

10-5 The Normal Distribution

A normal distribution has a mean of 416 and a standard deviation of 55.

1. Find the range of values that represent the middle 99.7% of the distribution.

SOLUTION:

The middle 99.7% of data in a normal distribution is the range from $\mu - 3\sigma$ to $\mu + 3\sigma$. The standard deviation is 55, so $3\sigma = 3 \cdot 55$ or 165.

$$416 + 165 = 581 \text{ and } 416 - 165 = 251$$

Therefore, the range of values in the middle 99.7% is $251 < X < 581$.

3. **CCSS TOOLS** The number of texts sent per day by a sample of 811 teens is normally distributed with a mean of 38 and a standard deviation of 7.

a. About how many teens sent between 24 and 38 texts?

b. What is the probability that a teen selected at random sent less than 31 texts?

SOLUTION:

a. 24 is 2σ away from the mean, so the range of values are between $\mu - 2\sigma$ and μ . This represents half of 95% of the distribution, or 47.5%.

$$47.5\% \text{ of } 811 = 385.225$$

Therefore, about 386 teens sent between 24 and 38 texts.

b. 31 is σ away from the mean, so the range of values are less than $\mu + \sigma$. The range of values from $\mu - \sigma$ to $\mu + \sigma$ represent 68% of the data, so the range from μ to $\mu + \sigma$ represents half of these values, or 34% of the data. 50% of the data are less than μ , so the percentage of data values less than $\mu + \sigma$ is 50% + 34% or 84%. Therefore, the probability that a teen selected at random sent less than 31 texts is 84%.

Find the missing variable. Indicate the position of X in the distribution.

5. z if $\mu = 13.3$, $X = 17.2$, and $\sigma = 1.9$

SOLUTION:

$$\begin{aligned} z &= \frac{X - \mu}{\sigma} && \text{Formula for } z\text{-values} \\ z &= \frac{17.2 - 13.3}{1.9} && X = 17.2, \mu = 13.3, \text{ and } \sigma = 1.9 \\ &\approx 2.05 && \text{Simplify.} \end{aligned}$$

The z -value that corresponds to $X = 17.2$ is approximately 2.05 standard deviations more than the mean.

7. σ if $\mu = 21.1$, $X = 13.7$, and $z = -2.40$

SOLUTION:

$$\begin{aligned} z &= \frac{X - \mu}{\sigma} && \text{Formula for } z\text{-values} \\ -2.40 &= \frac{13.7 - 21.1}{\sigma} && z = -2.40, X = 13.7, \text{ and } \mu = 21.1 \\ -2.40 &= \frac{-7.4}{\sigma} && \text{Subtract.} \\ -2.4\sigma &= -7.4 && \text{Multiply each side by } \sigma. \\ \sigma &= \frac{-7.4}{-2.4} && \text{Divide each side by } -2.4. \\ \sigma &\approx 3.08 && \text{Simplify.} \end{aligned}$$

The z -value that corresponds to $X = 13.7$ is approximately 2.4 standard deviations less than the mean.

A normal distribution has a mean of 29.3 and a standard deviation of 6.7.

9. Find the range of values that represent the outside 5% of the distribution.

SOLUTION:

The outside 5% of data in a normal distribution is equal to 1 minus the middle 95%.

The middle 95% is the range from $\mu - 2\sigma$ to $\mu + 2\sigma$. The standard deviation is 6.7, so $\sigma = 2 \cdot 6.7$ or 13.4

$$29.3 - 13.4 = 15.9 \text{ and } 29.3 + 13.4 = 42.7$$

The range of values in the middle 95% is $15.9 < X < 42.7$. Therefore, the outside 5% is represented by $X < 15.9$ or $X > 42.7$.

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11. **GYMS** The number of visits to a gym per year by a sample of 522 members is normally distributed with a mean of 88 and a standard deviation of 19.

- a. About how many members went to the gym at least 50 times?
b. What is the probability that a member selected at random went to the gym more than 145 times?

SOLUTION:

a. 50 is 2σ away from the mean of 88, so the range of values are greater than $\mu - 2\sigma$. $\mu - 2\sigma$ to μ represents half of 95% of the distribution, or 47.5%. 50% of the distribution is greater than μ , so everything greater than $\mu - 2\sigma$ is $47.5\% + 50\%$ or 97.5%.

$$97.5\% \text{ of } 522 = 508.95$$

Therefore, about 509 members went to the gym at least 50 times.

b. 145 is 3σ away from the mean, so the range of values are more than $\mu + 3\sigma$. The range of values from $\mu - 3\sigma$ to $\mu + 3\sigma$ represent 99.7% of the data, so the range from μ to $\mu + 3\sigma$ represents half of these values, or 49.85% of the data. The outer tail of the distribution is $50\% - 49.85\%$ or 0.15% of the data. Therefore, the probability that a member went to the gym more than 145 times is 0.15%.

Find the missing variable. Indicate the position of X in the distribution.

