has degree n. **The sum of the exponents of each term is n.**
4. If ax^7y^b is a term from the expansion of $(x + y)^{12}$, **describe** how to determine its coefficient a and missing exponent b without writing the entire expansion. **See margin.**

Guided Practice

5. Use Pascal's triangle to expand $(c + d)^5$. $c^5 + 5c^4d + 10c^3d^2 + 10c^2d^3 + 5cd^4 + d$

Use the Binomial Theorem to expand each binomial. 6–8. See margin.

6. $(a + 3)^6$
7. $(5 - y)^3$
8. $(3p - 2q)^4$

Find the designated term of each binomial expansion.

9. 6th term of $(a - b)^7$ $-21a^2b^5$
10. 4th term of $(x + \sqrt{3})^9$ $252\sqrt{3}x^6$
11. **Coins** A coin is flipped five times. Find the number of possible sets of heads and tails that have each of the following.
    a. 0 heads 1
    b. 2 heads 10
    c. at least 4 heads 6
    d. at most 3 heads 26

EXERCISES

Practice

A

Use Pascal's triangle to expand each binomial. 12–14. See margin.

12. $(a + b)^8$
13. $(n - 4)^6$
14. $(3c - d)^4$

15. Expand $(2 + a)^9$ using Pascal's triangle. $512 + 2304a + 4608a^2 + 5376a^3 + 4032a^4 + 2016a^5 + 672a^6 + 144a^7 + 18a^8 + a^9$

Use the Binomial Theorem to expand each binomial. 16–24. See margin.

B

16. $(d + 2)^7$
17. $(3 - x)^5$
18. $(4a + b)^4$
19. $(2x - 3y)^3$
20. $(3m + \sqrt{2})^4$
21. $(\sqrt{c} - 1)^6$
22. $\left(\frac{1}{2}n + 2\right)^5$
23. $\left(3a + \frac{2}{3}b\right)^4$
24. $(p^2 + q)^8$

25. Expand $(xy - 2z^3)^6$ using the Binomial Theorem. $x^6y^6 - 12x^5y^5z^3 + 60x^4y^4z^6 - 160x^3y^3z^9 + 240x^2y^2z^{12} - 192xyz^{15} + 64z^{18}$

Find the designated term of each binomial expansion.

26. 5th term of $(x + y)^9$ $126x^5y^4$
27. 4th term of $(a - \sqrt{2})^8$ $-112\sqrt{2}a^5$
28. 4th term of $(2a - b)^7$ $-560a^4b^3$
29. 7th term of $(3c + 2d)^9$ $145{,}152c^3d^6$

C

30. 8th term of $\left(\frac{1}{2}x - y\right)^{10}$ $-15x^3y^7$
31. 6th term of $(2p - 3q)^{11}$ $-7{,}185{,}024p^6q^5$
32. Find the middle term of the expansion of $(\sqrt{x} + \sqrt{y})^8$. $70x^2y^2$

pplications nd Problem olving

33. **Business** A company decides to form a recycling committee to find a more efficient means of recycling waste paper. The committee is to be composed of eight employees. Of these eight employees, at least four women are to be on the committee. How many of the possible groups of men and women have at least four women? **163**

34. **Critical Thinking** Describe a strategy that uses the Binomial Theorem to expand $(a + b + c)^{12}$. **See margin.**

35. **Education** Rafael is taking a test that contains a section of 12 true-false questions.
   a. How many of the possible groups of answers to these questions have exactly 8 correct answers of false? **495**
   b. How many of the possible groups of answers to these questions have at least 6 correct answers of true? **2510**

36. **Critical Thinking** Find a term in the expansion of $\left(3x^2 - \frac{1}{4x}\right)^6$ that does not contain the variable x. **See margin.**

7b. Sample nswer: .04060401

7c. 1.04060401, ie two values are qual.

37. **Numerical Analysis** Before the invention of modern calculators and computers, mathematicians searched for ways to shorten lengthy calculations such as $(1.01)^4$.
   a. Express 1.01 as a binomial. **Sample answer: 1 + 0.01**
   b. Use the binomial representation of 1.01 found in part **a** and the Binomial Theorem to calculate the value of $(1.01)^4$ to eight decimal places.
   c. Use a calculator to estimate $(1.01)^4$ to eight decimal places. Compare this value to the value found in part **b**.

ixed Review

8. 1 + (−1) + −3) + (−5) + −7) + (−9); -24

38. Write $\sum_{k=2}^{7} 5 - 2k$ in expanded form and then find the sum. *(Lesson 12-5)*

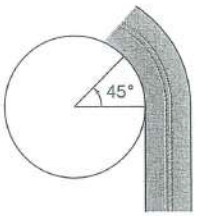
39. Use the ratio test to determine whether the series $2 + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + \cdots$ is *convergent* or *divergent*. *(Lesson 12-4)* **convergent**

40. Find the sum of $\frac{2}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \cdots$ or explain why one does not exist. $1\frac{1}{3}$ *(Lesson 12-3)*

