Assignment Guide

Core: 15–45 odd, 47–49, 51–59
Enriched: 14–44 even, 46–59

Teaching Tip For Exercise 45, an alternate method of finding the sum of the series would be to rewrite $\frac{8n^3-2n^2+5}{n^4}$ as $\frac{8}{n}-\frac{2}{n^2}+\frac{5}{n^4}$ and find each separate summation.

Additional Answers

8. $\sum_{n=1}^{5} 5n$

9. $\sum_{n=0}^{3} (3^k + 1)$

10. $\sum_{n=1}^{4} (8 - 6n)$

11. $\sum_{n=2}^{\infty} 3\left(\frac{1}{2}\right)^n$

12. $\sum_{n=1}^{\infty} (-3)^n$

14. $(-5) + (-3) + (-1) + 1$

15. $10 + 15 + 20 + 25$

16. $(-6) + (-10) + (-14) + (-18) + (-22) + (-26)$

17. $6 + 12 + 20 + 30 + 42$

18. $5 + 3 + \frac{7}{3} + 2$

19. $16 + 32 + 64 + 128 + 256$

20. $\frac{1}{3} + 1 + 3 + 9$

21. $4\frac{1}{2} + 16\frac{1}{2} + 64\frac{1}{2}$

22. $8 + 16 + 32$

23. $6 + 24 + 120 + 720 + 5040$

24. $4 + 3 + 2.25 + 1.6875 + \cdots$

25. $\frac{8}{5} + \frac{16}{25} + \frac{32}{125} + \cdots$

27. $\sum_{k=1}^{4} (3k + 3)$

28. $\sum_{k=0}^{4} 4^k$

29. $\sum_{k=4}^{12} 2k$

30. $\sum_{k=0}^{3} (-2)^{3-k}$

31. $\sum_{k=1}^{4} 2 \cdot 5^k$

32. $\sum_{k=0}^{3} (13 - 4k)$

Answers 12.5

Express each series using sigma notation. 8–12. See margin.

8. $5 + 10 + 15 + 20 + 25$

9. $2 + 4 + 10 + 28$

10. $2 - 4 - 10 - 16$

11. $\frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{32} + \cdots$

12. $-3 + 9 - 27 + \cdots$

13. **Aviation** Each October Albuquerque, New Mexico, hosts the Balloon Fiesta. In 1998, 873 hot air balloons participated in the opening day festivities. One of these balloons rose 389 feet aft 1 minute. Because the air in the balloon was not reheated, eac succeeding minute the balloon rose 63% as far as it did the previous minute. **b. about 1051 ft**

a. Use sigma notation to represent the height of the balloon above the ground after one hour. Then calculate the total height of the balloon after one hour to the nearest foot.

b. What was the maximum height achieved by this balloon?

13a. $\sum_{n=1}^{60} 389(0.63)^{n-1}$; about 1051

EXERCISES

Practice

14–25. See margin for expanded form.

A

Write each expression in expanded form and then find the sum.

14. $\sum_{n=1}^{4} (2n - 7)$ **−8**

15. $\sum_{a=2}^{5} 5a$ **70**

16. $\sum_{b=3}^{8} (6 - 4b)$ **−96**

17. $\sum_{k=2}^{6} (k + k^2)$ **110**

18. $\sum_{n=5}^{8} \frac{n}{n-4}$ $12\frac{1}{3}$

19. $\sum_{j=4}^{8} 2^j$ **496**

20. $\sum_{m=0}^{3} 3^{m-1}$ $13\frac{1}{3}$

21. $\sum_{r=1}^{3} \left(\frac{1}{2} + 4^r\right)$ $85\frac{1}{2}$

22. $\sum_{i=3}^{5} (0.5)^{-i}$ **56**

23. $\sum_{k=3}^{7} k!$ **5910**

24. $\sum_{p=0}^{\infty} 4(0.75)^p$ **16**

25. $\sum_{n=1}^{\infty} 4\left(\frac{2}{5}\right)^n$ $2\frac{2}{3}$

B

26. Write $\sum_{n=2}^{5} n + i^n$ in expanded form. Then find the sum. $(2 + i^2) + (3 + i^3) + (4 + i^4) + (5 + i^5)$;

Express each series using sigma notation. 27–40. See margin.

