**PRE-CALCULUS: by Finney, Demana, Watts and Kennedy**

**Chapter 1: Functions and Graphs** **1.2: Functions and their properties**

What you'll Learn About

Find the domain of the function algebraically.
Support your answer graphically

A) $f(x) = x^2 - 9$

B) $f(x) = \frac{3}{x} + \frac{7}{x-1}$

C) $f(x) = \frac{x}{x^2 + 2x - 3}$

D) $f(x) = \frac{x}{x^2 + 2x}$

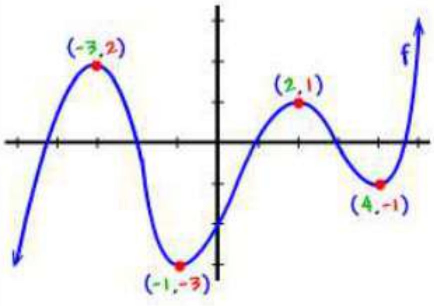
E) $f(x) = \frac{\sqrt{9 - x^2}}{x - 5}$

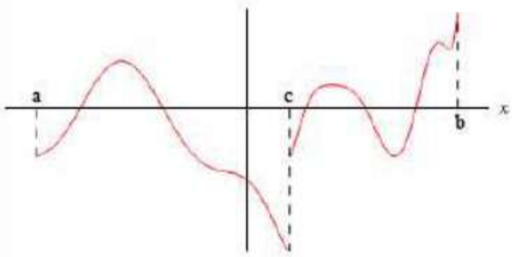
F) $f(x)=\frac{\sqrt{1-x}}{(x-2)(x^2+4)}$

G) $f(x)=\sqrt{x^3-4x}$

| | |
|---|---|
| | Determine the range of the function and remember how to determine if the graph is a function.<br>A) $f(x)=4+x^2$ B) $f(x)=2+\sqrt{9-x}$<br><br>C) $f(x)=\frac{x^2}{4-x^2}$ D) $f(x)=\frac{3-2x^2}{4+x^2}$ |
| Reminder: Sometimes a value of x that seems to be a vertical asymptote is actually a hole | Graph the function and tell whether or not the function has a point of discontinuity at $x=0$. If there is a discontinuity, tell whether the discontinuity is removable or non-removeable.<br>A) $f(x)=\frac{5}{x}$ B) $f(x)=\frac{x^2+x}{x}$<br><br>C) $f(x)=\frac{\lvert 5x \rvert}{x}$ D) $f(x)=\frac{2x}{x-4}$ |

State whether each labeled point identifies a local minimum, a local maximum, or neither. Identify intervals on which the function is decreasing and increasing.



Graph the function and identify intervals on which the function is increasing, decreasing or constant.

30) $f(x)=|x+1|+|x-1|-3$

33) $g(x) = 3 - (x - 1)^2$

Use your calculator to find all local maxima and minima and the values of x where they occur.

43. $h(x) = -x^3 + 2x - 3$

45) $f(x)=x^2\sqrt{x+4}$

State whether the function is odd, even, or neither. Support graphically and confirm algebraically.

A) $f(x)=4x^2$

B) $f(x)=3x^3$

C) $f(x)=\sqrt{x^4+1}$

D) $f(x)=4x+x^3$

E) $f(x)=4x+x^2$

Find all horizontal and vertical asymptotes

A) $f(x)=\frac{x+1}{x}$

B) $f(x)=2^x$

C) $f(x)=\frac{-3x^2+1}{x^2-1}$

D) $f(x)=\frac{3x-9}{x^2-9}$

E) $f(x)=\frac{3x^3+3}{x^2+1}$

F) $f(x)=\frac{x+5}{x^3-27}$

Determine if each function is continuous. If the function is not continuous, find the x-axis location of each discontinuity and classify each discontinuity as infinite or removable. Also find any horizontal asymptotes.

