12.1 Answers

1. The number houses sold must be negative.

3. Refer to Example 6.
   a. **Explain** why −24.58 is *not* a possible answer.
   b. **Determine** how much money Ms. Gonzales will make if she sells 10 houses. $60,000

4. **Describe** the common difference for an arithmetic sequence in which the terms are decreasing. negative

5. **You Decide** Ms. Brooks defined two sequences, $a_n = (-1)^n$ and $b_n = (-2)^n$, for her class. She asked the class to determine if they were arithmetic sequences. Latonya said the second was an arithmetic sequence and that the first was not. Diana thought the reverse was true. Who is correct? Explain. See margin.

Guided Practice

Find the next four terms in each arithmetic sequence.

6. 6, 11, 16, ... 21, 26, 31, 36
7. −15, −7, 1, ... 9, 17, 25, 33
8. $a - 6, a - 2, a + 2, \ldots$ $a + 6, a + 10, a + 14, a + 18$

For Exercises 9-15, assume that each sequence or series is arithmetic.

9. Find the 17th term in the sequence for which $a_1 = 10$ and $d = -3$. −38
10. Find n for the sequence for which $a_n = 37$, $a_1 = -13$, and $d = 5$. 11
11. What is the first term in the sequence for which $d = -2$ and $a_7 = 3$? 15
12. Find d for the sequence for which $a_1 = 100$ and $a_{12} = 34$. −6
13. Write a sequence that has two arithmetic means between 9 and 24. 9, 14, 19, 24
14. What is the sum of the first 35 terms in the series $7 + 9 + 11 + \cdots$? 1435
15. Find n for a series for which $a_1 = 30$, $d = -4$, and $S_n = -210$. 21
16. **Theater Design** The right side of the orchestra section of the Nederlander Theater in New York City has 19 rows, and the last row has 27 seats. The numbers of seats in each row increase by 1 as you move toward the back of the section. How many seats are in this section of the theater? 342 seats

EXERCISES

Practice

Find the next four terms in each arithmetic sequence. 17–25. See margin.

A
17. 5, −1, −7, ...
18. −18, −7, 4, ...
19. 3, 4.5, 6, ...
20. 5.6, 3.8, 2, ...
21. $b, b + 4, b + 8, \ldots$
22. $-x, 0, x, \ldots$
23. $5n, -n, -7n, \ldots$
24. $5 + k, 5, 5 - k, \ldots$
25. $2a - 5, 2a + 2, 2a + 9, \ldots$

B
26. Determine the common difference and find the next three terms of the arithmetic sequence $3 + \sqrt{7}, 5, 7 - \sqrt{7}, \ldots$. $2 - \sqrt{7}$; $9 - 2\sqrt{7}, 11 - 3\sqrt{7}, 13 - 4\sqrt{7}$

For Exercises 27-34, assume that each sequence or series is arithmetic.

27. Find the 25th term in the sequence for which $a_1 = 8$ and $d = 3$. 80
28. Find the 18th term in the sequence for which $a_1 = 1.4$ and $d = 0.5$. 9.9
29. Find n for the sequence for which $a_n = -41$, $a_1 = 19$, and $d = -5$. 13
30. Find n for the sequence for which $a_n = 138$, $a_1 = -2$, and $d = 7$. 21
31. What is the first term in the sequence for which $d = -3$, and $a_{15} = 38$? 80
32. What is the first term in the sequence for which $d = \frac{1}{3}$ and $a_7 = 10\frac{2}{3}$? $8\frac{2}{3}$
33. Find d for the sequence in which $a_1 = 6$ and $a_{14} = 58$. 4
34. Find d for the sequence in which $a_1 = 8$ and $a_{11} = 26$. $1\frac{4}{5}$

Assignment Guide

Core: 17–57 odd, 58–69
Enriched: 18–48 even, 50–69

Additional Answers

5. Neither student is correct, since neither sequence has a common difference. The difference fluctuates between −1 and 1. The second sequence has a difference that fluctuates between 2 and −2.
17. −13, −19, −25, −31
18. 15, 26, 37, 48
19. 7.5, 9, 10.5, 12
20. 0.2, −1.6, −3.4, −5.2
21. $b + 12, b + 16, b + 20, b + 24$
22. $2x, 3x, 4x, 5x$
23. $-13n, -19n, -25n, -31n$
24. $5 - 2k, 5 - 3k, 5 - 4k, 5 - 5k$
25. $2a + 16, 2a + 23, 2a + 30, 2a + 37$

#17 -35.00

Additional Answers
(pp. 764–765)

51. Let d be the common difference. Then, $y = x + d$, $z = x + 2d$, and $w = x + 3d$. Substitute these values into the expression $x + w - y$ and simplify.
$x + (x + 3d) - (x + d) = x + 2d$ or z.