27. $6 + 9 + 12 + 15$

28. $1 + 4 + 16 + \cdots + 256$

29. $8 + 10 + 12 + \cdots + 24$

30. $-8 + 4 - 2 + 1$

31. $10 + 50 + 250 + 1250$

32. $13 + 9 + 5 + 1$

33. $\frac{1}{9} + \frac{1}{14} + \frac{1}{19} + \cdots + \frac{1}{49}$

34. $\frac{2}{3} + \frac{4}{5} + \frac{8}{7} + \frac{16}{9} + \cdots$

35. $4 - 9 + 16 - 25 + \cdots$

36. $5 + 5 + \frac{5}{2} + \frac{5}{6} + \frac{5}{24} + \cdots$

37. $-32 + 16 - 8 + 4 - \cdots$

38. $2 + \frac{6}{2} + \frac{24}{3} + \frac{120}{4} + \cdots$

C

39. $\frac{1}{5} + \frac{2}{7} + \frac{3}{11} + \frac{4}{19} + \frac{5}{35} + \cdots$

40. $\frac{3}{9 \cdot 2} + \frac{8}{27 \cdot 6} + \frac{15}{81 \cdot 24} + \cdots$

41. $\sum_{k=1}^{\infty} \frac{2^{\frac{k}{2}}}{3k!}$

41. Express the series $\frac{\sqrt{2}}{3} + \frac{2}{6} + \frac{\sqrt{8}}{18} + \frac{4}{72} + \frac{\sqrt{32}}{360} + \cdots$ using sigma notation.

43. $a(a + 1)(a - 1)$

Simplify. Assume that n and m are positive integers, $a > b$, and $a > 2$.

42. $\frac{(a-2)!}{a!}$ $\frac{1}{a(a-1)}$

43. $\frac{(a+1)!}{(a-2)!}$

44. $\frac{(a+b)!}{(a+b-1)!}$ $a + b$

Graphing Calculator

45. Use a graphing calculator to find the sum of $\sum_{n=1}^{100} \frac{8n^3-2n^2+5}{n^4}$. Round to the nearest hundredth. **43.64**

Additional Answers

33. $\sum_{k=2}^{10} \frac{1}{5k-1}$

34. $\sum_{k=1}^{\infty} \frac{2^k}{2k+1}$

35. $\sum_{k=2}^{\infty} (-1)^k k^2$

36. $\sum_{k=0}^{\infty} \frac{5}{k!}$

37. $\sum_{n=0}^{\infty} \left[(-1)^{n+1} \frac{32}{2^n}\right]$

38. $\sum_{k=1}^{\infty} \frac{(k+1)!}{k}$

39. $\sum_{k=1}^{\infty} \frac{k}{2^k + 3}$

40. $\sum_{k=2}^{\infty} \frac{k^2 - 1}{3^k k!}$

Applications and Problem Solving

46. **Advertising** A popular shoe manufacturer is planning to market a new style of tennis shoe in a city of 500,000 people. Using a prominent professional athlete as their spokesperson, the company's ad agency hopes to induce 35% of the people to buy the product. The ad agency estimates that these satisfied customers will convince 35% of 35% of 500,000 to buy a pair of shoes, and those will persuade 35% of 35% of 35% of 500,000, and so on.
   a. Model this situation using sigma notation. $\sum_{n=1}^{\infty} 500{,}000(0.35)^n$
   b. Find the total number of people that will buy the product as a result of the advertising campaign. **269,230 people**
   c. What percentage of the population is this? **about 53.8%**
   d. What important assumption does the advertising agency make in proposing the figure found in part **b** to the shoe manufacturer?

46d. The ad agency assumes that the people who buy the tennis shoes will be satisfied with their purchase.