A) $f(x)=\dfrac{3x^2+15x}{x+5}$

B) $f(x)=\dfrac{x^2+3x}{x+2}$

C) $f(x)=\dfrac{9x+6}{x^2-4}$

D) $f(x)=\dfrac{9x+18}{x^2-4}$

E) $f(x)=\dfrac{x-5}{x^2-4x-5}$

Summary of the Characteristics of Graphs of Rational Functions

Horizontal Asymptote

- If the degree of the numerator is greater than the degree of the denominator, then there is no horizontal asymptote

  $$f(x)=\frac{x^2+2x+1}{x+3}$$

- If the degree of the numerator is less than the degree of the denominator, then the horizontal asymptote is y = 0

  $$f(x)=\frac{3x+1}{x^2+3x+1}$$

- If the degree of the numerator and denominator are the same, the horizontal asymptote will be y = the coefficients of the highest power divided by each other

  $$f(x)=\frac{5x^2+2x+1}{2x^2+3}$$

Vertical Asymptote (non-removeable discontinuities)

- Set the denominator equal to 0
- Make sure the values you get are asymptotes and not holes
- Substitute the value back into the function.
- $\frac{a}{0}$ , where is any number but 0, then there is a vertical asymptote

Holes (removeable discontinuities)

- Set the denominator equal to zero
- Make sure the values you get are holes and not vertical asymptotes
- Substitute the value back into the function.
  - $\frac{0}{0}$ is a hole
- To find where the hole is
  - Simplify the original function by factoring
  - Substitute the value from the domain into the cancelled function to find the y-value of the hole

$$f(x)=\frac{x^2+4x+3}{x+1}=\frac{(x+3)(x+1)}{(x+1)}=(x+3)$$

Oblique/Slant Asymptote

- If the degree of the numerator is greater than the degree of the denominator, use long division or synthetic division to find the asymptote (y =    )
  - Ignore the remainder
  - If there is no remainder then there is not a slant asymptote

$$f(x)=\frac{x^2+2x+1}{x+3}$$

X-intercept

- Set the function = 0 (y = 0)
  - You only need to worry about the numerator in a rational function

Y-intercept

- Set the x = 0
  - The values without x will give you your y-intercept

Even/Odd/Neither

- Plug a number to the left of x = 0 into the function and a number to the right of x = 0 into the function (x = -1 and x = 1 usually work.
  - If f(-1) = f(1) the function is even
    - Even functions also reflect over the y-axis
  - If f(-1) is the opposite of f(1) then the function is odd
    - Odd functions have rotational symmetry about the origin
  - If the f(-1) and f(1) are not the same or opposites the function is neither even or odd

Domain:

- If you have a fraction, set the denominator = 0
- If you have a square root in the numerator, set what is under the square root greater than or equal to 0
- If there is a square root in the denominator, set what is under the square roo greater than zero.

Range:

- Look at the graph for complicated functions, but be familiar with the ranges for the function given below

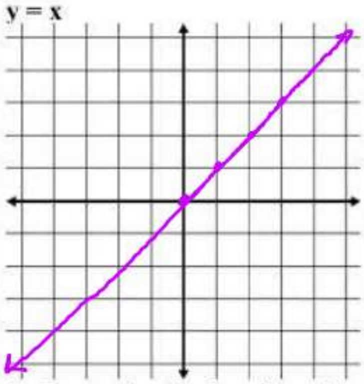
The domain and range of a linear/cubic function are always $(-\infty,\infty)$.

The domain of a quadratic function is always $(-\infty,\infty)$.
The range of a quadratic function that opens up is $[\text{y value of the min},\infty)$.
The range of a quadratic function that opens down is $(-\infty, \text{y value of the max}]$

The domain of sine and cosine are always $(-\infty,\infty)$.
The range of sine of and cosine are always [minimum y-value, maximum y-value].

Sketch a graph of the following functions

$y = x$



1) Determine the domain and range

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

2) Is the function even, odd or neither

odd

3) Intervals of Increase or Decrease

Inc: $(-\infty, \infty)$

4) Find any extrema.

Extrema
- Max
- Min

None

5) Determine the end behavior

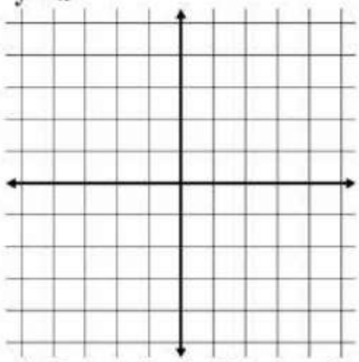
Right: $\lim_{x \to \infty} f(x) = \infty$ ← y-values

Left: $\lim_{x \to -\infty} f(x) = -\infty$

6) Find any asymptotes

None

$y = x^2$



1) Determine the domain and range

2) Is the function even, odd or neither

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4) Find any extrema.