13. z if $\mu = 19.9$, $X = 18.7$, and $\sigma = 0.9$

SOLUTION:

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for } z\text{-values}$$
$$z = \frac{18.7 - 19.9}{0.9} \quad X = 18.7, \mu = 19.9, \text{ and } \sigma = 0.9$$
$$\approx -1.33 \quad \text{Simplify.}$$

The z -value that corresponds to $X = 18$ is approximately 1.33 standard deviations less than the mean.

15. X if $\mu = 138.8$, $\sigma = 22.5$, and $z = 1.73$

SOLUTION:

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for } z\text{-val}$$
$$1.73 = \frac{X - 138.8}{22.5} \quad \mu = 138.8, z = 1.7$$
$$38.925 = X - 138.8 \quad \text{Multiply each side by 22.5}$$
$$177.725 = X \quad \text{Add 138.8 to each side}$$

The z -value that corresponds to $X = 177.725$ is approximately 1.73 standard deviations more than the mean.

17. **CAR BATTERIES** The useful life of a certain car battery is normally distributed with a mean of 113,627 miles and a standard deviation of 14,266 miles. The company makes 20,000 batteries a month.

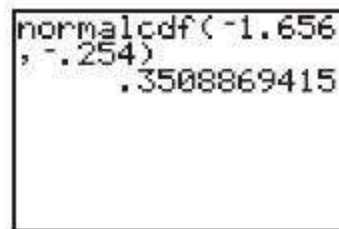
- a. About how many batteries will last between 90,000 and 110,000 miles?
b. About how many batteries will last more than 125,000 miles?
c. What is the probability that if you buy a car battery at random, it will last less than 100,000 miles?

SOLUTION:

- a. Find the z -values associated with 90,000 and 110,000. The mean μ is 113,627 and the standard deviation σ is 14,266.

$$z = \frac{X - \mu}{\sigma}$$
$$= \frac{90,000 - 113,627}{14,266}$$
$$\approx -1.656$$
$$z = \frac{X - \mu}{\sigma}$$
$$= \frac{110,000 - 113,627}{14,266}$$
$$\approx -0.254$$

Use a graphing calculator to find the area between the z -values.



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normalcdf(-1.656, -0.254)
.3508869415
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The value 0.35 is the percentage of batteries that will

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last between 90,000 and 110,000 miles. The total number of batteries in this group is $0.35 \times 20,000$ or 7000.

b. Find the z-value associated with 125,000.

$$z = \frac{X - \mu}{\sigma}$$
$$= \frac{125,000 - 113,627}{14,266}$$
$$\approx 0.797$$

We are looking for values greater than 125,000, so we can use a graphing calculator to find the area between $z = 0.797$ and $z = 4$.



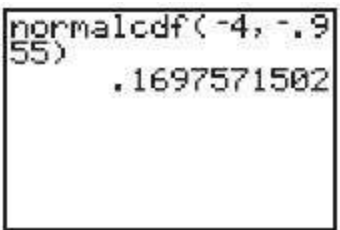
normalcdf(.797, 4)
.2126937641

The value 0.21 is the percentage of batteries that will last between more than 125,000 miles. The total number of batteries in this group is $0.21 \times 20,000$ or 4200.

c. Find the z-value associated with 100,000.

$$z = \frac{X - \mu}{\sigma}$$
$$= \frac{100,000 - 113,627}{14,266}$$
$$\approx -0.955$$

We are looking for values less than 100,000, so we can use a graphing calculator to find the area between $z = -4$ and $z = -0.955$.



normalcdf(-4, -.955)
.1697571502

The probability that if you buy a car battery at random, it will last less than 100,000 miles is about 17.0%.

19. **FINANCIAL LITERACY** The insurance industry uses various factors including age, type of car driven, and driving record to determine an individual's insurance rate. Suppose insurance rates for a sample population are normally distributed.

a. If the mean annual cost per person is \$829 and the standard deviation is \$115, what is the range of rates you would expect the middle 68% of the population to pay annually?

b. If 900 people were sampled, how many would you expect to pay more than \$1000 annually?

c. Where on the distribution would you expect a person with several traffic citations to lie? Explain your reasoning.

d. How do you think auto insurance companies use each factor to calculate an individual's insurance rate?

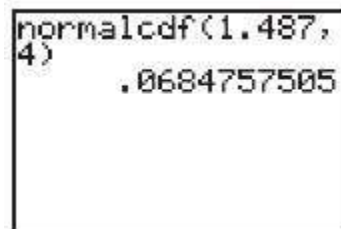
SOLUTION:

a. The middle 68% represents all data values within one standard deviation of the mean. Add $\pm \$115$ to \$829. The range of rates is \$714 to \$944.

b. Find the z-value associated with 1000.

$$z = \frac{X - \mu}{\sigma}$$
$$= \frac{1000 - 829}{115}$$
$$\approx 1.487$$

We are looking for values more than 1000, so we can use a graphing calculator to find the area between $z = 1.487$ and $z = 4$.



normalcdf(1.487, 4)
.0684757505

The probability that a customer selected at random will pay more than \$1000 is about 6.8%. Out of 900 people, about $0.06847575 \cdot 900$ or 62 people will pay more than \$1000.

c. Sample answer: I would expect people with several traffic citations to lie to the far right of the distribution where insurance costs are highest, because I think insurance companies would charge

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them more.

d. Sample answer: I think auto insurance companies would charge younger people more than older people because they have not been driving as long. I think they would charge more for expensive cars and sports cars and less for cars that have good safety ratings. I think they would charge a person less if they have a good driving record and more if they have had tickets and accidents.