54b. The common difference between each consecutive term in the sequence is 180, therefore the sequence is arithmetic.

55c. Conjecture: the sum of the first n terms of the sequence of natural numbers is n^2.

Proof:

Let $a_n = 2n - 1$. The first term of the sequence of natural numbers is 1, so $a_1 = 1$.

Then, using the formula for the sum of an arithmetic series,

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{n}{2}[1 + (2n - 1)]$

$= \frac{n}{2}(2n)$ or n^2

For Exercises 35-49, assume that each sequence or series is arithmetic.

35. What is the eighth term in the sequence $-4 + \sqrt{5}, -1 + \sqrt{5}, 2 + \sqrt{5}, \ldots$? 35. $17 + \sqrt{5}$

36. What is the twelfth term in the sequence $5 - i, 6, 7 + i, \ldots$? $16 + 10i$

37. Find the 33rd term in the sequence 12.2, 10.5, 8.8, ... -42.2

38. Find the 79th term in the sequence $-7, -4, -1, \ldots$. 227

39. Write a sequence that has one arithmetic mean between 12 and 21. 39. 12, 16.5, 21

40. Write a sequence that has two arithmetic means between -5 and 4. 40. $-5, -2, 1, 4$

41. Write a sequence that has two arithmetic means between $\sqrt{3}$ and 12. 41. $\sqrt{3}$, $\frac{12 + 2\sqrt{3}}{3}$, $\frac{24 + \sqrt{3}}{3}$, 12

42. Write a sequence that has three arithmetic means between 2 and 5. 42. 2, 2.75, 3.5, 4.25, 5

43. Find the sum of the first 11 terms in the series $\frac{3}{2} + 1 + \frac{1}{2} + \ldots$. -11

44. Find the sum of the first 100 terms in the series $-5 - 4.8 - 4.6 - \ldots$. 490

45. Find the sum of the first 26 terms in the series $-19 - 13 - 7 - \ldots$. 1456

46. Find n for a series for which $a_1 = -7$, $d = 1.5$, and $S_n = -14$. 8

47. Find n for a series for which $a_1 = -3$, $d = 2.5$, and $S_n = 31.5$. 7

48. Write an expression for the nth term of the sequence 5, 7, 9, ... $2n + 3$

49. Write an expression for the nth term of the sequence 6, -2, -10, ... $-8n + 14$

Applications and Problem Solving

50. **Keyboarding** Antonio has found that he can input statistical data into his computer at the rate of 2 data items faster each half hour he works. One Monday, he starts work at 9:00 A.M., inputting at a rate of 3 data items per minute. At what rate will Antonio be inputting data into the computer by lunchtime (noon)? 15 data items per min

51. **Critical Thinking** Show that if x, y, z, and w are the first four terms of an arithmetic sequence, then $x + w - y = z$. See margin.

52. **Construction** The Arroyos are planning to build a brick patio that approximates the shape of a trapezoid. The shorter base of the trapezoid needs to start with a row of 5 bricks, and each row must increase by 2 bricks on each side until there are 25 rows. How many bricks do the Arroyos need to buy? 1325 bricks

53. **Critical Thinking** The measures of the angles of a convex polygon form an arithmetic sequence. The least measurement in the sequence is 85°. The greatest measurement is 215°. Find the number of sides in this polygon. 12

54. **Geometry** The sum of the interior angles of a triangle is 180°.
a. What are the sums of the interior angles of polygons with 4, 5, 6, and 7 sides? 54a. 360°, 540°, 720°, 900°
b. Show that these sums (beginning with the triangle) form an arithmetic sequence. See margin.
c. Find the sum of the interior angles of a 35-sided polygon. 5940°

Extra Credit

Give an example of an arithmetic sequence whose terms are all multiples of 3 and 5, but not multiples of 4 or 6.
Sample answer: 15, 45, 75, 105, . . .