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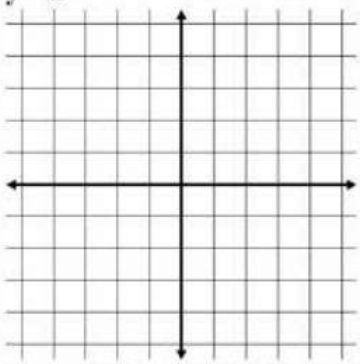
6) Find any asymptotes

$y = x^3$



1) Determine the domain and range

2) Is the function even, odd or neither

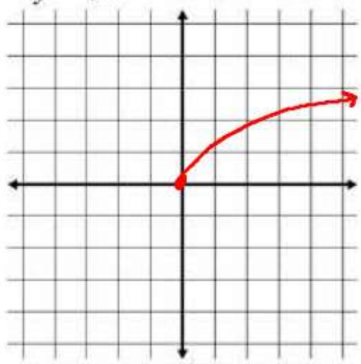
3) Intervals of Increase or Decrease

4) Find any extrema.

5) Determine the end behavior

6) Find any asymptotes

$y = \sqrt{x}$



1) Determine the domain and range

2) Is the function even, odd or neither

3) Intervals of Increase or Decrease

4) Find any extrema.

5) Determine the end behavior

Right $\lim_{x \to \infty} f(x) = \infty$

Left $\lim_{x \to 0} f(x) = 0$

6) Find any asymptotes

# Piecewise Functions

less than or =

Determine the value given.

$$f(x)=\begin{cases} x^2 & x\le 0 \\ \sqrt{x} & x>0 \end{cases}$$

$y = x^2 \quad x \le 0$

$y = \sqrt{x} \quad x > 0$

1. f(-2) = $(-2)^2 = 4$
2. f(2) = $\sqrt{2}$

Graph each piecewise-defined function and give any points of discontinuity. Then find the domain.

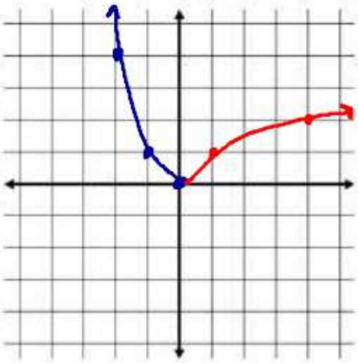
$$f(x)=\begin{cases} x^2 & x\le 0 \\ \sqrt{x} & x>0 \end{cases}$$

$x \le 0$

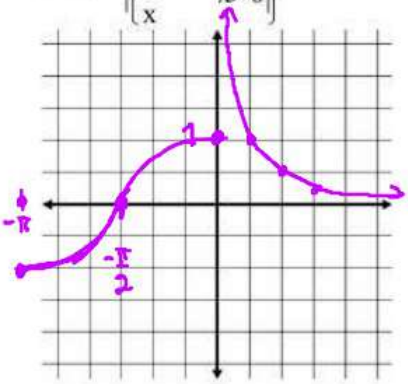
| x | $y=x^2$ |
|---|---|
| 0 | 0 |
| -1 | 1 |
| -2 | 4 |

$x > 0$

| x | $y=\sqrt{x}$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 4 | 2 |



$$f(x)=\begin{cases} \cos x & x\le 0 \\ \frac{1}{x} & x>0 \end{cases}$$




$x \le 0$

| x | $y=\cos x$ |
|---|---|
| 0 | 1 |
| $-\frac{\pi}{2}$ | 0 |

$x > 0$

| x | $y=\frac{1}{x}$ |
|---|---|
| 0 | $\frac{1}{0}$ VA |
| 1 | 1 |
| 2 | 1/2 |
| 3 | 1/3 |

$$f(x)=\begin{cases} x^2 & x\ge 2 \\ \sqrt{x} & x<2 \end{cases}$$

$x \ge 2$

| x | $y=x^2$ |
|---|---|
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |

$x < 2$

| x | $y=\sqrt{x}$ |
|---|---|
| 2 | $\sqrt{2} = 1.41$ |
| 1 | 1 |
| 0 | 0 |