21. **CCSS CRITIQUE** A set of normally distributed tree diameters have mean 11.5 cm, standard deviation 2.5, and range 3.6 to 19.8. Monica and Hiroko are to find the range that represents the middle 68% of the data. Is either of them correct? Explain.

Monica

The data span 16.2 cm. 68% of 16.2 is about 11 cm. Center this 11-cm range around the mean of 11.5 cm. This 68% group will range from about 6 cm to about 17 cm.

Hiroko

The middle 68% span from $\mu + \sigma$ to $\mu - \sigma$. So we move 2.5 cm below 11.5 and then 2.5 cm above 11.5. The 68% group will range from 9 cm to 14 cm.

SOLUTION:

Hiroko; Monica's solution would work with a uniform distribution.

23. **REASONING** The term six sigma process comes from the notion that if one has six standard deviations between the mean of a process and the nearest specification limit, there will be practically no items that fail to meet the specifications. Is this a true assumption? Explain.

SOLUTION:

Sample answer: True; according to the Empirical Rule, 99% of the data lie within 3 standard deviations of the mean. Therefore, only 1% will fall outside of three sigma. An infinitesimally small amount will fall outside of six-sigma.

25. **OPEN ENDED** Find a set of real-world data that appears to be normally distributed. Calculate the range of values that represent the middle 68%, the middle 95%, and the middle 99.7% of the distribution.

SOLUTION:

Sample answer: The scores per team in each game of the first round of the 2010 NBA playoffs. The mean is 96.56 and the standard deviation is 11.06. The middle 68% of the distribution is $85.50 < X < 107.62$. The middle 95% is $74.44 < X < 118.68$. The middle 99.7% is $63.38 < X < 129.74$.

27. The lifetimes of 10,000 light bulbs are normally distributed. The mean lifetime is 300 days, and the standard deviation is 40 days. How many light bulbs will last between 260 and 340 days?

- A 2500
B 3400
C 5000
D 6800

SOLUTION:

Given $S = 10,000$, $\mu = 300$ and $\sigma = 40$

The probability that a randomly selected value in the distribution is between $\mu - \sigma$ and $\mu + \sigma$, that is, between $300 - 40$ or 260 and $300 + 40$ or 340.

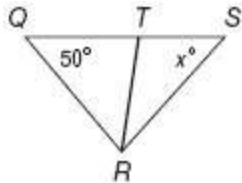
$$P(260 < x < 340) = 34\% + 34\% = 68\%$$

68% of 10000 is 6800

Option D is the correct answer.

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29. **SHORT RESPONSE** In the figure below, $RT = TS$ and $QR = QT$. What is the value of x ?



SOLUTION:

Given $RT = TS$ and $QR = QT$.

Since $RT = TS$, $m\angle SRT = x$.

Since $QR = QT$, $m\angle QRT = m\angle QTR$.

$$\begin{aligned} m\angle QRT &= \frac{180 - 50}{2} \\ &= 65 \end{aligned}$$

$$\begin{aligned} m\angle QRT + m\angle TRS + m\angle RST + m\angle TQR &= 180 \\ 65 + x + x + 50 &= 180 \\ 2x &= 65 \\ x &= 32.5 \end{aligned}$$

31. **SNOW** There is a 25% chance that it snows each day. What is the probability that it snows 3 out of the next 7 days.

SOLUTION:

$$\begin{aligned} P(X) &= {}_n C_X p^X q^{n-X} && \text{Binomial Pr} \\ P(3) &= {}_7 C_3 (0.25)^3 (0.75)^{7-3} && n = 7, X = 3 \\ &\approx 0.173 && \text{Simplify.} \end{aligned}$$

The probability that it will snow in 3 of the next 7 days is approximately 0.173.

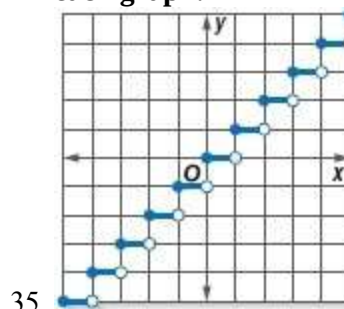
Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

33. the amount of precipitation in a city per month

SOLUTION:

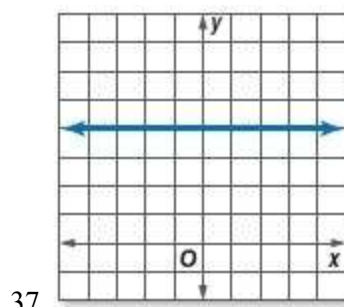
The amount of precipitation in a city is continuous because it can take on any value.

Identify the type of function represented by each graph.



SOLUTION:

greatest integer



SOLUTION:

constant