47. **Critical Thinking** Solve each equation for x.
   a. $\sum_{n=1}^{6}(x - 3n) = -3$ **10**
   b. $\sum_{n=0}^{5} n(n - x) = 25$ **2**

48. **Critical Thinking** Determine whether each equation is *true* or *false*. Explain your answer. **See margin for explanations.**
   a. $\sum_{k=3}^{7} 3^k + \sum_{b=7}^{9} 3^b = \sum_{a=3}^{9} 3^a$ **false**
   b. $\sum_{n=2}^{8}(2n - 3) = \sum_{m=3}^{9}(2m - 5)$ **true**
   c. $2\sum_{n=3}^{7} n^2 = \sum_{n=3}^{7} 2n^2$ **true**
   d. $\sum_{k=1}^{10}(5 + k) = \sum_{p=0}^{9}(4 + p)$ **false**

49. **Word Play** An *anagram* is a word or phrase that is made by rearranging the letters of another word or phrase. Consider the word "SILENT."
   a. How many different arrangements of the letters in this word are possible? Write this number as a factorial. (*Hint:* First solve a simpler problem to see a pattern, such as how many different arrangements are there of just 2 letters? 3 letters?) **6!**
   b. If a friend gives you a hint and tells you that an anagram of this word starts with "L," how many different arrangements still remain? **120**
   c. Your friend gives you one more hint. The last letter in the anagram is "N." Determine how many possible arrangements remain and then determine the anagram your friend is suggesting. **24, "LISTEN"**

50b. 1-6 × 6, 16-5 × 5, 25-4 × 4, 36-3 × 3, 49-2 × 2, and 64-1 × 1

50. **Chess** A standard chess board contains 64 small black or white squares. These squares make up many other larger squares of various sizes.

   a. How many 8 × 8 squares are there on a standard 8 × 8 chessboard? How many 7 × 7 squares? **1-8 × 8, 4-7 × 7**
   b. Continue this list until you have accounted for all 8 sizes of squares.
   c. Use sigma notation to represent the total number of squares found on an 8 × 8 chessboard. Then calculate this sum. $\sum_{n=1}^{8} n^2$; **204**

Mixed Review

51. Use the comparison test to determine whether the series $\frac{3}{3} + \frac{3}{4} + \frac{3}{5} + \frac{3}{6} + \cdots$ is convergent or divergent. *(Lesson 12-4)* **divergent**

52. **Chemistry** A vacuum pump removes 20% of the air in a sealed jar on each stroke of its piston. The jar contains 21 liters of air before the pump starts. After how many strokes will only 42% of the air remain? *(Lesson 12-3)* **4 strokes**

Extra Practice See p. A49.

Extra Credit

Use sigma notation to express the total number of unit cubes needed to form a series of ever-larger cubes from 1 unit on each edge to n units on each edge.

$\sum_{b=1}^{n} b^3$

Additional Answers

48a. $\sum_{k=3}^{7} 3^k = 3^3 + 3^4 + \cdots + 3^7$

$\sum_{b=7}^{9} 3^b = 3^7 + 3^8 + 3^9$

Since there are two 3^7 terms, $\sum_{k=3}^{7} 3^k + \sum_{b=7}^{9} 3^b \neq \sum_{a=3}^{9} 3^a$.

48b. $\sum_{n=2}^{8}(2n - 3) = 1 + 3 + \cdots + 13 = 49$

$\sum_{m=3}^{9}(2m - 5) = 1 + 3 + \cdots + 13 = 49$

Since $49 = 49$, $\sum_{n=2}^{8}(2n - 3) = \sum_{m=3}^{9}(2m - 5)$.

48c. $2\sum_{n=3}^{7} n^2 = 18 + 32 + \cdots + 98 = 270$

$\sum_{n=3}^{7} 2n^2 = 18 + 32 + \cdots + 98 = 270$

Since $270 = 270$, $2\sum_{n=3}^{7} n^2 = \sum_{n=3}^{7} 2n^2$.

48d. $\sum_{k=1}^{10}(5 + n) = 6 + 7 + \cdots + 15 = 105$

$\sum_{p=0}^{9}(4 + p) = 4 + 5 + \cdots + 13 = 85$

Since $85 \neq 105$, $\sum_{k=1}^{10}(5 + n) \neq \sum_{p=0}^{9}(4 + p)$.