41. **Finance** A bank offers a home mortgage for an annual interest rate of 8%. If a family decides to mortgage a $150,000 home over 30 years with this bank, what will the monthly payment for the principal and interest on their mortgage be? *(Lesson 11-2)* **$1100.65**

42. Write $\overline{MK}$ as the sum of unit vectors if $M(-2, 6, 3)$ and $K(4, 8, -2)$. *(Lesson 8-3)* $6\vec{i} + 2\vec{j} - 5\vec{k}$

43. **Construction** A highway curve, in the shape of an arc of a circle, is 0.25 mile. The direction of the highway changes 45° from one end of the curve to the other. Find the radius of the circle in feet that the curve follows. *(Lesson 6-1)* **1681 feet**



44. **SAT/ACT Practice** If b is a prime integer such that $3b > 10 > \frac{5}{6}b$, which of the following is a possible value of b? **D**

A 2 B 3 C 4 D 11 E 13

Extra Practice See p. A50.

Extra Credit

Use the Binomial Theorem to find the value of $(0.97)^6$. Round to the nearest hundredth. **0.83**

4 ASSESS

Open-Ended Assessment

Modeling Have students use a poster and color markers to demonstrate the binomial expansion to the 10th power. Then, have them use the poster to help evaluate $(1.05)^6$ by finding $(1 + 0.05)^6$ using the expansion.

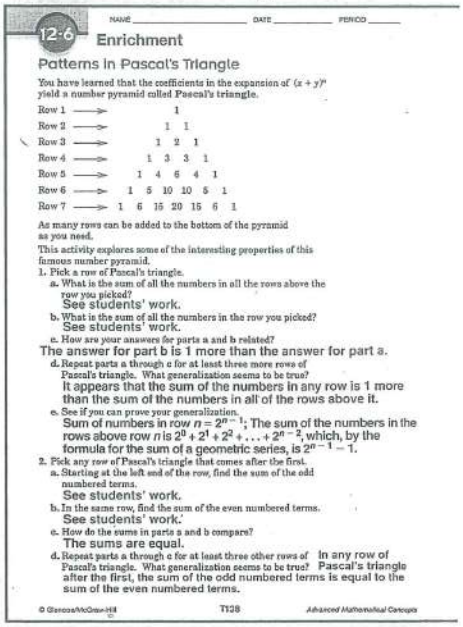
Additional Answers

34. Sample answer: treat $a + b$ as a single term and expand $[(a + b) + c]^{12}$ using the Binomial Theorem. Then evaluate each $(a + b)^n$ term in the expansion using the Binomial Theorem.

36. Find the term for which both x's have the same exponent. This will occur for the middle term of the expansion, the 4th term when $n = 6$. Use the Binomial Theorem to find the 4th term for the expansion of $\left(3x - \frac{1}{4x}\right)^6$.

$$\frac{6!}{3!3!}(3x)^3\left(-\frac{1}{4x}\right)^3 = -\frac{135}{16}$$

Enrichment Masters, p. 138



NAME _____ DATE _____ PERIOD _____

12-6 Enrichment

Patterns in Pascal's Triangle

You have learned that the coefficients in the expansion of $(x + y)^n$ yield a number pyramid called Pascal's triangle.

Row 1 → 1
Row 2 → 1 1
Row 3 → 1 2 1
Row 4 → 1 3 3 1
Row 5 → 1 4 6 4 1
Row 6 → 1 5 10 10 5 1
Row 7 → 1 6 15 20 15 6 1

As many rows can be added to the bottom of the pyramid as you need.

This activity explores some of the interesting properties of this famous number pyramid.

1. Pick a row of Pascal's triangle.
   a. What is the sum of all the numbers in all the rows above the row you picked?
      See students' work.
   b. What is the sum of all the numbers in the row you picked?
      See students' work.
   c. How are your answers for parts a and b related?
      The answer for part b is 1 more than the answer for part a.
   d. Repeat parts a through c for at least three more rows of Pascal's triangle. What generalization seems to be true?
      It appears that the sum of the numbers in any row is 1 more than the sum of the numbers in all of the rows above it.
   e. See if you can prove your generalization.
      Sum of numbers in row $n = 2^{n-1}$; The sum of the numbers in the rows above row n is $2^0 + 2^1 + 2^2 + \ldots + 2^{n-2}$, which, by the formula for the sum of a geometric series, is $2^{n-1} - 1$.
2. Pick any row of Pascal's triangle that comes after the first.
   a. Starting at the left end of the row, find the sum of the odd numbered terms.
      See students' work.
   b. In the same row, find the sum of the even numbered terms.
      See students' work.
   c. How do the sums in parts a and b compare?
      The sums are equal.
   d. Repeat parts a through c for at least three other rows of Pascal's triangle. What generalization seems to be true?
      In any row of Pascal's triangle after the first, the sum of the odd numbered terms is equal to the sum of the even numbered terms.

 T128 *Advanced Mathematical Concepts*